The Operational Amplifier Abstraction (Op Amp)
Two big aha moments in this sequence

- Power of abstraction (gate level abstraction)
- Feedback

- LMD
- Static discipline for noise immunity in digital design
- Small signal method
- Impedance method
Review our Previous Amplifier Abstraction

- MOSFET amplifier — 3 ports
- Amplifier abstraction
Amplifier Building Block

- We could use an abstract amplifier as a building block for more complex circuits.
- But our most interesting amplifier is actually not a great abstract amplifier block because we need to be careful about what we connect to it, since its behavior changes.
- Today, introduce a load and would change more powerful small signal amplifier abstraction gain!
Operational Amplifier Op Amp

Learn to work with abstraction without really knowing what's inside.

Optional: page 382 of A&L - see how to build a differential amp which is a key component of the op amp.
Circuit model of Op Amp (ideal):

\[ v_+ = 0 \quad v_- = 0 \]

\[ i_i = 0 \quad i_o = 0 \]

That is:
- 0 output resistance
- 00 input resistance
- "A" virtually \( \infty \)
- No saturation (i.e. \( v_o \) can take on any value)
Using it...

\[ V_S = 12V \]

\[ V_o = \frac{V_S}{R_L} \]

**Demo**

\[ i^+ = 0 \]

\[ i^- = 0 \]

**Active region**

\[ 12V \]

\[ -10\mu V \]

\[ 10\mu V \]

\[ A \sim 10^6 \]

**Saturation**

\[ -12V \]

\[ -10\mu V \]

\[ v_I \]

\[ v_o \]

\[ v_o \sim \text{high, but unstable.} \]

\[ A \text{ could be temp dependent.} \]

**Note:** Watch out for possible confusion with MOSFET saturation! This saturation is different.
Let us build a circuit: noninverting amplifier

Abstract Op Amp

Equivalent circuit of Op Amp

Equivalent circuit model
Let us analyze the circuit:

Find \( v_O \) in terms of \( v_I \)

Generally a good starting point is to write

\[
    v_O = A (v^+ - v^-)
\]

\[
    v_I \downarrow \quad v_O \quad R_2 \quad \overbrace{R_{1+R_2}}
\]

\[
    v_O = A (v_I - \frac{v_O R_2}{R_{1+R_2}})
\]

\[
    v_O (1 + A \frac{R_2}{R_{1+R_2}}) = A v_I
\]

\[
    v_O = A v_I \quad \frac{1 + A \frac{R_2}{R_{1+R_2}}}{1 + A \frac{R_2}{R_{1+R_2}}}
\]

What happens when “A” is very large?
Let's see... When A is large

\[ V_D = \frac{A V_I}{1 + \frac{A R_2}{R_1+R_2}} \]

\[ V_D \approx V_I \cdot \frac{R_1+R_2}{R_2} \]

Suppose

\[ A = 10^6, \quad R_1 = 9R, \quad R_2 = R \]

\[ V_D = \frac{10^6 V_I}{1 + \frac{10^6 R}{9R + R}} = \frac{10^6 V_I}{1 + 10^6} \]

A is gone! gain

\[ \text{Gain} \text{ determined by resistors only} \]

\[ \text{external component} \leq 10 \cdot V_I \]

\[ \text{Temperature, for variation...} \]

Demo
Why did this (temp insensitivity) happen? Our next Aha!

Insight:

\[ V_0 = V_I \frac{R+R}{R} = 2V_I \]

Suppose I perturb the circuit...

(eg. force \( V_I \) momentarily to 12V somehow)

Stable point when \( V_{+} = V_{-} \)

Stabilizing feedback

opamp tries to maintain this under

Key: negative feedback:

portion of error output fed back to the -ve input

\( \text{eq. for anti-lock} \)

brakes
Question: How to control a high-strung device?

Antilock brakes
Another cool op amp insight:

Observe, under negative feedback,

\[ V_D = A (V^+ - V^-) \]

\[ V^+ - V^- = \frac{V_o}{A} = \frac{V_I}{R_1 + R_2} \]

\[ \approx 0 \]

Virtual short:

\[ V^+ - V^- \rightarrow 0 \]

\[ V \approx V^- \]

We also know:

\[ i^+ = 0 \]
\[ i^- = 0 \]

\[ \rightarrow \text{yields an easier analysis method \ (under negative feedback). Called virtual short method, or virtual ground method} \]
Insightful analysis method under negative feedback – the virtual short method

\[ i^+ \approx 0 \]
\[ i^- \approx 0 \]
\[ v^+ \approx v^- \]

\[ v_o = v_i + \frac{v_i \cdot R_1}{R_2} \]

\[ i = 0 \]
\[ v \text{ drop across } R_1 \]

Introduce this virtual short constraint to simplify our analysis under negative feedback.
Question: What does this circuit do?

Alternatively, $v_o = v_z \frac{R_1}{R_2 + \frac{R_1 R_2}{R_o}} = v_z$.

$v_o = v_i$

Source follower.
Why is this circuit useful?

Buffer
- Voltage gain = 1
- Input impedance = ∞
- Output impedance = 0
- Current gain = ∞
- Power gain = ∞

\[ v_O \approx v_I \]
Inverting amplifier

\[ v = \frac{v_{1}}{R_{1}} + \frac{v_{2}}{R_{2}} \]

\[ i_{0} = -i_{-} = i_{+} = 0 \]

Analyze this circuit with virtual short method.
Inverting amplifier

\[ v^- + v^+ = v_I - R_1 - R_2 \]

\[ v^- + v^+ = v_o + R_1 - R_2 \]

Abstract Op Amp

\[ i^+ = 0 \]
\[ i^- = 0 \]

Equivalent circuit model
Let us analyze the circuit:

Find $v_o$ in terms of $v_i$

Start by $v_o = A(v^+ - v^-)$

$$v_o = A \left[ 1 + \frac{A}{R_2} \frac{R_2}{R_1 + R_2} \right] = -A \frac{v_i}{R_1 + R_2}$$

$$v_o = -A \frac{v_i R_1}{R_1 + R_2} + \frac{A R_2}{R_1 + R_2 + A R_2}$$
Inverting Amplifier analysis using virtual short

\[ v^+ \approx v^- \]

\[ i^+ \approx 0 \]
\[ i^- \approx 0 \]

\[ v_0 = v_i - \frac{R_1}{R_2} \]

\[ v_0 = -\frac{v_1 R_1}{R_2} \]

\[ R_1 = 1 \Omega \]

\[ v_0 = -2V \]

\[ v_i = 2V \]
Inverting amplifier input resistance

\[ R_{\text{in}} = \frac{V_I}{I_I} \]

IDEA: apply a test \( V_I \) at the input, and measure the resulting current \( I_I \). Then \( R_{\text{in}} = \frac{V_I}{I_I} \).
Inverting amplifier input resistance

\[ R_{\text{in}} = \frac{\nu_I}{i_I} \]

Hard way!

\[ R_{\text{in}} = \frac{\nu_I}{i_I} \]

\[ i_I = \frac{\nu_I - \nu_o}{R_1 + R_2} \]

\[ i_I = \left( \frac{\nu_I + A \frac{\nu_I R_1}{R_1 + R_2 + AR_2}}{R_1 + R_2 + AR_2} \right) \frac{1}{R_1 + R_2} \]

\[ \frac{1}{R_{\text{in}}} = \frac{i_I}{\nu_I} = \left( 1 + \frac{A R_1}{R_1 + R_2 + AR_2} \right) \frac{1}{R_1 + R_2 + AR_2} \approx \left( \frac{R_2 + R_1}{R_2} \right) \frac{1}{R_1 R_2} = \frac{1}{R_2} \]

\[ R_{\text{in}} \approx R_2 \]
Inverting amplifier input resistance using **virtual short method**

Easy way!

\[ R_{\text{in}} = \frac{v_i}{i_1} \]

\[ R_{\text{in}} = \frac{\sqrt{2}}{R_2} \]

Virtual short trick anointed as our rest aha moment!