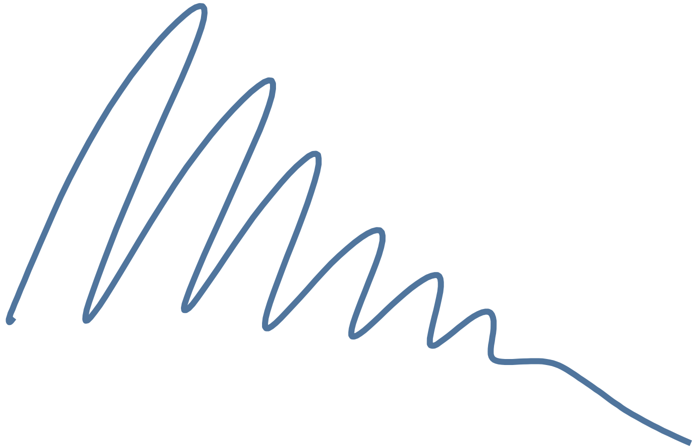
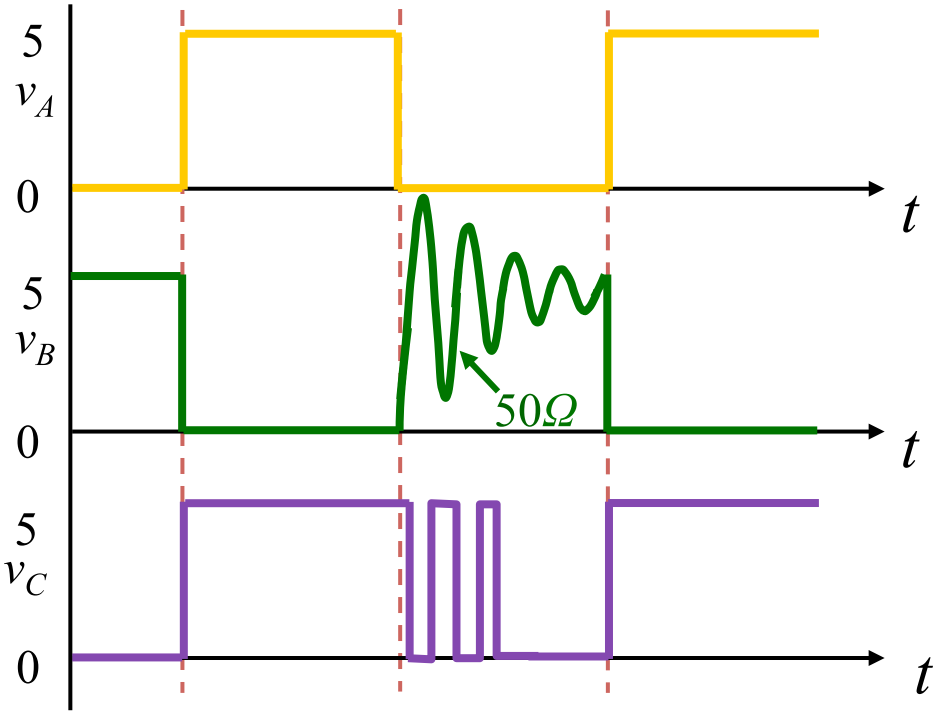
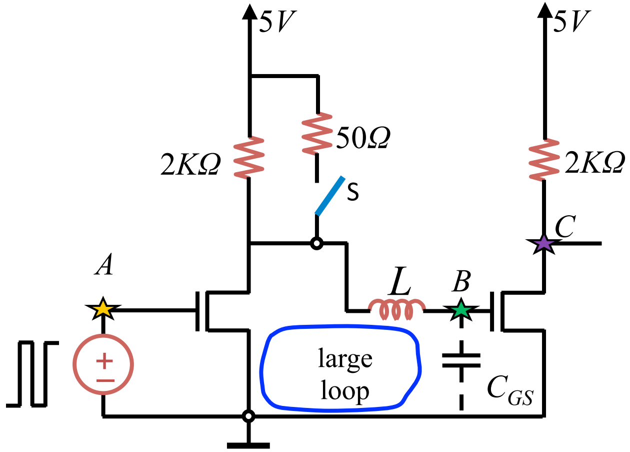
6.002x CIRCUITS AND ELECTRONICS

Damped Second-Order Systems



Review





In the last lecture, we started by analyzing the simpler LC circuit to build intuition

In the last lecture... LC circuit

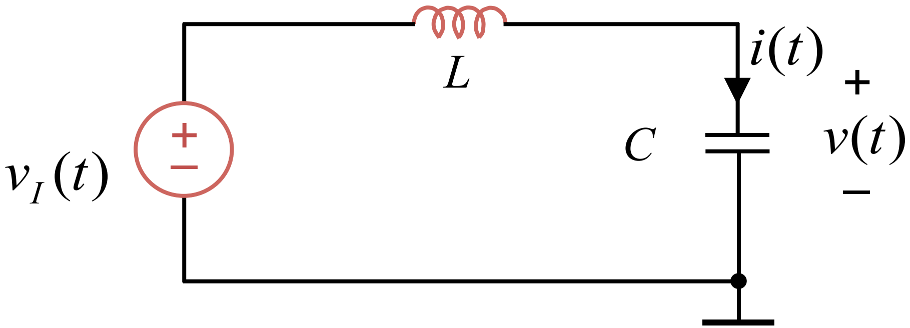
We solved

For input

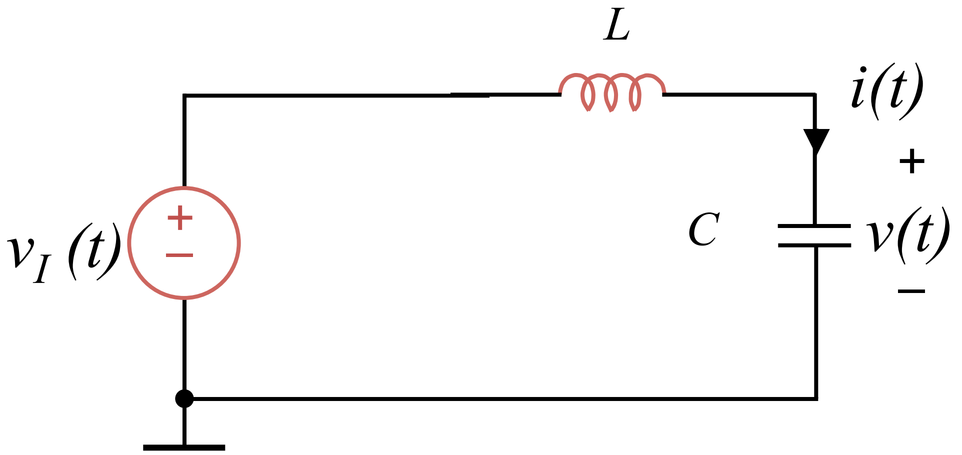
And for initial conditions

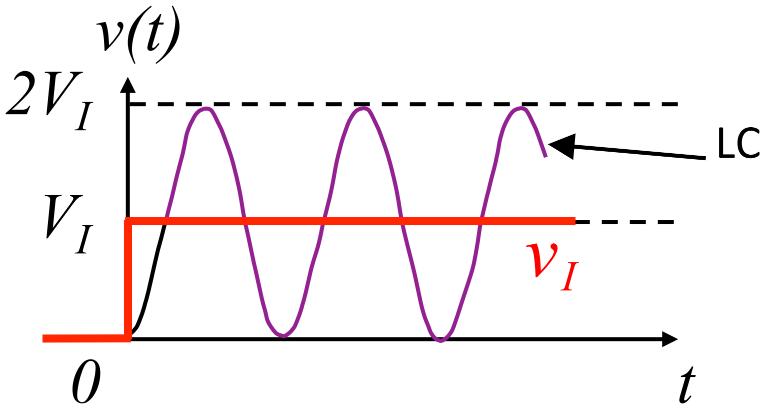
In the last lecture… LC circuit

Total solution



Today, we will close the loop on our observations in the demo by analyzing the RLC circuit





See A&L Section 13.6

Let's analyze the RLC network

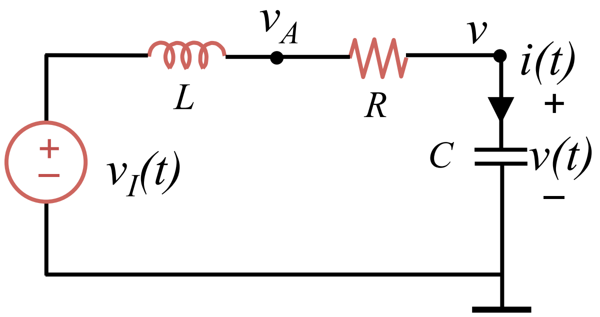
Recall element rules

L:

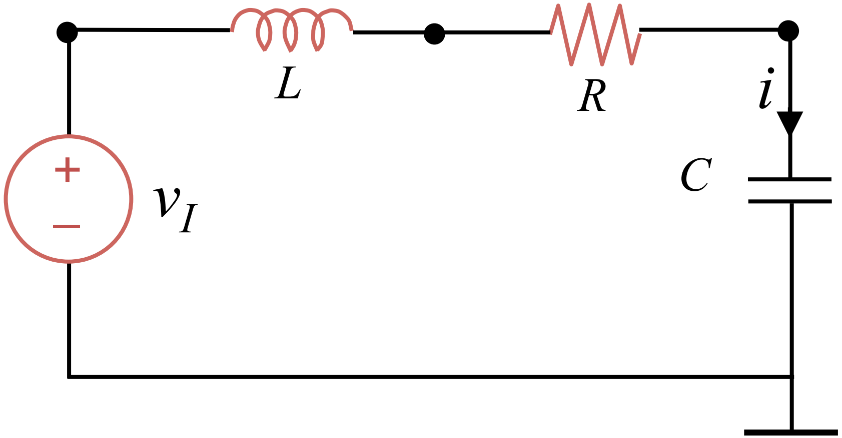
C:

Set up the differential equation

Need to get rid of v_A



Set up the differential equation differently



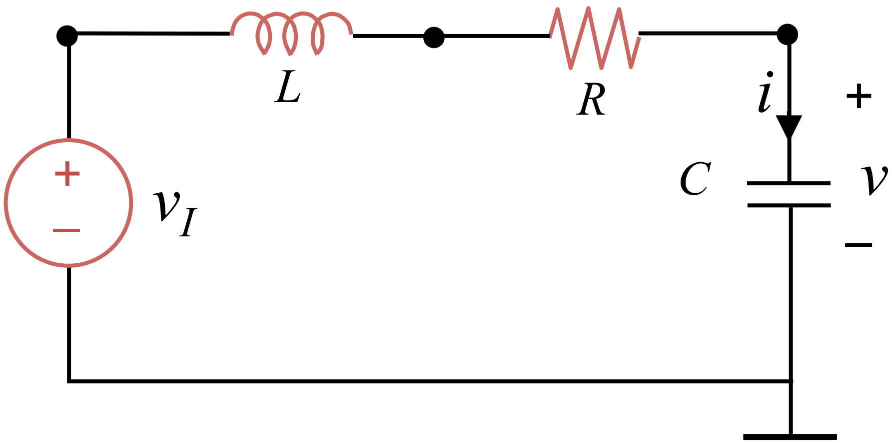
Solve

$$\frac{d^2v}{dt^2} + \frac{R}{L}\frac{dv}{dt} + \frac{1}{LC}v = \frac{1}{LC}v_I$$

Let's solve

$$\frac{d^2v}{dt^2} + \frac{R}{L}\frac{dv}{dt} + \frac{1}{LC}v = \frac{1}{LC}v_I$$

Set up the differential equation differently



1 Particular solution

$$\frac{d^2v}{dt^2} + \frac{R}{L}\frac{dv}{dt} + \frac{1}{LC}v = \frac{1}{LC}v_I$$

$$\frac{d^2v_P}{dt^2} + \frac{R}{L}\frac{dv_P}{dt} + \frac{1}{LC}v_P = \frac{1}{LC}V_I$$

② Homogeneous solution

$$\frac{d^2v}{dt^2} + \frac{R}{L}\frac{dv}{dt} + \frac{1}{LC}v = \frac{1}{LC}v_I$$

2 Homogeneous solution

Solution to $\frac{d^2 v_H}{dt^2} + \frac{R}{L}\frac{dv_H}{dt} + \frac{1}{LC} v_H = 0$

2A Assume solution of the form

$$v_H = Ae^{ST} \quad , \; A, s = ?$$

2 Homogeneous solution

Solution to $$\frac{d^2 v_H}{dt^2} + \frac{R}{L}\frac{dv_H}{dt} + \frac{1}{LC} v_H = 0$$

$$s^2 + \frac{R}{L}s + \frac{1}{LC} = 0$$

2 Homogeneous solution

Solution to $\dfrac{d^2 v_H}{dt^2} + \dfrac{R}{L}\dfrac{dv_H}{dt} + \dfrac{1}{LC} v_H = 0$

characteristic equation $s^2 + \dfrac{R}{L} s + \dfrac{1}{LC} = 0$

Roots $s_1 = -\alpha + \sqrt{\alpha^2 - \omega_o^2}$

$$s_2 = -\alpha - \sqrt{\alpha^2 - \omega_o^2}$$

$$\omega_o = \sqrt{\frac{1}{LC}}$$

$$\alpha = \frac{R}{2L}$$

3 Total solution

3 Let's stare at this a while longer…

$$v(t) = V_I + A_1 e^{\left(-\alpha+\sqrt{\alpha^2-\omega_o^2}\right)t} + A_2 e^{\left(-\alpha-\sqrt{\alpha^2-\omega_o^2}\right)t}$$

$$v(t) = V_I + A_1 e^{-\alpha t} e^{\left(\sqrt{\alpha^2-\omega_o^2}\right)t} + A_2 e^{-\alpha t} e^{\left(-\sqrt{\alpha^2-\omega_o^2}\right)t}$$

3 $\alpha > \omega_o$ Overdamped

$$v(t) = V_I + A_1 e^{-\alpha t} e^{\left(\sqrt{\alpha^2-\omega_o^2}\right)t} + A_2 e^{-\alpha t} e^{\left(-\sqrt{\alpha^2-\omega_o^2}\right)t}$$

$\alpha > \omega_o$ Overdamped

$\alpha < \omega_o$ Underdamped

$\alpha = \omega_o$ Critically damped

3 $\alpha < \omega_o$ Underdamped

$$v(t) = V_I + A_1 e^{-\alpha t} e^{\left(\sqrt{\alpha^2 - \omega_o^2}\right)t} + A_2 e^{-\alpha t} e^{\left(-\sqrt{\alpha^2 - \omega_o^2}\right)t}$$

$\alpha > \omega_o$ Overdamped

$\alpha < \omega_o$ Underdamped

$\alpha = \omega_o$ Critically damped

3 $\alpha < \omega_o$ Underdamped contd... $v(t) = V_I + K_1 e^{-\alpha t}\cos\omega_d t + K_2 e^{-\alpha t}\sin\omega_d t$

3 $\alpha < \omega_o$ Underdamped contd…

$$v(t) = V_I - V_I e^{-\alpha t}\cos\omega_d t - V_I \frac{\alpha}{\omega_d} e^{-\alpha t}\sin\omega_d t$$

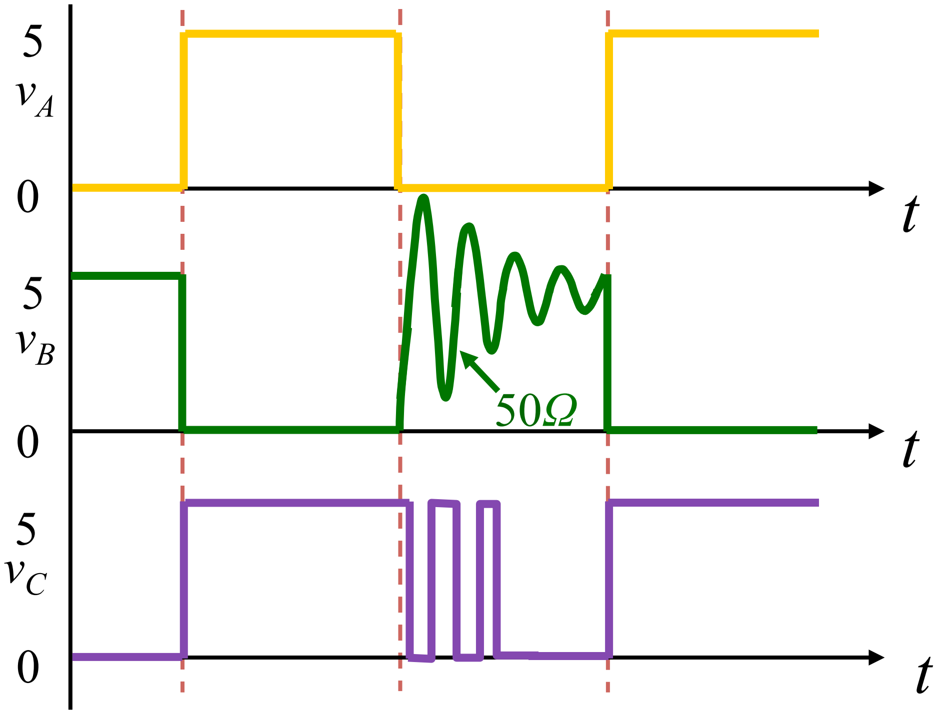
Remember, scaled sum of sines (of the same frequency) are also sines! -- Appendix B.7

$$A_1\cos\theta + A_2\sin\theta = \sqrt{A_1^2 + A_2^2}\cos\left(\theta - \tan^{-1}\frac{A_2}{A_1}\right)$$

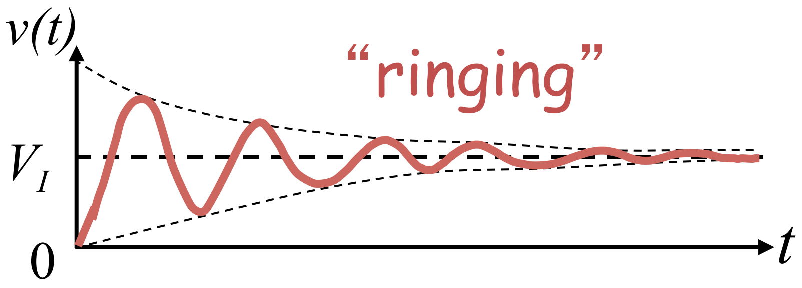
3 $\alpha = \omega_o$ Critically damped

Section 13.2.3

Remember this? We have now closed the loop…

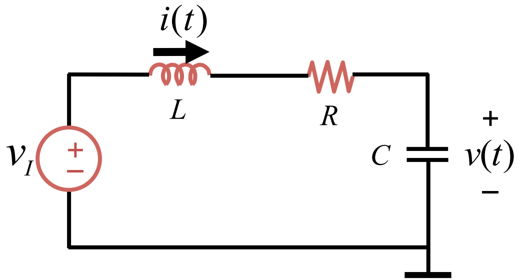


Easy way: Characteristic equation tells the whole story!



See Sec. 12.7 of A&L textbook

Intuitive Analysis

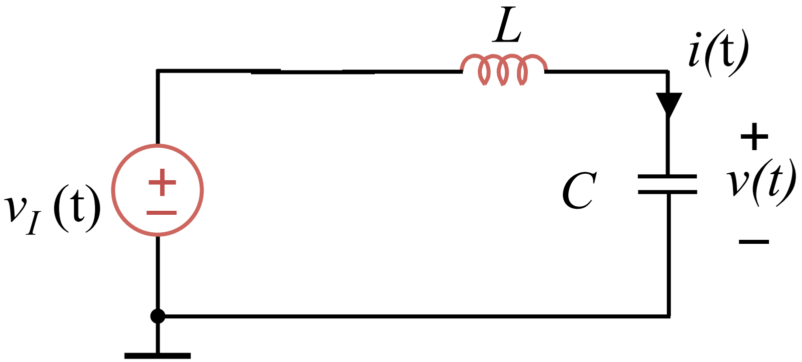


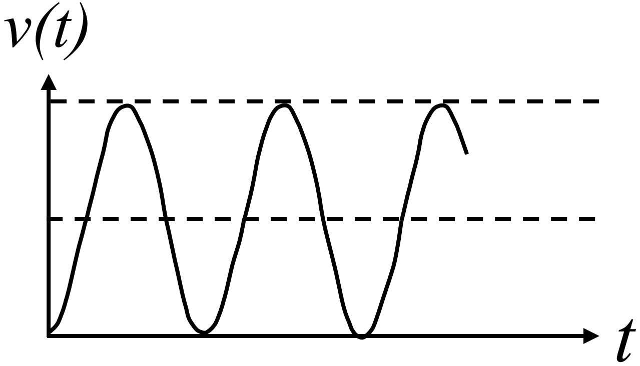
given

$i(0)$ -ve

$v(0)$ +ve

Next, introduce R: RLC Circuits





More in the next sequence! If you are impatient, see A&L Section 13.2

What about other variables?

Parallel RLC – Characteristic equation says it all