The Impedance Model

Reading: Section 13.3 from text
Sinusoidal Steady State (SSS) Reading 13.1, 13.2

\[ v_I = V_i \cos \omega t \]

Focus on sinusoids.
Focus on steady state, only care about \( v_p \) as \( v_H \) dies away.
Sinusoidal Steady State SSS Approach

The Sneaky Path

1. usual circuit model
2. set up DE
3. nightmare trig.
4. complex algebra
5. take real part
6. sneak in $V_i e^{j\omega t}$
7. drive $|V_p| \cos \omega t + V_p$

$V_p$ contains all the information we need:
- Amplitude of output cosine
- Phase

$V_p e^{j\omega t}$

$$\frac{V_i}{1 + j\omega RC}$$
Review

Frequency response - magnitude

\[ \frac{V_p}{V_i} = \frac{1}{1 + j\omega RC} = H(j\omega) \] transfer function

\[ V_p = \frac{V_i}{1 + j\omega RC} \]

\[ v = |V_p| \cos(\omega t + \angle V_p) \]
\[
\angle \frac{V_p}{V_i} = \tan^{-1}\left(\frac{-\omega RC}{1}\right)
\]

The Frequency View - phase

\[
\omega = \frac{1}{RC}
\]

The Frequency View
Is there an even simpler way to get $V_p$?

$$V_p = \frac{V_i}{1 + j\omega RC}$$
The Impedance Model

Is there an even simpler way to get $V_p$?

Consider resistor:
The Impedance Model -- Capacitor

Consider capacitor:

\[ C \]

\[ V_C \]

\[ i_C \]
The Impedance Model -- Inductor

Consider Inductor:

\[ \begin{align*}
  L & \quad \scriptstyle i_L \\
  v_L & \quad \scriptstyle L
\end{align*} \]
The Impedance Model -- Inductor

Consider Inductor:

\[ i_L = I_l e^{st} \]

\[ v_L = V_l e^{st} \]

\[ v_L = L \frac{di_L}{dt} \]
The Impedance Model -- Summary

capacitor

inductor

resistor
Back to RC example...

Is there an even simpler way to get $V_p$?
Back to RC example...

Impedance model:

$$Z_R = R$$

$$Z_C = \frac{1}{sC}$$

Is there an even simpler way to get $V_p$?
There you have it

Usual diff eqn. approach

sneaky approach

Super sneaky... no DEs!

Impedance method
Today
Signal Notation
Impedance Method Summary
Another example, recall series RLC

Remember, we want only the steady-state response to sinusoid (SSS)
Another example, recall series RLC

Remember, we want only the steady-state response to sinusoid (SSS)
The Big Picture...

\[ V_i \cos t \]

\[ |V_p| \cos t + V_p \]

usual circuit model

set up DE

nightmare trig.

\[ V_i e^{st} \]

drive

complex algebra

take real part

impedance-based circuit model

No D.E.s, No trig!
Let's study this transfer function

\[ \frac{V_r}{V_i} = \frac{j\omega R}{L} \left( \frac{1}{LC} + \frac{j\omega R}{L} - \omega^2 \right) \]

Review complex algebra in Appendix C of textbook
Graphically

\[
\frac{|V_r|}{|V_i|} = \frac{\omega RC}{\sqrt{(1 - \omega^2 LC)^2 + (\omega RC)^2}}
\]

Find limit

Low \(\omega\):

High \(\omega\):

\[
\omega = \frac{1}{\sqrt{LC}}
\]
Graphically

\[
\frac{|V_r|}{|V_i|} = \frac{\omega RC}{\sqrt{(1 - \omega^2 LC)^2 + (\omega RC)^2}}
\]

Observe

Low \( \omega \):

High \( \omega \):

\[
\omega = \frac{1}{\sqrt{LC}}
\]
Is there an even simpler way to get $V_p$?

$$V_p = \frac{V_i}{1 + j\omega RC}$$
The Impedance Model

Is there an even simpler way to get $V_p$?

Consider resistor:

\[ V_R \]
The Impedance Model -- Capacitor

Consider capacitor:
The Impedance Model -- Inductor

Consider Inductor:

\[ L \]

\[ i_L \]

\[ v_L \]

\[ L \]
The Impedance Model -- Inductor

Consider Inductor:

\[ i_L = I_i e^{st} \]

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\[ v_L = L \frac{di_L}{dt} \]
The Impedance Model -- Summary

capacitor  inductor  resistor
Is there an even simpler way to get $V_p$?
Back to RC example...

Impedance model:

$$Z_R = R$$

$$Z_C = \frac{1}{sC}$$

Is there an even simpler way to get $V_p$?
There you have it

Usual diff eqn. approach

agony

sneaky approach

easy

sneaky... no DEs!

Impedance method

Today
Signal Notation
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$V_i \cos t$

usual circuit model

set up DE

nightmare trig.

$|V_p| \cos t + V_p$

$V_i e^{st}$
drive

complex algebra

impedance-based circuit model

take real part

No D.E.s, No trig!
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\frac{V_r}{V_i} = \frac{j \omega R}{L} \frac{L}{LC} + \frac{j \omega R}{L} - \omega^2
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