Online Course on Debt Sustainability Analysis (DSAx)
DSAx Part 1:

Principles of Debt Sustainability

Instructor: Irina Yakadina, IMF

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Part 1 Unit 1:
Learning Objectives and Structure of Part 1
Learning Objectives of Part 1

- Learn about key concepts of debt sustainability analysis
- Understand the dangers of high debt
- Derive debt dynamics for different types of debt
- Discuss the role of fiscal adjustment and other macroeconomic policies
Structure of Part 1

Unit 1: Overview

Unit 2: Defining debt sustainability

Unit 3: Why is debt sustainability analysis important?

Unit 4: Public debt sustainability in a closed economy: part 1

Unit 5: Debt dynamics Public debt sustainability in a closed economy: part 2
Structure of Part 1

- Unit 6  Public debt sustainability in an open economy
- Unit 7  Chipping away at public debt (Adjustment paths and their implications)
- Unit 8  Role of macroeconomic policies
- Unit 9  External debt sustainability: part 1
- Unit 10  External debt sustainability: part 2
Part 1 Unit 2:
Defining Debt Sustainability
UNIT OBJECTIVES

- Understand the concept of sustainability
- Master the relevant terminology
- Identify relevant indicators of solvency and liquidity
- Familiarization with various types of DSA conducted by the IMF/World Bank
UNIT OUTLINE

✔ Debt sustainability from different angles
✔ Debt burden indicators
✔ Introduction to IMF/World Bank Debt Sustainability Analysis (DSA)
Part 1 Unit 2: Lecture 1

What is Sustainability?
Debt Sustainability from Different Angles:

- Academic
- Pragmatic
Intertemporal Solvency Condition

- **Initial debt**
- **Future stream of primary expenditure**
- **Future stream of income**

Future flows should be discounted

- Primary expenditure = Expenditure – Interest expenditure
- Discounting: refers to calculating the present discounted value
Intertemporal Solvency Condition

When a government, business or individual is solvent, it is able to service its current debt out of future income or surpluses. A person with a small debt and large future income is solvent.

Example: Consider a business with debt of $20,000 and a prospect of annual profits of $10,000. The business is solvent as this debt can be serviced from future profits.
Intertemporal Solvency Condition

A counter-example is a Ponzi-scheme. In such schemes, initial debt is serviced by relying on new investors, rather than serviced out of future surpluses.
Solvency is
“very much like honesty: it can never be fully certified, and proofs are slow to materialize.”

Guillermo Calvo
Academic Definition of Debt Sustainability

Debt is sustainable if the intertemporal solvency condition is satisfied, that is, if the expected present value of future primary balances covers the existing stock of debt.
The ability to postpone generating primary surpluses to cover for the existing debt obligations into the future makes solvency a relatively weak requirement.
Academic Perspective

Precise

Unobservable

Forecasting Future
Debt is sustainable if projected debt ratios are stable or decline, while also being sufficiently low.

Debt is unsustainable if projected debt ratios increase or remain high.
Pragmatic Definition of Debt Sustainability

Pragmatism consists in recognizing that the ratio of debt to capacity to pay is what matters in order to avoid a debt crisis.

To be sustainable, debt cannot grow faster than incomes and the capacity to repay it.
Pragmatic Definition of Debt Sustainability

Another aspect of pragmatism is to recognize that economies are subject to shocks.

A debt ratio which is declining but high can still be unsustainable if it associated with a high risk of default.
Example of Pragmatic View of Sustainable Debt

Debt/GDP vs. Time
Taking Solvency Condition to the Data

Solvency vs. Non-explosive Debt Ratios:

- Useful result when interest rates are higher than the growth rate:

  if the ratio of debt to GDP is either stable or declining in the long run, the solvency condition is automatically met.
Risks to Sustainability

The projected trajectory and the level of debt should be based on realistic assumptions about the underlying macroeconomic variables.

The resulting gross financing needs have to be evaluated.

The market perception of the sovereign risks has to be factored in based on debt maturity structure, its currency composition, its creditor base, etc.
Part 1 Unit 2: Lecture 2

Define Debt Sustainability
Debt Sustainability from Different Angles:

- Economic Policy Definition
Economic Policy Definition of Debt Sustainability

Debt is sustainable if the country (or its government) does NOT, in the future, need to default or renegotiate or restructure its debt or make implausibly large policy adjustments.
Argentina nears $100bn debt default agreement

By Adam Thomson

Argentina’s battle to get bondholders to accept the largest sovereign debt reduction in the history of capitalism is almost won.

Wall Street analysts and, privately, some bondholder representatives agree that most of the country’s creditors would accept a proposal to restructure about $100bn in defaulted debt that offered recovery values of a little over 30 cents in the dollar. That is just above the price of defaulted bonds on the secondary market.
Debt Restructuring

EU Economy

Greek debt swap support close to 95%
By Richard Milne in Oslo, Kevin Hope in Athens and Peter Spiegel in Brussels

The largest debt restructuring in history was heading for a successful outcome last night as Greece looked set to see a participation rate of close to 95 per cent for its €206bn bond exchange.

One person involved in the deal said that more than 90 per cent and possibly more than 95 per cent of investors had taken part, assuming collective action clauses (CACs) were used to bind dissenting holders of some bonds.

"PSI is a success. Will it exceed 95 per cent? There is a good chance," the person said.

A Greek cabinet minister said: “It will be good news tonight.”

The Greek government will announce the official results of its two-week campaign to win over private holders of its debt early on Friday morning. But late on Thursday night bankers and senior Greek officials briefed in the results said an overwhelming majority of investors had voted in favour of the deal, which will see them suffer

MARKETS INSIGHT

Greek deal will buy time but hard work lies ahead
By Richard Milne

It is as complex as it is big. Greece's €206bn debt restructuring has left people drowning in a sea of figures, struggling to make sense of the deal.

Investors will get up to 24 new securities for each existing bond, a 66 per cent threshold for use of so-called “collective action clauses” but a 50 per cent quorum, and a participation rate that needs to be 95 per cent – or is it 90 or 70 per cent instead? Confused?

The simple fact is that, for all its complexity, Greece has structured this deal so that it is likely to be reasonably successful. Retrospectively inserting collective action clauses, which allow the decision of a majority of bondholders to bind all investors, may seem unfair to holdouts. But it all but ensures that Greece will be able to get all its Greek law bondholders to take part, giving it about 86 per cent participation.

The question will then be to see how many international law bonds, which account for the other 14 per cent of the €206bn, are tendered in the exchange. It is unlikely enough will be tendered to reach the 95 per cent the International Monetary Fund has said is necessary to get Greece's debt down to 120 per cent of gross domestic product by 2020.

Greece has threatened not to pay any international law holdouts at all. That would open the way for a legal battle. But it also raises the prospect that Athens could raise more money than expected by simply not paying some bondholders.
Policy Adjustments

Athens narrowly passes austerity bill
By Kerin Hope in Athens

The Greek parliament narrowly approved a fresh austerity package on Wednesday night, opening the way for international lenders to transfer a long-delayed €31.5bn slice of funding and take steps to ease the terms of the country’s €174bn bailout.

Lawmakers voted for 128 votes against. There were 18 abstentions and parties forming the coalition government.

Antonis Samaras, the caretaker prime minister, has indicated that his government may have to cut spending and raise taxes to help finance the bailout.

The vote was held after a week of negotiations, including cuts in pensions and public sector salaries and increases in fuel and cigarettes, and in social spending and healthcare. The retailer for recent entrants to the market.

"Everyone here is in the same situation as we are - they can’t afford to go away for a holiday," says the 46-year-old local government worker on his annual leave.

His wife Anna, busy unpacking a cold box, adds: "Last year we spent a week in a self-catering apartment on an island … This year I lost my job and that means we can’t even eat at the beach cafe."
Economic Policy Definition of Debt Sustainability

Sustainability rules out any of the following situations:

- a debt restructuring is already needed
- the borrower accumulates debt at a rate faster than the growth in its capacity to service debt
- the borrower lives beyond its means by accumulating debt in the knowledge that a major retrenchment will be needed to service these debts
Intertemporal solvency condition is weaker than the economic policy definition of sustainability.

We call debt **sustainable** if a country or a government is able to service its debts without the need for implausibly large policy adjustments; renegotiating the terms of debt; or simply defaulting.
Part 1 Unit 2: Lecture 3

Debt Burden Indicators for Solvency and Liquidity: Commonly Used Ratios
The concept of liquidity
Debt burden indicators and their role
- Key solvency indicators
- Key liquidity indicators
Liquidity

We define an entity as **liquid** if, regardless of whether it satisfies the solvency condition, its liquid assets and available financing are sufficient to meet or roll-over its maturing liabilities.
What to Watch for to Minimize Liquidity Risks

The projected trajectory AND the level of debt should be based on realistic assumptions.

Risk factors include:

- Market perception of the sovereign
- Debt maturity structure
- The currency composition of debt
- The availability of liquid assets
- The creditor base (notably, the share of non-resident creditors)
Insolvency vs. Illiquidity

Sometimes it can be difficult to distinguish between insolvency and illiquidity situations.

- Liquidity problems are often symptoms of underlying solvency problems: creditors refuse to roll over maturing debt because of solvency concerns.
- Liquidity problems may give rise to insolvency, by raising interest rates or pressuring the exchange rate.
Vulnerability

When we talk about debt sustainability, vulnerability is defined as a risk that the liquidity or solvency conditions are violated and the borrower enters a crisis.
How Do We Assess the Debt Burden?

By examining the projected evolution of a set of debt burden indicators over time

What are the indicators?
Debt Burden Indicators

- Ratios of the debt stock or debt service relative to what we define as measures of the ability to service debt (repayment capacity), e.g.
  - GDP
  - export proceeds
  - fiscal revenue
  - Other

- Gross financing needs, either in level or scaled by the above measures
Debt Burden Indicators as Measures of Solvency and Liquidity

✓ Ratios of debt stock relative to repayment capacity are indicators of solvency

✓ Ratios of debt service are indicators of potential liquidity problems

✓ Gross financing needs is an indicator of potential liquidity problems
Definitions of Gross Financing Needs

✓ Gross financing needs are the amount of financing necessary to cover the deficit plus amortization of debt

\[ GFN = \text{Deficit} + \text{Amortization} \]

\[ GFN = \text{Primary Deficit} + \text{Debt Service} \]

✓ GFN can be positive or negative
Definitions of Debt Service and Amortization

Debt service

\[ DS = \text{Interest} + \text{Amortization} \]

Amortization = principal payments coming due on medium- and long-term debt plus short-term debt coming due (maturity of 1 year or less)
Illustration: debt/GDP vs. debt/exports

Debt Ratios for an Open Economy (exports/GDP=60%)

Debt Ratios for a Closed Economy (exports/GDP=10%)

Both countries have the same debt/GDP ratio, but very different debt/exports ratio.
### Illustration: Gross Financing Needs and Other Debt Burden Indicators

<table>
<thead>
<tr>
<th>Description</th>
<th>STD=10% total debt</th>
<th>STD=60% total debt</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Gross Financing Needs (deficit plus amortization)</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Deficit</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Primary deficit</td>
<td>22</td>
<td>22</td>
</tr>
<tr>
<td>Interest payments</td>
<td>108</td>
<td>108</td>
</tr>
<tr>
<td><strong>Amortization Payments</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Short-term debt</td>
<td>121</td>
<td>728</td>
</tr>
<tr>
<td>Medium and long-term debt</td>
<td>109</td>
<td>49</td>
</tr>
<tr>
<td><strong>Debt service (interest plus amortization)</strong></td>
<td>338</td>
<td>884</td>
</tr>
<tr>
<td><strong>Gross financing needs-to-GDP</strong></td>
<td>18%</td>
<td>44%</td>
</tr>
<tr>
<td><strong>Gross financing needs-to-Revenue</strong></td>
<td>73%</td>
<td>184%</td>
</tr>
<tr>
<td><strong>Debt service-to-GDP</strong></td>
<td>17%</td>
<td>43%</td>
</tr>
<tr>
<td>Debt service-to-Revenue</td>
<td>69%</td>
<td>180%</td>
</tr>
<tr>
<td><strong>Total public debt-to-GDP</strong></td>
<td>66%</td>
<td>66%</td>
</tr>
<tr>
<td><strong>Total public debt-to-Revenue</strong></td>
<td>273%</td>
<td>273%</td>
</tr>
</tbody>
</table>
RECAP

✔ Concept of Liquidity
✔ Debt Burden Indicators:
  - Repayment Capacity (the ability to service debt).
  - Solvency and Liquidity
Part 1 Unit 2: Lecture 4

Scope of Debt
Sustainability Analysis
Scope of the IMF/World Bank Debt Sustainability Analysis (DSA)

DSA for Different Types of Debt
DSA and DSF

- A DSA is produced for a particular country.
- The Debt Sustainability Framework (DSF) is the framework within which DSAs are produced.
- The DSF is needed to ensure comparability across DSAs produced for different countries.
MAC DSA

For advanced and emerging economies with access to financial markets, we use the Framework for Fiscal Policy and Public Debt Sustainability Analysis in Market-Access Countries (MAC DSA)
Debt Sustainability Analysis for Market-Access Countries

The Fund’s approach to debt sustainability analysis differentiate between market-access countries, that typically have significant access to international capital markets, and low-income countries, which meet their external financings needs mostly through concessional resources. The assessments of public and external debt sustainability are conducted in the context of both IMF program design and reviews, and Article IV surveillance. The assessments are performed through standardized templates.

The framework for public debt sustainability analysis was reformed in 2011 and guidance to staff on the implementation of the new framework was introduced in May 2013.

The assessment of external debt sustainability continues to be anchored by the framework introduced in June 2002 (see "Assessing Sustainability"). This framework was subsequently refined in June 2003 and July 2005 (see "Sustainability Assessments—Review of Application and Methodological Refinements" and Information Note On Modifications To The Fund’s Debt Sustainability Assessment Framework For Market-Access Countries").
LIC DSF

The DSF originated as the framework to assess debt sustainability in low-income countries (LICs)
Low-income countries have often struggled with large external debts. Debt burdens have been reduced, thanks in large part to international debt relief initiatives. As part of the Millennium Development Goals (MDGs), the IMF and the World Bank have developed a framework to help guide countries and donors in mobilizing the financing of low-income countries’ development needs, while reducing the chances of an excessive build-up of debt in the future. The joint World Bank–International Monetary Fund (IMF) Debt Sustainability Framework (DSF) was introduced in April 2005, and is periodically reviewed, to address this challenge. The most recent review was discussed by the Executive Boards of the International Development Association and the IMF in February 2012.
External DSA

Public and publicly guaranteed (PPG) external debt

Public domestic debt

Private external debt (non-guaranteed)
SECOND REVIEW UNDER THE EXTENDED ARRANGEMENT UNDER THE EXTENDED FUND FACILITY AND REQUEST FOR

4. Balance of Payments, 2008-20

5. External Financing Requirements and Sources, 2012-20

6. Monetary Survey, 2008-20

7. Indicators of Fund Credit, 2012-20

8. Schedule of Reviews and Purchases

9. Restrictive Measures on Transactions

10. Selected Reforms—Measures Completed

11. MEFP Commitments for the Financial Sector

12. MEFP Commitments for the Fiscal Sector

ANNEXES

1. Public Sector Debt Sustainability Analysis

2. External Sector Debt Sustainability Analysis
Scope of Public DSA

The public DSA (also called the fiscal DSA) covers total debt of the public sector, to external and domestic creditors.

- MAC DSA for public debt covers only public debt, not publicly guaranteed debt (PPG).
- LIC DSA for public debt covers public and publicly guaranteed debt (PPG).
Scope of External DSA

The external DSA covers external debt in the economy.

- MAC DSA for external debt covers debt owed by both the public sector and the private sector.

- LIC DSA for external debt covers public and publicly guaranteed (PPG) external debt.
Part 1 Unit 2: Lecture 5

Debt Sustainability Frameworks
Components of the Debt Sustainability Framework

- Public sector external debt
- Public domestic debt
- Private external debt
- External DSA

Public DSA

+ +
Elements of the Debt Sustainability Analysis

- Baseline scenario (macroframework)
- Assessment of the realism of macroeconomic assumptions
- Stress tests
Two frameworks: MACs and LICs

**MAC DSA**
- Used for: countries with sustained access to international capital markets
- Horizon: 5 years
- Indicators: debt level in nominal terms, gross financing needs, debt profile
- Benchmarks: differ for advanced and emerging market economies, are used to assess the level of risks

**LIC DSF**
- Used for: countries relying on concessional resources
- Horizon: 20 years
- Indicators: solvency and liquidity debt burden indicators expressed in present value terms
- Thresholds: policy-dependent, are used to determine risk rating
RECAP

- Scope of Debt Sustainability Analysis (DSA)
  - MAC or LIC
- DSA for Different Types of Debt
  - Public or External
Part 1 Unit 3:
Why is Debt Sustainability Analysis Important?
UNIT
OBJECTIVES

☑ Understanding the costs of high debt
☑ Learn the definition and origin of debt and other crises
☑ Understanding the mechanism of debt crisis
Unit Outline

- Costs Associated with High Debt
- Types of Economic Crisis
- Mechanism of Debt Crisis
Part 1 Unit 3: Lecture 1

Costs Associated with High Debt
Consequences of High Debt

- Vulnerability to Sudden Stops
- Crowding out of private investment
- Loss of policy flexibility
- Debt Overhang
- Debt Restructuring
Consequences of High Debt

For both public debt and total external debt:

- Vulnerability to a sudden stop in financing (official or private flows)
Impact of Sudden Stops

External debt:

- current and capital account restrictions
- currency crisis, banking crisis, recession, default

Public debt

- drastic reduction in primary spending
- currency crisis, banking crisis, recession, default
Consequences of High Debt

For public debt, consequences include:

- Higher interest rates and crowding out of private investment
- Less flexibility to conduct countercyclical policy
- Debt overhang
Debt Overhang

Definition: The expected tax burden to finance debt is so high that it is a disincentive to current investment/consumption and hence a drag on the economic activity

Consequences:
- lower growth, lower government revenues
- insufficient funds for primary expenditures
- higher chance of default
Debt Overhang

Worries about debt sustainability

Rising risk premium

Concerns about future financing of the sovereign

Worries about fiscal deficits

Concerns about economic growth
Consequences of Sovereign Debt Restructuring

✓ Political and economic penalties

✓ Spillovers across segments of the economy (especially if banks are major holders of government debt)

✓ Contagion to other countries

✓ The 1998 Russian sovereign default and fears in 2010 of a possible Greek default are examples of strong contagion to other countries
Costs Associated with High Debt:

- Vulnerability
- Crowding out
- Loss of flexibility
- Debt Overhang
- Debt Restructuring
Part 1 Unit 3: Lecture 2

Types of Economic Crises
Types of Economic Crises
  - Currency Crises
  - Banking Crises
  - External Debt Crises
  - Sovereign Debt Crises
Currency Crises

- What: an attack in a country’s currency results in one, or a combination, of the following
  - large devaluation
  - sharp depreciation
  - large increase in interest rates
  - large fall in reserves

- When: concerns about the viability of the exchange rate regime or the level of the exchange rate
Currency Crises

Why:

- market expects that foreign exchange (FX) reserves will run out because of inconsistent policies or be insufficient to cover short-term debt
- market expects government to devalue in order to address a policy goal, such as improved competitiveness
Banking Crises

- **What:** run on banks or large-scale government intervention to rescue banks

- **When:** concerns about solvency and liquidity of banks
Banking Crises

Why:

- bursting bubble in equity or real estate prices
- interest rate, exchange rate, or growth shocks
- bust typically follows lending booms (stimulated by financial liberalization/capital inflows)
Debt Crises

Debt crises can be associated with either sovereign (public) or commercial (private) debt.
Sovereign Debt Crises

- What: defaults, involuntary restructuring of sovereign debt, or belief that this is about to occur

- When: often combined (or immediately following) banking crises:
  - this was true for over 60 percent of all sovereign debt crises after 1970 (Rogoff and Reinhart, 2010)
Sovereign Debt Crises

Why:
- financial rescue packages
- extended period of low growth
- fiscal profligacy (including war finance)
- failed state-owned enterprises
- natural disasters
- etc.
External Debt Crises

What: payment arrears on a substantial fraction of external debt

When: cash flow problems or difficulties obtaining foreign exchange

Why:
- “sudden stops” following capital inflow episode
- interest rate, exchange rate, or growth shocks
Different Types of Economic Crises:

- Currency Crises
- Banking Crises
- External Debt Crises
- Sovereign Debt Crises
Please watch the video on Latvia:

Debt and BoP crisis with internal devaluation and fiscal adjustment
Part 1 Unit 3: Lecture 3

Mechanism of Sovereign Debt Crises
Sample Mechanism of Debt Crisis

Bank-sovereign Interdependence
A Sample Mechanism of a Sovereign Debt Crisis

✔ Financial sector rescue packages weigh on public debt and the deficit

✔ Economic activity nosedives

✔ Fiscal revenue collapses while expenditures skyrocket
A Sample Mechanism of a Sovereign Debt Crisis

✓ The resulting spike in deficits and debt causes concerns about the fiscal balance and debt sustainability

✓ Costs of borrowing for the sovereign increase

✓ Fiscal position further worsens
Bank-Sovereign Interdependence

- Damage to bank balance sheets $\rightarrow$ bailout costs and increase in sovereign debt

- Increase in sovereign debt $\rightarrow$ higher possibility of sovereign default, lower ratings $\rightarrow$ damage to bank balance sheets
Bank-Sovereign Interdependence

Lower sovereign ratings

Recession
✓ Sample mechanism of debt crisis
✓ Bank-sovereign Interdependence
Part 1 Unit 4

Public Debt Sustainability in a Closed Economy
Learn how to derive the law of motion for public debt

Learn how to derive the formal solvency condition for public debt
Closed economy:

- Law of motion for public debt
- Solvency condition for public debt
Part 1 Unit 4: Lecture 1

Law of Motion for Public Debt
OUTLINE

- The debt-deficit relationship
- The primary balance
- The government budget constraint
The Relationship between Deficit and Public Debt

Current Stock of Debt = Past Stock of Debt + Deficit + Other Flows + Exchange Rate Valuation

In the closed economy, we assume debt is issued in local currency, so that there is no contribution of exchange rate valuation. In the open economy, we allow for debt issued in local currency and in foreign currency.
Vicious Circle of Debt and Deficit

1. Increase in debt
2. Borrowing
3. Deficit
4. Higher interest payments
Relationship between Deficit and Public Debt

- Budget deficit can be financed by borrowing or other means (e.g., printing money or selling assets)
- Net new borrowing necessary to finance budget deficit adds to the current stock of debt
PUBLIC DEBT DYNAMICS

CLOSED ECONOMY
**Notation**

\[ D_t = D_{t-1} + \Delta D_t \]

\[ I_t = i_t D_{t-1} \]

\[ R_t \] government revenues

\[ G_t \] government primary (non-interest) spending

\[ PB_t = R_t - G_t \]

\[ i_t \] nominal interest rate

\[ r_t \] real interest rate

\[ \pi_t \] inflation rate

\[ P_t Y_t \] nominal GDP

\[ g_t \] real GDP growth rate

\[ P_t Y_t = (1+\pi_t)(1+g_t)P_{t-1}Y_{t-1} \]
Flow Budget Constraint

\[ G_t + i_tD_{t-1} - R_t + OT_t = (D_t - D_{t-1}) \]

- Spending
- Revenues
- Other flows, e.g. bank recapitalization
- Change in debt
Note

Other flows include

- asset purchases and expenditure items not included in $G$
  - bank recapitalization
  - assumption of guaranteed state enterprise debt
- non-debt sources of financing
  - asset sales such as privatization revenues
  - seigniorage

Such non-debt sources of financing enter with a negative sign.
Focusing on Primary Balance:

Substitute the primary balance definition:

\[-PB_t + i_t D_{t-1} + OT_t = D_t - D_{t-1}\]

Assume other flows are zero: \(OT_t = 0\)
Equation for Debt Dynamics:

\[ D_t = (1 + i_t)D_{t-1} - PB_t \]
Example: Evolution of Debt over Time

\[ D_t = (1 + i_t)D_{t-1} - PB_t \]

- \( D_{t-1} = 100 \)
- \( i_t = 2\% \)
- \( PB_t = -2 \)
Part 1 Unit 4: Lecture 2

Deriving the Solvency Condition
Deriving the Solvency Condition from the Flow Budget Constraint:

- Derive the Intertemporal Budget Constraint
- Impose Transversality
- Obtain Solvency Condition
Checkpoint: Where Are We?

- We talked about the debt-deficit relationship
- We derived the government budget constraint
- We derived one-period law of motion for public debt
Checkpoint: what’s next?

* We will start with the flow budget constraint.
* We will use forward substitution to derive the intertemporal budget constraint.
* Which we need to obtain solvency condition (in present-value terms).
Flow Budget Constraint

\[ D_t = (1 + i_t) D_{t-1} + PB_t \]
Intertemporal Budget Constraint for t=2

\[ D_1 = (1+i)D_0 - PB_1 \]
\[ D_2 = (1+i)D_1 - PB_2 \]
\[ = (1+i)((1+i)D_0 - PB_1) - PB_2 \]
\[ = D_1 \]
\[ D_2 = (1+i)^2D_0 - (1+i)PB_1 - PB_2 \]
Intertemporal Budget Constraint for $t=N$

$$D_N = (1+i)^N D_0 - \sum_{j=1}^{N} (1+i)^{N-j} PB_j$$
By dividing both sides by \((1+i)^N\) and putting \(D_0\) on the other side, we have the following expression:

\[
D_0 = \sum_{j=1}^{N} \left( \frac{1}{1+i} \right)^j pB_j + \left( \frac{1}{1+i} \right)^N D_N
\]
**Transversality (no-Ponzi Scheme) Condition**

\[
\lim_{N \to \infty} \left( \frac{1}{1+i} \right)^N D_N = 0
\]

**Meaning of Transversality Condition:** It prohibits issuing more and more debt without ever repaying principal and interest on the previously accumulated stocks.
Solvency Condition

Solvency: Assuming transversality (no-Ponzi scheme), the outstanding initial debt should be covered by the present value of future primary balances:

\[ D_0 = \sum_{j=1}^{\infty} \left( \frac{1}{1+i} \right)^j PB_j \]
Part 1 Unit 5

Public Debt Sustainability
In Closed Economy: Part 2
Learn how to derive the law of motion for the ratio of public debt-to-GDP for a closed economy

Analyze contributions of key macroeconomic variables to debt dynamics

Obtain the debt-stabilizing primary balance
Closed economy:

- Law of motion for public-debt-to GDP ratio
- Automatic debt dynamics
- Debt-stabilizing primary balance
Part 1 Unit 5: Lecture 1

Law of Motion for Public Debt-to-GDP in a Closed Economy
Law of motion for public-debt-to GDP ratio

Key macroeconomic variables affect debt sustainability

- Primary balance
- Initial level of debt
- Growth
- Real interest rate
Checkpoint: Where Are We?

We are still in the case of a closed economy (to avoid worrying about the exchange rate-induced variations in debt)
Checkpoint: what’s next?

We are about to get pragmatic and do some derivations in terms of ratios of debt stock to the economy’s capacity (GDP).

Law-of-motion for debt-to-GDP in hand, we will look at the impact of the key macroeconomic variables on debt dynamics.
Law of Motion: the Debt-to-GDP Ratio

Evolution of government debt at time $t$

$$D_t = (1+i_t) D_{t-1} - PB_t \quad (1)$$

By dividing (1) by nominal GDP, $P_t Y_t$

$$\frac{D_t}{P_t Y_t} = \frac{(1+i_t)}{(1+\pi_t) (1+g_t)} \quad \frac{D_{t-1}}{P_{t-1} Y_{t-1}} - \frac{PB_t}{P_t Y_t}$$
Dynamics of Debt-to-GDP Ratio

Evolution of the debt-to-GDP ratio using the definition of the real interest rate

\[
\phi_t = \frac{(1+i_t)}{(1+\pi_t)} \frac{(1+r_t)}{(1+g_t)} = \frac{(1+r_t)}{(1+g_t)}
\]

\[
d_t = \frac{(1+r_t)}{(1+g_t)} d_{t-1} - pb_t
\]
Change in Debt-to-GDP Ratio

Deducting past debt from both sides:

\[ d_t - d_{t-1} = \left( \frac{1 + r_t}{1 + g_t} \right) d_{t-1} - \phi_t - pb_t \]  (2)

Automatic debt dynamics
Law-of-motion for the debt-to-GDP ratio

Evolution of the debt-to-GDP ratio

\[
d_t = \frac{1+r_t}{1+g_t} d_{t-1} - pb_t
\]  

(2)

A higher primary balance \(\rightarrow\) a lower debt/GDP ratio
A higher initial debt \(\rightarrow\) a higher debt/GDP ratio
A higher growth rate \(\rightarrow\) a lower debt/GDP ratio
A higher real interest rate \(\rightarrow\) a higher debt/GDP ratio
Part 1 Unit 5: Lecture 2

Stabilizing Debt in a Closed Economy
Automatic debt dynamics
Stability of debt
Debt stabilizing primary balance
The danger of debt momentum
We can distinguish favorable from unfavorable automatic debt dynamics:

\[ d_t - d_{t-1} = \left( \frac{r_t - g_t}{1 + g_t} \right) d_{t-1} - pb_t \]  

**Automatic debt dynamics**

\( r < g \): favorable debt dynamics

\( r > g \): unfavorable debt dynamics
Stable/Sustainable Debt Dynamics

\[ d_t = \phi_t d_{t-1} - pb_t \]

where \( \phi_t < 1 \) or \( r_t < g_t \)

45° line

\( d_t \)
\( -pb_t \)
\( d^* \)
\( d_0 \)
\( d_{t-1} \)
EXPLOSIVE DEBT DYNAMICS

\[ d_t = \phi_t d_{t-1} - pb_t \]

where \( \phi_t > 1 \) or \( r_t > g_t \)
Primary Balance to Stabilize Debt:

Assuming \( d \) is constant, one can solve for the debt stabilizing primary balance \( pb^* \):

\[
pb_t^* = \frac{r_t - g_t}{1 + g_t} d_{t-1}
\]

Automatic debt dynamics
Primary Balance to Stabilize Debt:

- The primary surplus needed to keep the debt/GDP constant equals the debt dynamics. It is proportionate to the gap between real interest rate and real growth rate.

- The primary balance needed to keep the debt/GDP constant will rise directly with the size of the initial debt/GDP, if \( r > g \).

- We can also interpret the equation as telling us the level of debt which can be sustained for a given primary balance.
The Danger of Debt Momentum:

- The primary surplus needed to keep the debt/GDP constant will rise directly with the size of the initial debt/GDP.
- The higher is the initial debt stock, the more difficult it is to stabilize the debt/GDP ratio.
- Danger of built in momentum, the higher debt-to-GDP ratio gets, the less likely it is to run a sufficiently large primary surplus as debt rises.
- Thus, vulnerability rises with debt-to-GDP ratio.
Illustration: Evolution of Debt/GDP over Time:

\[ d_t = \left(1 + \frac{r_t}{1 + g_t}\right) d_{t-1} - pb_t \]

\[ d_{t-1} = 100\% \quad r = 2\% \quad g = 5\% \quad pb_t = -2\% \]
UNIT RECAP

- Law of motion for public-debt-to GDP ratio
- Automatic debt dynamics
- Debt-stabilizing primary balance
Part 1 Unit 6

Public Debt Sustainability
In Open Economy
Learn how to derive the law of motion for public debt for an open economy

Obtain the debt-stabilizing primary balance

Analyze contributions of key macroeconomic variables to debt dynamics
Open economy:

- Law of motion for public debt
- Debt-stabilizing primary balance
- Comparative statics
Law of Motion for Public Debt in an Open Economy
Flow budget constraint with external financing

Law of motion for the debt-to-GDP ratio

Automatic debt dynamics
Public Debt Dynamics

Open Economy
**NOTATION**

- \( D_t^D \): stock of domestic currency denominated debt (domestic debt)
- \( D_t^F \): stock of foreign currency denominated debt (foreign debt)
- \( i_t^D \): nominal interest rate on domestic debt
- \( i_t^F \): nominal interest rate on foreign debt
- \( i_t^{\text{eff}} \): effective nominal interest rate
- \( e_t \): nominal exchange rate (domestic currency per $)
- \( \varepsilon_t \): rate of exchange rate depreciation
- \( \alpha_t \): share of foreign currency denominated debt
Now suppose the government can borrow from abroad as well as domestically.

\[ D_t = D^D_t + e_t D^F_t \]

- Domestic currency denominated debt
- Foreign currency (\$) denominated debt
- Nominal exchange rate (local currency per \$)
The flow budget constraint becomes:

$$\begin{align*}
D_t^d + e_t D_t^f &= (1+i_t^d) D_{t-1}^d + (1+i_t^f) e_t D_{t-1}^f - PB_t + OT_t \\
&= (1) 
\end{align*}$$
Flow Budget Constraint

This can be re-expressed as:

$$D_t = (1+i_t^d)(1-\alpha_{t-1})D_{t-1} + (1+i_t^f)\alpha_{t-1}(1+\varepsilon_t)D_{t-1} - PB_t + OT_t$$

where we introduced the share of foreign currency denominated debt $\alpha$ and $\varepsilon$ is the rate of exchange rate depreciation

$$\alpha_{t-1} = \frac{e_{t-1}}{D_{t-1}} \cdot \frac{D_{t-1}^f}{D_{t-1}}$$
Dividing both sides by GDP at date $t$:

$$d_t = \frac{\left[ 1+i^w_t + \alpha_{t-1} \varepsilon_t (1+i^f_t) \right]}{(1+g_t)(1+\pi_t)} d_{t-1} - pb_t + ot_t$$

$i^w_t$ weighted average of domestic and foreign nominal interest rates

$i^f_t$ nominal interest rates in foreign-currency denominated debt

$\varepsilon_t$ change in the exchange rate (local currency per U.S. dollar)

$\alpha_{t-1}$ share of foreign-currency denominated public debt
Debt Law of Motion: Linking to the Real Interest Rate

Another way of expressing the debt-law-of motion:

\[ d_t = \frac{(1+r_t^d)}{(1+g_t)} d_{t-1}^d + \frac{(1+i_t^f)(1+\xi_t)}{(1+g_t)(1+\pi_t)} d_{t-1}^f - pb_t + ot_t \]

(2')

\[ (1+r_t^d) = \frac{(1+i_t^d)}{(1+\pi)} \]
Change in Debt to GDP Ratio

\[ d_t = \phi_t d_{t-1} - pb_t + ot_t \]

Subtract \( d_{t-1} \) on both sides:

\[ \Delta d_t = (\phi_t - 1)d_{t-1} - pb_t + ot_t \quad (3) \]

Automatic debt dynamics
Coefficient on Automatic Debt Dynamics:

\[ \phi_t^* - 1 = \frac{i_t^w - \pi_t (1 + g_t) - g_t + \alpha_{t-1} \varepsilon_t (1 + i_t^f)}{(1 + g_t) (1 + \pi_t)} \] 

(3)
**Automatic Debt Dynamics:**

\[
d_t - d_{t-1} = \frac{i^w_t - \pi_t (1 + g_t)}{(1 + g_t)(1 + \pi_t)} d_{t-1} - \frac{g_t}{(1 + g_t)(1 + \pi_t)} d_{t-1} + \frac{\varepsilon_t \alpha_t (1 + f_t)}{(1 + g_t)(1 + \pi_t)} d_{t-1} - pb_t + o_t,
\]

- **Contribution of the effective real interest rate**
- **Contribution of the real GDP growth**
- **Contribution of exchange rate depreciation**
Coefficient on Automatic Debt Dynamics

\[ \phi^*_t - 1 = \left[ \frac{i^w_t - \pi_t (1 + g_t) - g_t + \alpha_{t-1} e_t (1 + i^f_t)}{(1 + g_t)(1 + \pi_t)} \right] \]
Automatic Debt Dynamics

\[ d_t - d_{t-1} = \frac{i_t - \pi_t (1 + g_t)}{(1 + g_t)(1 + \pi_t)} d_{t-1} - \frac{g_t}{(1 + g_t)(1 + \pi_t)} d_{t-1} + \frac{\varepsilon_t \alpha_{t-1} (1 + i_t^f)}{(1 + g_t)(1 + \pi_t)} d_{t-1} - pb_t + ot_t \]

**Contribution of the effective real interest rate**

**Contribution of real GDP growth**

**Contribution of exchange rate depreciation**
Debt stabilizing primary balance

Comparative statics: role of key macroeconomic variables:
- Initial level of debt
- Growth
- Interest Rate
- Exchange Rate
Debt-Stabilizing Primary Balance in an Open Economy:

\[ \Delta d_t = (\phi_t^* - 1)d_{t-1} - pb_t + ot_t \] (3)

\[ d_t = d_{t-1} \]

\[ pb_t^* = (\phi_t^* - 1)d_t + ot_t \]
Debt-Stabilizing Primary Balance

\[ pb^* = \frac{(r^w_t - g_t) + \alpha_{t-1} \varepsilon_t^* (1 + r^f_t)}{(1 + g_t)} \]

\[ d_{t-1} + ot_t \]

Automatic debt dynamics

\[ r^w = ar^f + (1 - \alpha) r^d \]

\[ (1 + \varepsilon^*) = \frac{(1 + \varepsilon)(1 + \pi^*)}{(1 + \pi)} \]
Key Comparative Statics:

The required primary balance is higher when:

- The real interest rate - growth differential is large
- Other flows contribute to an increase in debt (e.g. financial sector support measures, nationalization of private pensions)
- There is exchange rate depreciation ($\varepsilon$) in countries with large foreign exchange denominated debt ($\alpha$)

Note: The last two are examples of “stock-flow adjustments” because they help reconcile the change in the value of debt with the deficit
# Illustration:

## Key Macro-Economic Variables

<table>
<thead>
<tr>
<th></th>
<th>Country W</th>
<th>Country X</th>
<th>Country Y</th>
<th>Country Z</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>(% change)</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( r^d )</td>
<td>5</td>
<td>4</td>
<td>5</td>
<td>5</td>
</tr>
<tr>
<td>( r^f )</td>
<td>..</td>
<td>..</td>
<td>..</td>
<td>5</td>
</tr>
<tr>
<td>( g )</td>
<td>4</td>
<td>5</td>
<td>4</td>
<td>4</td>
</tr>
<tr>
<td>( \varepsilon^* )</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>5</td>
</tr>
<tr>
<td><strong>(% GDP)</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( d^d )</td>
<td>100</td>
<td>100</td>
<td>20</td>
<td>50</td>
</tr>
<tr>
<td>( d^f )</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>50</td>
</tr>
<tr>
<td>( pb^* )</td>
<td>0.96</td>
<td>-0.95</td>
<td>0.19</td>
<td>3.49</td>
</tr>
</tbody>
</table>
Illustration:

- We can calculate $pb^*$ by applying the formula, while being careful to express interest rates and growth rates as a percent:

$$ pb^* = \frac{(r_t^w - g_t)d + \varepsilon^* \cdot (1 + r_t^f)d^f}{(1 + g_t)} $$

- For country Z for example:

$$ pb^* = \frac{(0.05 - 0.04) \cdot 1 + 0.05 \cdot (1 + 0.05) \cdot 0.5}{(1 + 0.04)} = 0.0349 = 3.49\% $$
UNIT RECAP

- Debt Law-of-Motion
- Debt-stabilizing Primary Balance
- Key Comparative Statics
Part 1 Unit 7

Chipping Away at Public Debt
UNIT OBJECTIVES

✓ Understand different adjustment paths and their implications

✓ Understand how fiscal adjustment may affect GDP and the risk premium on government debt
Adjustment Paths and Their Implications

Front-loaded and back-loaded adjustments

Fiscal Adjustment, the Business Cycle, and the Risk Premium

History of Past Fiscal Adjustments (video)
Part 1 Unit 7: Lecture 1

Adjustment Paths and Their Implications
Definition of front-loaded and back-loaded adjustment

Circumstances favoring each
Front-loaded and Back-loaded Adjustment:

- Front-loaded adjustment:

- Back-loaded adjustment:
Front-loaded and Back-loaded Adjustment:

✓ Front-loaded fiscal adjustment quickly raises the primary balance to the “targeted” level.

✓ Back-loaded adjustment phases in the adjustment over time.
Circumstances Affecting the Timing of Adjustment:

- **Front-loading may be necessary:**
  - when facing severe financing constraints
  - to build credibility
  - to seize opportunity of political support

- **Back-loading may be preferable to:**
  - support to economic activity
  - ensure quality of measures
Circumstances Affecting the Timing of Adjustment:

- Credibility is very important in the context of high debt, because of its effect on the risk premium and therefore debt dynamics.

- In the case of back-loading, credibility can be enhanced by institutional mechanisms, such as balanced budget rules and procedural rules.
Circumstances Affecting the Timing of Adjustment:

- If fiscal adjustment has a negative impact on growth, it may undermine debt sustainability
  - undertake fiscal adjustment in the upswing of the business cycle
  - undertake fiscal adjustment during worldwide recoveries
  - support fiscal adjustment with accommodating monetary policy
Front-loaded and Back-loaded Adjustment

In the Additional Resources we provide a formula for the fiscal adjustment necessary to reduce debt over a given number of periods. The formula allows one to distinguish front-loaded adjustment from back-loaded adjustment.
Reducing Debt in \( k \) Periods

First, we define the future debt target as a proportion of the actual debt:

\[
d^*_t = \gamma^* d_t \quad \text{with} \quad \gamma^* \leq 1
\]

Assuming a constant interest rate, GDP growth rate, and government primary balance (\( pb^* \)) we can express the solvency condition as:

\[
d_t = \left( \frac{1}{\phi} \right)^k \gamma^* d_t + pb^* \sum_{j=1}^{k} \left( \frac{1}{\phi} \right)^j
\]
Useful Formula for Geometric Series

\[ \sum_{j=1}^{k} ar^j = a \left( \frac{r - r^{k+1}}{1 - r} \right) \]
Reducing Debt in $k$ Periods

The needed primary balance ($pb^*$) to reduce the debt from $d$ to $d^*$ in $k$ periods corresponds to:

$$pb^* = \left( \frac{(\phi - 1)(Y^* - \phi^k)}{1 - \phi^k} \right) d_t$$

The lower $Y^*$ and/or $k$, the larger $pb^*$ would need to be to reach the debt target in the desired time.
This material took stock of:

- The primary balance necessary to reduce debt to certain level
- The potential impact on cost of funding
The tradeoff between front-loaded and back-loaded fiscal adjustment is

- front-loading to ease financing constraints and gain credibility vs.

- back-loading to support growth and work out quality measures
Part 1 Unit 7: Lecture 2

Fiscal adjustment and the business cycle
How the budget balance affects GDP
How the budget balance affects the risk premium
Fiscal Adjustment and the Business Cycle

Diagram showing the relationship between Direct effect, Consolidation, Interest rate, Automatic Stabilizers, Growth/inflation, Multipliers, and Direct effect.
Three main channels from primary balance to debt/GDP:

- directly via the primary balance in the debt dynamics
- via GDP through demand
- lower government spending and higher taxes reduce demand
- via interest rates through credibility (risk premium) and demand
- crowding out
Fiscal Adjustment and the Business Cycle

- Fiscal consolidation may lead to slower GDP growth
  - High multiplier (closed economy, high unemployment)
  - Coordinated consolidations in economic partners
- Fiscal consolidation may lead to lower interest rates
  - In high-debt countries credibility effects are particularly important
Front-loaded vs back-loaded adjustment

The speed at which debt can be reduced depends on how fiscal adjustment affects GDP and interest rates.
Please watch the video on Chipping Away At Public Debt
Part 1 Unit 8

Role of Macroeconomic Policies
UNIT OBJECTIVES

☑ Understand the role of monetary policy
☑ Understand the economic policy tradeoffs
Monetary Policy Stance and Debt

Policy Tradeoffs
Part 1 Unit 8: Lecture 1

Monetary Policy Stance and Debt
Expansionary monetary policy—possible effects

The effect of monetary policy on interest rates and inflation
Monetary policy stance and debt

- Interest rate
- Inflation
- Exchange rate
- Growth
Monetary policy stance and debt

- Expansionary monetary policy—possible effects
  - Lower nominal interest rates
  - Lower real interest rates
  - Higher inflation
  - Higher growth
  - Depreciated exchange rate
Monetary policy stance and debt

Use the debt dynamics equation and assuming at first no foreign currency debt:

if real interest rates fall and growth improves (therefore improving the primary balance), debt is reduced.

\[ d_t = \frac{(1 + r_t)}{(1 + g_t)} d_{t-1} - (pb_t + ot_t) \]
Monetary policy stance and debt

If there is foreign currency debt, the effect is no longer unambiguous. Debt/GDP could increase, in the case of less than complete pass-through of exchange rate depreciation to inflation.

\[ d_t = \frac{(1 + r_t^d)}{(1 + g_t^u)} d_{t-1}^d + \frac{(1 + i_t^f)(1 + \varepsilon_t^u)}{(1 + g_t^u)(1 + \pi_t^u)} d_{t-1}^f - (p b_t^u + o t_t) \]
**Illustration: Impact of Monetary Easing**

\[ d_t \approx (r^d_t - g_t) d^d_{t-1} + (i^f_t + \varepsilon_t - g_t - \pi_t) d^f_{t-1} - pb_t - \delta_t \]

<table>
<thead>
<tr>
<th>Easing of monetary policy</th>
<th>Depreciation</th>
</tr>
</thead>
<tbody>
<tr>
<td>No easing</td>
<td>No change in exchange rate</td>
</tr>
<tr>
<td>(% change)</td>
<td></td>
</tr>
<tr>
<td>( r^d )</td>
<td>6</td>
</tr>
<tr>
<td>( i^f )</td>
<td>3</td>
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<tr>
<td>( \varepsilon^* )</td>
<td>0</td>
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<tr>
<td>( \pi )</td>
<td>0</td>
</tr>
<tr>
<td>( g )</td>
<td>3</td>
</tr>
<tr>
<td>( pb )</td>
<td>-2</td>
</tr>
<tr>
<td>(% GDP)</td>
<td></td>
</tr>
<tr>
<td>( d^d_{t-1} )</td>
<td>50</td>
</tr>
<tr>
<td>( d^f_{t-1} )</td>
<td>50</td>
</tr>
<tr>
<td>( d_t )</td>
<td>105.0</td>
</tr>
</tbody>
</table>
Monetary Policy Stance and Debt

- The effect of monetary policy on interest rates and inflation needs to be qualified.

- Expansionary monetary policy tends to reduce short-term interest rates but increase long-term ones, reflecting expectations of future inflation.
Monetary Policy Stance and Debt

✓ Fischer equation: \( i \approx r^* + \pi^e \)

✓ Interest rates are set based on a required real return \( r^* \)

- Given \( r^* \), higher \( \pi^e \) translates into higher \( i \)

- Ex post \( r \) need not equal \( r^* \) if there is surprise inflation (in which case \( r < r^* \))
Part 1 Unit 8: Lecture 2

Policy Tradeoffs
Sustainable Debt vs. Inflation
Sustainable Debt vs. Competitiveness
Sustainable Debt vs. Fairness
Policy Tradeoffs

✓ Fiscal dominance
✓ Fear of floating
✓ Fairness/income distribution
Policy Tradeoffs: Sustainable Debt vs. Inflation

✅ Fiscal dominance: inability to conduct contractionary monetary policy because it would jeopardize government debt dynamics

✅ Contractionary monetary policy, which would result in
  - higher real interest rates
  - lower growth
  - higher debt/GDP
Policy Tradeoffs: Sustainable Debt vs. Competitiveness

✔ Fear of Floating: reluctance to allow a floating exchange rate to depreciate

✔ Loose monetary policy is helpful for competitiveness and growth, but will raise the value of foreign currency debt expressed in local currency (public and private) and may cause bankruptcies
Policy Tradeoffs: Sustainable Debt vs. Competitiveness

Fear of floating follows from the “original sin”—the inability of emerging markets to issue external debt in their own currency.
Policy Tradeoffs: Sustainable Debt vs. Fairness

- Inflation as default
  - There is a thin line separating inflation from default since inflation erodes away the value of debt (especially when there is “financial repression” capping nominal interest rates)
  - Inflation creates a redistribution of wealth from creditors to debtors
Part 1 Unit 9

External Debt Sustainability
UNIT OBJECTIVES

✓ Understanding similarities between external and fiscal sustainability

✓ Understanding external debt creating flows

✓ Understanding solvency condition for external debt
UNIT OUTLINE

- External DSA
- External debt creating flows
- Debt law-of-motion
- Solvency condition for external debt
Part 1 Unit 9: Lecture 1

External Debt Creating Flows
External DSA
External debt creating flows
The adjusted balance
External DSA:

- Similarities between external and fiscal sustainability apply similar methodologies
- Focus on external debt of the country (including the private sector debt)
- The current account balance of the balance of payments takes the place of the overall budget balance.
Key Differences with Public Debt:

- The government does not directly control the CAB
- In a healthy cycle, exports and CAB will improve over time, allowing for repayment of debt
- Exchange rate normally plays larger role in external sustainability
Our goal in this unit is to derive a law of motion for external debt, which links debt to past debt and the current account balance.

\[ D_t^f = (1 + i_t^f) D_{t-1}^f - AB_t \]

In the process we will define the adjusted balance, \( AB \), which is a modified current account balance.
Notation:

$CA_t$ current account balance
$AB_t$ adjusted balance
$l_t$ interest payments on external debt
$KA_t$ capital account
$FA_t$ financial account
$A_t$ external assets
$L_t$ external liabilities
$D_{tf}$ external debt liabilities
$E_t$ external equity liabilities

$(P_t Y_t)$ GDP expressed in USD = $P_t Y_t / e_t$

All variables are expressed in USD
The Adjusted Balance:

Define:

\[ AB_t = (CA_t + I_t) + (\Delta E_t - \Delta A_t) \]

The current account (the sum of net exports, income and current transfers) records interest payments on debt as a negative income item. Here we add interest back to obtain the non-interest CAB.
Part 1 Unit 9: Lecture 2

External Financing Constraint and Debt—Law—of—Motion
OUTLINE

☑️ External Financing Constraint
☑️ Debt Law-of-motion
Using the terminology of the BOP and IIP manual (6th edition), we write:

\[ CA_t + KA_t = FA_t \]

We assume for simplicity that the capital account \( KA \) (capital transfers for the most part) is zero.

\[ KA_t = 0 \]
External Financing Constraint

- The financial account of the BOP records the acquisition of assets and the incurrence of liabilities (e.g. as the result of external borrowing). These flows are called “transactions”.

- We assume for simplicity that valuation effects are zero, so that the change in the value of assets and liabilities is equal to these BOP transactions.

\[ CA_t = \Delta A_t - \Delta L_t \]
Liabilities can be either debt liabilities or equity liabilities:

\[ CA_t = \Delta A_t - (\Delta D_t^f + \Delta E_t) \]

\[ \Delta L_t \]

Debt includes debt securities, loans, currency and bank deposits. Equity includes shares and foreign direct investment.
Rewriting the previous equation in terms of current period debt and adding and deducting interest:

\[ D_{t}^{f} = D_{t-1}^{f} - CA_t - \Delta E_t + \Delta A_t - I_t + I_t - AB_t \]

Next assume as before:

\[ I_t = i_t^f D_{t-1}^f \]
Finally, using the definition of the adjusted balance and grouping terms involving lagged debt, we find the law-of-motion we had set out to find:

\[ D_t^f = (1 + i_t^f)D_{t-1}^f - AB_t \]
Part 1 Unit 9 Lecture 3:

Solvency condition for external debt
Solvency condition for external debt

✓ From the debt law-of-motion we can obtain the intertemporal budget constraint through repeated substitution, as we did for public debt.

✓ We then obtain the solvency condition for external debt by imposing the tranversality condition or No-Ponzi condition. Specifically, we require that the present discounted value of external debt at time infinity approaches zero.
Solvency Condition For External Debt

Using the same method as for public debt, the intertemporal budget condition extended to $N$ periods is:

\[ D_{0}^{f} = \sum_{j=1}^{N} \left( \frac{1}{1+i_{t}^{f}} \right)^{j} AB_{j} + \left( \frac{1}{1+i_{t}^{f}} \right)^{N} D_{N}^{f} \]

We extend this formula to time infinity and impose the transversality condition.
Solvency Condition

Solvency: the present value of all surpluses (of the adjusted balance) is equal to initial debt.

\[ D_0^f = \sum_{j=1}^{\infty} \left( \frac{1}{1+i_t^f} \right)^j AB_j \]
External Debt Sustainability Analysis
External Debt Creating Flows
Debt Law-of-motion
Solvency condition for external debt
Part 1 Unit 10 Lecture 1:
Deriving External Debt Law-of-Motion
Debt Law-of-motion: The Debt-To-GDP Ratio

Automatic Debt Dynamics
External Debt Law-of-Motion: the Debt-to-GDP Ratio

- Evolution of external debt at time $t$:
  \[ D_t^f = (1 + i_t^f)D_{t-1}^f - AB_t \]
- We divide by GDP expressed in USD, $(P_t Y_t)^*$ since external debt is expressed in USD.
- Using $(P_t Y_t)^* = \frac{P_t Y_t}{e_t}$
  \[ \frac{D_t^f}{(P_t Y_t)^*} = \frac{(1 + i_t^f)D_{t-1}^f}{(1 + g_t)(1 + \pi_t)P_{t-1}Y_{t-1} / e_t} - \frac{AB_t}{(P_t Y_t)^*} \]
  \[ P_t Y_t \]
We convert GDP in local currency back to GDP in USD and use $1+\varepsilon_t = e_t/e_{t-1}$

$$\frac{D_t^f}{(P_tY_t)^*} = \frac{(1+i_t^f)(1+\varepsilon_t)}{(1+g_t)(1+\pi)} \frac{D_{t-1}^f}{(P_{t-1}Y_{t-1})^*} - \frac{AB_t}{(P_tY_t)^*}$$

$D_t^f$  $d_t^f$  $D_{t-1}^f$  $d_{t-1}^f$  $AB_t$  $ab_t$
Debt Law-of-Motion: the Debt-to-GDP Ratio

It is easy to show that

\[ \frac{(1 + i_t^f)(1 + \varepsilon_t)}{(1 + \pi_t)} = (1 + r_t^f)(1 + \varepsilon^*_t) \]

where

\[ (1 + r_t^f) = \frac{(1 + i_t^f)}{(1 + \pi^*_t)} \]
\[ (1 + \varepsilon^*_t) = \frac{(1 + \varepsilon)(1 + \pi^*)}{(1 + \pi)} \]

\( \pi^* = \) foreign inflation rate
\( r_t^f = \) real interest rate on foreign debt
\( \varepsilon^* = \) real exchange rate depreciation
Debt Law-of-Motion: Analytical Representation

Evolution of the debt-to-GDP ratio

\[ d_t^f = \left( \frac{1 + r_t^f}{1 + \varepsilon^*_t} \right) (1 + g_t) d_{t-1}^f - ab_t \]

For \( ab_t = 0 \) and \( d_{t-1}^f > 0 \):

- If \( \phi_t < 1 \), debt converges to zero
- If \( \phi_t > 1 \), debt explodes
Deducting past debt from both sides and simplifying:

\[ d_t^f - d_{t-1}^f = \frac{(1 + r_t^f)(1 + \varepsilon_t^*)}{(1 + g_t)} d_{t-1}^f - d_{t-1}^f - ab_t \]

\[ d_t^f - d_{t-1}^f = \frac{r_t^f - g_t + \varepsilon_t^* (1 + r_t^f)}{(1 + g_t)} d_{t-1}^f - ab_t \]
Debt-Stabilizing Adjusted Balance

What level of ab keeps debt constant? To find the answer, simply set $d_t = d_{t-1}$ in the previous expression (denote the constant debt level by $d^*$)

$$ab^* = \frac{r_t^f - g_t + \varepsilon^*_t (1 + r_t^f)}{(1 + g_t)} d^*$$

$ab^*$ is the debt-stabilizing primary surplus. Note that it equals the automatic debt dynamics.

If $ab > ab^*$ debt falls continuously; if $ab < ab^*$, debt explodes and is therefore unsustainable.
Debt dynamics are affected by:

- Real interest rate
- Growth rate of the economy
- Current level of indebtedness
- Net exports
- Long-term level of other flows
- Real exchange rate changes
### Illustration

<table>
<thead>
<tr>
<th></th>
<th>Country X</th>
<th>Country Y</th>
<th>Country Z</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>(%) change</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$r^f$</td>
<td>5</td>
<td>5</td>
<td>5</td>
</tr>
<tr>
<td>$g$</td>
<td>4</td>
<td>4</td>
<td>4</td>
</tr>
<tr>
<td>$\epsilon$</td>
<td>0</td>
<td>0</td>
<td>-2</td>
</tr>
<tr>
<td><strong>(%) GDP</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$d^f$</td>
<td>100</td>
<td>20</td>
<td>100</td>
</tr>
</tbody>
</table>

*ab*
We can calculate \( ab^* \) by applying the formula, while being careful to divide the parameters by 100:

\[
ab^* = \frac{r_t^f - g_t + \epsilon_{t}^* (1 + r_t^f)}{(1 + g_t)} d^*
\]

For country Z for example:

\[
ab^* = \frac{0.05 - 0.04 - 0.02(1 + 0.05)}{(1 + 0.04)} \cdot 1 = -0.0106 = -1.06\%
\]
Summary
Key Takeaways for External Debt
Key Equations

Debt law-of-motion

\[ D_t^f = (1 + i_t^f)D_{t-1}^f - AB_t \]

Adjusted balance

\[ AB_t = (CA_t + I_t) + (\Delta E_t - \Delta A_t) \]

Debt law-of-motion (%GDP)

\[ d_t^f - d_{t-1}^f = \frac{r_t^f - g_t + \varepsilon_t^* (1 + r_t^f)}{(1 + g_t)}d_{t-1}^f - ab_t \]
Debt dynamics

Debt dynamics are affected by:
- Real interest rate
- Growth rate of the economy
- Current level of indebtedness
- Net exports
- Long-term level of other flows
- Real exchange rate changes