Neuronal Dynamics: Computational Neuroscience of Single Neurons

Week 7 – Optimizing Neuron Models For Coding and Decoding

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7.1 What is a good neuron model?
- Models and data

7.2 AdEx model
- Firing patterns and analysis

7.3 Spike Response Model (SRM)
- Integral formulation

7.4 Generalized Linear Model
- Adding noise to the SRM

7.5 Parameter Estimation
- Quadratic and convex optimization

7.6 Modeling in vitro data
- how long lasts the effect of a spike?

7.7. Helping Humans

Week 7 – part 3: Spike Response Model (SRM)
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7.7. Helping Humans
Exponential versus Leaky Integrate-and-Fire

\[ \tau \frac{du}{dt} = -(u - u_{\text{rest}}) + \Delta \exp \left( \frac{u - \vartheta}{\Delta} \right) + RI(t) \]

Reset if \( u = \vartheta \)

Leaky Integrate-and-Fire

\[ \tau \frac{du}{dt} = -(u - u_{\text{rest}}) + RI(t) \]


\[ \Delta = 2\text{mV} \]
Neuronal Dynamics – 7.3 Adaptive leaky integrate-and-fire

\[
\begin{align*}
\tau \frac{du}{dt} &= -(u - u_{rest}) - R \sum_k w_k + RI(t) \\
\tau_k \frac{dw_k}{dt} &= a_k (u - u_{rest}) - w_k + b_k \tau_k \sum_f \delta(t - t^f)
\end{align*}
\]

SPIKE AND RESET

\begin{align*}
\text{after each spike} \\
& w_k \text{ jumps by an amount } b_k \\
\text{If } u = \theta(t) \text{ then reset to } u = u_r
\end{align*}

Dynamic threshold
Neuronal Dynamics – 7.3 Adaptive leaky I&F and SRM

\[
\tau \frac{du}{dt} = -(u - u_{\text{rest}}) - R \sum_k w_k + RI(t)
\]
\[
\tau_k \frac{dw_k}{dt} = a_k (u - u_{\text{rest}}) - w_k + b_k \tau_k \sum_f \delta(t - t^f)
\]

Linear equation \(\rightarrow\) can be integrated!

\[
u(t) = \sum_f \eta (t - t^f) + \int_0^\infty ds \kappa(s) I(t - s)
\]
\[
\vartheta(t) = \theta_0 + \sum_f \theta_1(t - t^f)
\]

Adaptive leaky I&F

Spike Response Model (SRM)

Gerstner et al. (1996)
Neuronal Dynamics – 7.3 Spike Response Model (SRM)

Gerstner et al., 1993, 1996

\[ \vartheta(t) = \theta_0 + \sum_{t'} \theta_1(t - t') \]

\[ u(t) = \sum_t \eta(t - t') + \int_0^\infty \kappa(s) I(t - s) ds + u_{rest} \]

Spike emission

Arbitrary Linear filters

Potential
Neuronal Dynamics – 7.3 Bursting in the SRM

SRM with appropriate $\eta$ leads to bursting

\[ u(t) = \sum_{f} \eta (t - t^f) + \int_{0}^{\infty} ds \kappa(s) I(t - s) + u_{\text{rest}} \]
Neuronal Dynamics – 7.3 Spike Response Model (SRM)

Input $I(t)$

Potential $u(t) = \sum_{t'} \eta(t - t') + \int_0^{\infty} \kappa(s) I(t - s) ds + u_{\text{rest}}$

Threshold $
\vartheta(t) = \vartheta_0 + \sum \theta_1(t - t')$}

Firing if $u(t) = \vartheta(t)$

Gerstner et al., 1993, 1996

$S(t)$
Neuronal Dynamics – 7.3 Spike Response Model (SRM)

potential

\[ u(t) = \sum_{t'} \eta(t - t') + \int_0^{\infty} \kappa(s) I(t - s) ds + u_{\text{rest}} \]

threshold

\[ \mathcal{H}(t) = \theta_0 + \sum_{t'} \theta_1(t - t') \]

Linear filters for
- input
- threshold
- refractoriness
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- input
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