Default Probabilities
Credit Risk Plus

Dr. Pasquale Cirillo
Credit Risk Plus (CR+)

- Introduced in 1997 by Credit Suisse Financial Products.

- It is based on well-known tools of actuarial mathematics.

- It is a powerful but complex model. Here we just sketch the very basic idea.
Suppose that a financial institution has $n$ loans of a given type.

For simplicity we assume these loans to be homogeneous in terms of risk, so that we can say that the 1-year PD of each loan is $p$.

$p$ can be obtained from external or internal credit ratings, for example.
Let $\mu$ be the expected number of defaults for the whole portfolio of loans.

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If we assume defaults to be independent, the probability of observing \( m \) defaults over the total of \( n \) loans will be like the probability of tossing a (possibly biased) coin \( n \) times and observing \( m \) heads, when the probability of getting a head is \( p \).
If you are familiar with basic probability, you know that such a probability is

\[
\frac{n!}{m!(n - m)!} p^m (1 - p)^{n-m}
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Binomial distribution
If we assume $p$ to be small and $n$ large, the Binomial distribution is well approximated by a Poisson distribution.

The probability of observing $m$ defaults thus becomes

$$e^{-\mu} \frac{\mu^m}{m!}$$
The previous information about the probability of observing a certain number of defaults can be combined with the probability distribution for the losses experienced when a certain type of counterparty defaults.

This leads us to the computation of a probability distribution for the total losses from defaults.

On that distribution we can compute quantities such as VaR and ES.
The probability distribution for the losses from a counterparty, when it defaults, can be determined from historical data.

For example, from historical data about EADs and LGDs.
The simple approach we have just seen is just a very special and unrealistic version of CR+.

The model which is actually used by banks is much more complex from a mathematical point of view, because it introduces more realistic components, e.g.:

- Correlation/dependence among defaults;
- Variable default rates;
- Macroeconomic factors;
- etc.
An interesting characteristic of CR+ is the possibility of obtaining closed-form results, once we make some technical assumptions about the parameters of the model.

At the same time, CR+ is easy to simulate, and it can also be studied using computational techniques.
Thank You