TW3421x - An Introduction to Credit Risk Management

Default Probabilities

Credit Risk Plus

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Week 6

Lesson 3

- Introduced in 1997 by **Credit Suisse Financial Products.**
- It is based on well-known tools of actuarial mathematics.
- It is a powerful but complex model. Here we just sketch the very basic idea.

Basic Idea

- Suppose that a financial institution has n loans of a given type.
- For simplicity we assume these loans to be **homogeneous in terms of risk**, so that we can say that the 1-year PD of each loan is p.
- p can be obtained from **external or internal credit ratings**, for example.

Number of Defaults

- Let μ be the expected number of defaults for the whole portfolio of loans.
- Then we have that

$$\mu = np$$

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Number of Defaults

- If we assume defaults to be **independent**, the probability of observing m defaults over the total of n loans will be like the probability of tossing a (possibly biased) coin n times and observing m heads, when the probability of getting a head is p.

Number of Defaults

- If you are familiar with basic probability, you know that such a probability is

$$\frac{n!}{m!(n-m)!}p^m(1-p)^{n-m}$$

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Binomial distribution

Poisson Approximation

- If we assume p to be small and n large, the Binomial distribution is well approximated by a **Poisson distribution**.
- The probability of observing m defaults thus becomes

$$\frac{e^{-\mu}\mu^m}{m!}$$

Total distribution of losses

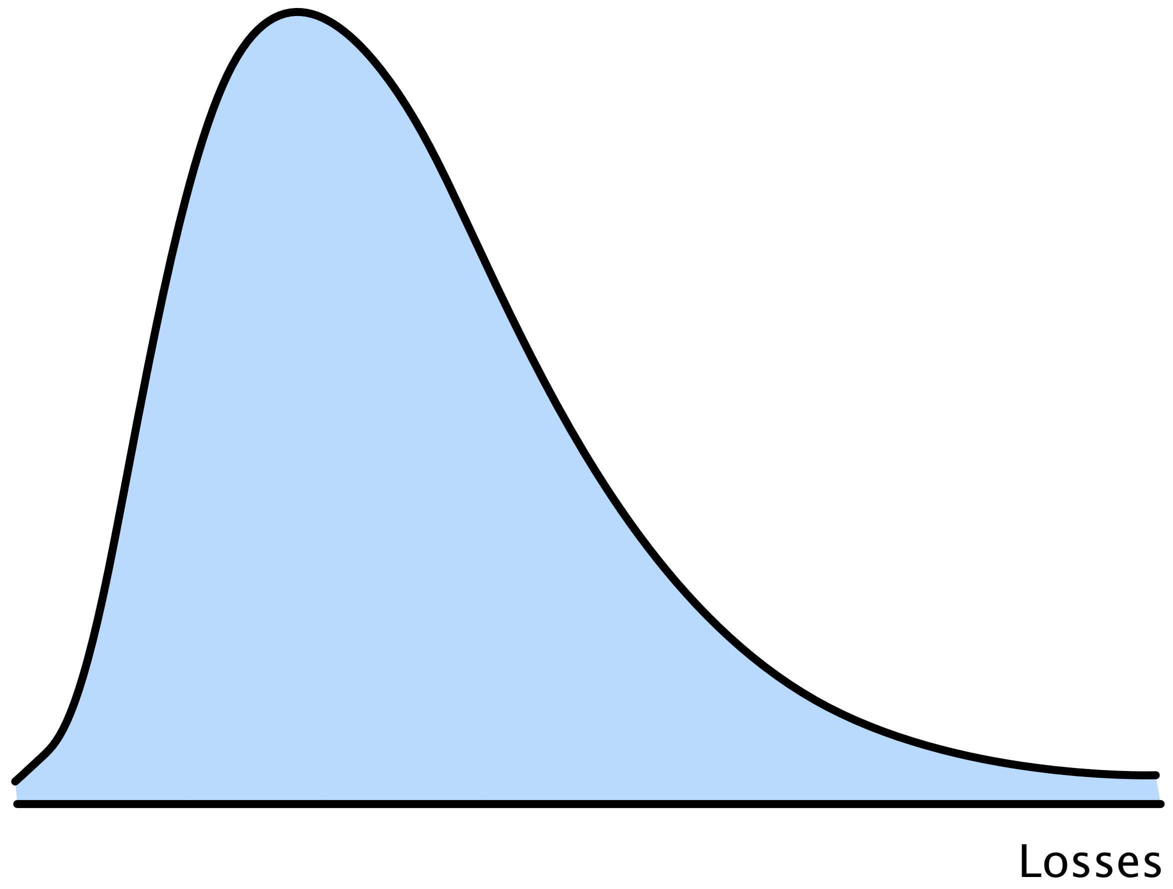
- The previous information about the probability of observing a certain number of defaults can be combined with the probability distribution for the losses experienced when a certain type of counterparty defaults.

- This leads us to the computation of a **probability distribution for the total losses from defaults**.

- On that distribution we can compute quantities such as VaR and ES.

Losses from a counterparty

- The probability distribution for the losses from a counterparty, when it defaults, can be determined from historical data.
- For example, from historical data about EADs and LGDs.



The “real” CR+

- The simple approach we have just seen is just a very special and unrealistic version of CR+.
- The model which is actually used by banks is much more complex from a mathematical point of view, because it introduces more realistic components, e.g. :
    - Correlation/dependence among defaults;
    - Variable default rates;
    - Macroeconomic factors;
    - etc.

Practical use of CR+

- An interesting characteristic of CR+ is the possibility of obtaining closed-form results, once we make some technical assumptions about the parameters of the model.

- At the same time, CR+ is easy to simulate, and it can also be studied using computational techniques.

Thank You