TW3421x - An Introduction to Credit Risk Management

Default Probabilities

CreditMetrics

Dr. Pasquale Cirillo

Week 6

Lesson 1

CreditMetrics™

JPMorgan Chase & Co.

- Introduced in 1997 by J.P. Morgan & Co.
- It is a structural model of default, which **also takes into account the risk of credit deterioration**.
- As Moody's KMV, it can be seen as a Merton-like model.

The intuition

- In CreditMetrics thresholds are not given by liabilities, but defined **through credit ratings**.

Some details

- We consider a firm that has been assigned to some credit rating class (say B) at the beginning of period [0,T].
- This means that we also have information about the transition probabilities of our company from its rating class to another.

Initial Rating	Rating at the end of the period T (%)							
	AAA	AA	A	BBB	BB	B	CCC	DF
AAA	90.81	8.33	0.68	0.06	0.12	0.00	0.00	0.00
AA	0.70	90.65	7.79	0.64	0.06	0.14	0.02	0.00
A	0.09	2.27	91.05	5.52	0.74	0.26	0.01	0.06
BBB	0.02	0.33	5.85	86.9	5.30	1.1	0.32	0.18
BB	0.03	0.14	0.67	7.73	80.53	8.84	1.00	1.06
B	0.00	0.11	0.24	0.43	6.48	83.46	4.07	5.20
CCC	0	0.00	0.22	1.30	2.38	11.24	64.86	19.79

Example

Some details

- With $\bar{p}(j), 0 \leq j \leq n$, we indicate the probability that our company will be in rating class j at time T.
- We order the j's, so that $j=0$ means "default" and $j=n$ means that our company is in the best rating class, e.g. AAA.
- With $\bar{p}(0)$, we thus indicate the probability of default of the company.

Asset value

- Using the same notation of Merton's model, we indicate the value of the assets of our company in T as V_T.
- As we have seen with Merton, V_T can be linked to a Normal distribution function.

The thresholds

- We can then define a collection of thresholds

$$-\infty = \tilde{d}_0 < \tilde{d}_1 < \tilde{d}_2 < \cdots < \tilde{d}_n < \tilde{d}_{n+1} = \infty$$

such that

$$P\left(\tilde{d}_j < V_T \leq \tilde{d}_{j+1}\right) = \bar{p}(j), \ \forall j \in \{0, \ldots, n\}$$

- In this way we translate the transition probabilities into a series of thresholds for the asset value process.

The thresholds

- The threshold $\tilde{d}_1$ is clearly the default threshold.
- Larger thresholds represent the asset value levels that define the boundaries of the higher rating classes.
- In this setting, a firm belongs to rating class *j* at time T if and only if

$$\tilde{d}_j < V_T \leq \tilde{d}_{j+1}$$

Re-parametrization

- In CreditMetrics, to exploit the properties of the **standard Normal distribution**, the asset value V_T is transformed into

$$X_T := \frac{\log(V_T) - \log(V_0) - (r - \sigma_V^2/2)T}{\sigma_V \sqrt{T}}$$

- And the same for the thresholds:

$$d_j^* := \frac{\log(\tilde{d}_j) - \log(V_0) - (r - \sigma_V^2/2)T}{\sigma_V \sqrt{T}}$$

Re-parametrization

- As a consequence of this new parametrization, we have that a firm belongs to rating class j at time T if and only if

$$d_j^* < X_T \leq d_{j+1}^*$$

An example

- Consider a B-rated company over a 1-year period.
- And consider the following fictitious Moody's transition matrix

Initial Rating	Rating at the end of the 1 year (%)								
	Aaa	Aa	A	Baa	Ba	B	Caa	Ca-C	DF
Aaa	90.42	8.92	0.62	0.01	0.03	0.00	0.00	0.00	0.00
Aa	1.02	90.12	8.38	0.38	0.05	0.02	0.01	0.00	0.02
A	0.06	2.82	90.88	5.52	0.51	0.11	0.03	0.01	0.06
Baa	0.05	0.19	4.79	89.41	4.35	0.82	0.18	0.02	0.19
Ba	0.01	0.06	0.41	6.22	83.43	7.97	0.59	0.09	1.22
B	**0.01**	**0.04**	**0.14**	**0.38**	**5.32**	**82.19**	**6.45**	**0.74**	**4.73**
Caa	0.00	0.02	0.02	0.16	0.53	9.41	68.43	4.67	16.76
Ca-C	0.00	0.00	0.00	0.00	0.39	2.85	10.66	43.54	42.56

- Let X_B be the **rescaled asset value**, according to CreditMetrics, of our B-rated company at the end of the year.

- As anticipated, it can be shown that X_B is distributed as a Normal(0,1).

An example

- The probability that our B-rated company will become Aaa by the end of the year is 0.01% or 0.0001.

- The probability to move to the Aa class is 0.0004.

- The probability to default is 0.0473, and so on.

- Using the **quantile function** of a Normal(0,1), we can compute the threshold values we need for the CreditMetrics model.

An example

- Let's start from the lowest rating class, that is default. The probability of default is 0.0473. This corresponds to a quantile equal to

$$\Phi^{-1}(0.0473) = -1.6716$$

- If X_B is smaller than/equal to -1.6716, we have that our B-rated company has defaulted.

- -1.6716 is nothing more than d_1^* in the notation we have used before.

An example

- Let's start from the lowest rating class, that is default. The probability of default is 0.0473. This corresponds to a quantile equal to

```
> qnorm(0.0473)
[1] -1.671616
```

$$\Phi^{-1}(0.0473) = -1.6716$$

- If X_B is smaller than/equal to -1.6716, we have that our B-rated company has defaulted.

- -1.6716 is nothing more than d_1^* in the notation we have used before.

An example

- If X_B is larger than -1.6716, but smaller than/equal to -1.6009, our company is in the Ca-C class.

- -1.6009 is computed as $\Phi^{-1}(0.0473 + 0.0074)$

- Then we have the Caa class for $-1.6009 < X_B \leq -1.1790$

- And so on, until we have the Aaa class for $X_B > 3.7190 = \Phi^{-1}(0.9999)$

```
> qnorm(0.0473+0.0074+0.0645)
[1] -1.178995
```

Question for you

- What is the rating class, at the end of the year, of our B-rated company, if

$$X_B = 1.31?$$

- The complete CreditMetrics model also takes into account **dependence among defaults**.
- This is done using probabilistic tools called Copulas.
- In particular, in CreditMetrics a Gaussian copula is generally used.

CreditMetrics in R

- In R there is a nice package called CreditMetrics.
- You can find the link in the extra materials of this week.

Thank You