Default Probabilities

CreditMetrics

Dr. Pasquale Cirillo
• Introduced in 1997 by J.P. Morgan & Co.

• It is a structural model of default, which also takes into account the risk of credit deterioration.

• As Moody’s KMV, it can be seen as a Merton-like model.
In CreditMetrics thresholds are not given by liabilities, but defined through credit ratings.
We consider a firm that has been assigned to some credit rating class (say B) at the beginning of period \([0,T]\).

This means that we also have information about the transition probabilities of our company from its rating class to another.

<table>
<thead>
<tr>
<th>Initial Rating</th>
<th>AAA</th>
<th>AA</th>
<th>A</th>
<th>BBB</th>
<th>BB</th>
<th>B</th>
<th>CCC</th>
<th>DF</th>
</tr>
</thead>
<tbody>
<tr>
<td>AAA</td>
<td>90.81</td>
<td>8.33</td>
<td>0.68</td>
<td>0.06</td>
<td>0.12</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>AA</td>
<td>0.70</td>
<td>90.65</td>
<td>7.79</td>
<td>0.64</td>
<td>0.06</td>
<td>0.14</td>
<td>0.02</td>
<td>0.00</td>
</tr>
<tr>
<td>A</td>
<td>0.09</td>
<td>2.27</td>
<td>91.05</td>
<td>5.52</td>
<td>0.74</td>
<td>0.26</td>
<td>0.01</td>
<td>0.06</td>
</tr>
<tr>
<td>BBB</td>
<td>0.02</td>
<td>0.33</td>
<td>5.85</td>
<td>86.9</td>
<td>5.30</td>
<td>1.1</td>
<td>0.32</td>
<td>0.18</td>
</tr>
<tr>
<td>BB</td>
<td>0.03</td>
<td>0.14</td>
<td>0.67</td>
<td>7.73</td>
<td>80.53</td>
<td>8.84</td>
<td>1.00</td>
<td>1.06</td>
</tr>
<tr>
<td>B</td>
<td>0.00</td>
<td>0.11</td>
<td>0.24</td>
<td>0.43</td>
<td>6.48</td>
<td>83.46</td>
<td>4.07</td>
<td>5.20</td>
</tr>
<tr>
<td>CCC</td>
<td>0</td>
<td>0.00</td>
<td>0.22</td>
<td>1.30</td>
<td>2.38</td>
<td>11.24</td>
<td>64.86</td>
<td>19.79</td>
</tr>
</tbody>
</table>

Example
With $\bar{p}(j)$, $0 \leq j \leq n$, we indicate the probability that our company will be in rating class $j$ at time $T$.

We order the $j$'s, so that $j=0$ means “default” and $j=n$ means that our company is in the best rating class, e.g. AAA.

With $\bar{p}(0)$, we thus indicate the probability of default of the company.
Using the same notation of Merton’s model, we indicate the value of the assets of our company in $T$ as $V_T$.

As we have seen with Merton, $V_T$ can be linked to a Normal distribution function.
The thresholds

We can then define a collection of thresholds

\[-\infty = \tilde{d}_0 < \tilde{d}_1 < \tilde{d}_2 < \cdots < \tilde{d}_n < \tilde{d}_{n+1} = \infty\]

such that

\[P \left( \tilde{d}_j < V_T \leq \tilde{d}_{j+1} \right) = \bar{p}(j), \forall j \in \{0, \ldots, n\}\]

In this way we translate the transition probabilities into a series of thresholds for the asset value process.
* The threshold $\tilde{d}_1$ is clearly the default threshold.

* Larger thresholds represent the asset value levels that define the boundaries of the higher rating classes.

* In this setting, a firm belongs to rating class $j$ at time $T$ if and only if

$$\tilde{d}_j < V_T \leq \tilde{d}_{j+1}$$
In CreditMetrics, to exploit the properties of the standard Normal distribution, the asset value $V_T$ is transformed into

$$X_T := \frac{\log(V_T) - \log(V_0) - (r - \sigma^2_V/2)T}{\sigma_V \sqrt{T}}$$

And the same for the thresholds:

$$d^*_j := \frac{\log(\tilde{d}_j) - \log(V_0) - (r - \sigma^2_V/2)T}{\sigma_V \sqrt{T}}$$
As a consequence of this new parametrization, we have that a firm belongs to rating class $j$ at time $T$ if and only if

$$d_j^* < X_T \leq d_{j+1}^*$$
Consider a B-rated company over a 1-year period.

And consider the following fictitious Moody’s transition matrix:

<table>
<thead>
<tr>
<th>Initial Rating</th>
<th>Aaa</th>
<th>Aa</th>
<th>A</th>
<th>Baa</th>
<th>Ba</th>
<th>B</th>
<th>Caa</th>
<th>Ca-C</th>
<th>DF</th>
</tr>
</thead>
<tbody>
<tr>
<td>Aaa</td>
<td>90.42</td>
<td>8.92</td>
<td>0.62</td>
<td>0.01</td>
<td>0.03</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>Aa</td>
<td>1.02</td>
<td>90.12</td>
<td>8.38</td>
<td>0.38</td>
<td>0.05</td>
<td>0.02</td>
<td>0.01</td>
<td>0.00</td>
<td>0.02</td>
</tr>
<tr>
<td>A</td>
<td>0.06</td>
<td>2.82</td>
<td>90.88</td>
<td>5.52</td>
<td>0.51</td>
<td>0.11</td>
<td>0.03</td>
<td>0.01</td>
<td>0.06</td>
</tr>
<tr>
<td>Baa</td>
<td>0.05</td>
<td>0.19</td>
<td>4.79</td>
<td>89.41</td>
<td>4.35</td>
<td>0.82</td>
<td>0.18</td>
<td>0.02</td>
<td>0.19</td>
</tr>
<tr>
<td>Ba</td>
<td>0.01</td>
<td>0.06</td>
<td>0.41</td>
<td>6.22</td>
<td>83.43</td>
<td>7.97</td>
<td>0.59</td>
<td>0.09</td>
<td>1.22</td>
</tr>
<tr>
<td>B</td>
<td>0.01</td>
<td>0.04</td>
<td>0.14</td>
<td>0.38</td>
<td>5.32</td>
<td>82.19</td>
<td>6.45</td>
<td>0.74</td>
<td>4.73</td>
</tr>
<tr>
<td>Caa</td>
<td>0.00</td>
<td>0.02</td>
<td>0.02</td>
<td>0.16</td>
<td>0.53</td>
<td>9.41</td>
<td>68.43</td>
<td>4.67</td>
<td>16.76</td>
</tr>
<tr>
<td>Ca-C</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.39</td>
<td>2.85</td>
<td>10.66</td>
<td>43.54</td>
<td>42.56</td>
</tr>
</tbody>
</table>
Let $X_B$ be the rescaled asset value, according to CreditMetrics, of our B-rated company at the end of the year.

As anticipated, it can be shown that $X_B$ is distributed as a Normal(0,1).
The probability that our B-rated company will become Aaa by the end of the year is 0.01% or 0.0001.

The probability to move to the Aa class is 0.0004.

The probability to default is 0.0473, and so on.

Using the quantile function of a Normal(0,1), we can compute the threshold values we need for the CreditMetrics model.
Let’s start from the lowest rating class, that is default. The probability of default is 0.0473. This corresponds to a quantile equal to

\[ \Phi^{-1}(0.0473) = -1.6716 \]

If \( X_B \) is smaller than/equal to -1.6716, we have that our B-rated company has defaulted.

-1.6716 is nothing more than \( d_1^* \) in the notation we have used before.
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An example

- If $X_B$ is larger than -1.6716, but smaller than/equal to -1.6009, our company is in the Ca-C class.

- -1.6009 is computed as $\Phi^{-1}(0.0473 + 0.0074)$

- Then we have the Caa class for $-1.6009 < X_B \leq -1.1790$

- And so on, until we have the Aaa class for $X_B > 3.7190 = \Phi^{-1}(0.9999)$

\[ > \text{qnorm}(0.0473+0.0074+0.0645) \]

\[ [1] \quad -1.178995 \]
What is the rating class, at the end of the year, of our B-rated company, if $X_B = 1.31$?
The complete CreditMetrics model also takes into account dependence among defaults.

This is done using probabilistic tools called Copulas.

In particular, in CreditMetrics a Gaussian copula is generally used.
In R there is a nice package called CreditMetrics.

You can find the link in the extra materials of this week.
Thank You