TW3421x - An Introduction to Credit Risk Management

Default Probabilities

The KMV Model

Dr. Pasquale Cirillo

Week 5
Lesson 3

Introduction

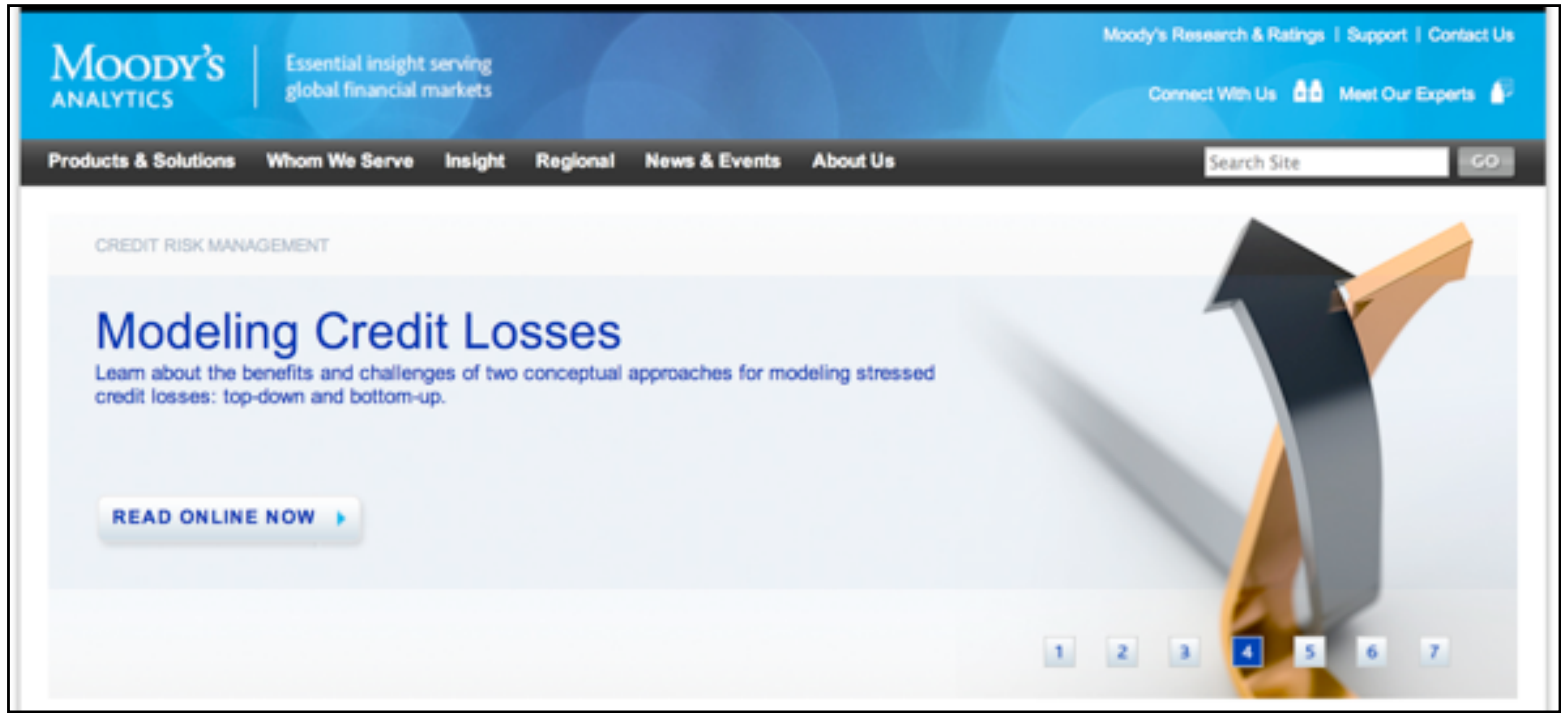
- The KMV model is an important example of industry model derived from Merton's one.
- It was first introduced in the late 80's by KMV, a leading provider of quantitative credit analysis tools.
- A large number of world financial institutions are subscribers of the model.

Introduction

- From a purely theoretical point of view, the differences between KMV and Merton's models are not dramatic.

- The KMV model, however, relies on an extensive empirical testing and it is implemented using a very large proprietary database.

- The model is now maintained and developed on a continuous basis by Moody's KMV, a division of Moody's Analytics.
  Moody's Analytics acquired KMV in 2002.



www.moodysanalytics.com
© Copyright 2014 Moody's Analytics, Inc. and / or its licensors and affiliates.
All Rights Reserved.

The EDF

- A fundamental quantity in the KMV model is the *Expected Default Frequency*, or **EDF™**.

- EDF™ is a registered trademark.
  However, we will simply use "EDF" to simplify notation.

- The EDF is nothing but the probability that a given firm will default within 1 year according to the KMV methodology.

- In order to understand the way in which KMV obtains the EDF, we can use Merton's model.

- As for all industry models, some specific parameters and implementations of the model are not completely known, and we can simply rely on what the company allows us to discover.

The EDF

- In Merton's model the 1-year PD of a firm is given by the probability that in 1 year the asset value V_1 will be below threshold B.

- On the basis of what we have seen so far, we have that

$$P(V_1 \leq B) = \Phi\left(\frac{\log(B) - \log(V_0) - (r - \sigma_V^2/2)}{\sigma_V}\right)$$

The EDF

- From the symmetry property of the Gaussian distribution, we know that

$$\Phi(x) = 1 - \Phi(-x) = \bar{\Phi}(-x)$$

- Hence we can re-write $P(V_1 \leq B)$ as

$$P(V_1 \leq B) = \bar{\Phi}\left(\frac{\log(V_0) - \log(B) + \left(r - \sigma_V^2/2\right)}{\sigma_V}\right)$$

The EDF

- From the symmetry property of the Gaussian distribution, we know that

$$\Phi(x) = 1 - \Phi(-x) = \bar{\Phi}(-x)$$

- Hence we can re-write $P(V_1 \leq B)$ as

$$P(V_1 \leq B) = \bar{\Phi}\left(\frac{\log(V_0) - \log(B) + \left(r - \sigma_V^2/2\right)}{\sigma_V}\right)$$

"Merton's EDF"

The EDF

- From the symmetry property of the Gaussian distribution, we know that

$$\Phi(x) = 1 - \Phi(-x) = \bar{\Phi}(-x)$$

- Hence we can re-write $P(V_1 \leq B)$ as

$$P(V_1 \leq B) = \bar{\Phi}\left(\frac{\log(V_0) - \log(B) + \left(r - \sigma_V^2/2\right)}{\sigma_V}\right)$$

"Merton's EDF"

- From the symmetry property of the Gaussian distribution, we know that

$$\Phi(x) = 1 - \Phi(-x) = \bar{\Phi}(-x)$$

- Hence we can re-write $P(V_1 \leq B)$ as

$$\underbrace{P(V_1 \leq B)}_{\text{“Merton's EDF”}} = \bar{\Phi}\left(\underbrace{\boxed{\frac{\log(V_0) - \log(B) + \left(r - \sigma_V^2/2\right)}{\sigma_V}}}_{A}\right)$$

The KMV Approach

- The EDF in KMV model is very similar, but $\bar{\Phi}$ is replaced by some decreasing function that is empirically estimated.
- B is replaced by another threshold $\tilde{B}$ that better reflects the structure of the firm's liabilities.
- Then *the argument A of* $\bar{\Phi}$ is substituted by a simpler expression.

The KMV Approach

- The EDF in KMV model is very similar, but $\bar{\Phi}$ is replaced by some decreasing function that is empirically estimated.
- B is replaced by another threshold $\tilde{B}$ that better reflects the structure of the firm's liabilities.
- Then *the argument A of* $\bar{\Phi}$ is substituted by a simpler expression.

Liabilities payable within 1 year

The KMV Approach

- The EDF in KMV model is very similar, but $\bar{\Phi}$ is replaced by some decreasing function that is empirically estimated.
- B is replaced by another threshold $\tilde{B}$ that better reflects the structure of the firm's liabilities.
- Then *the argument A of* $\bar{\Phi}$ is substituted by a simpler expression.

The Distance-to-Default

- An important quantity of the KMV approach is the so-called *Distance to Default* (DD):

$$DD := \frac{V_0 - \tilde{B}}{\sigma_V V_0}$$

- DD is often referred to as "the number of standard deviations from default" by practitioners.

The Distance-to-Default

P.J. Crosbie and J.R. Bohn (2002), Modeling default risk, *KMV Working paper.*

- Now, let us come back to

$$A = \frac{\log(V_0) - \log(B) + \left(r - \sigma_V^2/2\right)}{\sigma_V}$$

- Empirical research shows that the quantity $r - \sigma_V^2/2$ is usually very close to zero (see Crosbie and Bohn, 2002), so that we can consider it to be irrelevant.

- Given this we have

$$\frac{\log(V_0) - \log(B) + \left(r - \sigma_V^2/2\right)}{\sigma_V} \approx \frac{\log(V_0) - \log(B)}{\sigma_V} \approx \frac{V_0 - B}{\sigma_V V_0} \approx \frac{V_0 - \tilde{B}}{\sigma_V V_0} = DD$$

The Empirical Function

- Using a huge historical data set, KMV has estimated (updates are very frequent), for virtually every meaningful time horizon, and for many small intervals of DD values (“cells”), the proportion of firms that in each cell have defaulted within a given time horizon.
- This information is used to substitute $\bar{\Phi}$, with an empirical function, which we can call $\bar{F}_{KMV}$.
- The exact form of $\bar{F}_{KMV}$ is proprietary to Moody’s KMV and protected by patents.
- The use of this function allows Moody’s KMV to overcome one of the points of weakness of Merton’s Model: the Gaussianity assumption.
- Under $\bar{F}_{KMV}$, extreme events are probably better represented.

The KMV EDF

- Summarizing we have that

$$EDF = \bar{F}_{KMV}(DD) = \bar{F}_{KMV}\left(\frac{V_0 - \tilde{B}}{\sigma_V V_0}\right)$$

- This new PD overcomes some of the weaknesses of Merton's model, namely:
  - Gaussianity.
  - The fact that default only happens only in T.

Thank You