TW3421x - An Introduction to Credit Risk Management

Default Probabilities

Merton’s Model - Part 2

Dr. Pasquale Cirillo

Week 5
Lesson 2

From last time

- In the last lesson, we have seen that, according to Merton's model,

$$S_T = (V_T - B)^+$$

- This means that the value of the firm's equity at time T corresponds to the payoff of a European call option on V_T.

- The famous Black-Scholes-Merton formula tells us that the value of the equity today is

$$S_0 = V_0 \Phi(d_1) - Be^{-rT}\Phi(d_2) \qquad \textbf{EQUATION 1}$$

with

$$d_1 = \frac{\log(V_0/B) + (r + \sigma_V^2/2)T}{\sigma_V \sqrt{T}} \qquad d_2 = d_1 - \sigma_V \sqrt{T}$$

and where $\Phi(\cdot)$ is the cumulative distribution function of a standard Gaussian (i.e. N(0,1)).

Black-Scholes-Merton

- The famous Black-Scholes-Merton formula tells us that the value of the equity today is

$$S_0 = V_0 \Phi(d_1) - Be^{-rT}\Phi(d_2) \qquad \textbf{EQUATION 1}$$

with

risk-free rate on the market (r)

Volatility of assets (assumed constant) (σ_V)

$$d_1 = \frac{\log(V_0/B) + (r + \sigma_V^2/2)T}{\sigma_V \sqrt{T}} \qquad d_2 = d_1 - \sigma_V \sqrt{T}$$

and where $\Phi(\cdot)$ is the cumulative distribution function of a standard Gaussian (i.e. N(0,1)).

- The famous Black-Scholes-Merton formula tells us that the value of the equity today is

$$S_0 = V_0 \Phi(d_1) - Be^{-rT} \Phi(d_2) \qquad \textbf{EQUATION 1}$$

with

$$d_1 = \frac{\log(V_0/B) + (r + \sigma_V^2/2)T}{\sigma_V \sqrt{T}} \qquad d_2 = d_1 - \sigma_V \sqrt{T}$$

and where $\Phi(\cdot)$ is the cumulative distribution function of a standard Gaussian (i.e. N(0,1)).

→ $pnorm(\cdot)$

The Probability of Default

- Under Merton's model, a company defaults if, at maturity T, the value of its assets falls below the liabilities' threshold B, that is to say when

$$V_T \leq B$$

- With some maths, it can be show that the probability that $V_T \leq B$, i.e $P(V_T \leq B)$ is equal to

$$P(V_T \leq B) = \Phi\left(\frac{\log(B/V_0) - (r - \sigma_V^2/2)T}{\sigma_V\sqrt{T}}\right) = \Phi(-d_2)$$

V_0 and σ_V

- We have just seen that the probability of default in T (and only in T, not before!) is

$$P(V_T \leq B) = \Phi\left(\frac{\log(B/V_0) - (r - \sigma_V^2/2)T}{\sigma_V\sqrt{T}}\right) = \Phi(-d_2)$$

- This probability depends on several different quantities, but only two of them are really problematic for us: V_0 and σ_V .

- These two quantities cannot be observed directly. They need to be somehow inferred from data.

V_0 and σ_V (continues)

- In particular, for the market value of the company's assets, we have the following:
  - Market value can strongly differ from the value of a company as measured by accountancy rules.
  - The market value of a company corresponds to the sum of the market values of its equity and debt. But only a relatively small part of firm's debt (i.e. issued bonds) is fully knowable.

V_0 and σ_V (continues)

- If the company we are analyzing is publicly traded, we can observe S_0 .

- It can be shown that the value of equity today must satisfy the following equation

$$S_0 = \frac{\sigma_V}{\sigma_S} \Phi(d_1) V_0$$

EQUATION 2

instantaneous volatility of equity (σ_S)

- This equation, together with equation 1, can be used to find V_0 and σ_V .

Computing V_0 and σ_V

- Let us rearrange equations 1 and 2, so that

$$F(V_0, \sigma_V) = V_0 \Phi(d_1) - Be^{-rT}\Phi(d_2) - S_0 = 0$$

$$G(V_0, \sigma_V) = \frac{\sigma_V}{\sigma_S}\Phi(d_1)V_0 - S_0 = 0$$

- V_0 and σ_V can be found by minimizing

$$[F(V_0, \sigma_V)]^2 + [G(V_0, \sigma_V)]^2$$

- This can be done numerically, for example with R.

Exercise

- Consider a company whose equity is 3 million euros. The volatility of equity is 0.80. The company's debt is equal to 10 million euros and it must be paid in one year. The risk-free rate on the market is 5% per annum.

- This means that

$$S_0 = 3, \sigma_S = 0.8, r = 0.05, T = 1, B = 10$$

Question 1: What are the values of V_0 and σ_V ?

Question 2: What is the probability of default of the company in one year?

Exercise

- Consider a company whose equity is 3 million euros. The volatility of equity is 0.80. The company's debt is equal to 10 million euros and it must be paid in one year. The risk-free rate on the market is 5% per annum.

- This means that

$$S_0 = 3, \sigma_S = 0.8, r = 0.05, T = 1, B = 10$$

Question 1: What are the values of V_0 and σ_V ?

Question 2: What is the probability of default of the company in one year?

We will use R to solve this exercise.

Summarizing

- Merton's model in a nutshell.

Pros	Cons
Simple yet able to obtain meaningful results.	Default only in T.
Effective view of the potential conflict of interest between shareholders and debt holders in a company (see $P(V_T \leq B)$, slide 6)	Default = Liquidation.
"Easily" computable.	Assumes that the world is "Gaussian".

Thank You