TW3421x - An Introduction to Credit Risk Management

Default Probabilities

Merton's Model - Part 1

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Week 5

Lesson 1

Introduction

- Introduced by R.C. Merton (1974). On the pricing of corporate debt: the risk structure of interest rates. *Journal of Finance* **29**: 449-470.

Prof. Dr. R. C. Merton

- Born 31 July 1944.
- Nobel laureate in Economics (1997), together with Scholes.
- Professor at the MIT Sloan School of Management.
- Many fundamental contributions in credit risk modeling and continuous-time finance.

Source: Wikipedia. Author: Digarnick.

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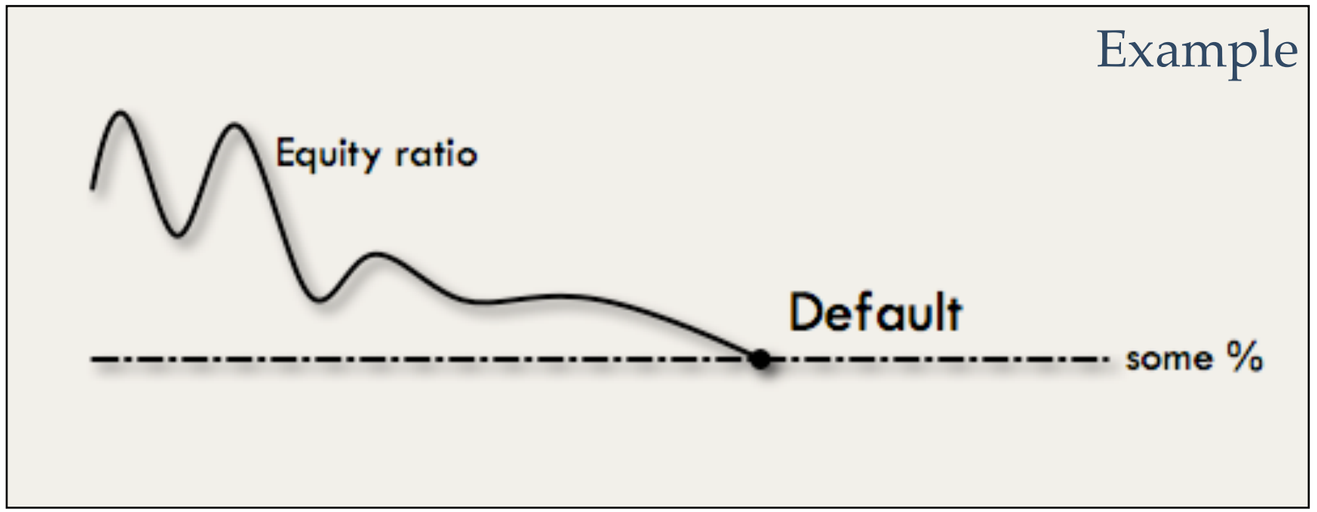
- It is the prototype of the class of structural models of default.

Structural Models

- Default occurs whenever a stochastic variable (or process) representing some asset value falls below a given threshold representing liabilities.



- Sometimes they are also known as threshold models.

- Merton's model and all its derivations (KMV, CreditMetrics, etc.) belong to this class.

Introduction

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- It is the prototype of the class of structural models of default.
- In the Basel II-III framework, it is an internal rating-based tool.
- Many extensions have been proposed in the literature.
- It represents the basis for many influential industry solutions, such as the KMV model.
- Despite its simplicity (and flaws), it is still a very popular benchmark model for practitioners.
- "It is the Black-Scholes model for credit risk".

The “ingredients”

- We consider a limited (Ltd) company whose asset value follows some stochastic process (V_t).
- The firm can finance itself with equity or with debt.
- Debt is represented by one single debt obligation, or zero-coupon bond, with face value B and maturity T.
- S_t denotes the value of equity at time *t*, while B_t represents debt.
- Markets are assumed frictionless, hence the value of the firm’s assets at time *t* is given by

$$V_t = S_t + B_t, \ 0 \leq t \leq T$$

The “ingredients”

- An important assumption of Merton’s model is that a firm cannot pay dividends or issue new debt (especially to pay old debt) until time T.
- Default occurs if the firm is not able to pay debt holders, i.e. by missing a payment on debt.
- In the basic model this may only happen at maturity T.

Two Possibilities

- Hence we have two different possible scenarios at time T:
  - The value of the firm's assets exceeds the one of liabilities, i.e. $V_T > B$.

    In this case debt holders receive B, while shareholders receive the residual value, so that

    $$B_T = B, \; S_T = V_T - B$$

  - The value of the firm's assets is less than the liabilities and the firm is not able to meet its obligations. In other terms $V_T < B$.
    Shareholders do not have any interest in providing new capital, since it would go directly to debt holders. Instead they hand over control of the firm to debt holders by exercising the limited liability option.
    Debt holders thus liquidate the company and distribute the revenues among them. Hence:

    $$B_T = V_T, \; S_T = 0$$

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    The case $V_T = B$ can be collected under this second scenario.

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OK

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KO
(Default)

- The value of the firm's assets is less than the liabilities and the firm is not able to meet its obligations. In other terms $V_T \leq B$.
  Shareholders do not have any interest in providing new capital, since it would go directly to debt holders. Instead they hand over control of the firm to debt holders by exercising the limited liability option.
  Debt holders thus liquidate the company and distribute the revenues among them. Hence:

$$B_T = V_T, \; S_T = 0$$

Black & Scholes?

- We can summarize all this as:

$$S_T = \max(V_T - B, 0) = (V_T - B)^+$$
$$B_T = \min(V_T, B) = B - (B - V_T)^+$$

- This implies that the value of the firm's equity at time T corresponds to the payoff of a European call option on V_T, while the value of the firm's debt at maturity is equal to the nominal value of liabilities minus the payoff of a European put option on V_T with exercise price equal to B.

- All this makes us think of Black & Scholes...

Next time

- In the next lesson, we will see how the probability of default (PD) is actually obtained in the Merton's model.
- We will also consider some exercises.

Thank You