Week 5 - Lesson 3: Moody's KMV

Hi, welcome everybody.

Today, we will consider the KMV model, which is a very important industry model, derived from Merton's one.

Ok, let's start.

The KMV model has been introduced in the late 80's by KMV, a research driven company that soon became a leading provider of quantitative credit analysis tools.

At present the great majority of world financial institutions are subscribers of the model. Something like 70% of top 50 banks, for example.

From a theoretical point of view, the KMV model is not that different from Merton's one. As we will see, the KMV model essentially tries to overcome some of the flaws of Merton's model, by making an extensive use of empirical data.

The model, and the data set on which it relies, is now maintained and developed by Moody's Analytics, which acquired KMV (now Moody's KMV) in 2002.

A fundamental quantity in the KMV model is the so called Expected Default Frequency, or EDF, if we use the acronym, a registered trademark.

The Expected Default Frequency is essentially the probability that a given company will default within 1 year according to the KMV methodology.

In order to understand the way in which KMV obtains the EDF, we can use Merton's model.

As you can imagine, some specific parameters and implementations of the model are not completely known to the public, and we have to rely on what the company allows us to know.

In Merton's model the 1-year PD of a company is given by the probability that in 1 year the asset value V1, because T=1, will be below B.

On the basis of what we have seen so far, we can write the equation you see on your screen.

Using the symmetry property of the normal distribution, we can re-write that probability in terms of the so-called survival function.

If we have Phi(x), this corresponds to Phi bar of -x. In words, for a Gaussian distribution, the probability of observing something that is smaller than x is equal to the probability of observing something which is greater then -x.

Using this, we can re-write our probability of default as the survival function of a standard Gaussian computed in…d2, yes: this is exactly d2, when T=1.

This is the EDF under Merton's model.

Once again notice that it is expressed in terms of the survival function of a standard Gaussian.

Let us call the argument of Phi bar A.

The real EDF of KMV is quite similar, from a philosophical point of view. What changes is that Phi bar is substituted with a decreasing function estimated from data.

The threshold level B is replaced by another quantity that we can call B tilde.

This quantity better reflects the liabilities' structure of the company. Typically, B tilde represents the liabilities that are payable within one year.

As we will see this will have an impact on the model, because now default can happen at every time before T.

Finally, the argument A of Phi bar is replaced by another simpler expression.

A fundamental quantity of the KMV approach is the so-called Distance to Default (DD).

This is the ratio between V_0 minus B tilde, and the product of sigmaV and V0.

As a curiosity, practitioners often refer to the DD as the number of standard deviations to default.

Now, let us come back to A, the argument of our Phi bar.

Empirical research shows that the quantity r-sigmaV^2/2 is usually very close to zero, so that we can consider it to be irrelevant in our computations.

But this means that A is approximately equal to the difference of log(V0) and log(B), over sigmaV. But the difference of log(V0) and log(B) is approximately equal to V0-B over V0. If we substitute B with B tilde, which is the threshold used by Moody's KMV, we have that our term A can be approximated by the DD!

The next step, in the KMV approach, is to substitute, as we have anticipated, Phi bar with an empirical function, which we can call F bar KMV. This decreasing function is obtained using historical data.

KMV has in fact estimated, for virtually every meaningful time horizon, and for many small intervals of DD values that we call “cells”, the proportion of firms that in each cell have defaulted within a given time horizon.

Please notice that the use of this empirical F overcomes one of the weaknesses of Merton's model, that's Gaussianity. The use of empirical data allows Moody's KMV to better represent extreme events, and defaults are extreme events. At least if we assume that what we have observed in the past is representative of what we can expect from the future.

All in all, the EDF is given by this empirical F computed in the DD.

An important assumption of the KMV model is that companies having the same DD have the same probability to default, i.e. the same EDF.

For completeness, we have to say that Moody's KMV uses a proprietary algorithm to obtain V0 and sigmaV. In other words, they do not use the minimization procedure we have seen under Merton's model. However, this is rather technical and we do not enter into details.

The KMV model is hence able to overcome two problems of Merton's one: Gaussianity and the fact that default only happens in T. In fact, now, thanks to B tilde, the possibility of defaulting within T is taken into consideration.

Ok, let's now see some little graphics, to better understand the KMV model as a whole.

This is our Merton's model, once we express the probability of default in 1-year in terms of the survival function of a standard Gaussian.

We now want to transform this model into the KMV one.

This means that now we are interested in the EDF of our company.

For this reason we substitute Phi bar with a decreasing function F, empirically estimated from a huge data set of companies and defaults.

Then we compute this function in the DD of the company. V0 and sigmaV are estimated using a special algorithm developed by KMV.

And this is it. This is how to transform Merton's model into the KMV one.