## Week 5 - Lesson 2: Merton P2

Hi there, welcome back.

Today we continue our discussion on Merton's model.

We will see some more mathematical stuff. However, do not be frustrated. If you have problems, if you do not understand something, if you just have curiosities, you can always ask for help in the forum or via Twitter.

Ok, let's start.

Last time we have seen the basics of Merton's model.

We have seen that according to that model, the value of equity at maturity is equal to the payoff of a European call on VT, the market value of the company's assets.

If you had the time to look for it, you know that a call is a financial contract between two parties, the buyer and the seller. By paying a fee, called premium, the buyer of the call has the right, but not the obligation to buy an agreed quantity of a given commodity or security, which we call the underlying, from the seller of the option, at a certain time in the future (maturity), for a certain price (the strike price). The seller, also known as the writer of the call, is obliged to sell the commodity or security to the buyer, if the buyer exercises the option.

As you probably know, given your interest in finance, the price of a European call option is obtained using the famous Black-Scholes formula. The mathematical treatment of that formula is beyond the scope of these lectures, hence we do not enter into much details. At TU Delft, the complete derivation and analysis of Black-Scholes formula and Merton's model is part of my master course in Financial Mathematics.

According to Black-Scholes-Merton, the value of equity today is equal to V0 times the cumulative distribution function of a standard Gaussian, computed in d1, minus the actual value of B times, again, the cumulative distribution function of a standard Gaussian computed in d2. This is equation 1.

d1 and d2 are explicitly given in the slides.

As we can see, d1 and d2 depend on different quantities. Most of them are already known to us.

The two quantities we have not introduced yet are: the risk-free rate r, and the volatility of assets, sigmaV, that is to say the standard deviation of assets, which we assume to be constant.

Phi, the cumulative distribution function of a standard Gaussian, can be computed using the pnorm function in R, normdist in Excel or even with the Normal tables.

We have seen that, in Merton's model, default happens, at maturity T, if the value of the assets of the company falls below the liabilities' threshold B.

Using some advanced maths, it can be shown that the probability of default is equal to the cumulative distribution function of a standard Gaussian computed in -d2.

This is rather interesting.

According to Merton's model, the PD increases in the amount of debt B, decreases in V0 (the value of the company today) and it increases in the volatility of assets, for V0 greater than B.

All this is clearly in accordance with the economic intuition that excessive debt, or the excessive riskiness of assets, make default more plausible.

The probability of default is therefore function of several different quantities. Two of them are particularly problematic for us, since they cannot be directly observed: V0 and sigmaV.

In particular, for what concerns the market value of the company, when dealing with real-life and actual data, it is not possible to exactly observe it.

This is due to the fact that market value can strongly differ from the value of a company as measured by accountancy rules. This is rather common for service companies. Think for example of dot.com companies.

Moreover, the market value of a company corresponds to the sum of the market values of its equity and its debt.

But while the market value of equities is known, if the company is publicly traded, for what concerns liabilities, only a relatively small part of firm's debt is fully known. For example, we may know issued bonds, but other types of debt are more difficult to observe (loans, commercial debts, etc.)

If we assume that the company we are observing is publicly traded, S0, the value of equity today, is known. This fact can be exploited to obtain V0 and sigmaV using optimization.

In fact S0 must satisfy the following equation, coming from stochastic calculus. This equation that we call equation 2, together with equation 1, can be used to find V0 and sigmaV.

The quantity sigmaS is the instantaneous volatility of equity; something that we can observe on the market.

In fact, V0 and sigmaV can be found numerically, by minimizing the sum of the squares of functions F and G, which are two functions in V0 and sigmaV, which we obtain by rearranging equations 1 and 2.

Let's consider the following exercise, in order to see how Merton's model works in practice.

We have a company whose equity is 3 million euros. The volatility of equity is 0.8. The company's debt is equal to 10 million euros and it must be paid in one year. The risk-free rate on the market is 5% per annum.

Question 1: What are the values of V0 and sigmaV?

Question 2: What is the probability of default of the company in one year?

To solve this exercise, we will make use of R. The code we are going to use is available in the course materials.

We start by defining the quantities, whose value is given in the text of the exercise. These include, among the others, the value of equity at time zero, that is to say today, equal to 3 million euros; the risk-free rate, sigmaS, maturity equal to 1 year and liabilities equal to 10 million euros.

We then define the function that we minimize in order to obtain V0, the value of the company's assets today, and sigmaV, assets' volatility.

This function is available in the materials of the course, and I will not enter into much detail here.

However, remember that you can always ask for help in the forum.

In the function, we necessarily define all the quantities of interest: d1, d2, F and G. And then we define the function that we want to minimize: F^2+G^2.

To minimize this function, we use the optima routine in R. We have to provide two initial guesses for V0 and sigmaV, and we choose 13 and 0.5.

What we get are two values for V0 and sigmaV that are 12.39 and 0.2124 respectively. We now compute d1 and d2 explicitly, and thanks to pnorm(-d2) we find that PD in one year is 12.7%.

What happens if the value of equity today increases to 5 million euros?

To answer this question, we re-run the code, after changing S0 to 5 million euros.

And we discover that the new probability is 10.5%. So, it decreases as expected.

Let's conclude this class, by summarizing the points of strength and the points of weakness of Merton's model.

Surely it is an intuitive model that, despite its simplicity, is able to obtain very meaningful results. This is the reason why many important companies have developed their models starting from Merton's one, trying to correct some of its flaws.

In the next class, we will consider the KMV model.

It is very interesting to see how Merton's model is able to depict the potential conflict of interest between shareholders and debt holders.

The first ones have an interest in the firm investing in risky projects, that increase the volatility of the underlying "security", but may guarantee higher returns; while the second ones prefer a less volatile and less risky assets' value.

Regarding the points of weakness, Merton's model (as Black-Scholes) essentially assumes we are in a Gaussian world, in which rare and extreme events, black swans in the terminology of Nicholas Taleb, are not taken into consideration.

Moreover, in Merton's model, default can only happen at maturity, say in one year, and not within that period, say within one year. This is obviously not realistic.

Finally, in this model, default corresponds to liquidation, that is to say to the complete disappearance of the company from the market. But we know that in most legislations, this is not the case. For example, in the US, the so-called Chapter 11 allows companies under bankruptcy to try a reorganization, in order to try to become profitable again. In that case we do not have immediate liquidation, which, always in the US, is regulated by Chapter 7.

That was our last slide.

Today we had a rather long and difficult lecture. I hope you enjoyed it.

Thank you for your patience. See you soon. Bye!