Week 5 - Lesson 1: Merton P1

Welcome everybody. In this class we continue our discussion about the probability of default.

Today we introduce Merton's model.

This model has been developed by Dr. Robert Merton, Professor at MIT Sloan School of Management.

In 1997, for his important contributions in risk modeling and continuous-time finance, Merton received the Nobel Memorial Prize in Economic Sciences, together with Prof. Scholes. Prof. Black had died in 1995, hence he could not receive the prize.

The strict link between Merton's Model and the famous Black-Scholes formula will be clear in the next slides.

But let's come back to the model.

Merton's Model is the prototype of the class of structural models of default.

Structural models are an important family of models, in which the default of a company happens as soon as a stochastic variable representing some asset value of the company falls below some given threshold, often representing liabilities.

Look at the picture. In the picture, the variable of interest is the equity ratio, but other quantities can be taken into consideration.

In Merton's model, for example, we will consider the total value of the firm's assets.

Given the presence of the threshold, structural models are sometimes called threshold models.

Merton's model, the KMV model and many others all belong to this large family.

Looking at the Basel framework, Merton's model falls in the class of internal rating-based models. In fact, as we will see, it clearly does not belong to the standardized approaches we have seen so far.

Many extensions of Merton's model have been proposed in the literature, and it also represents the basis for many important industry solutions, such as the KMV and the CreditMetrics models.

It is such an influential model that it is still used as a benchmark, even if some of the assumptions of the model are not really plausible.

The model is strictly connected to Black-Scholes formula (also known as the Black-Scholes-Merton formula). In the field of credit risk it plays the same role of the Black-Scholes' model in option pricing.

Let us consider a limited company whose asset value follows some stochastic process Vt. A limited company is a company in which the liability of shareholders is limited to what they have invested or guaranteed to the company.

We assume that the firm can finance itself with equity or with debt.

For what concerns debt, we assume it is represented by one single debt obligation, such as a zero-coupon bond, with face value B and maturity T.

Let St denote the value of equity at time t, while Bt represents debt at time t.

Markets are assumed to be frictionless (actually perfect in the paper by Merton), therefore the value of the company's assets at time t, Vt, for every t from 0 (i.e. today) and T (maturity), is given by the sum of St and Bt.

An important assumption of Merton's model is that a company cannot pay dividends or issue new debt (especially to pay old debt) until time T.

Default occurs if the firm is not able to pay debt holders, i.e. by missing a payment on debt.

In the basic model this may only happen at time T, that is to say at maturity.

As we will see, this is one of the points of weakness of the model.

Given all the elements we have just seen, at maturity, we can have two scenarios.

In the first, the value of the firm's assets VT is greater than B, the value of liabilities. In this case the company is able to repay debt and everything is fine. Debt holders receive their repayment: this means that BT=B. Shareholders receive the rest, so that ST=VT-B.

But...what happens if the company is not able to repay debt? This means that at time T, VT<B.

In this case shareholders have no interest in providing new capital, since it would go directly to debt holders. Therefore they hand over control of the company to debt holders by exercising the limited liability option. And here we see the importance of assuming that we deal with a limited company.

Debt holders thus liquidate the company and distribute the revenues among them. In a sense we are assuming that recovery rate is 100%.

Hence we have BT=VT and ST=0.

In reality we have a third scenario: VT=B (the value of the company is exactly the value of liabilities). However we can include this under the second one, so that we consider the case VT smaller than/equal to B. In fact, it is not a strong assumption to say that a company cannot survive when its equity is equal to zero.

Hence when VT>B, the company is safe.

When VT<=B the company defaults.

This two scenarios can be summarized as follows.

ST, the value of equity at maturity, is equal to the maximum between VT-B (when the company is safe), and zero (when the company is liquidated). In finance we represent this with the formula (VT-B) plus.

BT, the value of debt at maturity, what debt holders receive, is the minimum between VT and B. This can also be seen as B minus the maximum between B-VT and zero.

If you are familiar with European options, all this implies that the value of the firm's equity at time T corresponds to the payoff of a European call option on VT, while the value of the firm's debt at maturity is equal to the nominal value of liabilities, B, minus the payoff of a European put option on VT, with exercise price equal to B.

All this reminds us of Black-Scholes' model.

In the next lesson, we will see how the probability of default is actually obtained in the Merton's model.

We will also consider some exercises.

If you are not familiar with Black-Scholes, it may be worth to have a look at some book or reference. The Wikipedia page on Black-Scholes' model can be a good starting point.

Thank you, and see you next time.