TW3421x - An Introduction to Credit Risk Management

The VaR and its derivations

Special VaRs and the Expected Shortfall

Dr. Pasquale Cirillo

Week 3
Lesson 2

An exercise to start with

- A 1-year project has a 94% chance of leading to a gain of €5 million, a 3% chance of a gain of €2 million, a 2% chance of leading to a loss of €3 million and a 1% chance of producing a loss of €8 million.
  What is the VaR for α=0.98? And for α=0.99?

The Mean-VaR

- Let **μ** be the mean of the loss distribution. The mean-VaR is defined as

$$VaR_{\alpha}^{mean} = VaR_{\alpha} - \mu$$

- The distinction between VaR and mean-VaR is often negligible in risk management, especially for short time horizons.

- For longer time periods (e.g. 1-year), however, the distinction is much more important. In credit risk management, **mean-VaR** is used to determine economic capital against losses in loans.

Distribution-specific VaRs

- Suppose that the loss distribution is Gaussian with mean μ and standard deviation σ. Let us fix α in the interval (0,1). Then

$$VaR_\alpha = \mu + \sigma\Phi^{-1}(\alpha) \qquad VaR_\alpha^{mean} = \sigma\Phi^{-1}(\alpha)$$

- Where $\Phi^{-1}(\alpha)$ is the α-quantile of a standard normal.

Distribution-specific VaRs

- Suppose now that losses L are such that

$$\frac{L-\mu}{\sigma} \sim t(\nu)$$

- In other terms, $L \sim t(\nu; \mu, \sigma)$.

- Notice that σ is not the standard deviation of the distribution, since

$$var(L) = \frac{\nu\sigma^2}{\nu - 2}, \quad \nu > 2$$

- Concerning the VaR, we have

$$VaR_\alpha = \mu + \sigma t_\nu^{-1}(\alpha)$$

Exercise

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  What is the 95% VaR? And the 98%?

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- Using the standard normal tables or a function such as *qnorm* in R, we easily find that

$$\Phi^{-1}(0.95) = 1.6448 \qquad \Phi^{-1}(0.98) = 2.0537$$

```
> qnorm(0.95)
[1] 1.644854
> qnorm(0.95,10,5)
[1] 18.22427
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- Hence:

$$VaR_{0.95}(L) = 10 + 5 \times 1.6448 = 18.2243$$

$$VaR_{0.98}(L) = 10 + 5 \times 2.0537 = 20.2687$$

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The Expected Shortfall

- Expected shortfall, aka conditional value at risk, answers to the question

“If things go bad, what is the expected loss?”

- It is a measure of risk with many interesting properties.

The Expected Shortfall

- From a statistical point of view, the expected shortfall is a sort of mean excess function, i.e. the average value of all the values exceeding a special threshold, the **VaR**!

$$ES_\alpha = E[L|L \geq VaR_\alpha]$$

- Why is it important?

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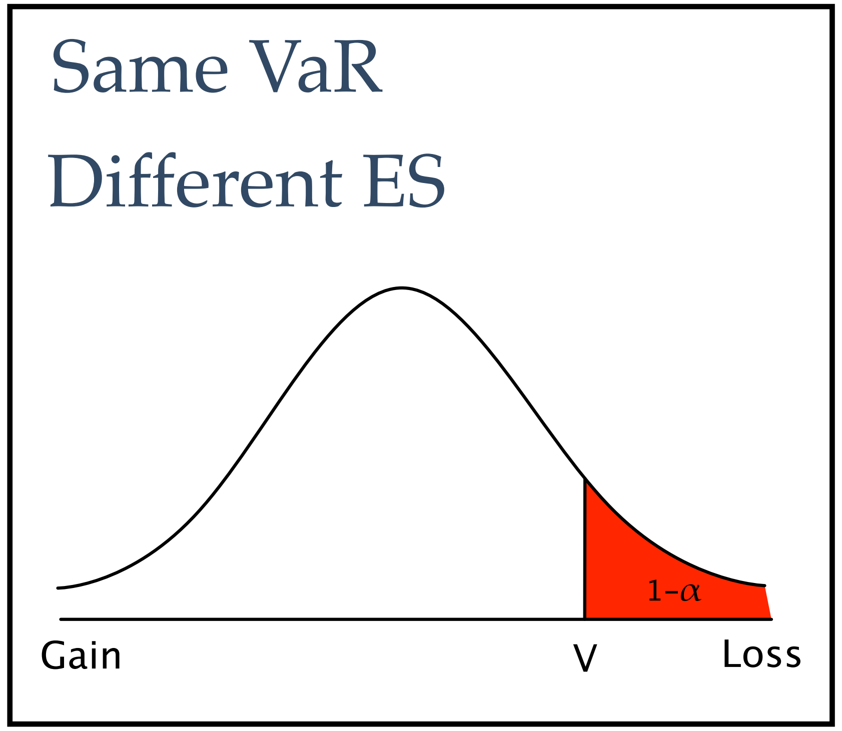
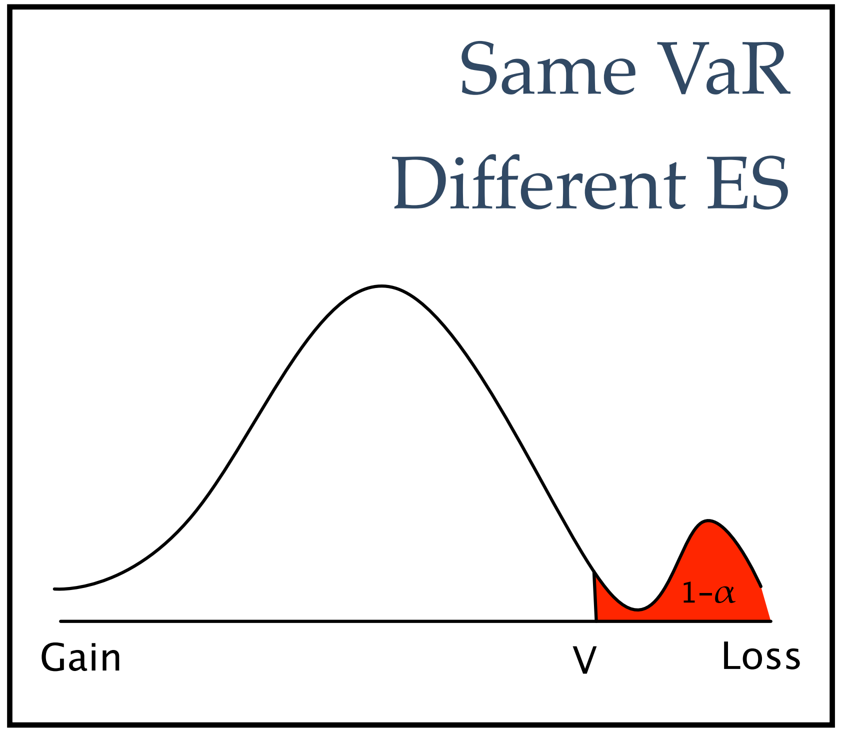
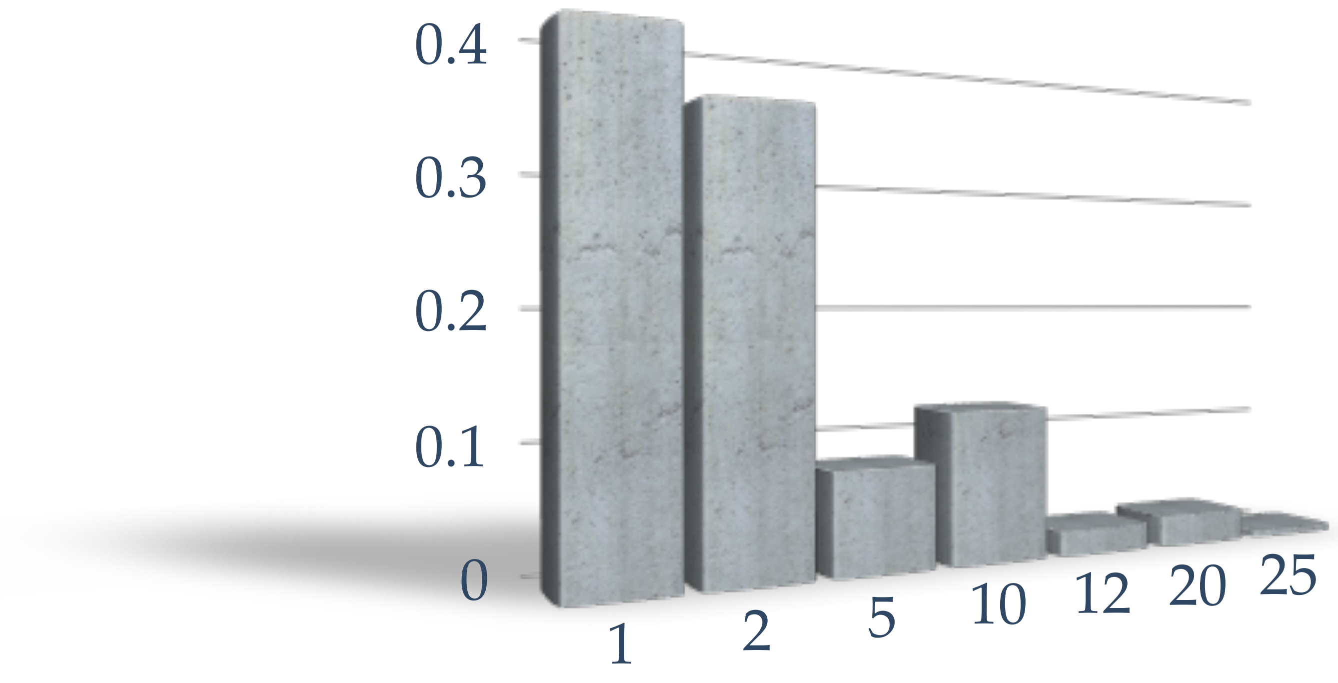




Exercise

- A portfolio of loans may lead to the losses in the table.
- What is the expected shortfall for α=0.95? And α=0.99?



Loss (\$ 10^6)	Probability
1	40%
2	35%
5	8%
10	12%
12	2%
20	2.5%
25	0.5%

Exercise

- In the first case we have:

$$ES_{0.95} = \frac{12 * 0.02 + 20 * 0.025 + 25 * 0.005}{0.05} = 17.3$$

- In the second:

$$ES_{0.99} = \frac{20 * 0.005 + 25 * 0.005}{0.01} = 22.5$$

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Trick: move in this direction

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- In the second:

They sum to 1-α

$$ES_{0.99} = \frac{20 * 0.005 + 25 * 0.005}{0.01} = 22.5$$

1-α

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1	40%
2	35%
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1%

Special ES

- Even for the expected shortfall, it may be useful to compute some special cases depending on well-known distributions.

- For example, in the case of a normal with mean μ and standard deviation σ, we have

$$ES_\alpha = \mu + \sigma \frac{\phi(\Phi^{-1}(\alpha))}{1-\alpha}$$

Thank You