TW3421x - An Introduction to Credit Risk Management

The VaR and its derivations

Introducing the Value-at-Risk

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Week 3
Lesson 1

A thing called VaR

- In very simple terms, the VaR is a measure of risk that tries to answer the following question:

"How bad can things get?"

- In more probabilistic terms, we look for a measure that allows us to say:

"With probability α we will not lose more than V euros in time T"

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V is the VaR

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Confidence level

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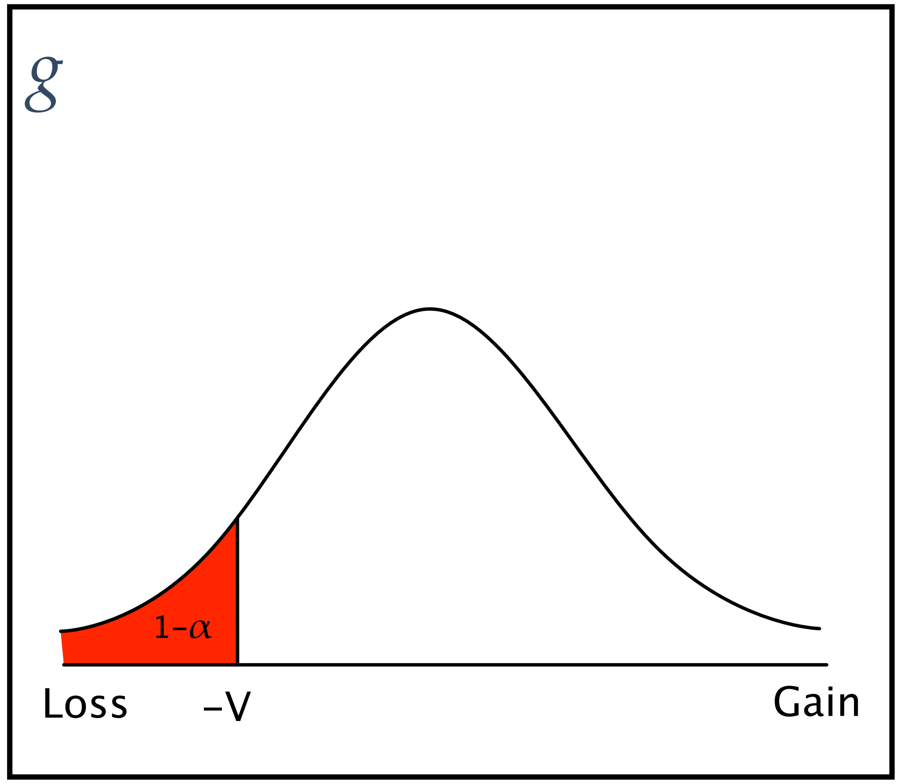
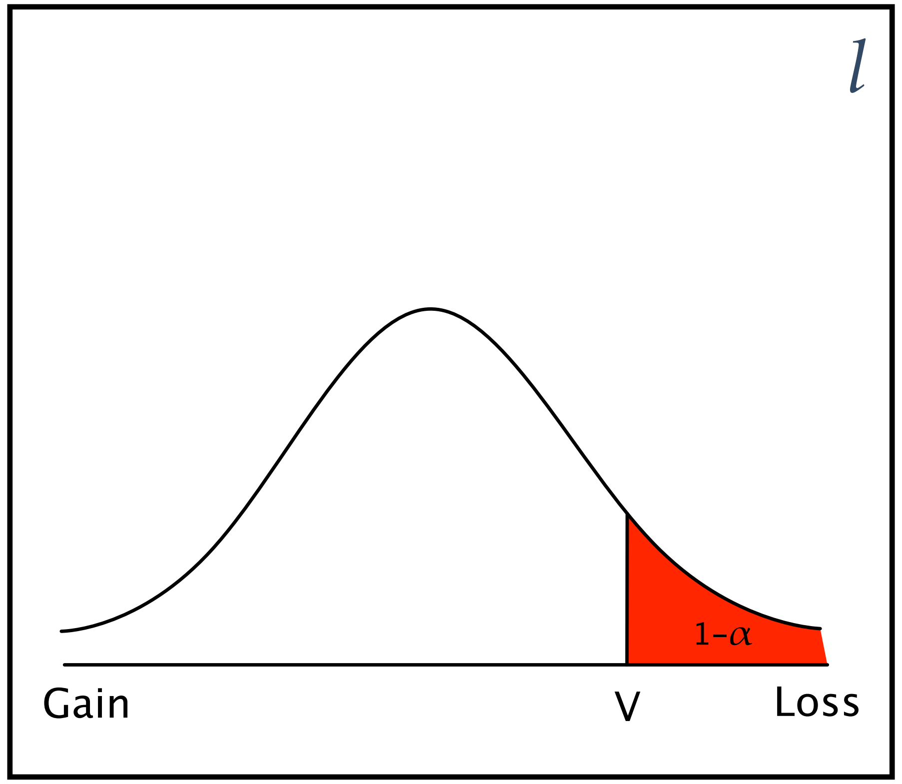
“With probability α we will not lose more than V euros in time T”

Time horizon

The reference distribution

- VaR can be computed using two different distributions:
  - the *distribution of gains* (g), where a loss is a negative gain.
  - the *distribution of losses* (l), where a gain is a negative loss.





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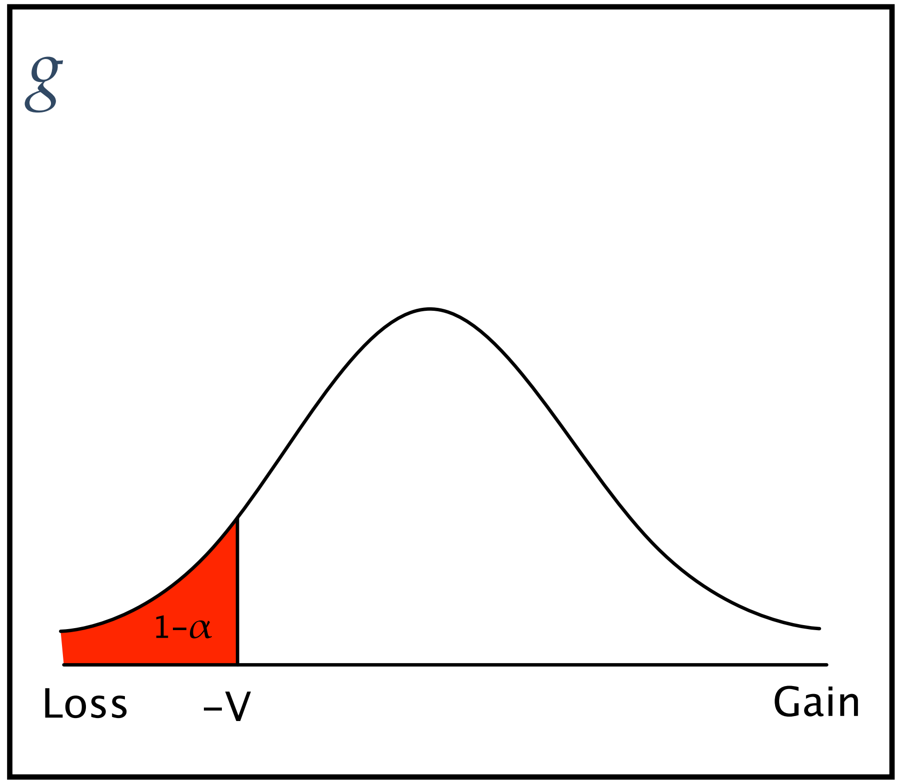
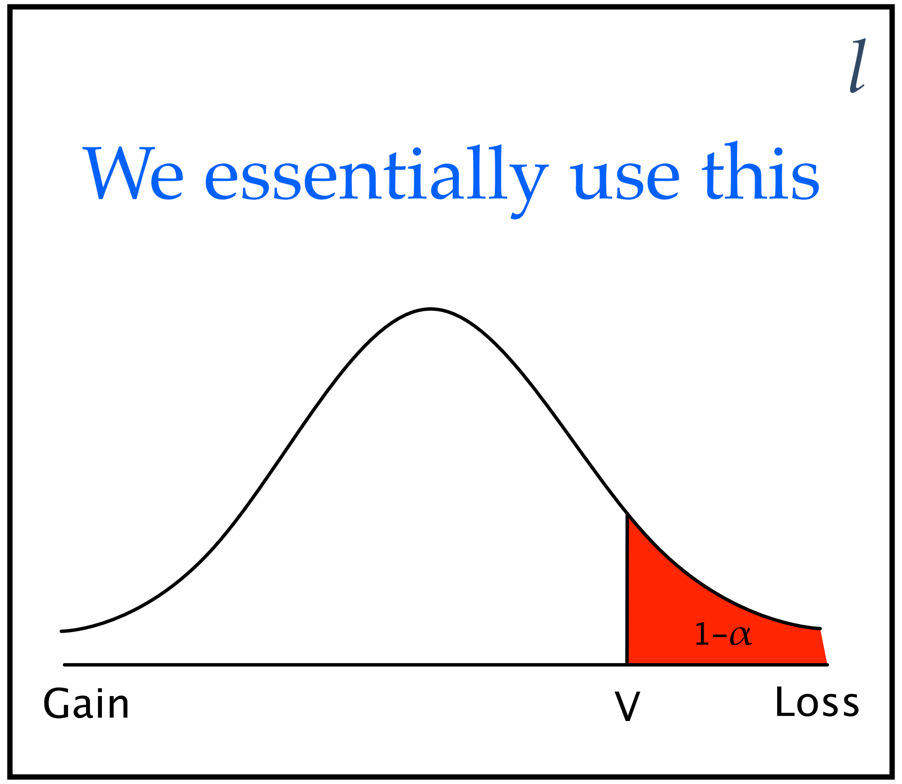


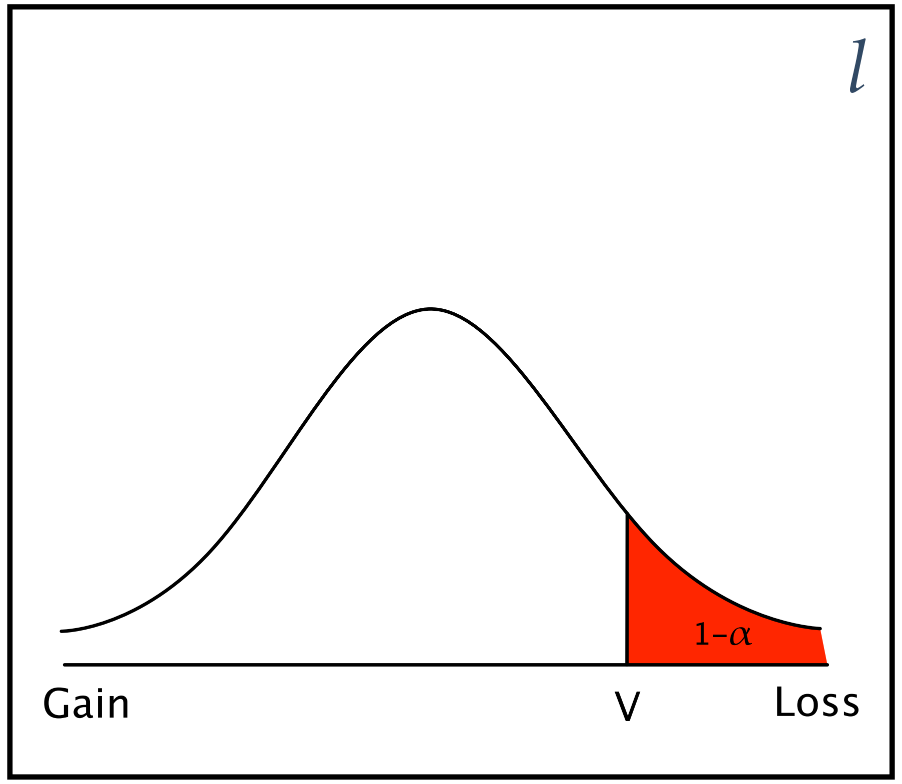


The formal definition

- Using some more formality, the VaR is nothing but a quantile of the loss distribution, and in particular the α-quantile for which

$$VaR_\alpha(L) = \inf\{l \in \mathbb{R} : P(L > l) \leq 1 - \alpha\} = \inf\{l \in \mathbb{R} : F_L(l) \geq \alpha\}$$

In words: *the VaR is the "loss value for which the probability of observing a larger loss, given the available information, is equal to 1-α".*



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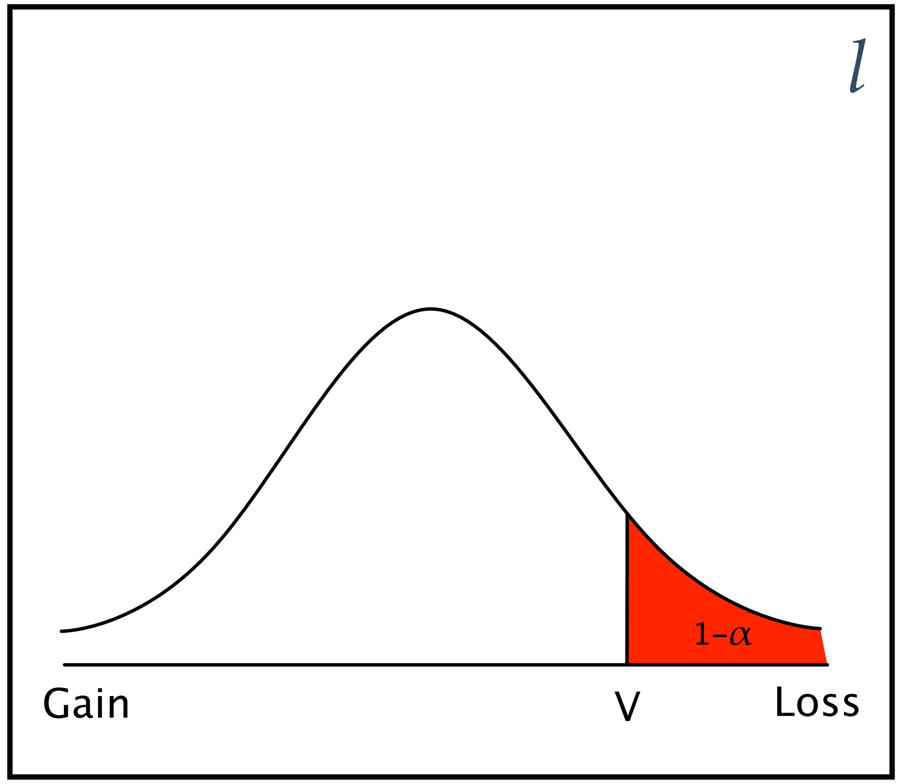

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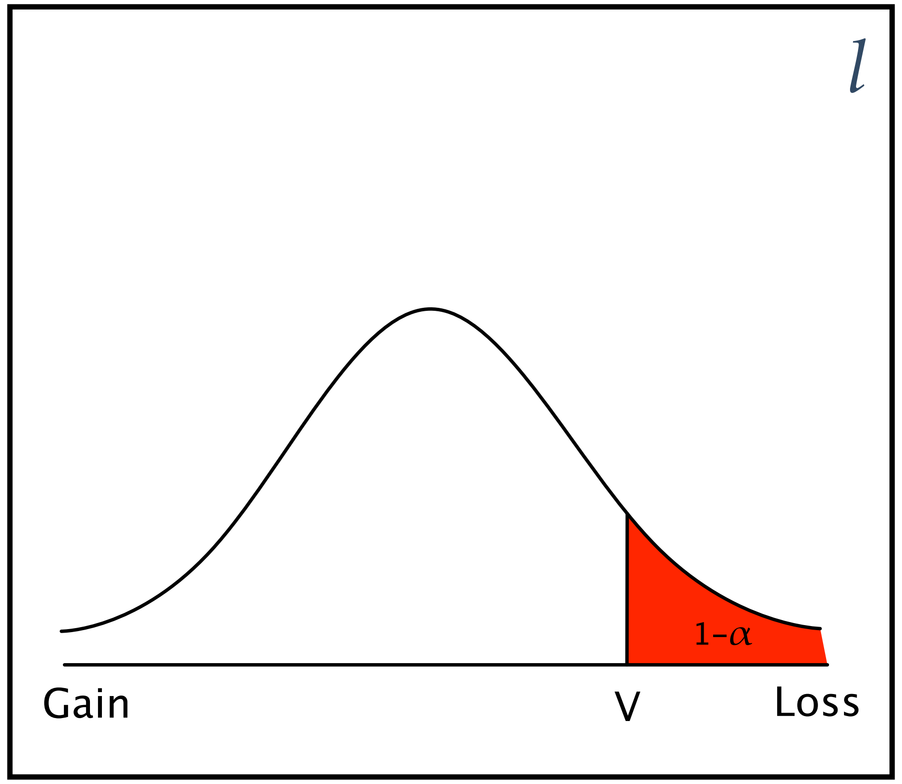

In the field of Credit Risk, the VaR is often referred to as **C-VaR**, as we will see later in this course.

Key elements

- The (credit) Value-at-Risk essentially depends on 2 elements:
  - the loss distribution;
  - the α value.

The loss distribution

- The loss distribution is always expressed over a time horizon T and it can be empirical or theoretical.
- In the first case, it is the so-called historical distribution, i.e. the distribution that emerges from the observation of reality.
- In the second case it can be whatever distribution (normal, lognormal, Pareto, etc.) and it is essentially used for modeling purposes.

The α value

- In theory, the α value may be freely chosen by the risk manager.
- In reality, it is often determined by the law or other prescriptions (e.g. Basel II-III).
- Common values are 0.95, 0.99, 0.995 and 0.999.
- For Credit Risk, we are usually interested in

$$VaR_{0.99}^{1-day} = VaR_{99\%}^{1-day} \qquad VaR_{0.99}^{10-day} \qquad VaR_{0.99}^{1-year} \qquad VaR_{0.999}^{1-year}$$

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Exercise

- Suppose that, for a 1-year project, all the outcomes between a gain of €80 million and a loss of €20 million are considered equally likely. What is the 1-year VaR for α=0.90?

Thank You