The VaR and its derivations
Introducing the Value-at-Risk

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In very simple terms, the VaR is a measure of risk that tries to answer the following question:

“How bad can things get?”
In more probabilistic terms, we look for a measure that allows us to say:

“With probability $\alpha$ we will not lose more than $V$ euros in time $T$”
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- the *distribution of losses* \((l)\), where a gain is a negative loss.
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- the *distribution of gains* ($g$), where a loss is a negative gain.
- the *distribution of losses* ($l$), where a gain is a negative loss.
Using some more formality, the VaR is nothing but a quantile of the loss distribution, and in particular the $\alpha$-quantile for which

$$\text{VaR}_\alpha(L) = \inf\{l \in \mathbb{R} : P(L > l) \leq 1 - \alpha\} = \inf\{l \in \mathbb{R} : F_L(l) \geq \alpha\}$$

In words: the VaR is the “loss value for which the probability of observing a larger loss, given the available information, is equal to $1-\alpha$”.
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In the field of Credit Risk, the VaR is often referred to as C-VaR, as we will see later in this course.
The (credit) Value-at-Risk essentially depends on 2 elements:

- the loss distribution;
- the $\alpha$ value.
The loss distribution is always expressed over a time horizon $T$ and it can be empirical or theoretical.

In the first case, it is the so-called historical distribution, i.e. the distribution that emerges from the observation of reality.

In the second case it can be whatever distribution (normal, lognormal, Pareto, etc.) and it is essentially used for modeling purposes.
In theory, the $\alpha$ value may be freely chosen by the risk manager.

In reality, it is often determined by the law or other prescriptions (e.g. Basel II-III).

Common values are 0.95, 0.99, 0.995 and 0.999.

For Credit Risk, we are usually interested in

$$\begin{align*}
    \text{VaR}_{0.99}^{1\text{--day}} & \quad \text{VaR}_{0.99}^{10\text{--day}} & \quad \text{VaR}_{0.99}^{1\text{--year}} & \quad \text{VaR}_{0.999}^{1\text{--year}} \\
    \text{VaR}_{0.99\%}^{1\text{--day}} & \quad \text{VaR}_{0.99\%}^{1\text{--day}}
\end{align*}$$
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= & \quad VaR_{99\%}^{1\text{--}day} & &\
\end{align*}
\]
Suppose that, for a 1-year project, all the outcomes between a gain of €80 million and a loss of €20 million are considered equally likely. What is the 1-year VaR for $\alpha=0.90$?
Thank You