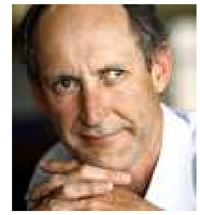
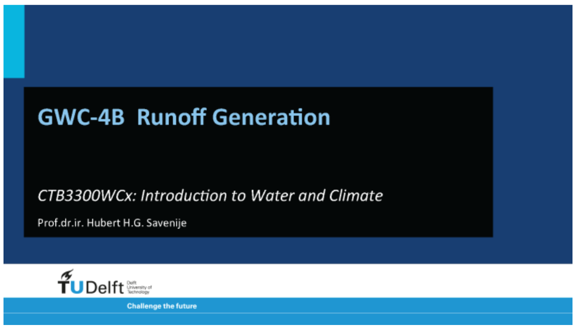
CTB3300WCx – Introduction to Water and Climate

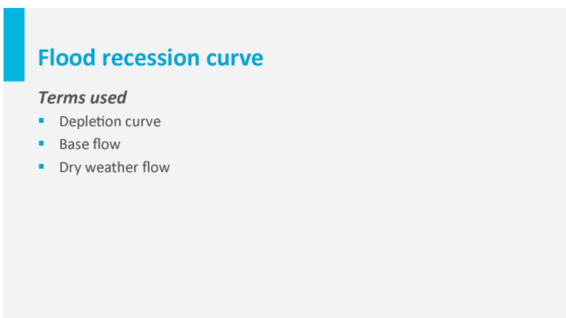
GWC 4b – Runoff Generation



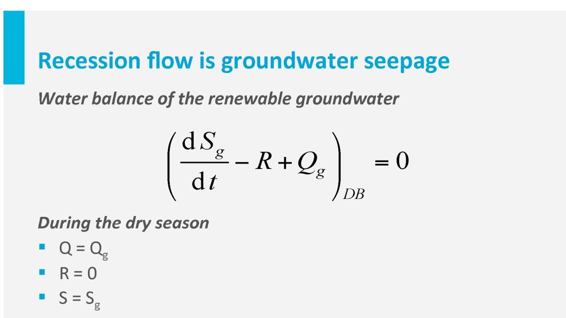
Hubert Savenije



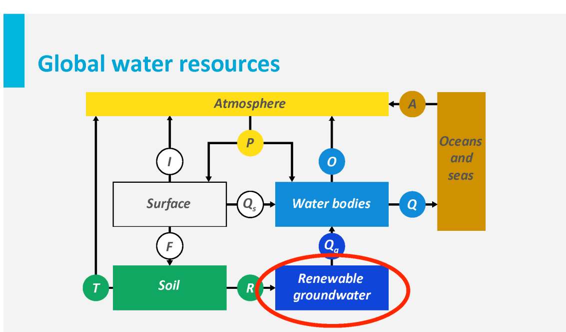
Welcome! My name is Hubert Savenije and I am a hydrologist. In this Module we'll discuss the runoff generating mechanisms.



Maybe the most predictable part of a hydrograph is the recession curve which represents the slow groundwater depletion during the base flow of a river.



The recession flow obeys the water balance of the renewable (deep blue) groundwater In the dry season the deep blue groundwater has no recharge and in the river there is no surface runoff. So the catchment discharge consists of groundwater and the active storage is the groundwater storage.



It is the water balance of the dark blue box.

Simplified water balance equation

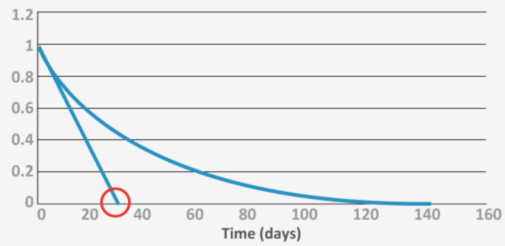
$$\frac{\mathrm{d}S}{\mathrm{d}t} = -Q$$

Linear reservoir $S = kQ$

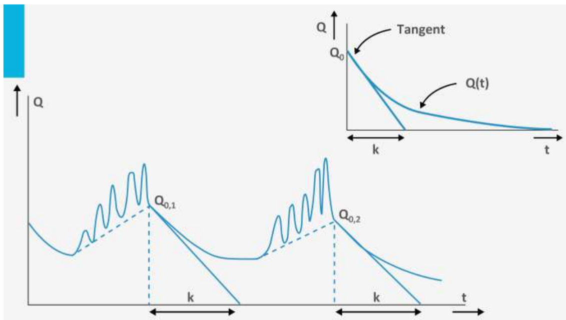
Solution $Q_t = Q_0 \exp\left(-\frac{t}{k}\right)$

This results in a very simple water balance for dry weather. On top of that , we may assume that the groundwater discharge is directly proportional to the storage. The proportionality coefficient k represents the average residence time of the water in the groundwater reservoir. Mathematically , it can be easily shown that the solution of these two equations is an exponential function (please see the example and do it yourself).

Residence time (k=30 days)



One can easily recognise the residence time k from the graph, because it is the time where the tangent to the curve hits the time axis.



Here you see this illustrated.

Special properties of $Q_t = Q_0 \exp\left(-\frac{t}{k}\right)$

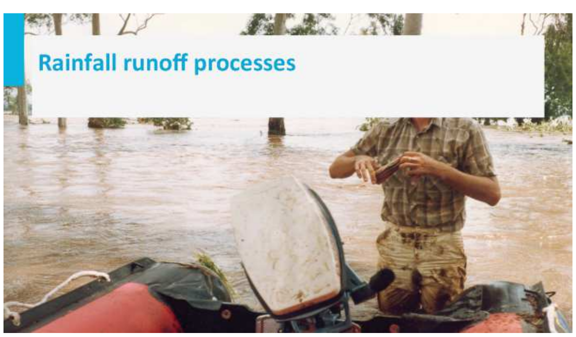
Derivative of the exponential function

$$\frac{\mathrm{d}Q_t}{\mathrm{d}t} = -\frac{1}{k} Q_0 \exp\left(-\frac{t}{k}\right) = -\frac{Q_t}{k}$$

Integral of the exponential function

$$\int_t^{\infty} Q_t \,\mathrm{d}t = \int_t^{\infty} Q_0 \exp\left(-\frac{t}{k}\right) \mathrm{d}t = -k \left| Q_0 \exp\left(-\frac{t}{k}\right) \right|_t^{\infty} = -k\left(0 - Q_t\right) = kQ_t$$

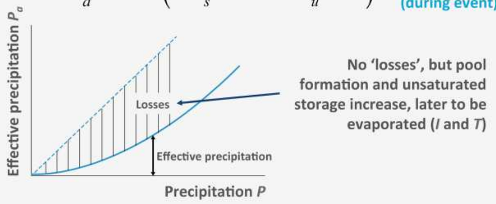
The exponential function has special properties: the time derivative of Q is –Q/k and the integral of Q (being the remaining groundwater storage) is k*Q.
Check it yourself!



Now let's discuss the flood generating mechanisms.

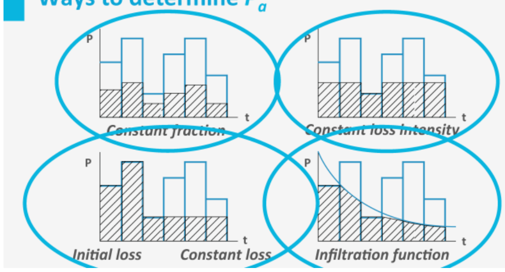
Effective precipitation

$$P_a = P - \left(\mathrm{d}S_s/\mathrm{d}t + \mathrm{d}S_u/\mathrm{d}t\right) - \underset{\text{(during event)}}{0}$$



When we want to determine the runoff as a result of a large precipitation event, we only should consider that part of the rain which contributed to runoff: the effective precipitation Pa. We thus have to subtract that part of the precipitation that replenishes the soil moisture or that fills stagnant pools, later to be evaporated. The evaporation itself during a large precipitation events we may neglect.

Ways to determine P_a



So how do we subtract this increase of temporary storage from the precipitation? There are different methods. Some subtract a certain percentage. Some subtract a fixed threshold. Some distinguish between a short term buffer (pool formation) and a longer term threshold (soil moisture storage). And others assume a sort of maximum infiltration capacity.
We generally take the fixed threshold for its simplicity.

The storage principle

$S = kQ$ *Q discharge per unit surface area [L/T]*

$$\frac{\mathrm{d}S}{\mathrm{d}t} = P_a - Q \quad \textit{hence:} \quad k\frac{\mathrm{d}Q}{\mathrm{d}t} = P_a - Q$$

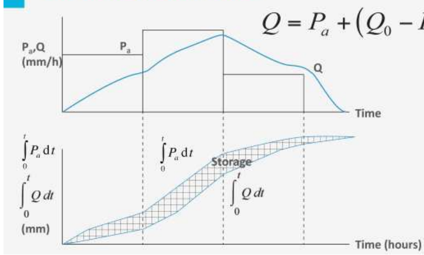
$$\frac{\mathrm{d}Q}{Q - P_a} = -\frac{1}{k}\mathrm{d}t \quad \textit{hence:} \quad \ln\left(Q - P_a\right) = -\frac{t}{k} + C$$

If t=0, then Q=Q₀ *hence:* $C = \ln\left(Q_0 - P_a\right)$

We saw that groundwater depletion can be simulated as a linear reservoir. If groundwater is dominant in a catchment, we could simulate both the fill and the depletion by a linear reservoir. We call that the storage principle. The solution is an exponential function with a fixed time scale k.

The storage principle

$$Q = P_a + \left(Q_0 - P_a\right)\exp\left(-\frac{t}{k}\right)$$



For every time increment, we can apply this principle, whereby the runoff tends exponentially to the effective precipitation Pa, starting from the discharge in the previous time step.

The storage principle

Analytical: $Q = P_a + \left(Q_0 - P_a\right)\exp\left(-\frac{t}{k}\right)$

Numerical:

$$\Delta S = \left(P_a - \bar{Q}\right)\Delta t$$

$$S_2 = S_1 + \left(P_a - \left(\frac{Q_1 + Q_2}{2}\right)\right)\Delta t$$

$S_1 = kQ_1$ *and* $S_2 = kQ_2$

We can do this analytically, but also numerically. The numerical approach can be easily done in a spreadsheet (please look at the example).

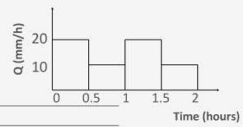
The storage principle

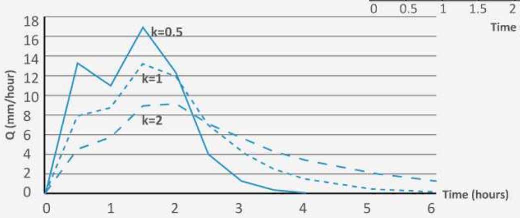
$$Q_2 = \frac{k - 0.5\Delta t}{k + 0.5\Delta t} Q_1 + \frac{\Delta t}{k + 0.5\Delta t} P_a$$

More appropriate for groundwater dominated catchments

The numerical solution is that the discharge Q2 at the next time step is a function of the effective precipitation and the discharge in the previous time step. The coefficients depend on both the time scale and the time step. It is easy to see that the sum of the two coefficients is one.
Why is that?

Example storage principle





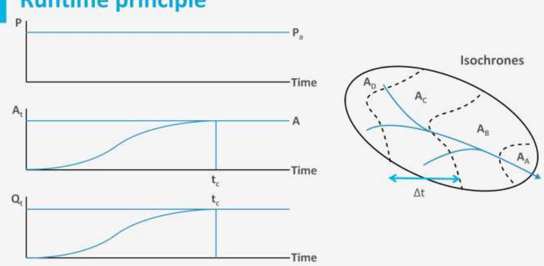
Here is what the result looks like for different values of k.

Runtime principle

- *Rational method*
- *Runoff is proportional to the contributing surface area, which increases in time*
- *Runoff is proportional to the effective precipitation, hence linearity*
- *Analogy with ping-pong balls*

If a catchment is not groundwater dominated, but rather dominated by fast runoff (for instance a paved area, or relatively impervious area, or a hillslope with underlying impervious hard rock) then the runtime principle can be very useful. This approach is also called the Rational method, whereby the runoff is proportional to the surface area contributing to the runoff. This area increases over time as an ever larger area of the catchment contributes to the runoff, depending on the time needed for the water to reach the outfall. Imagine you are in an amphitheater and ping-pong balls come falling from the sky at a continuous rate. The balls that fall closest to the stage discharge first. Only when the balls from the highest seats in the amphitheater have reached the stage, is the discharge equal to the flux coming from the sky.

Runtime principle



This is how it looks. The top graph shows the precipitation rate The middle one the contributing area and the lower one the discharge. At the end, the discharge equals the precipitation rate.

Assumptions of runtime principle

- *Runtime is time invariant (stationarity)*
- *Runtime is proportional to contributing area*
- *Linearity between Q and P_a*
- *Precipitation is equally distributed in space*
- *Often used in urban environments (impervious surfaces, urban drains, small impervious catchments)*

Of course this is only correct under a range of limiting assumptions, but in relatively small catchments in urban or impervious areas, it may work very well.

Runtime principle equations

$$Q(t) = P_a \min\left(A, A_t\right) = P_a A \min\left(1, \frac{A_t}{A}\right)$$

- *Q in [L^3T^{-1}], q in [LT^{-1}]*
- *P_a assumed constant over an interval*
- *When A_t equals A (total surface area of catchment), then $t=t_c$ (time of concentration)*
- *Superposition*

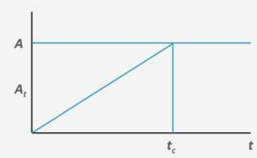
$$q(t) = \frac{Q(t)}{A} = P_a \min\left(1, \frac{A_t}{A}\right)$$

Mathematically it can be described by a simple threshold function, described bij the MIN operator.

Runtime principle

Linear increase of A_t:

$$A_t = \frac{A}{t_c} t$$



$$q(t) = P_a \min\left(1, \frac{A_t}{A}\right) = P_a \min\left(1, \frac{t}{t_c}\right)$$

And the interesting part is that the runoff can be described purely as a function of only effective precipitation and the so-called time of concentration: the time it takes for the most remote part of the catchment to contribute to the discharge

Runtime principle numerical equations

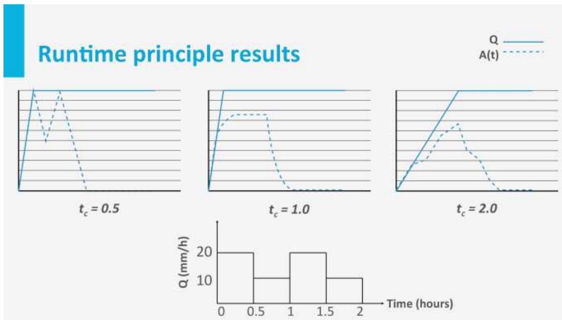
Superposition of subsequent events:

$$Q(t) = \sum_{i=1}^{n} \Delta P_{a,i} \cdot A \min\left(1, \frac{\max\left(A_{t-\Delta t(i-1)}, 0\right)}{A}\right)$$

$$q(t) = \sum_{i=1}^{n} \Delta P_{a,i} \min\left(1, \frac{\max\left(t - \Delta t(i-1), 0\right)}{t_c}\right)$$

If you have subsequent precipitation events of fixed time steps, then we can also transfer this solution into a numerical scheme which can be readily incorporated in a spreadsheet.

Runtime principle results



Here you see outputs of the spreadsheet for the same effective precipitation event, but with different times of concentration.

Runtime principle questions

- *Why must the time step Δt be smaller than t_c?*
- *What happens if Δt is equal to t_c?*
- *Under which conditions can we apply the runtime principle?*

Here are some questions for you to reflect on.

Applicability of the different methods

Storage principle
- 'Flat areas'
- Groundwater dominated catchments

Runtime principle
- Steep impervious catchments
- Surface runoff

'Real' catchments
- Unit hydrograph
- Hydrological modelling

The storage principle and the runtime principle are primarily educational tools, although engineers may use them for particular circumstances, as long as they realise the limitations. But real catchments are more complex.

Challenge

- *This was treacherously simple*
- *Real catchments are more complex*
- *There still is a world to discover in hydrology*

Scientific hydrologists still don't exactly know how water moves through the terrestrial system. I plan to make a follow-up course in hydrological modeling to help you to further explore runoff generating processes.
"There still is a world to discover in Hydrology".

GWC-4B Runoff Generation

CTB3300WCx: Introduction to Water and Climate

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Challenge the future