ITMO UNIVERSITY

How to Win Coding Competitions: Secrets of Champions

Week 4: Algorithms on Graphs
Lecture 3: Introduction to Depth First Search

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Saint Petersburg 2016

Example problem

Recall: an undirected graph is connected if for every pair of vertices a and b there is a path between them

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Idea 1: Reduce the all-to-all problem to one-to-all problem

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- If there is a path from a and b, and between b and c, then there is a path between a and c

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  - Just concatenate these paths!

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Idea 2: Solve the one-to-all problem

- Traverse the graph, starting from some vertex
- Required property: if you visit a vertex, you also visit all adjacent vertices
- Meet Depth First Search!

Depth First Search: Simple version

```
$G = \langle V, E \rangle$                                  ▷ the graph
$U \leftarrow \emptyset$                                    ▷ set of visited vertices

procedure $\mathrm{DFS}(v)$
    $U \leftarrow U \cup \{v\}$
    for $(v, u) \in E$ do
        if $u \notin U$ then $\mathrm{DFS}(u)$ end if
    end for
end procedure
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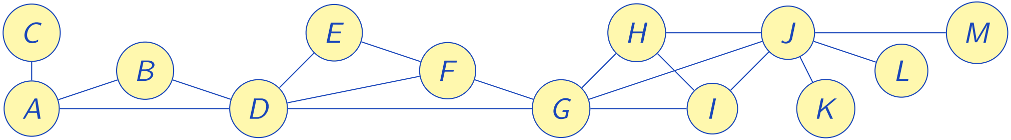

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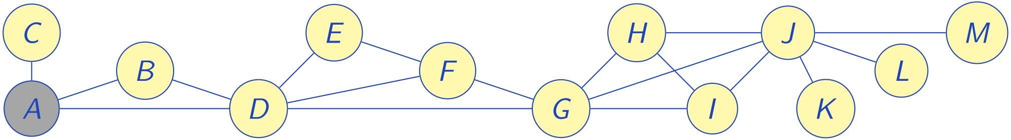

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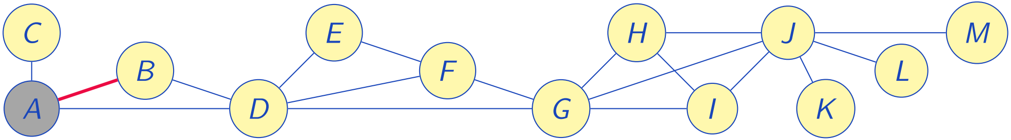

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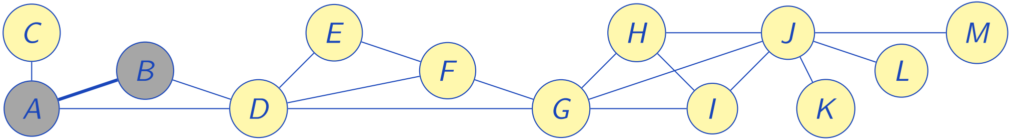

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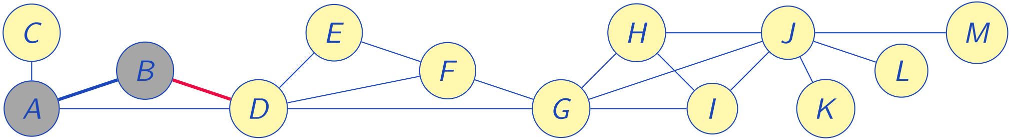

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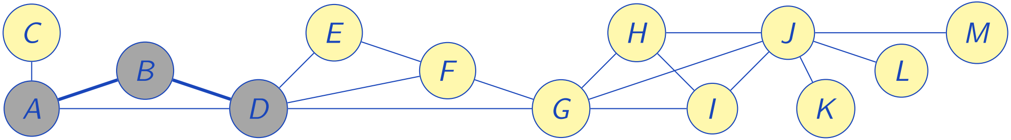

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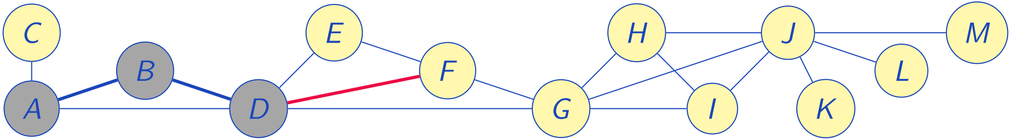

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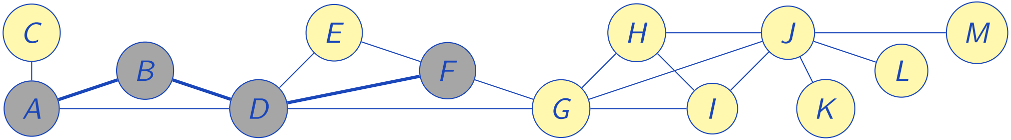

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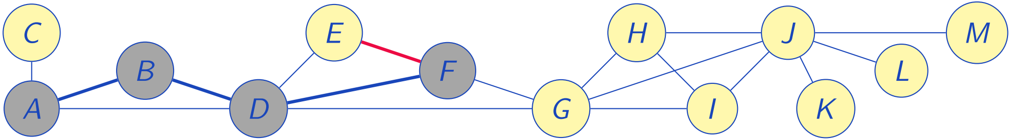

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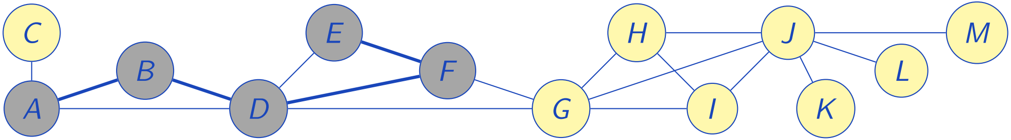

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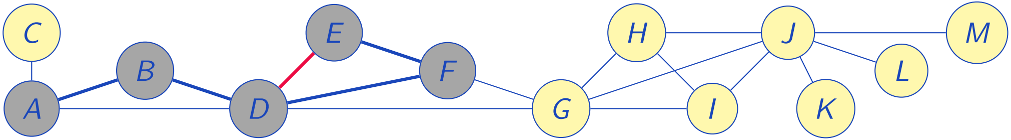

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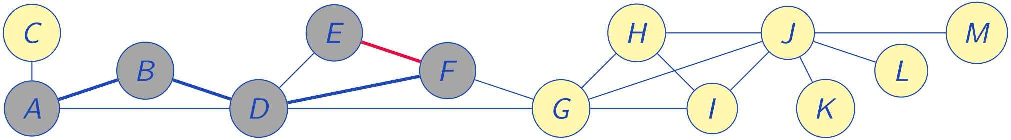

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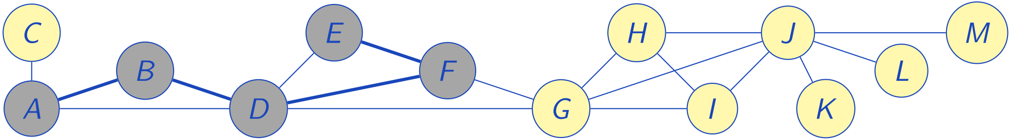

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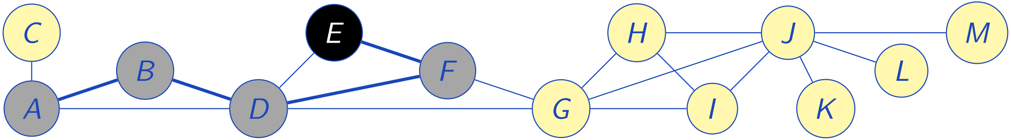

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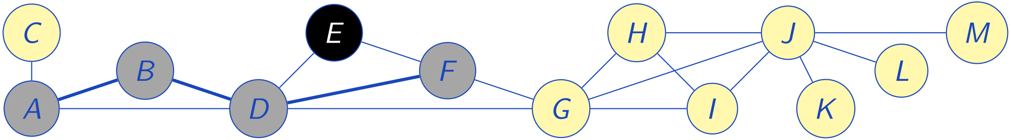

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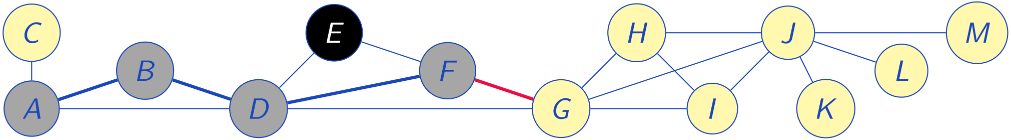

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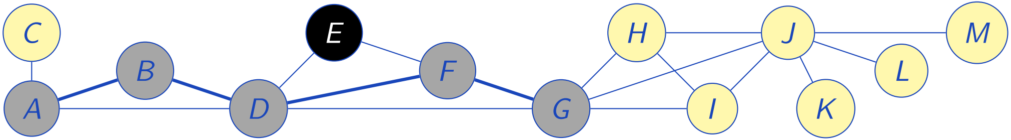

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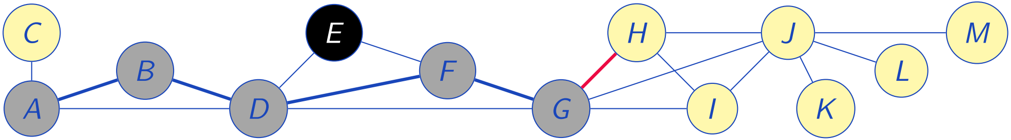

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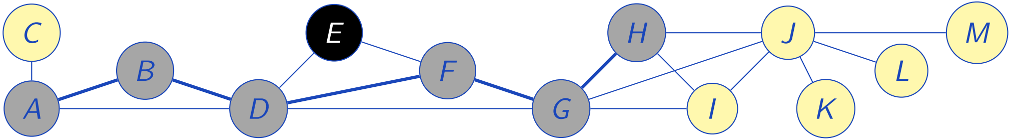

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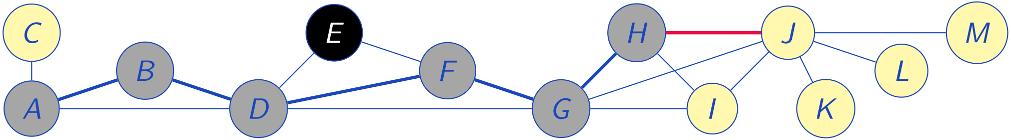

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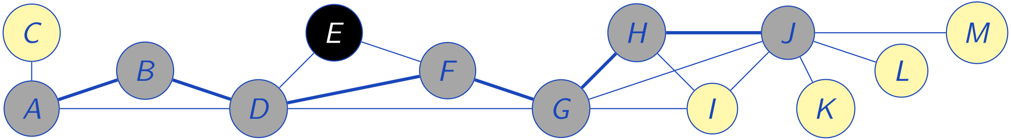

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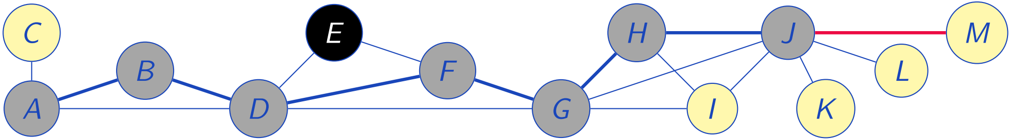

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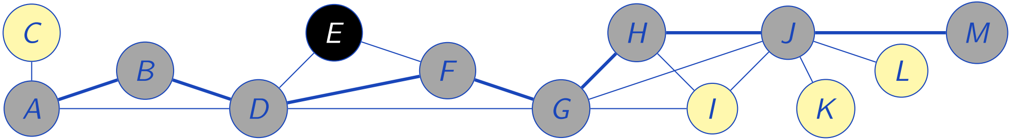

Depth First Search: Simple version

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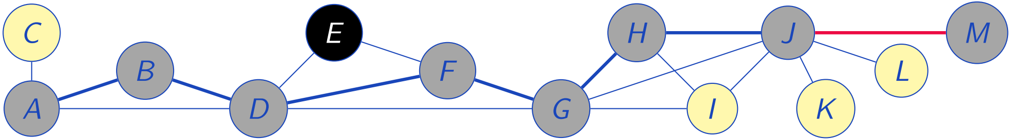

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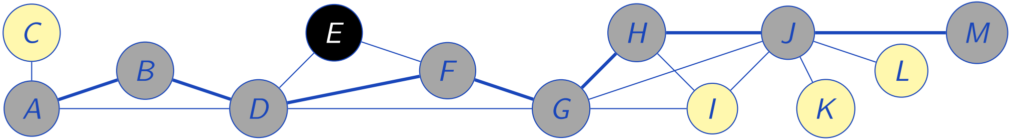

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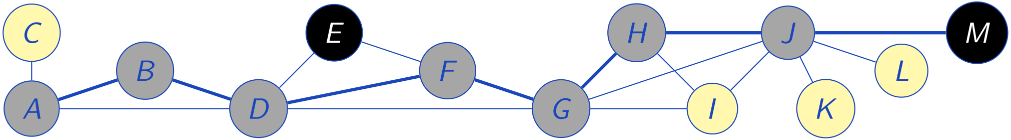

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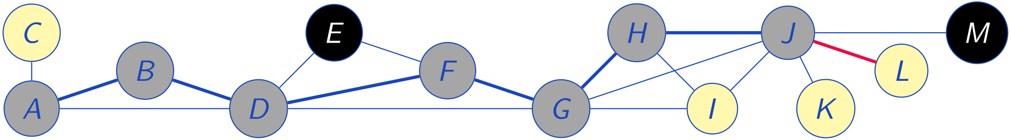

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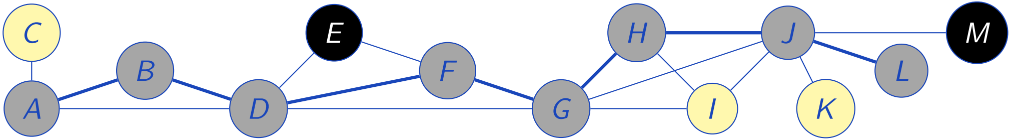

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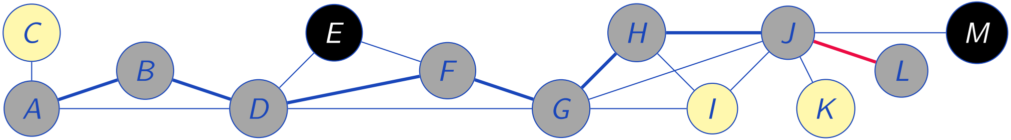

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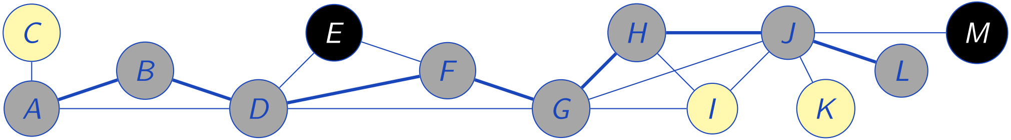

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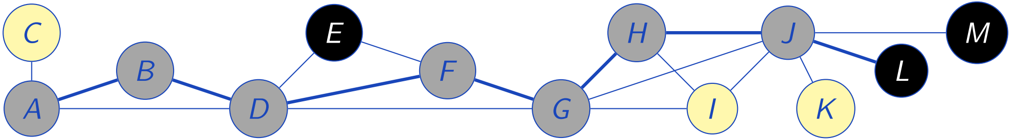

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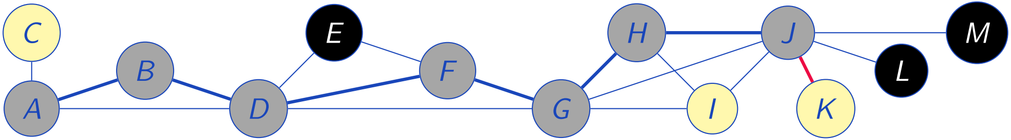

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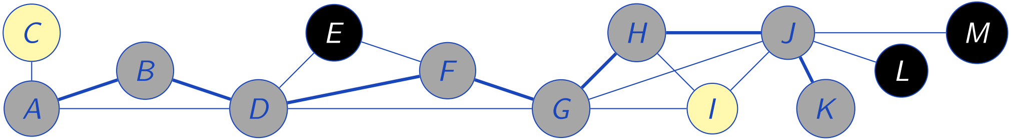

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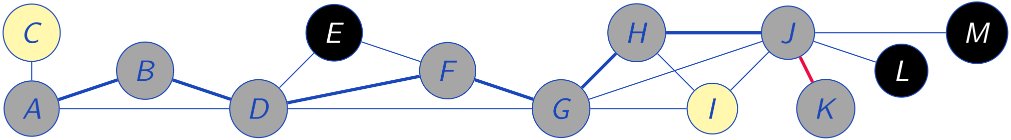

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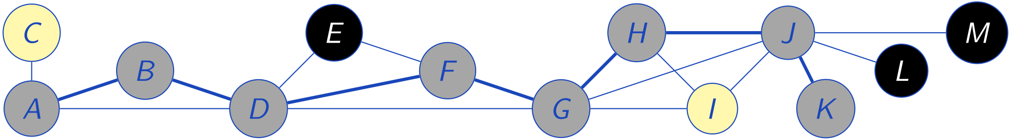

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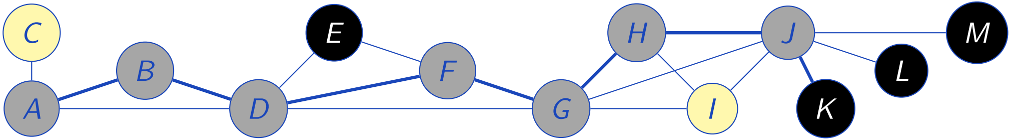

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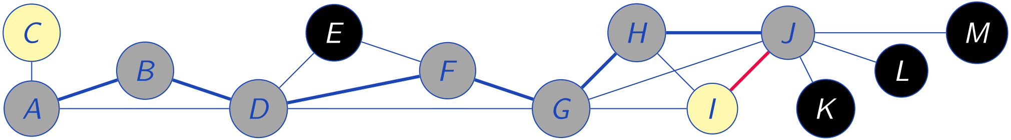

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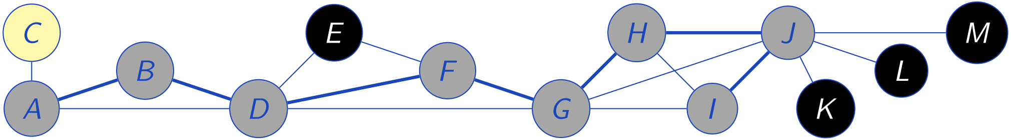

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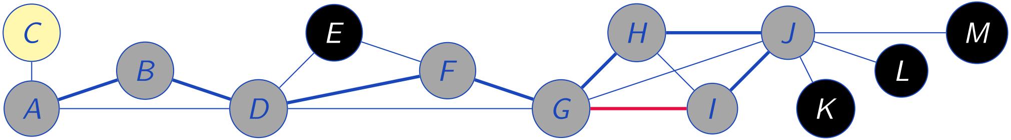

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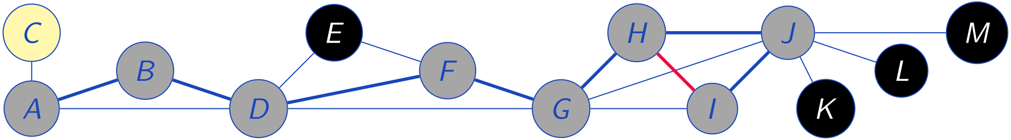

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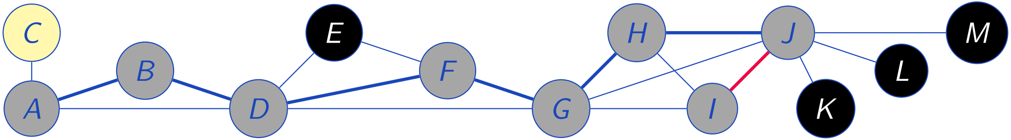

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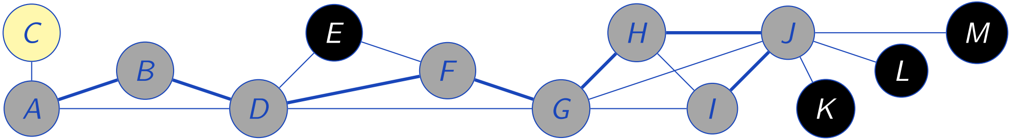

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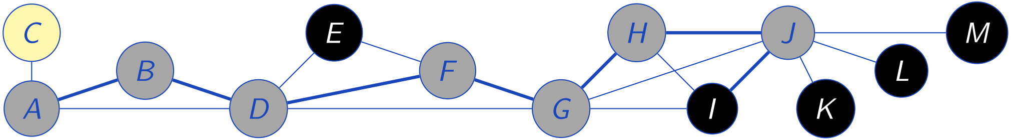

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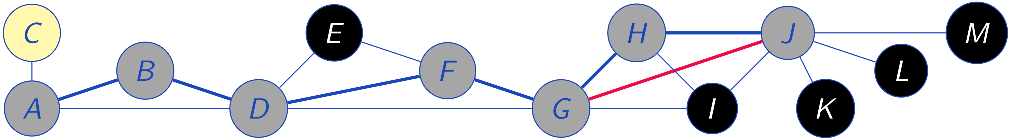

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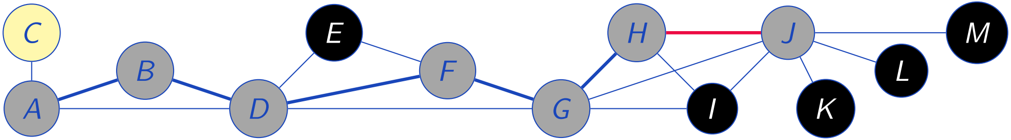

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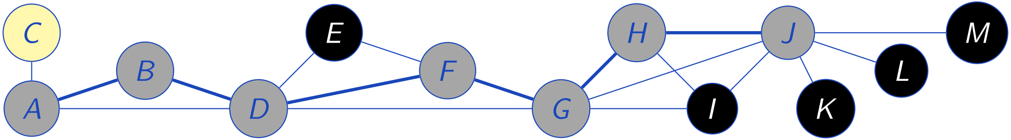

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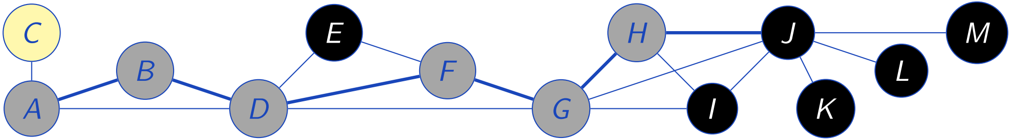

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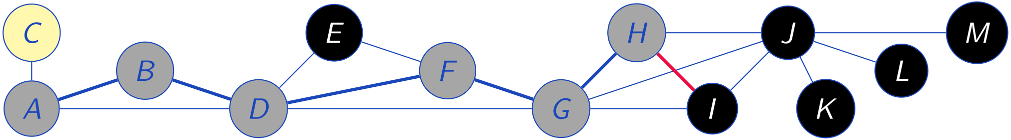

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Depth First Search: Simple version

```
$G = \langle V, E \rangle$                                    ▷ the graph
$U \leftarrow \emptyset$                                      ▷ set of visited vertices
$A(v) = \{u \mid (v, u) \in E\}$      ▷ Adjacent vertex function: for free with adjacency lists
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    $U \leftarrow U \cup \{v\}$                          ▷ marking current vertex visited
    for $u \in A(v)$ do                       ▷ visiting all outgoing edges: more efficient
        if $u \notin U$ then $\mathrm{DFS}(u)$ end if   ▷ if target is not visited, calling recursively
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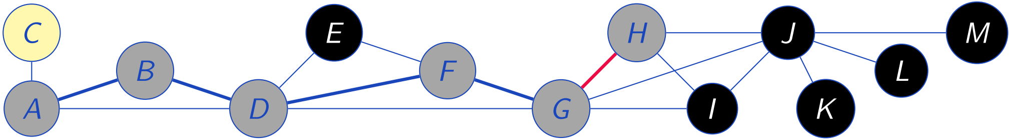

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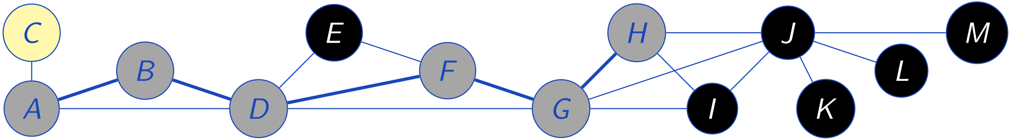

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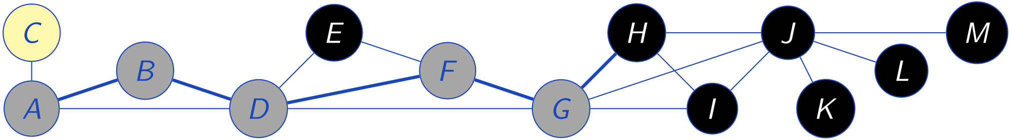

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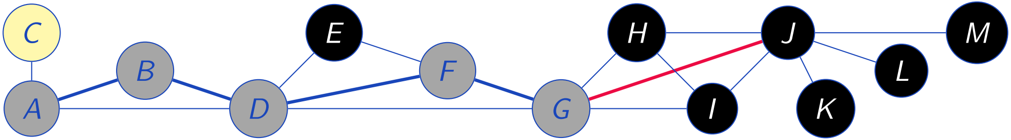

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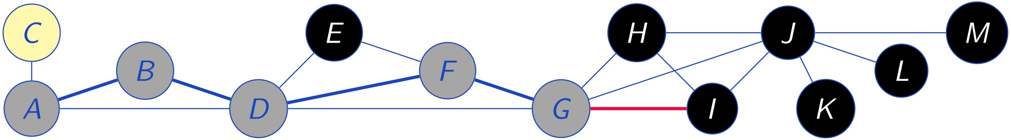

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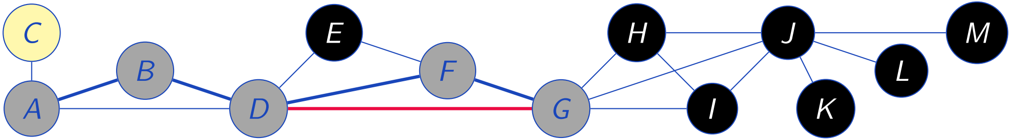

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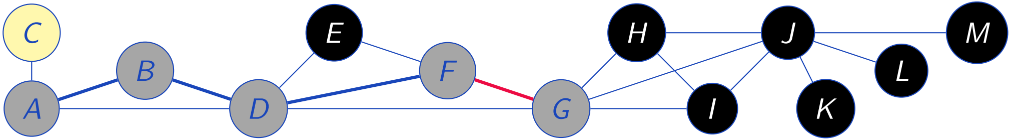

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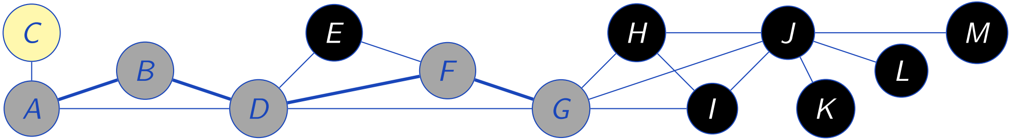

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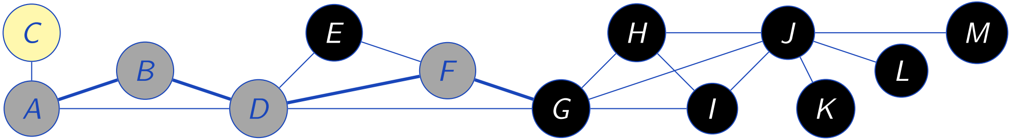

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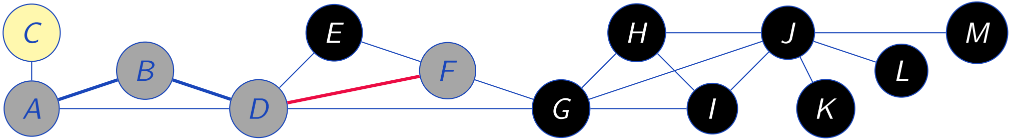

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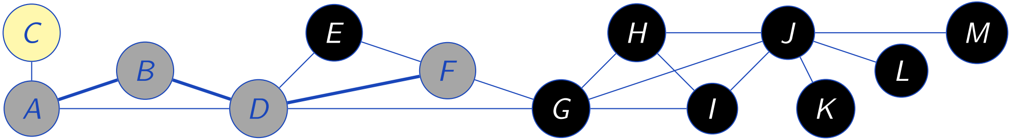

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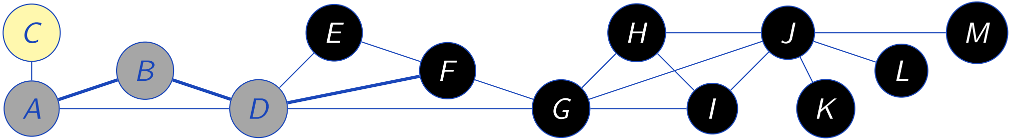

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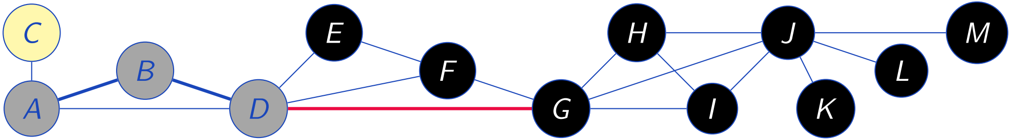

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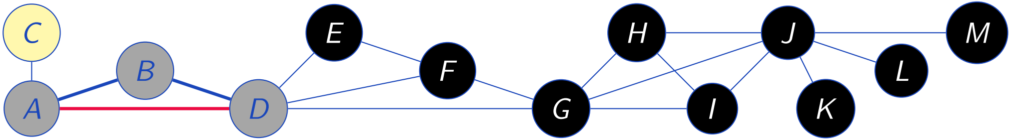

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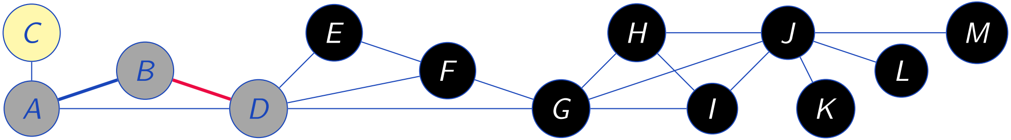

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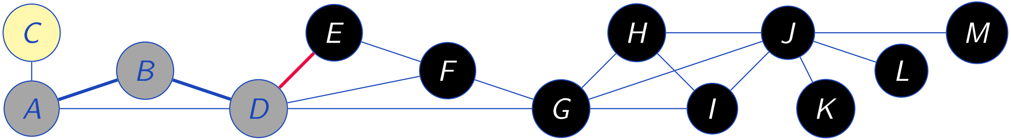

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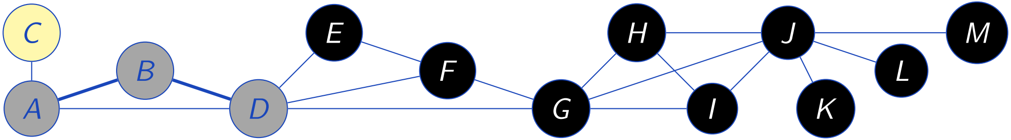

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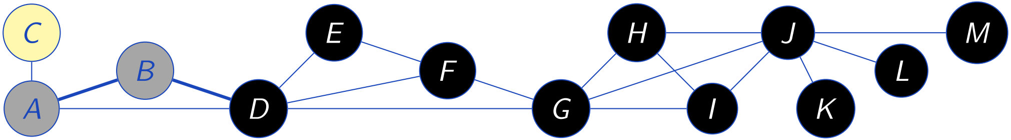

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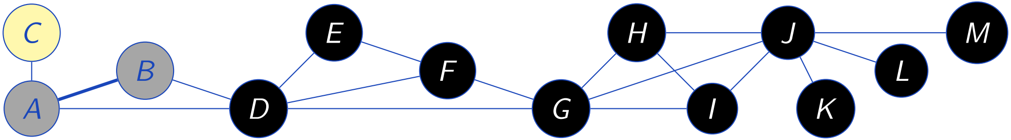

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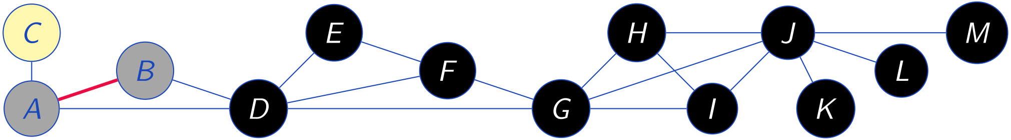

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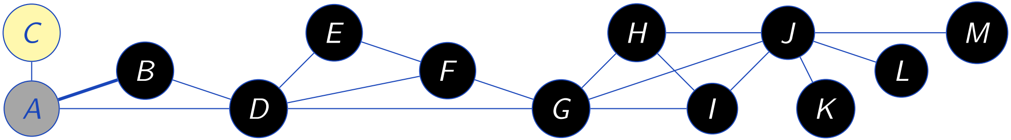

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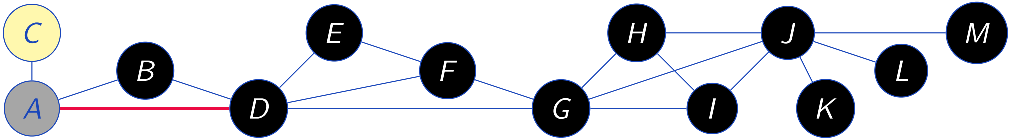

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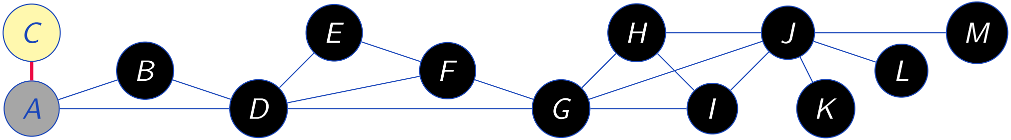

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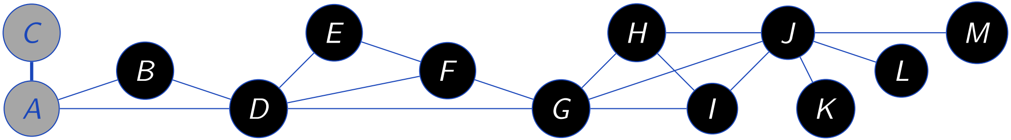

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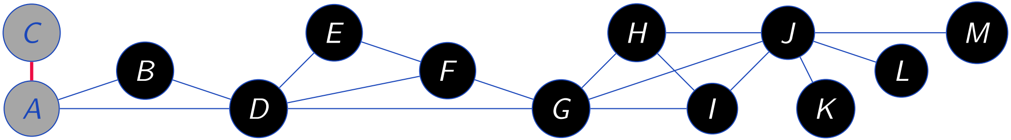

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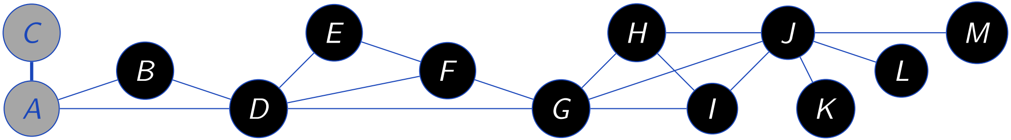

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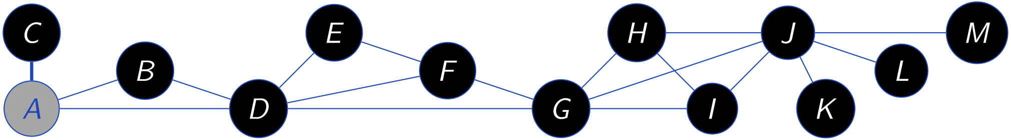

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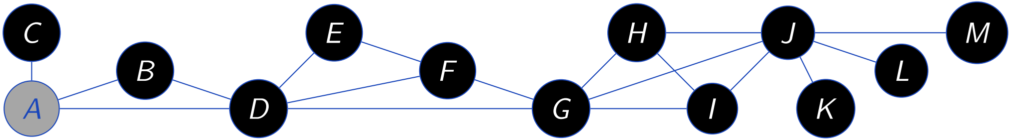
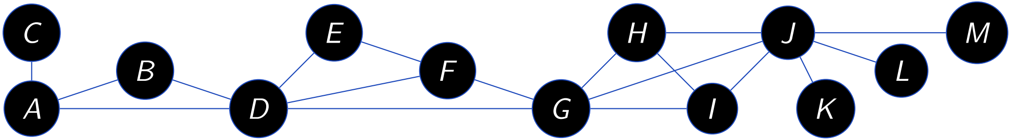
$U \leftarrow \emptyset$                                      ▷ set of visited vertices
$A(v) = \{u \mid (v, u) \in E\}$    ▷ Adjacent vertex function: for free with adjacency lists
procedure $\mathrm{DFS}(v)$          ▷ recursive procedure, argument: current vertex
    $U \leftarrow U \cup \{v\}$                         ▷ marking current vertex visited
    for $u \in A(v)$ do                   ▷ visiting all outgoing edges: more efficient
        if $u \notin U$ then $\mathrm{DFS}(u)$ end if     ▷ if target is not visited, calling recursively
    end for
end procedure
```



Depth First Search: Simple version

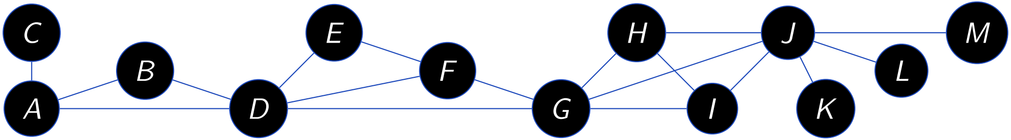
```
$G = \langle V, E \rangle$                          ▷ the graph
$U \leftarrow \emptyset$                            ▷ set of visited vertices
$A(v) = \{u \mid (v, u) \in E\}$                    ▷ Adjacent vertex function: for free with adjacency lists
procedure $\mathrm{DFS}(v)$                         ▷ recursive procedure, argument: current vertex
    $U \leftarrow U \cup \{v\}$                     ▷ marking current vertex visited
    for $u \in A(v)$ do                             ▷ visiting all outgoing edges: more efficient
        if $u \notin U$ then $\mathrm{DFS}(u)$ end if   ▷ if target is not visited, calling recursively
    end for
end procedure
```



Depth First Search: Simple version

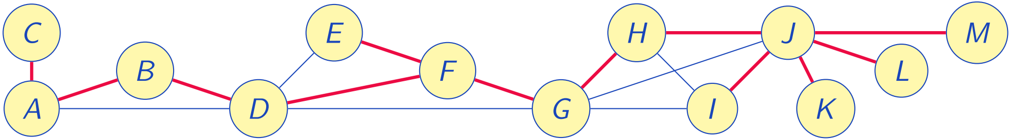
```
$G = \langle V, E \rangle$
$U \leftarrow \emptyset$
$A(v) = \{u \mid (v, u) \in E\}$
procedure $\mathrm{DFS}(v)$
    $U \leftarrow U \cup \{v\}$
    for $u \in A(v)$ do
        if $u \notin U$ then $\mathrm{DFS}(u)$ end if
    end for
end procedure
```

```
procedure $\textsc{IsConnected}(V, E)$
    $\mathrm{DFS}($arbitrary vertex from $V)$
    return $U = V$
end procedure
```



```
$G = \langle V, E\rangle$
$U \leftarrow \emptyset$
$A(v) = \{u \mid (v, u) \in E\}$
procedure $\mathrm{DFS}(v)$
    $U \leftarrow U \cup \{v\}$
    for $u \in A(v)$ do
        if $u \notin U$ then $\mathrm{DFS}(u)$ end if
    end for
end procedure
```

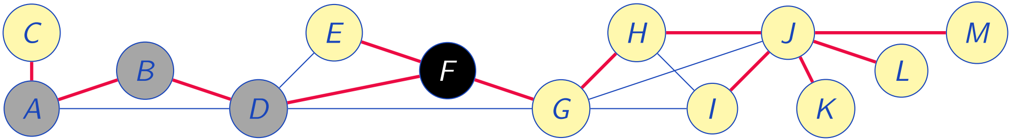
- DFS tree: all traversed edges



Depth First Search: Simple version

```
$G = \langle V, E \rangle$
$U \leftarrow \emptyset$
$A(v) = \{u \mid (v, u) \in E\}$
procedure $\mathrm{DFS}(v)$
    $U \leftarrow U \cup \{v\}$
    for $u \in A(v)$ do
        if $u \notin U$ then $\mathrm{DFS}(u)$ end if
    end for
end procedure
```

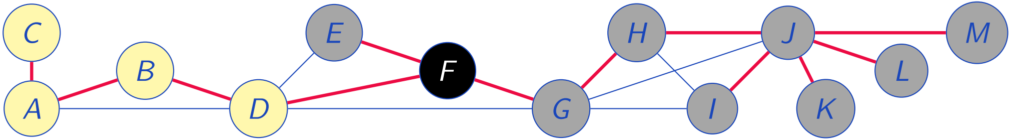
- DFS tree: all traversed edges
- Ancestors of v: all vertices up the DFS tree from v



Depth First Search: Simple version

```
$G = \langle V, E \rangle$
$U \leftarrow \emptyset$
$A(v) = \{u \mid (v, u) \in E\}$
procedure $\mathrm{DFS}(v)$
    $U \leftarrow U \cup \{v\}$
    for $u \in A(v)$ do
        if $u \notin U$ then $\mathrm{DFS}(u)$ end if
    end for
end procedure
```

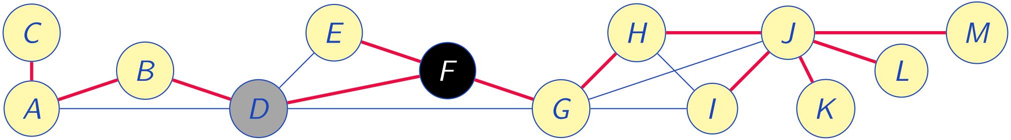
- DFS tree: all traversed edges
- Ancestors of v: all vertices up the DFS tree from v
- Descendants of v: all vertices down the DFS tree from v



Depth First Search: Simple version

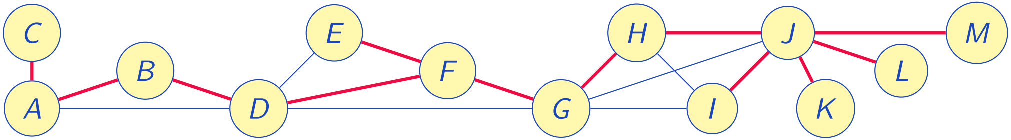
```
$G = \langle V, E \rangle$
$U \leftarrow \emptyset$
$A(v) = \{u \mid (v, u) \in E\}$
procedure $\mathrm{DFS}(v)$
    $U \leftarrow U \cup \{v\}$
    for $u \in A(v)$ do
        if $u \notin U$ then $\mathrm{DFS}(u)$ end if
    end for
end procedure
```

- DFS tree: all traversed edges
- Ancestors of v: all vertices up the DFS tree from v
- Descendants of v: all vertices down the DFS tree from v
- Parent of v: the immediate ancestor of v



```
$G = \langle V, E \rangle$
$U \leftarrow \emptyset$
$A(v) = \{u \mid (v, u) \in E\}$
procedure $\mathrm{DFS}(v)$
    $U \leftarrow U \cup \{v\}$
    for $u \in A(v)$ do
        if $u \notin U$ then $\mathrm{DFS}(u)$ end if
    end for
end procedure
```

- DFS tree: all traversed edges
- Ancestors of v: all vertices up the DFS tree from v
- Descendants of v: all vertices down the DFS tree from v
- Parent of v: the immediate ancestor of v
- Undirected: Non-DFS-tree edges connect vertices with ancestors or descendants



Depth First Search: Cycle detection

```
$G = \langle V, E \rangle$
$U \leftarrow \emptyset$
$A(v) = \{u \mid (v, u) \in E\}$
procedure $\mathrm{DFS}(v)$
    $U \leftarrow U \cup \{v\}$
    for $u \in A(v)$ do
        if $u \notin U$ then $\mathrm{DFS}(u)$ end if
    end for
end procedure
```

Depth First Search: Cycle detection

```
$G = \langle V, E \rangle$
$U \leftarrow \emptyset$, $X \leftarrow \emptyset$                                ▷ $X$: the set of exited vertices
$A(v) = \{u \mid (v, u) \in E\}$
procedure $\mathrm{DFS}(v)$
    $U \leftarrow U \cup \{v\}$
    for $u \in A(v)$ do
        if $u \in U$ and $u \notin X$ then
            return true          ▷ If hitting a visited and not exited vertex, found a cycle
        end if
        if $u \notin U$ and $\mathrm{DFS}(u)$ then return true end if
    end for
    $X \leftarrow X \cup \{v\}$
    return false
end procedure
```

Depth First Search: Cycle detection

```
$G = \langle V, E \rangle$
$U \leftarrow \emptyset$, $X \leftarrow \emptyset$                         ▷ $X$: the set of exited vertices
$A(v) = \{u \mid (v, u) \in E\}$        ▷ $U$ and $X$ are typically implemented as a single array
procedure $\mathrm{DFS}(v)$                          ▷ color[v] = 0: $v \notin U$, $v \notin X$
    $U \leftarrow U \cup \{v\}$                                  ▷ color[v] = 1: $v \in U$, $v \notin X$
    for $u \in A(v)$ do                             ▷ color[v] = 2: $v \in U$, $v \in X$
        if $u \in U$ and $u \notin X$ then
            return true        ▷ If hitting a visited and not exited vertex, found a cycle
        end if
        if $u \notin U$ and $\mathrm{DFS}(u)$ then return true end if
    end for
    $X \leftarrow X \cup \{v\}$
    return false
end procedure
```