Figure 1 *Cables of The George Washington Bridge,[1] New York City*

Suspension Bridges II: Cables

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January 2019

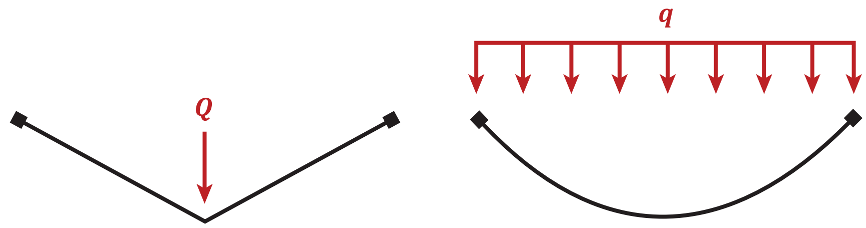
In ***Structural Studies: Suspension Bridges I,*** we learned about the components of a suspension bridge (See **Visual Glossary)** and the forces acting in the towers and anchors. Here we focus on the forces in the cables[2].

1. Form: The Cable

Unlike cantilevers and columns, which are straight and stiff, cables are flexible structural elements. A flexible structural element can resist only ***axial tensile forces***; it cannot resist compression, shear or bending. Because of their flexibility, cables can change their shape so that any applied load will only create tensile forces in them. For example, a cable that has a single point load, Q, applied at midspan takes on the shape shown in **Figure 2, left**.

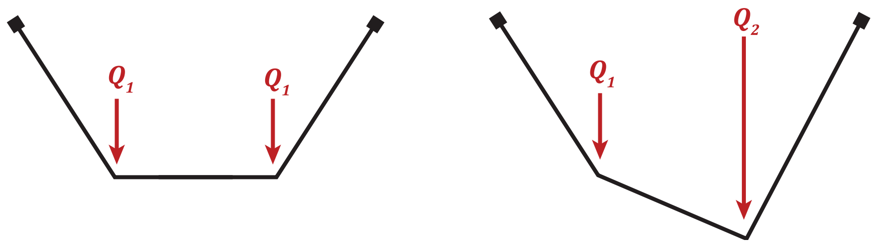
Figure 2, right shows the form (a parabola) that a cable takes on under a uniformly distributed load, q, like that of a suspension bridge.

Figure 2
Left: *Cable Shape Under a Point Load*
Right: *Cable Shape Under a Uniformly Distributed Load*



These forms taken by cables under certain loading conditions are called ***funicular forms.*** A distinct funicular form exists for each unique loading condition. The main cable of a suspension bridge is an example of a funicular form. Due to the nature of the distributed load, the cables develop the parabolic curvature that is characteristic of suspension bridges.

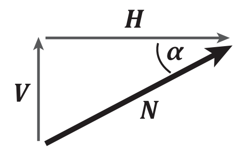
Figure 3
Left: *Cable with 2 equal point loads applied.*
Right: *Cable with 2 unequal point loads applied*



2. Forces: Axial Tension

The tensile force that develops in the cable acts along the axis of the cable (N), which can be separated into a horizontal (H) and vertical (V) component (**Figure 4**). In the previous study, we illustrated how to find the vertical and horizontal forces at the tower due to main span and back span. Thus, we can use these values at the tower to find the axial force in the cable.

Figure 4
Slope of The Cable (α)



The axial force can be found by using trigonometric relationships of horizontal and vertical force components. First, the angle of the cable at a particular point is found by the following relationship:

Equation 1
Slope of cable

$$\tan(\alpha) = \frac{V}{H}$$

where V is the vertical force component and H is the horizontal force component acting on the cable. The resulting axial force in the cable, N (lbs or k), is determined with the formula:

Equation 2
Axial force in cable

$$N = H\cos(\alpha) + V\sin(\alpha)$$

where α is the slope of the cable. Alternatively, one can use Pythagorean theorem:

Equation 3
Axial force in cable (alternative)

$$N = \sqrt{H^2 + V^2}$$

Example 1

Axial Force in the Cables of the George Washington Bridge in the midspan

Determine:
The axial force in the cable of the George Washington Bridge at the towers on the main span side and at midspan (See **Figure 5**).

Given: (from previous calculations in ***Structural Studies: Suspension Bridges I***)
Vertical Reaction at the Tower, $V_M = 82{,}300\ \mathrm{k}$
Horizontal Reaction at the Tower, $H_M = 221{,}400\ \mathrm{k}$
Vertical Reaction at Midspan, $V = 0\ \mathrm{k}$
Horizontal Reaction at Midspan, $H = 221{,}400\ \mathrm{k}$

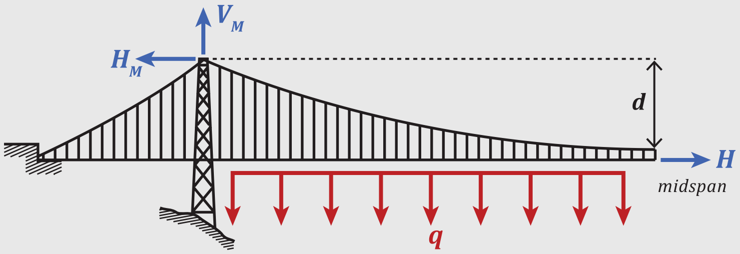


Figure 5
Main span free-body diagram of bridge from tower to midspan

Solution

Step 1: Calculate the slope of the cable at the tower and at midspan.

$$\tan(\alpha) = \frac{V}{H} \qquad \alpha = \tan^{-1}\left(\frac{V}{H}\right)$$

At the tower:

$$\alpha = \tan^{-1}\left(\frac{V}{H}\right)$$

At the midspan:

$$\alpha_{midspan} = \tan^{-1}\left(\frac{0\ \text{k}}{221{,}400\ \text{k}}\right) = 0°$$

The cable is horizontal at midspan, therefore it makes sense that $\alpha = 0$.

Step 2: Calculate the axial force of the cable at the tower and at midspan.

$$N = H\cos(\alpha) + V\sin(\alpha)$$

At the tower:

$$N_{tower\ main\ span} = 221{,}400\ \text{k} \cdot \cos(20°) + 82{,}300\ \text{k} \cdot \sin(20°)$$

$$N_{tower\ main\ span} \approx 236{,}200\ \text{k}$$

At the midspan:

$$N_{midspan} = 221{,}400\ \text{k} \cdot \cos(0°) + 0\ \text{k} \cdot \sin(0°)$$

$$N_{midspan} = 221{,}400\ \text{k}$$

3. Tensile Stress

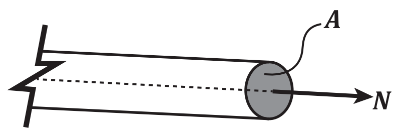
Recall from ***Structural Study: Basics of Structural Analysis*** that stress is computed by dividing the axial force by the cross-sectional area. For cables the axial loads are tensile. The resulting stresses are also tensile and we'll refer to them as ***tensile stresses***, f_t.

Equation 4
Tensile Stress

$$f_t = \frac{N}{A}$$

At the midspan of the bridge the cable is horizontal and therefore the tensile force, N, is in the horizontal direction. This tensile force is equal (in magnitude and direction) to the horizontal reaction, H, at the tower as seen in the previous example. We will use this tensile force to calculate the ***tensile stress***, f_t in the cable (**Figure 6**).

Figure 6
Cable Under Tension



Example 2

Stress in the Cables of the George Washington Bridge

Determine:
The stress in the cable of the George Washington Bridge ***at midspan*** and calculate its efficiency.

Given:
There are four cables each with an area, $A = 800 \text{ in}^2$
Recall the tensile force, $H = 221{,}400 \text{ k}$
Ammann used an allowable stress, $f_{allow} = 82 \text{ ksi}$ for steel cable.

Solution

Step 1: Calculate the total cross-sectional area of the cables

$$A = number\ of\ cables \cdot A_{one-cable}$$

$$A = 4 \cdot 800 \text{ in}^2 = 3{,}200 \text{ in}^2$$

Step 2: Calculate the tensile stress in each cable[3]:

$$f_t = \frac{N_{midspan}}{A} = \frac{H}{A} = \frac{221{,}400 \text{ k}}{3{,}200 \text{ in}^2} = 69.2 \text{ ksi}$$

Step 3: Calculate the efficiency of the cables [3]:

$$Efficiency = \frac{f_t}{f_{allow}} = \frac{69.2 \text{ ksi}}{82 \text{ ksi}} = 0.84$$

4. Conclusions

Notice from the previous example that the axial force at midspan is equal to the horizontal force in the towers. At midspan, the slope is 0°. As we move along the cable towards the tower support, the cable slope increases. As the cable slope increases, the horizontal components of force remain constant, but the vertical force components increases, thus increasing the overall axial force in the cable. The tensile force is therefore greatest at the tower supports where the slope is the greatest.

$H = qL^2/8d$ is the main equation for a suspension bridge. It represents the transformation of vertical forces into horizontal reactions and is dependent on the chosen form. The load applied to the bridge is defined by qL while the form of the bridge is defined by the ratio of the main span to the sag[4], L/d. While the equation permits you to calculate the tensile force in a cable given certain parameters span (L), sag (d), and the distributed load (q), the choice of these parameters, is ultimately made by the designer. These choices dictate how the forces are resisted in the structure and influence the aesthetic of the bridge.

Summary of Terms

Term	Description	Unit
A	cross sectional area	in^2
H	horizontal reaction at tower tensile force in cable	lbs or k
V	vertical reaction at tower	lbs or k
α	slope of the cable	degrees
N	axial force in the cable	lbs or k

Summary of Equations

Slope of Cable

$$\tan(\alpha) = \frac{V}{H}$$

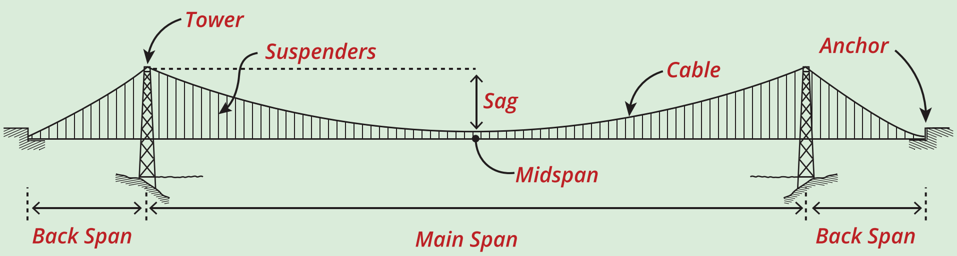
Axial Force in Cable

$$N = H\cos(\alpha) + V\sin(\alpha)$$

Tensile Stress

$$f_t = \frac{N}{A}$$

Visual Glossary



Notes

1 Image obtained from Flickr from author Kathleen Bence licensed under the Creative Commons Attribution-NonCommercial 2.0 Generic.

2 Parts of this Structural Study are based upon Structures and the Urban Environment, Structural Studies, by David P, Billington and Robert Mark, 1983, with contributions from Tracy Huynh.

3 Ammann calculated the maximum tensile force, N, in the cable (which occurs at the tower closer to the New York side) to be $261{,}000\ \mathrm{k}$ (which is similar to the value we would obtain if we used the back span values for V and N as approximated in the previous structural study). Using this force (rather than the force at midspan) and $f_{allow} = 82\ \mathrm{ksi}$ he calculated a required area of $3{,}190\ \mathrm{in}^2$ (rounded to $3{,}200\ \mathrm{in}^2$).

$$f_{allow} = \frac{T}{A}$$

therefore,

$$A = \frac{F}{f_{allow}} = \frac{261{,}000\ \mathrm{k}}{82\ \mathrm{ksi}} \approx 3{,}200\ \mathrm{in}^2$$

Since the area required comes directly from the allowable stress in the cables, $f_t = f_{allow}$ and the efficiency is 1.

4 The Innovators; The Engineering Pioneers who Made America Modern, by David P. Billington, John Wiley & Sons, Inc., New York, 1996, p. 7, 9 and 12.