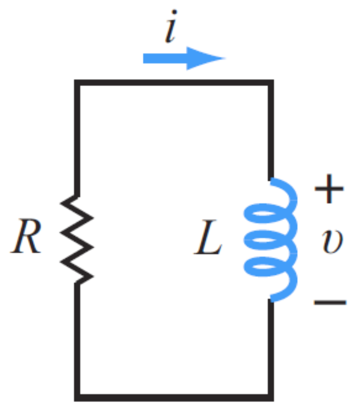
Natural Response of an RL Circuit

Consider the circuit below. Assume we know that the inductor, L, has an initial current i(0) through it. What is the current, i, through L, for t ≥ 0?



Applying KVL, we can write:

$$Ri + L\frac{di}{dt} = 0$$

We can clean this up a bit by dividing by L:

$$\frac{di}{dt} + ai = 0$$

Where:

$$a = \frac{R}{L}$$

This is a differential equation. Just as in the RC case, it turns out the solution to the differential equation in the blue box above is:

$$i(t) = i(0)\, e^{-t/\tau} \qquad (\text{for } t \geq 0),$$

where:

$$\tau = \frac{1}{a} = \frac{L}{R}.$$

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Once we know the current, i, we can also determine:

- the voltage across the inductor, v (since v = L*di/dt for an inductor)
- the power being absorbed or injected by the inductor (since P = i*v)
- the energy stored in the inductor at any time (since U = ∫P or ½*Li^2)

What does the *time constant*, τ, tell us?

Just as with the RC circuit, the magnitude of the time constant τ is a measure of how fast or how slowly a circuit responds to a sudden change.

- Notice that the units of τ are seconds (that is henrys / ohms = seconds).
- After 1 τ, the capacitor has discharged to 0.37 of the initial value.
- After about 5τ, v(t) has dropped to <1% of its original value. Engineers assume 5τ is long enough for particular RL circuit to charge or discharge to its final value.

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