Foundations of Computer Graphics

Online Lecture 9: Ray Tracing 1

History and Basic Ray Casting

Ravi Ramamoorthi

Effects needed for Realism

- (Soft) Shadows
- Reflections (Mirrors and Glossy)
- Transparency (Water, Glass)
- Interreflections (Color Bleeding)
- Complex Illumination (Natural, Area Light)
- Realistic Materials (Velvet, Paints, Glass)
- …

Ray Tracing

- Different Approach to Image Synthesis as compared to Hardware pipeline (OpenGL)
- Pixel by Pixel instead of Object by Object
- Easy to compute shadows/transparency/etc

Outline

- *History*
- Basic Ray Casting (instead of rasterization)
  - Comparison to hardware scan conversion
- Shadows / Reflections (core algorithm)
- Ray-Surface Intersection
- Optimizations

Ray Tracing: History

- Appel 68
- Whitted 80 [recursive ray tracing]
  - Landmark in computer graphics
- Lots of work on various geometric primitives
- Lots of work on accelerations
- Current Research
  - Real-Time raytracing (historically, slow technique)
  - Ray tracing architecture

Ray Tracing History

- "An improved illumination model for shaded display" by T. Whitted, CACM 1980
- 512x512, VAX 11/780
- 74 min, today real-time

Turner Whitted 1980.
Spheres and Checkerboard

Outline in Code

```
Image Raytrace (Camera cam, Scene scene, int width, int height)
{
    Image image = new Image (width, height) ;
    for (int i = 0 ; i < height ; i++)
        for (int j = 0 ; j < width ; j++) {
            Ray ray = RayThruPixel (cam, i, j) ;
            Intersection hit = Intersect (ray, scene) ;
            image[i][j] = FindColor (hit) ;
            }
    return image ;
}
```

Outline

- History
- *Basic Ray Casting (instead of rasterization)*
  - Comparison to hardware scan conversion
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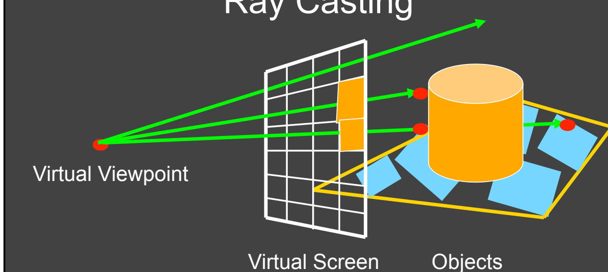
Ray Casting

Produce same images as with OpenGL
- Visibility per pixel instead of Z-buffer
- Find nearest object by shooting rays into scene
- Shade it as in standard OpenGL

Ray Casting



Comparison to hardware scan-line

- Per-pixel evaluation, per-pixel rays (not scan-convert each object). On face of it, costly
- But good for walkthroughs of extremely large models (amortize preprocessing, low complexity)
- More complex shading, lighting effects possible

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Core Algorithm: Shadows and Reflections

Ravi Ramamoorthi

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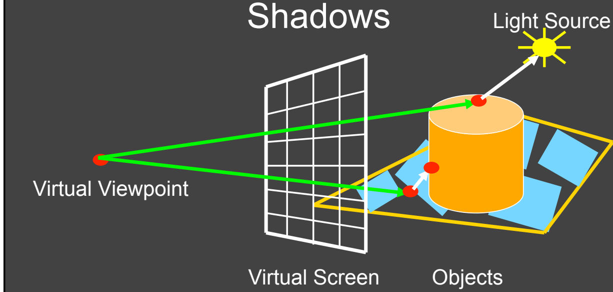
Shadows

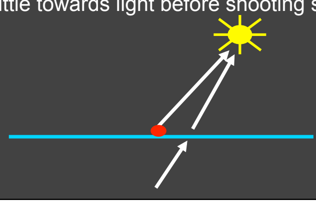


Shadow ray to light is blocked: object in shadow

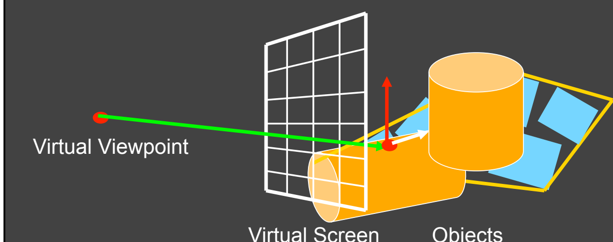
Shadow ray to light is unblocked: object visible

Shadows: Numerical Issues

- Numerical inaccuracy may cause intersection to be below surface (effect exaggerated in figure)
- Causing surface to incorrectly shadow itself
- Move a little towards light before shooting shadow ray



Mirror Reflections/Refractions



Generate reflected ray in mirror direction,
Get reflections and refractions of objects

Recursive Ray Tracing

For each pixel

- Trace Primary Eye Ray, find intersection
- Trace Secondary Shadow Ray(s) to all light(s)
  - Color = Visible ? Illumination Model : 0 ;
- Trace Reflected Ray
  - Color += reflectivity * Color of reflected ray

Problems with Recursion

- Reflection rays may be traced forever
- Generally, set maximum recursion depth
- Same for transmitted rays (take refraction into account)

Effects needed for Realism

- (Soft) Shadows
- Reflections (Mirrors and Glossy)
- Transparency (Water, Glass)
- Interreflections (Color Bleeding)
- Complex Illumination (Natural, Area Light)

Discussed in this lecture
Not discussed but possible with distribution ray tracing
Hard (but not impossible) with ray tracing; radiosity methods

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Ray-Surface Intersection

Ravi Ramamoorthi

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- *Ray-Surface Intersection*
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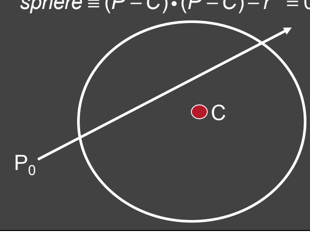
Ray/Object Intersections

- Heart of Ray Tracer
  - One of the main initial research areas
  - Optimized routines for wide variety of primitives
- Various types of info
  - Shadow rays: Intersection/No Intersection
  - Primary rays: Point of intersection, material, normals
  - Texture coordinates
- Work out examples
  - Triangle, sphere, polygon, general implicit surface

Ray-Sphere Intersection

$$ray \equiv \vec{P} = \vec{P}_0 + \vec{P}_1 t$$

$$sphere \equiv (\vec{P} - \vec{C}) \bullet (\vec{P} - \vec{C}) - r^2 = 0$$



Ray-Sphere Intersection

$$ray \equiv \vec{P} = \vec{P}_0 + \vec{P}_1 t$$

$$sphere \equiv (\vec{P} - \vec{C}) \bullet (\vec{P} - \vec{C}) - r^2 = 0$$

Substitute

Ray-Sphere Intersection

$$ray \equiv \vec{P} = \vec{P}_0 + \vec{P}_1 t$$
$$sphere \equiv (\vec{P} - \vec{C}) \cdot (\vec{P} - \vec{C}) - r^2 = 0$$

Substitute

$$ray \equiv \vec{P} = \vec{P}_0 + \vec{P}_1 t$$
$$sphere \equiv (\vec{P}_0 + \vec{P}_1 t - \vec{C}) \cdot (\vec{P}_0 + \vec{P}_1 t - \vec{C}) - r^2 = 0$$

Simplify

Ray-Sphere Intersection

$$ray \equiv \vec{P} = \vec{P}_0 + \vec{P}_1 t$$
$$sphere \equiv (\vec{P} - \vec{C}) \cdot (\vec{P} - \vec{C}) - r^2 = 0$$

Substitute

$$ray \equiv \vec{P} = \vec{P}_0 + \vec{P}_1 t$$
$$sphere \equiv (\vec{P}_0 + \vec{P}_1 t - \vec{C}) \cdot (\vec{P}_0 + \vec{P}_1 t - \vec{C}) - r^2 = 0$$

Simplify

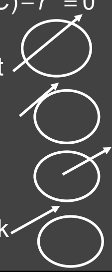
$$t^2(\vec{P}_1 \cdot \vec{P}_1) + 2t\, \vec{P}_1 \cdot (\vec{P}_0 - \vec{C}) + (\vec{P}_0 - \vec{C}) \cdot (\vec{P}_0 - \vec{C}) - r^2 = 0$$

Ray-Sphere Intersection

$$t^2(\vec{P}_1 \cdot \vec{P}_1) + 2t\, \vec{P}_1 \cdot (\vec{P}_0 - \vec{C}) + (\vec{P}_0 - \vec{C}) \cdot (\vec{P}_0 - \vec{C}) - r^2 = 0$$

Solve quadratic equations for t

- 2 real positive roots: pick smaller root
- Both roots same: tangent to sphere
- One positive, one negative root: ray origin inside sphere (pick + root)
- Complex roots: no intersection (check discriminant of equation first)



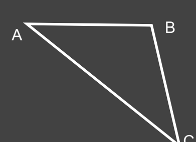
Ray-Sphere Intersection

- Intersection point: $ray \equiv \vec{P} = \vec{P}_0 + \vec{P}_1 t$
- Normal (for sphere, this is same as coordinates in sphere frame of reference, useful other tasks)

$$normal = \frac{\vec{P} - \vec{C}}{\left|\vec{P} - \vec{C}\right|}$$

Ray-Triangle Intersection

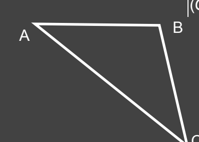
- One approach: Ray-Plane intersection, then check if inside triangle
- Plane equation:



Ray-Triangle Intersection

- One approach: Ray-Plane intersection, then check if inside triangle
- Plane equation:

$$n = \frac{(C - A) \times (B - A)}{\left|(C - A) \times (B - A)\right|}$$

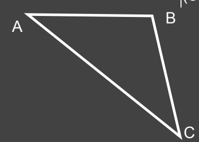


Ray-Triangle Intersection

- One approach: Ray-Plane intersection, then check if inside triangle
- Plane equation:

$$plane \equiv \vec{P} \bullet \vec{n} - \vec{A} \bullet \vec{n} = 0$$

$$n = \frac{(C-A)\times(B-A)}{\left|(C-A)\times(B-A)\right|}$$



Ray-Triangle Intersection

- One approach: Ray-Plane intersection, then check if inside triangle
- Plane equation:

$$plane \equiv \vec{P} \bullet \vec{n} - \vec{A} \bullet \vec{n} = 0$$

$$n = \frac{(C-A)\times(B-A)}{\left|(C-A)\times(B-A)\right|}$$

- Combine with ray equation

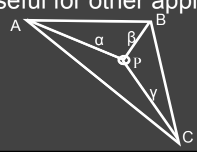
$$ray \equiv \vec{P} = \vec{P}_0 + \vec{P}_1 t$$

$$(\vec{P}_0 + \vec{P}_1 t) \bullet \vec{n} = \vec{A} \bullet \vec{n}$$

$$t = \frac{\vec{A} \bullet \vec{n} - \vec{P}_0 \bullet \vec{n}}{\vec{P}_1 \bullet \vec{n}}$$

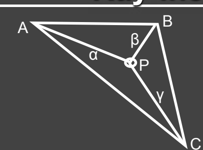
Ray inside Triangle

- Once intersect with plane, need to find if in triangle
- Many possibilities for triangles, general polygons
- We find parametrically [barycentric coordinates]. Also useful for other applications (texture mapping)



$$P = \alpha A + \beta B + \gamma C$$
$$\alpha \geq 0, \beta \geq 0, \gamma \geq 0$$
$$\alpha + \beta + \gamma = 1$$

Ray inside Triangle



$$P = \alpha A + \beta B + \gamma C$$
$$\alpha \geq 0, \beta \geq 0, \gamma \geq 0$$
$$\alpha + \beta + \gamma = 1$$

$$P - A = \beta(B-A) + \gamma(C-A)$$
$$0 \leq \beta \leq 1 \;,\; 0 \leq \gamma \leq 1$$
$$\beta + \gamma \leq 1$$

Other primitives

- Much early work in ray tracing focused on ray-primitive intersection tests
- Cones, cylinders, ellipsoids
- Boxes (especially useful for bounding boxes)
- General planar polygons
- Many more

Ray-Tracing Transformed Objects

We have an optimized ray-sphere test

- But we want to ray trace an ellipsoid…

Solution: Ellipsoid transforms sphere

- Apply inverse transform to ray, use ray-sphere
- Allows for instancing (traffic jam of cars)

Mathematical details worked out next

Transformed Objects

Transformed Objects

- Consider a general 4x4 transform M (matrix stacks)
- Apply inverse transform M^{-1} to ray
  - Locations stored and transform in homogeneous coordinates
  - Vectors (ray directions) have homogeneous coordinate set to 0 [so there is no action because of translations]
- Do standard ray-surface intersection as modified
- Transform intersection back to actual coordinates
  - Intersection point p transforms as Mp
  - Normals n transform as $M^{-t}n$. Do all this before lighting

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Optimizations

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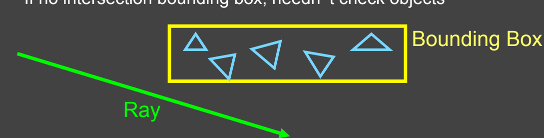
- *Optimizations*
- Current Research

Acceleration

Testing each object for each ray is slow
- Fewer Rays
  Adaptive sampling, depth control
- Generalized Rays
  Beam tracing, cone tracing, pencil tracing etc.
- Faster Intersections (more on this later)
  - Optimized Ray-Object Intersections
  - ***Fewer Intersections***

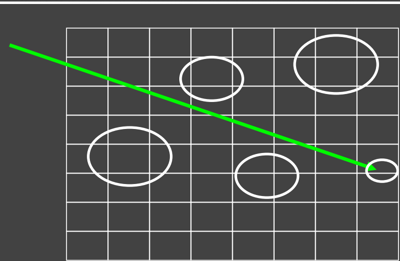
Acceleration Structures

Bounding boxes (possibly hierarchical)
If no intersection bounding box, needn't check objects



Spatial Hierarchies (Oct-trees, kd trees, BSP trees)

Acceleration Structures: Grids



Acceleration and Regular Grids

- Simplest acceleration, for example 5x5x5 grid
- For each grid cell, store overlapping triangles
- March ray along grid (need to be careful with this), test against each triangle in grid cell
- More sophisticated: kd-tree, oct-tree bsp-tree
- Or use (hierarchical) bounding boxes