Nodal Analysis

You may wonder if there exist standardized ways to solve an arbitrary linear circuit (beyond adhoc application of KVL, KCL and device rules like Ohm's Law). There are! One such method is **Nodal Analysis**.

The basic idea behind this (and other) methods is to use an algorithm to reliably arrive at a set of equations which can be solved using any method to arrive at the voltages and currents that define the behavior of the circuit. You'll need to know some algebra to follow the method below.

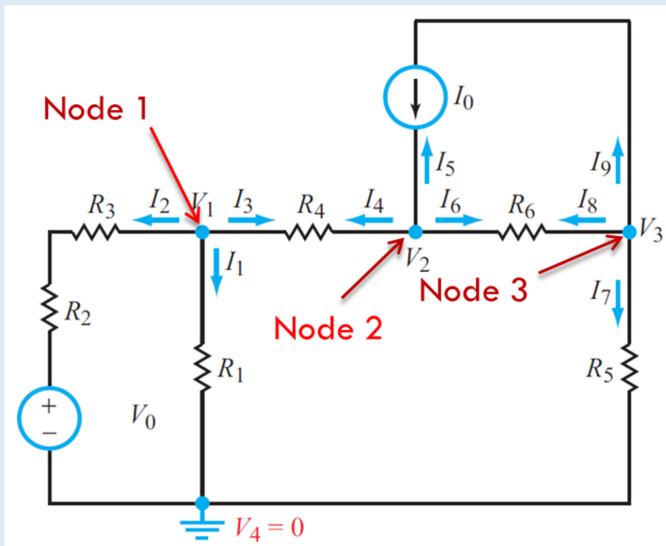
Here is the algorithm for Nodal Analysis:

- **Step 1:** Identify all nodes with three or more branches (we will call these *extraordinary nodes*), select one of them as a reference node (call it ground), and then assign node voltage labels to the remaining extraordinary nodes. You now have (n – 1) unknown voltages (since one of the extraordinary nodes is grounded and its voltage is 0).
- **Step 2:** At each of the non-ground extraordinary nodes, apply KCL. This will yield (n – 1) equations.
- **Step 3:** Solve the (n − 1) independent simultaneous equations to determine the unknown node voltages.

Example:

Step 1:

- The circuit at right has 4 extraordinary nodes.
- We label the bottommost node, V_4, ground. This defines $V_4 = 0$.
- The other three nodes are labeled V_1, V_2 and V_3. These variables represent three unknown voltages. Note that voltages are defined across two points, so these are the voltages of the three labeled nodes *with respect to ground*.



Step 2:

- Write a KCL equation for each node. These will initially be in terms of currents, but we will replace these currents with voltages using Ohm's Law.

Node 1:

$$I_1 + I_2 + I_3 = 0$$

Now apply Ohm's Law to replace each of the I's:

$$\frac{V_1 - 0}{R_1} + \frac{V_1 - V_0}{R_2 + R_3} + \frac{V_1 - V_2}{R_4} = 0$$

Node 2:

$$I_4 + I_5 + I_6 = 0$$

$$\frac{V_2 - V_1}{R_4} - I_0 + \frac{V_2 - V_3}{R_6} = 0$$

Node 3:

$$I_7 + I_8 + I_9 = 0$$

$$\frac{V_3 - 0}{R_5} + \frac{V_3 - V_2}{R_6} + I_0 = 0$$