ITMO UNIVERSITY

How to Win Coding Competitions: Secrets of Champions

Week 3: Sorting and Search Algorithms
Lecture 1: Introduction to Sorting

Maxim Buzdalov
Saint Petersburg 2016

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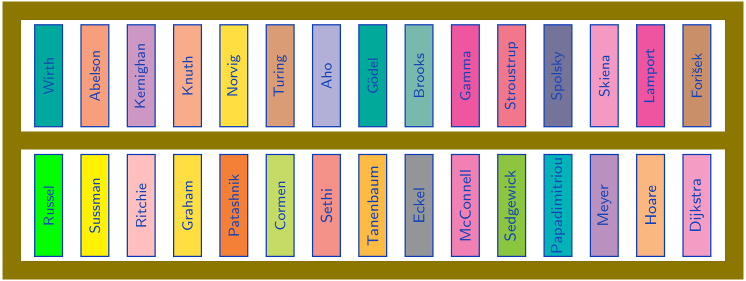

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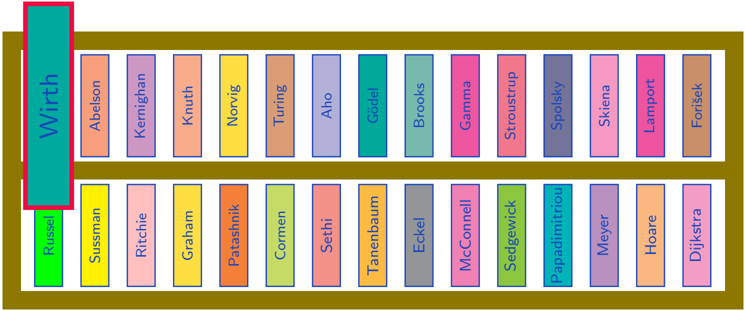

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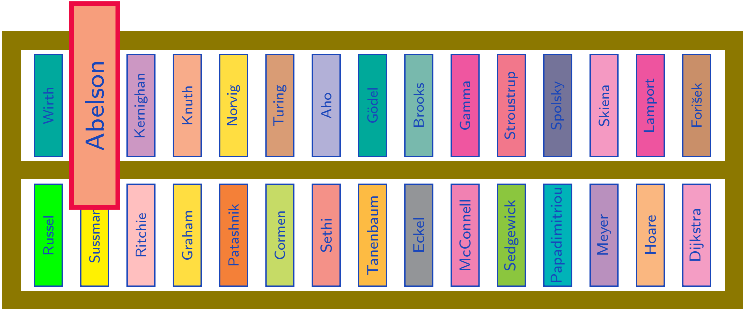

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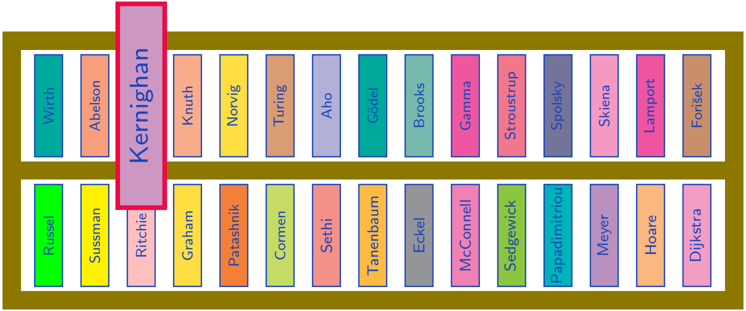

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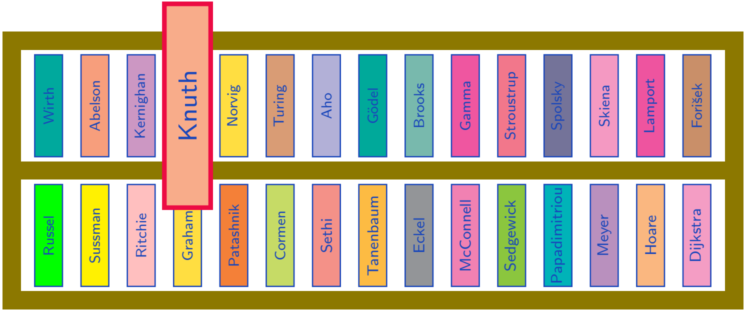

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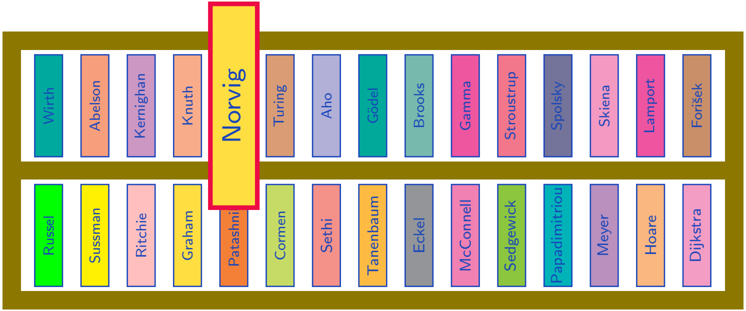

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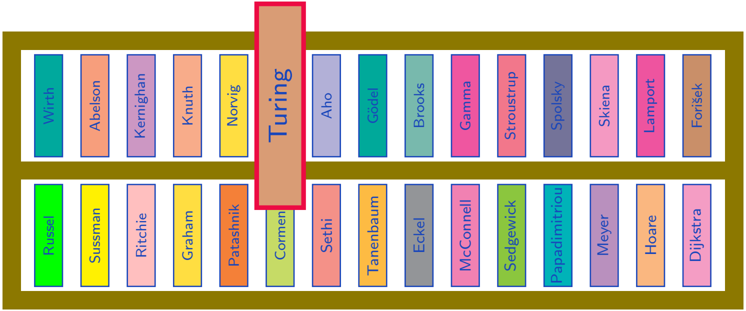

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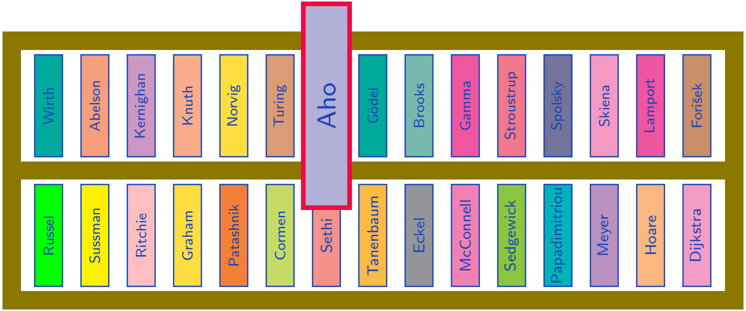

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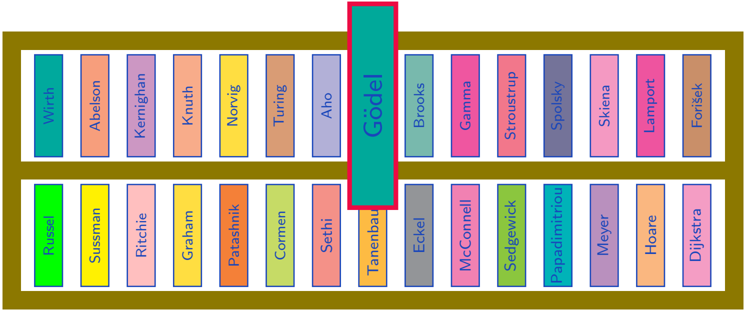

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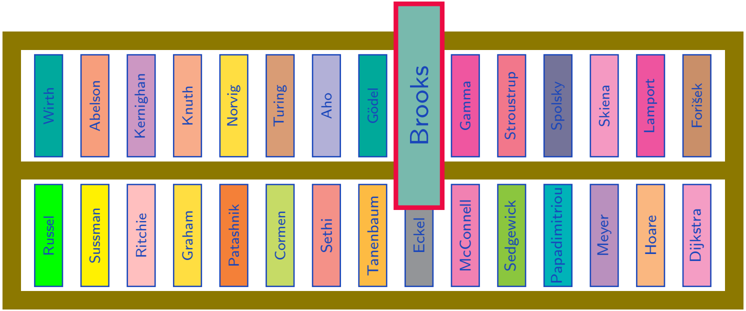

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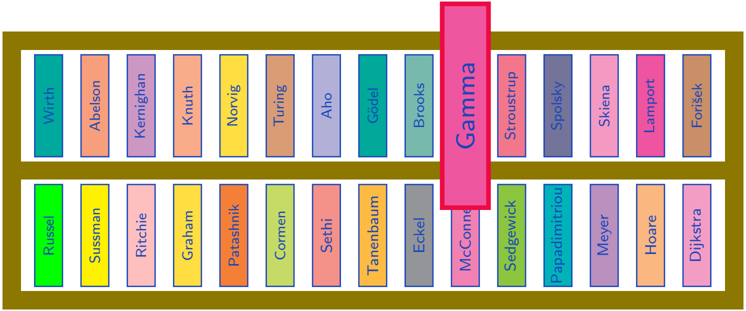

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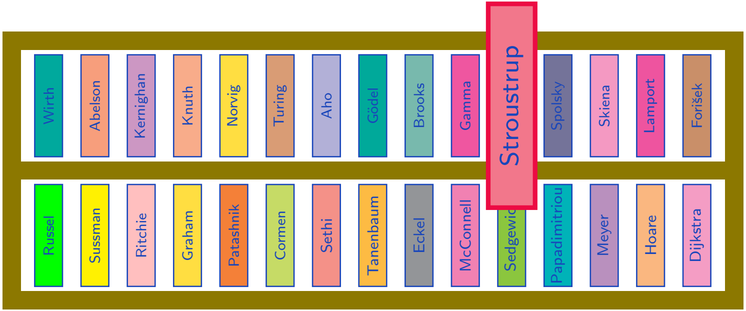

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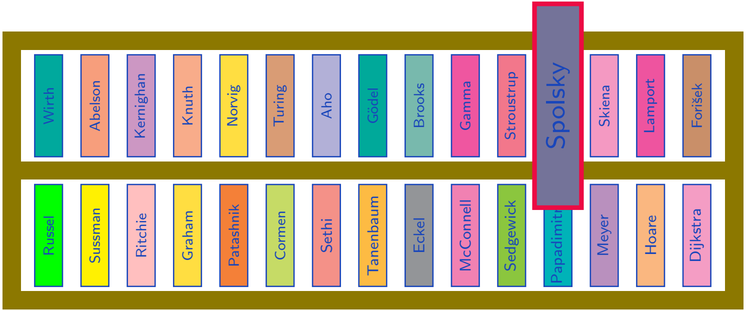

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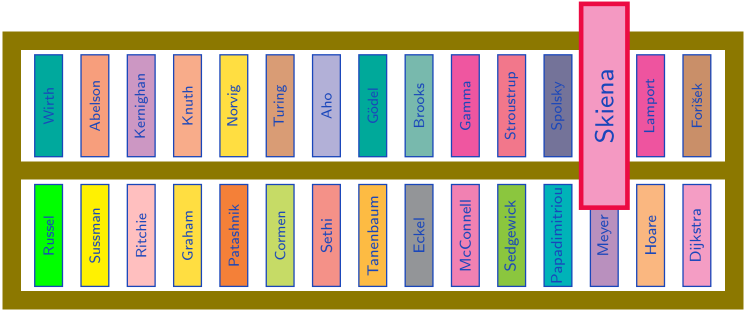

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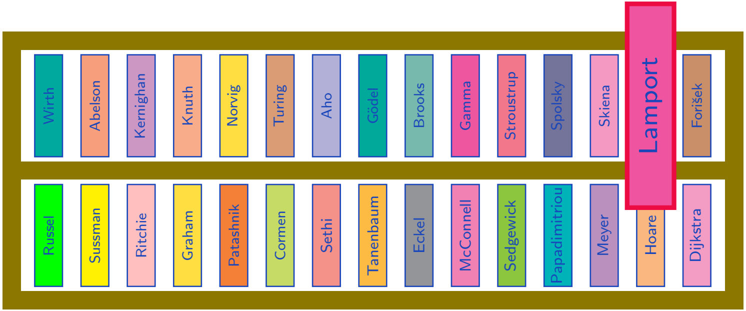

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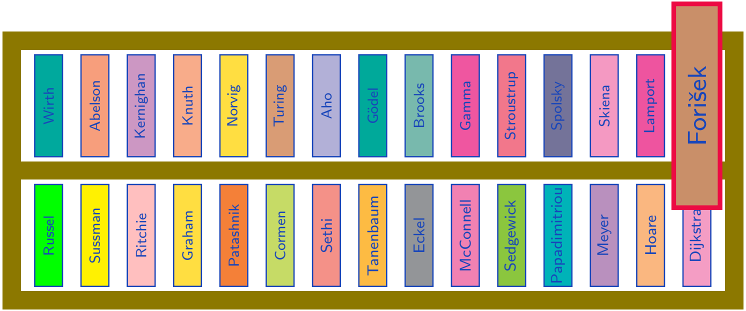

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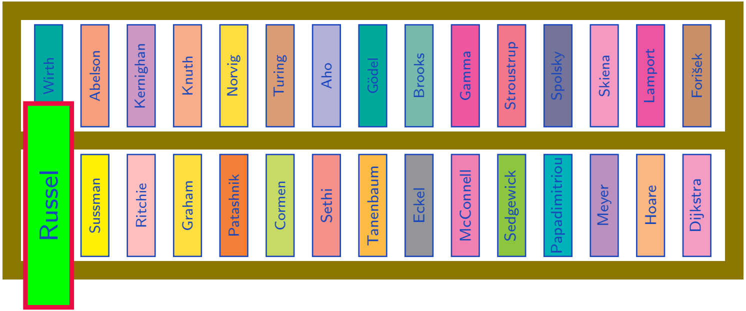

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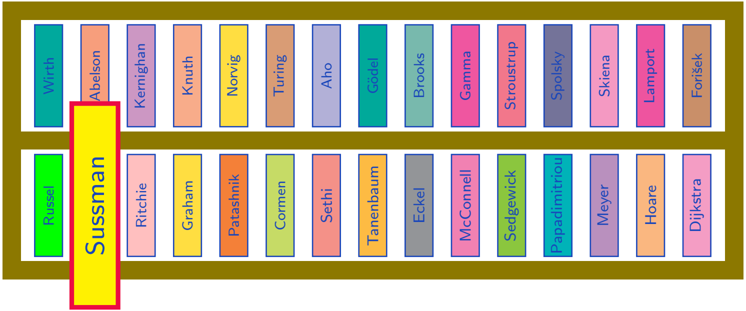

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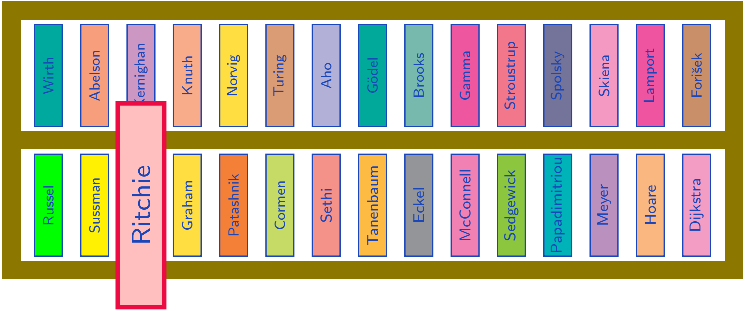

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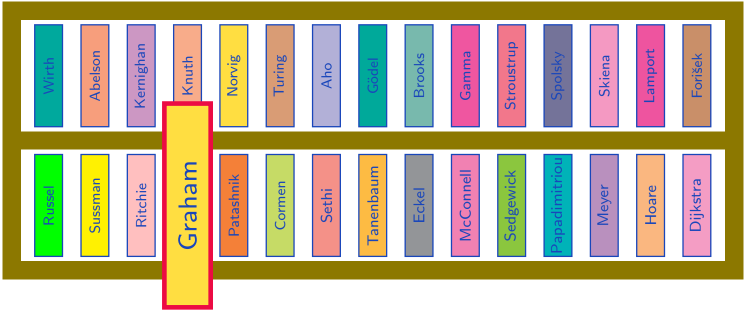

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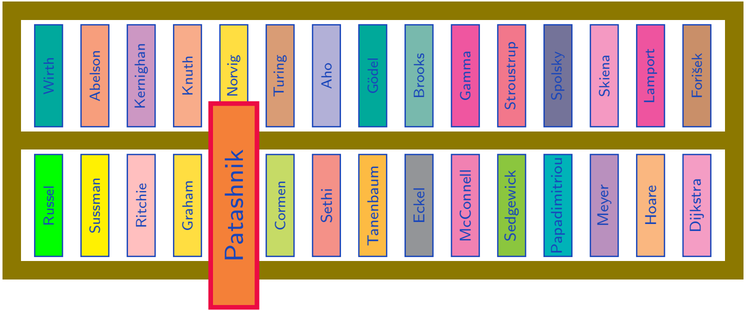

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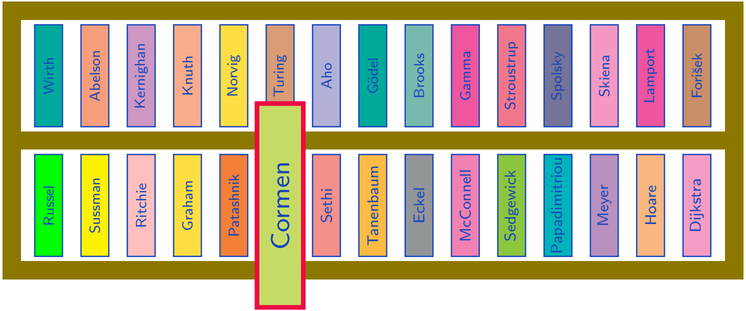

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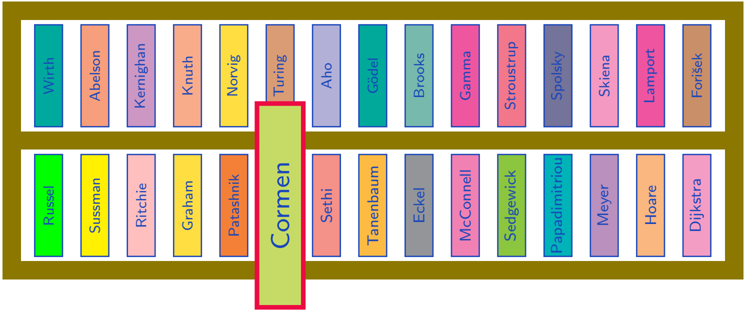
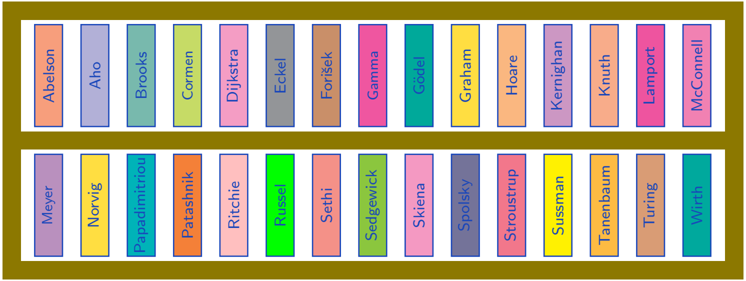


Too slow to be practical :/

Find Cormen when books are in alphabetical order



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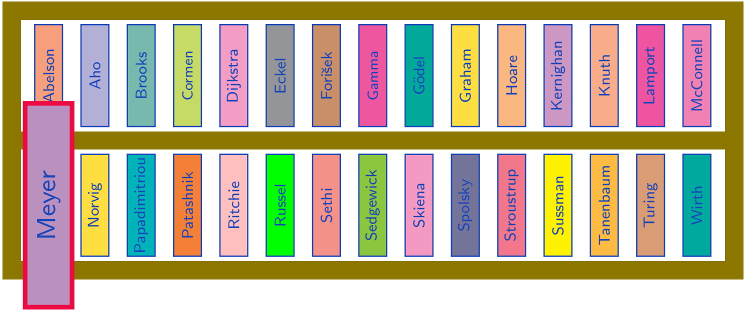

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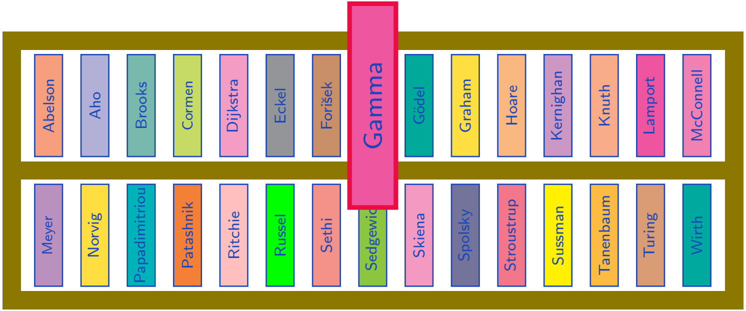

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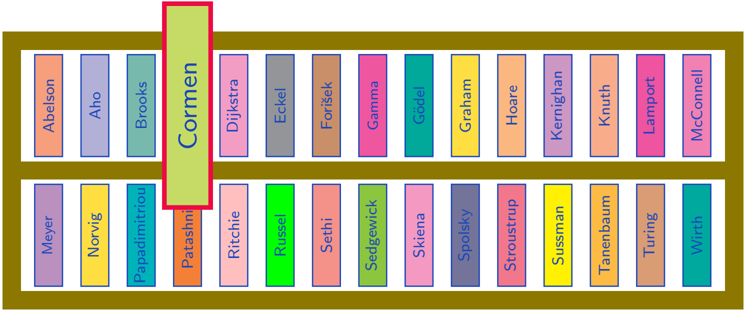

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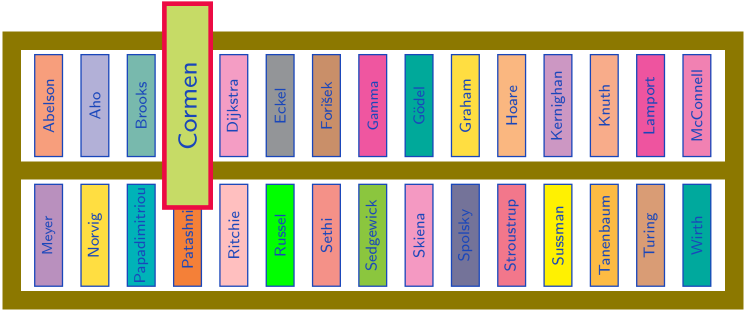

Much faster if sorted! :)

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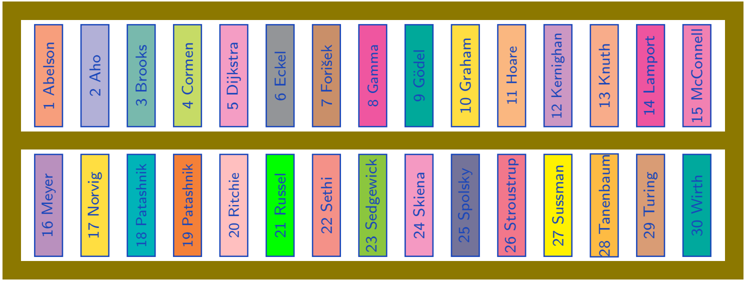
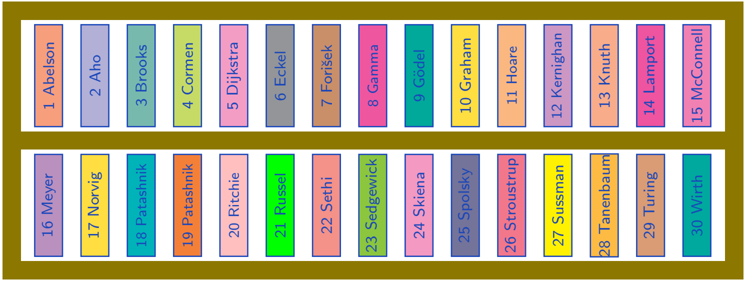
  - For numbers $(\mathbb{S} = \mathbb{Z}, \mathbb{R}, \ldots)$, $\preceq$ is often $\leq$
  - We will denote $\preceq$ as $\leq$, and $\prec$ as $<$, in the rest of the week materials

Why do we need sorting? – Prepare to fast query answering



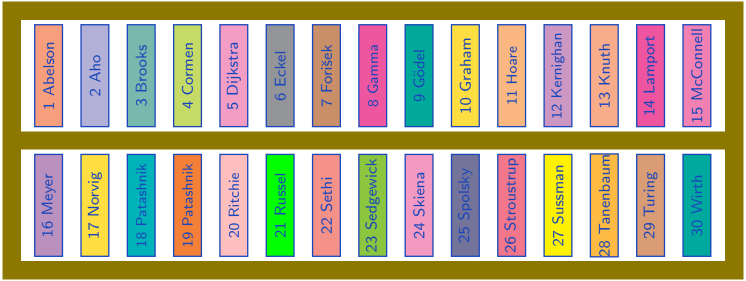
Why do we need sorting? – Prepare to fast query answering

1. Does this book exist?



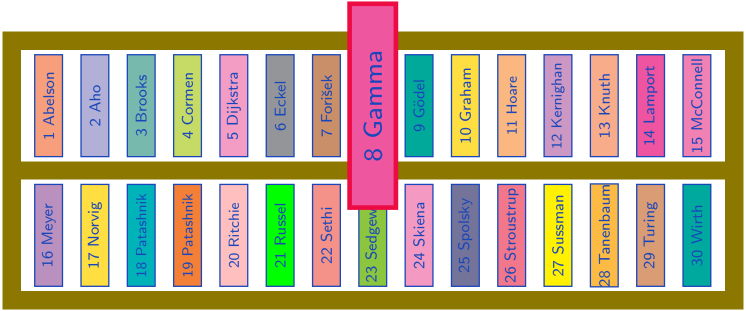
Why do we need sorting? – Prepare to fast query answering

1. Does this book exist? Gamma



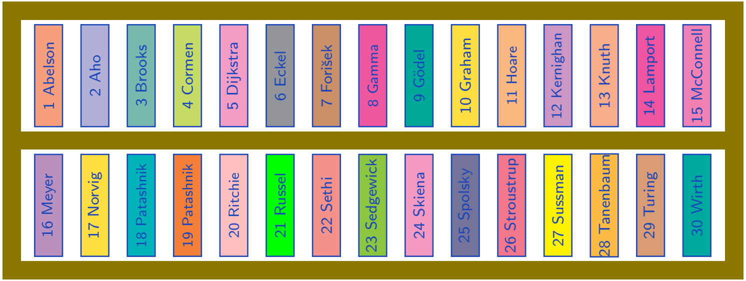
Why do we need sorting? – Prepare to fast query answering

1. Does this book exist? Gamma → YES



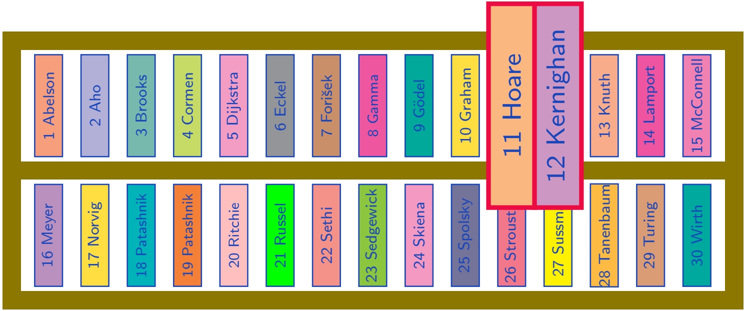
Why do we need sorting? – Prepare to fast query answering

1. Does this book exist? Kant



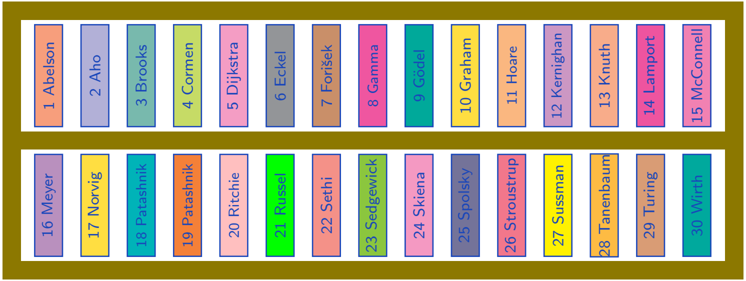
Why do we need sorting? – Prepare to fast query answering

1. Does this book exist? Kant → NO



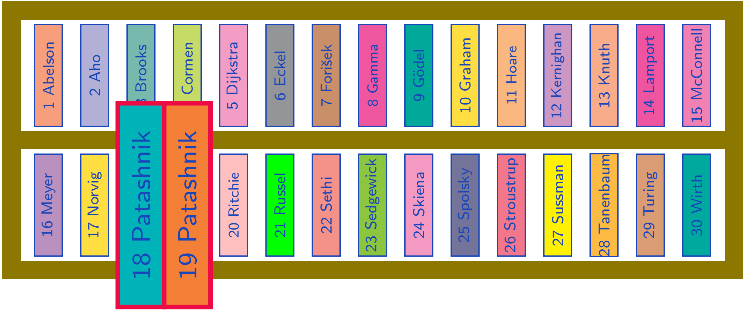
Why do we need sorting? – Prepare to fast query answering

2. How many books of Patashnik is there?



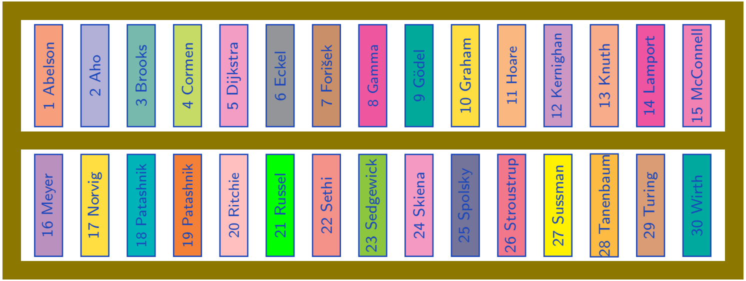
Why do we need sorting? – Prepare to fast query answering

2. How many books of Patashnik is there? Two



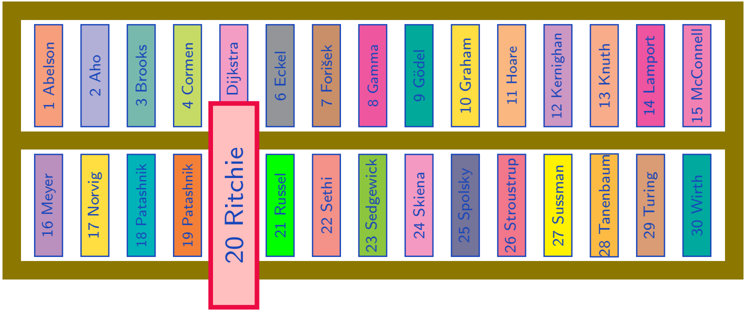
Why do we need sorting? – Prepare to fast query answering

3. How many books have a name smaller than Ritchie?



Why do we need sorting? – Prepare to fast query answering

3. How many books have a name smaller than Ritchie? 19 (the index minus 1)



Why do we need sorting? – Prepare to fast data processing

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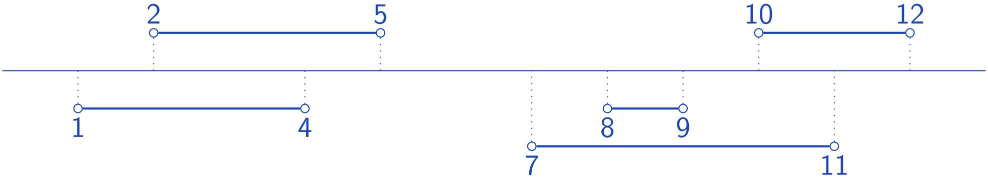
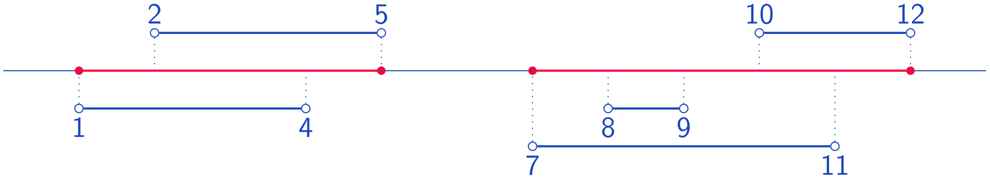
Given segment endpoints where paint was applied.



Why do we need sorting? – Prepare to fast data processing

Given segment endpoints where paint was applied. Find which length was painted



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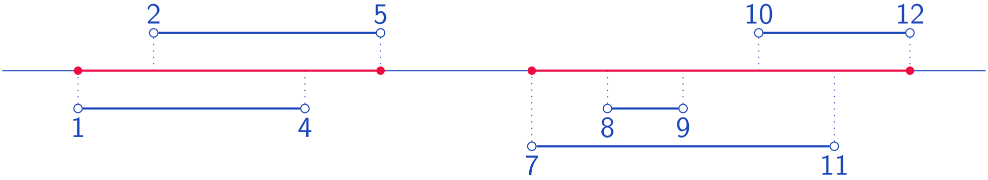

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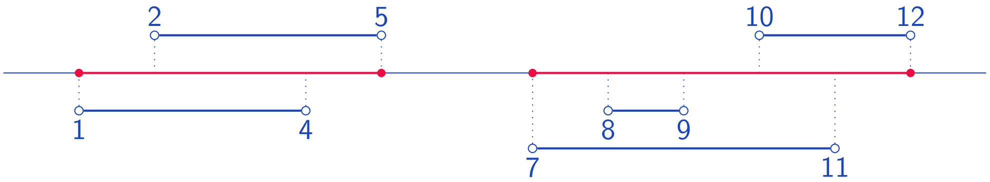

Solution:

- Sort segments by the left coordinate

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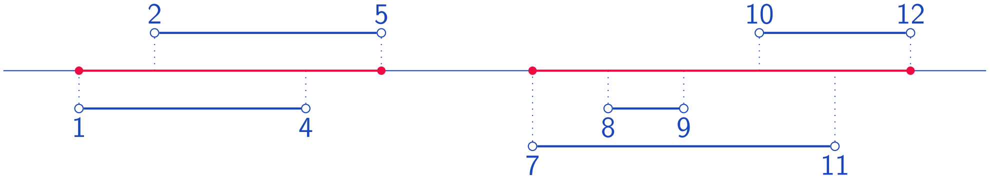

Solution:

- Sort segments by the left coordinate
- Traverse segments in the sorted order

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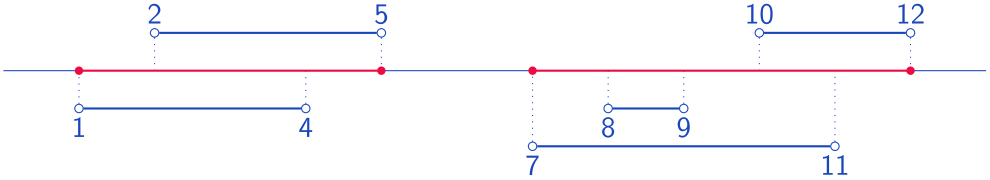

Solution:

- Sort segments by the left coordinate
- Traverse segments in the sorted order
  - Track the endpoint of the current segment cluster
  - Finish the cluster when the next segment is beyond this point

Why do we need sorting? – Prepare to fast data processing

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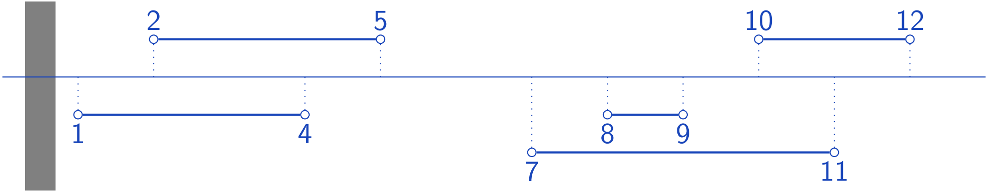

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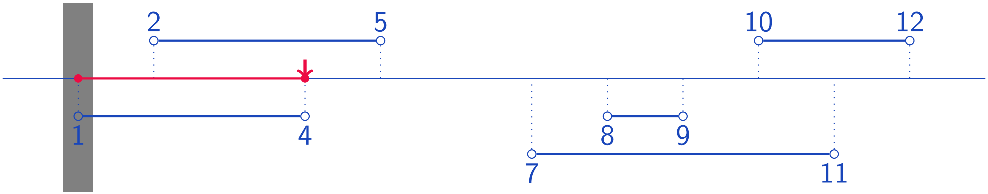

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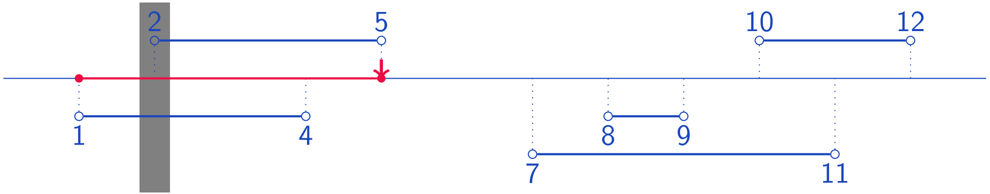

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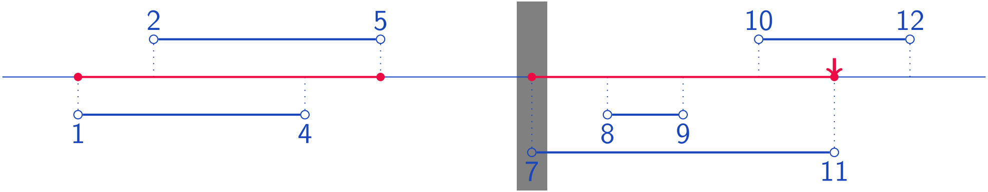
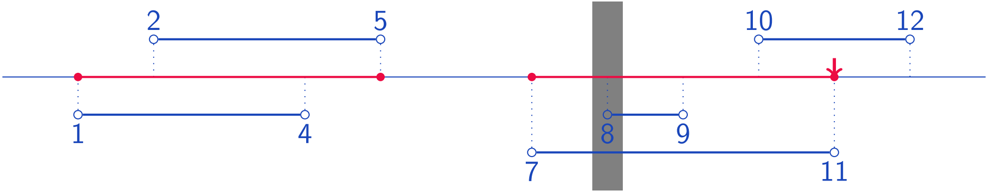


Solution:

- Sort segments by the left coordinate
- Traverse segments in the sorted order
  - Track the endpoint of the current segment cluster
  - Finish the cluster when the next segment is beyond this point

Why do we need sorting? – Prepare to fast data processing

Given segment endpoints where paint was applied. Find which length was painted



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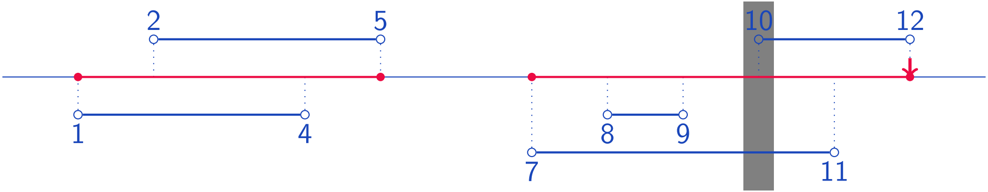

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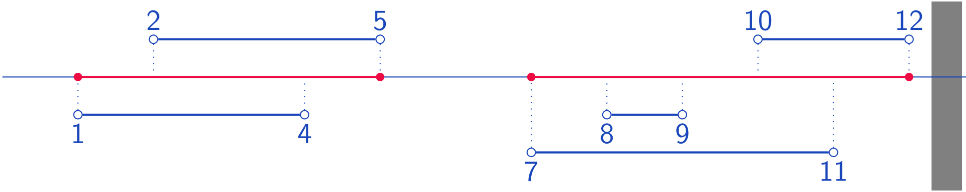

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  - Track the endpoint of the current segment cluster
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Why do we need sorting? – Construct an optimal solution

7	1	4	6	8	2	9	3	1	4	3	5	9	8	2
3	1	2	8	6	5	4	7	4	9	4	5	1	3	8

Why do we need sorting? – Construct an optimal solution

Example:

- Given two sequences $A = [A_1, \dots A_N]$ and $B = [B_1, \dots B_N]$
- Find permutations P and Q such that $\sum_{i=1}^{N} A_{P_i} \cdot B_{Q_i}$ is maximum possible

7	1	4	6	8	2	9	3	1	4	3	5	9	8	2
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Solution:

- Sort A in non-decreasing order

1	1	2	2	3	3	4	4	5	6	7	8	8	9	9
3	1	2	8	6	5	4	7	4	9	4	5	1	3	8

Why do we need sorting? – Construct an optimal solution

Example:

- Given two sequences $A = [A_1, \dots A_N]$ and $B = [B_1, \dots B_N]$
- Find permutations P and Q such that $\sum_{i=1}^{N} A_{P_i} \cdot B_{Q_i}$ is maximum possible

Solution:

- Sort A in non-decreasing order
- Sort B in non-decreasing order

1	1	2	2	3	3	4	4	5	6	7	8	8	9	9
1	1	2	3	3	4	4	4	5	5	6	7	8	8	9

Why do we need sorting? – Construct an optimal solution

Example:

- Given two sequences $A = [A_1, \ldots A_N]$ and $B = [B_1, \ldots B_N]$
- Find permutations P and Q such that $\sum_{i=1}^{N} A_{P_i} \cdot B_{Q_i}$ is minimum possible

Solution:

- Sort A in non-decreasing order
- Sort B in non-increasing order

1	1	2	2	3	3	4	4	5	6	7	8	8	9	9
9	8	8	7	6	5	5	4	4	4	3	3	2	1	1

Why do we need sorting? – Construct an optimal solution

Example:

- Given two sequences $A = [A_1, \dots A_N]$ and $B = [B_1, \dots B_N]$
- Find permutations P and Q such that $\sum_{i=1}^{N} A_{P_i} \cdot B_{Q_i}$ is minimum possible

Solution:

- Sort A in non-decreasing order
- Sort B in non-increasing order
- These facts will be proven in a this week's video

1	1	2	2	3	3	4	4	5	6	7	8	8	9	9
9	8	8	7	6	5	5	4	4	4	3	3	2	1	1

- Insertion sort: a simple sorting algorithm
- If a sorted sequence yields an optimal result, how to prove it?
- Quick sort: a very fast algorithm
- Merge sort: can never be too slow
- Stable and unstable sorting algorithms
- How to compare various objects
- An $\Omega(n \log n)$ bound on the complexity of comparison-based algorithms
- Bucket sort and radix sort: Linear non-comparison sorting algorithms
- Sorting algorithms in standard libraries