

Formula list

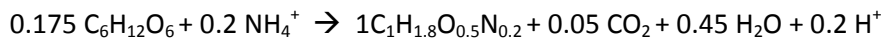
Week 2

2.2 Element compositions of biomass

1 Cmol cell dry mass:

- 4 elements: $C_1H_{1.8}O_{0.5}N_{0.2}$
- 8 elements: $C_1H_{1.8}O_{0.5}N_{0.2}P_{0.03}S_{0.02}K_{0.008}Mg_{0.01}$

Anabolic reaction for 1 mol biomass:



2.3 Total rates R_i : Process performance from broth balances

Balance compound i (non-volatile):

$$\frac{d(V_L c_i)}{dt} [mol\ i / h] = R_i [mol\ i / h] + F_{in} c_{i,in} [mol\ i / h] - F_{out} c_{i,out} [mol\ i / h]$$

Biomass production rate in steady state (mol X/h):

$$\frac{d(V_L c_X)}{dt} = R_X + F_{in} c_{X,in} - F_{out} c_{X,out} = 0 \rightarrow R_X = F_{out} c_{X,out} - F_{in} c_{X,in}$$

Substrate (non-volatile) consumption rate in steady state (mol S/h):

$$\frac{d(V_L c_s)}{dt} = R_s + F_{in} c_{s,in} - F_{out} c_{s,out} = 0 \rightarrow R_s = F_{out} c_{s,out} - F_{in} c_{s,in} \quad (\text{note: } R_s < 0)$$

Product (non-volatile) production rate in steady state (mol P/h):

$$\frac{d(V_L c_p)}{dt} = R_p + F_{in} c_{p,in} - F_{out} c_{p,out} = 0 \rightarrow R_p = F_{out} c_{p,out} - F_{in} c_{p,in}$$

2.4 Total rates R_i : process performance from broth and gas phase balances

Broth oxygen balance (mol O_2 /h):

$$\frac{d(V_L c_o)}{dt} = R_o + F_{in} c_{o,in} - F_{out} c_{o,out} + T_{N,o}$$

Gas phase oxygen balance (mol O_2 /h):

$$\frac{d(N_G y_o)}{dt} = 0 + F_{N,in} y_{o,in} - F_{N,out} y_{o,out} - T_{N,o} \rightarrow T_{N,o} = F_{N,in} y_{o,in} - F_{N,out} y_{o,out}$$

Combined O_2 -balance in steady state (mol O_2 /h):

$$\text{Broth: } \frac{d(V_L c_o)}{dt} = R_o + F_{in} c_{o,in} - F_{out} c_{o,out} + T_{N,o} = 0$$

$$\text{Gas: } \frac{d(N_G y_o)}{dt} = F_{N,in} y_{o,in} - F_{N,out} y_{o,out} + T_{N,o} = 0 \rightarrow T_{N,o} = F_{N,in} y_{o,in} - F_{N,out} y_{o,out}$$

Combined:

$$-R_o = \underbrace{F_{in} c_{o,in}}_{\text{Small}} - \underbrace{F_{out} c_{o,out}}_{\text{Small}} + \underbrace{F_{N,in} y_{o,in}}_{\text{Large}} - \underbrace{F_{N,out} y_{o,out}}_{\text{Large}}$$

Combined (broth+gas) CO₂-balance in steady state (mol CO₂/h):

$$R_c = \underbrace{-F_{in} c_{c,in}}_{\text{Small}} + \underbrace{F_{out} c_{c,out}}_{\text{Small}} - \underbrace{F_{N,in} y_{c,in}}_{\text{Small}} + \underbrace{F_{N,out} y_{c,out}}_{\text{Large}}$$

Calculations F_{N,out} by using N₂ gas balance and F_{N,in}:

$$F_{N,in} (1 - y_{o,in} - y_{c,in}) = F_{N,out} (1 - y_{o,out} - y_{c,out})$$

2.5

Biomass specific rates = q-rates, quantify performance of organism:

$$q - \text{rate definition : } q_i = \frac{R_i}{V_L c_X} \left[\frac{\text{mol } i / \text{h}}{\text{mol } X \text{ present in fermenter}} \right]$$

$$q_s = \frac{R_s}{V_L c_X} \left[\frac{\text{mol } s / \text{h}}{\text{mol } X \text{ present in fermenter}} \right]$$

$$\mu = \frac{R_X}{V_L c_X} \left[\frac{\text{mol } X / \text{h}}{\text{mol } X \text{ present in fermenter}} \right]$$

$$q_p = \frac{R_p}{V_L c_X} \left[\frac{\text{mol } p / \text{h}}{\text{mol } X \text{ present in fermenter}} \right]$$

$$q_{CO_2} = \frac{R_{CO_2}}{V_L c_X} \left[\frac{\text{mol } CO_2 / \text{h}}{\text{mol } X \text{ present in fermenter}} \right]$$

$$q_{O_2} = \frac{R_{O_2}}{V_L c_X} \left[\frac{\text{mol } O_2 / \text{h}}{\text{mol } X \text{ present in fermenter}} \right]$$

Product yield $\left(\frac{\text{Mol } P \text{ produced}}{\text{Mol } S \text{ consumed}} \right)$ **must be calculated from rates, not concentrations:**

$$\left| \frac{\text{mol } p}{\text{mol } s} \right| = \left| \frac{R_p}{R_s} \right| = \left| \frac{q_p}{q_s} \right| \neq \left| \frac{(c_{p,out} - c_{p,in})}{(c_{s,out} - c_{s,in})} \right|$$

Chemostat in steady state gives q-rates:

q-rates must be calculated from their balances and measurements.

μ from biomass balance (mol X/h):

$$\frac{d(V_L c_X)}{dt} = \mu V_L c_X + F_{in} c_{X,in} - F_{out} c_{X,out} = 0$$
$$\mu_{ss} = \frac{F_{out} c_{X,out} - F_{in} c_{X,in}}{V_L c_X} \left[\frac{\text{mol X produced / h}}{\text{mol X present in fermenter}} \right]$$

Only when $c_{X,in}=0$, $c_{X,out}=c_X$, $\mu_{ss}=F_{out}/V_L$

q_S from substrate balance, non-volatile substrate (mol S/h):

$$\frac{d(V_L c_S)}{dt} = q_S V_L c_X + F_{in} c_{S,in} - F_{out} c_{S,out} = 0$$
$$q_{S,ss} = \frac{F_{out} c_{S,out} - F_{in} c_{S,in}}{V_L c_X} \left[\frac{\text{mol S consumed / h}}{\text{mol X present in fermenter}} \right] \text{ note: } q_{S,ss} < 0$$

q_P from product balance, non-volatile product (mol P/h):

$$\frac{d(V_L c_P)}{dt} = q_P V_L c_X + F_{in} c_{P,in} - F_{out} c_{P,out} = 0$$
$$q_{P,ss} = \frac{F_{out} c_{P,out} - F_{in} c_{P,in}}{V_L c_X} \left[\frac{\text{mol P produced / h}}{\text{mol X present in fermenter}} \right]$$

2.6 q-rates from batch

μ is obtained from the biomass balance in batch (mol X/h) and measurements:

$$\frac{d(V_L(t)c_X(t))}{dt} = \mu(t)V_L(t)c_X(t) \rightarrow \frac{d(N_X(t))}{dt} = \mu(t)N_X(t) \text{ note: } N_X(t)=V_L(t)C_X(t)$$

Integration of the biomass balance gives the exponential biomass equation in batch (μ is time independent = μ_{max}):

$$N_X(t) = N_X(0)e^{\mu_{max}t} \rightarrow \text{measurement of } V_L(t)C_X(t)=N_X(t) \text{ at different time points gives } \mu_{max}$$

q_S is obtained from the Substrate balance (mol S/h) in batch and measurements:

$$\frac{d(V_L(t)c_S(t))}{dt} = q_S(t)V_L(t)c_X(t) \rightarrow \frac{d(N_S(t))}{dt} = q_S(t)N_X(t) \text{ (cannot be easily integrated)}$$

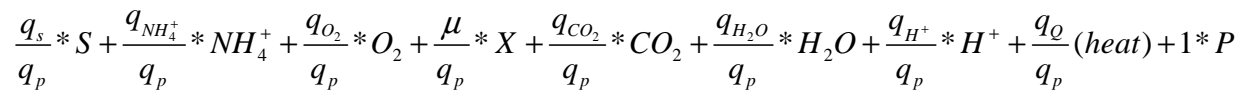
Combined (remove $N_X(t)$) biomass and substrate balance in batch:

$$(N_S(t) - N_S(0)) = \frac{q_{S,max}}{\mu_{max}} (N_X(t) - N_X(0)) \rightarrow N_S(t) \text{ and } N_X(t) \text{ measurements give } q_{S,max}/\mu_{max}, \text{ which}$$

gives (μ_{max} known) the $q_{S,max}$ value.

Week 3

3.1 Process reaction



3.2 BB-models: substrate limited condition

Substrate uptake rate :

$$q_s = q_{s,max} * \frac{c_s}{K_s + c_s}$$

3.3 BB-kinetic models: 1 free variable

Energy consuming products (aerobic):

Substrate uptake rate

$$q_s = q_{s,max} * \frac{c_s}{K_s + c_s}$$

Herbert-Pirt substrate distribution relation

$$q_s = a * \mu + b * q_p + m_s$$

Non-linear $q_p(\mu)$ relation (example)

$$q_p(\mu) = \frac{\alpha\mu}{\beta + \mu}$$

Energy producing products (anaerobic):

Glucose uptake rate

$$q_s = q_{s,max} * \frac{c_s}{K_s + c_s}$$

Herbert-Pirt substrate distribution relation

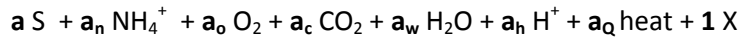
$$q_s = a * \mu + m_s$$

Linear $q_p(\mu)$ relation

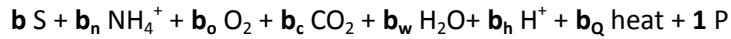
$$q_p = \alpha * \mu + \beta$$

3.5 Process reaction stoichiometry as function of μ

Biomass reaction (use parameter a from Herbert-Pirt) rate μ :



Product reaction (use parameter b from Herbert-Pirt) rate q_p :



Catabolic reaction rate ($-m_s$):

-1 substrate + acceptor + catabolic products

Note: Only α, β, m_s (substrate Herbert-Pirt) are known! All unknown stoichiometric coefficients in these reactions, follow from conservation principles.

Different Herbert-Pirt distribution relations:

$$q_s = a \mu + b q_p + m_s$$

$$q_{O_2} = a_o \mu + b_o q_p + m_o$$

$$q_w = a_w \mu + b_w q_p + m_w$$

$$q_Q = a_Q \mu + b_Q q_p + m_Q$$

$$q_n = a_n \mu + b_n q_p + m_n$$

$$q_h = a_h \mu + b_h q_p + m_h$$

$$q_{CO_2} = a_c \mu + b_c q_p + m_c$$

Growth Production Maintenance

Combining all Herbert-Pirt relations with $q_p(\mu)$ relation gives each $\frac{q_i}{q_p}$ stoichiometric coefficient of the process reaction as a function of μ only!

3.6 Fermenter, gas and liquid, in- and outputs

Total gas balance in steady state (in mol gas/h):

$$0 = F_{N,in} - F_{N,out} - T_{N,o} + T_{N,c} + T_{N,w}$$

Total broth balance in steady state (in kg/h):

$$0 = F_{m,in} + T_{N,o} MW_{O_2} * 10^{-3} + T_{N,NH_3} MW_{NH_3} * 10^{-3} - T_{N,c} MW_{CO_2} * 10^{-3} - T_{N,w} MW_{H_2O} * 10^{-3} - F_{m,out}$$

(MW = molecular weight of compound i in g/mol)

Week 4

4.3 Fermenter types

Biomass balance batch:

$$\frac{d(V_L(t)c_x(t))}{dt} = \mu(t)V_L(t)c_x(t) \rightarrow \frac{dN_x(t)}{dt} = \mu_{\max}N_x(t) \rightarrow N_x(t) = N_x(0)e^{\mu_{\max}t}$$

Biomass balance chemostat:

$$\frac{d(V_L c_x)}{dt} = \mu V_L c_x + F_{in} c_{x,in} - F_{out} c_{x,out} = 0 \rightarrow \mu_{ss} = \frac{F_{out}}{V_L} \text{ (only when } c_x = c_{x,out} \text{ and } c_{x,in} = 0 \text{)}$$

4.4 O₂ + CO₂

Partial pressure:

$$p_i = y_i P$$

Henry's law O₂ and CO₂

$$c_i^* = \alpha_i p_i$$

$$\alpha_c = 38 \text{ molCO}_2/(\text{m}^3 \text{ broth bar CO}_2)$$

$$\alpha_o = 1.25 \text{ molO}_2/(\text{m}^3 \text{ broth bar O}_2)$$

Oxygen transfer:

$$\text{Total rate } T_{N,o} = K_L A (c_o^* - c_o) \text{ molO}_2/\text{h}$$

$$\text{Volume specific } T_{N,o} / V_L = K_L a (c_o^* - c_o) \text{ molO}_2/(\text{m}^3 \text{ broth h}) \text{ where } a = A/V_L$$

$$\text{Volumetric O}_2 \text{ transfer capacity } K_L a c_o^* \text{ molO}_2/(\text{m}^3 \text{ broth h})$$

Carbon dioxide transfer rate:

$$\text{Total rate } T_{N,c} = K_L A (c_c - c_c^*) \text{ mol CO}_2/\text{h}$$

$$\text{Volume specific } T_{N,c} / V_L = K_L a (c_c - c_c^*) \text{ molCO}_2/(\text{m}^3 \text{ h})$$

$$\text{Volumetric CO}_2 \text{ transfer capacity } K_L a c_c^* \text{ molCO}_2/(\text{m}^3 \text{ h})$$

Superficial gas velocity:

$$v_{gs} (\text{m/s}) = \frac{F_g}{A_f} \left(\frac{\text{m}^3/\text{s}}{\text{m}^2} \right)$$

K_La for Coalescing bubbles:

$$\text{Stirred vessel } K_L a (\text{s}^{-1}) = 0.026 \left(\frac{P_s (\text{W})}{V_L (\text{m}^3)} \right)^{0.4} [v_{gs} (\text{m/s})]^{0.5}$$

Bubble column $K_L a (s^{-1}) = 0.32 \left[v_{gs} (m/s) \right]^{0.70}$

O₂-depletion in coalescing bubble columns

0.55 or 0.45% O₂/m broth height

K_La for non-Coalescing bubbles:

Stirred vessel $K_L a = 0.002 \left(\frac{P_s (W)}{V_L (m^3)} \right)^{0.7} \left(v_{gs} \frac{m}{s} \right)^{0.2}$

Average pressure p (t = top, b = bottom)

$$p = (p_t + p_b)/2$$

Bottom pressure (H broth height in m)

$$p_b = p_t + \rho_L gH \times 10^{-5}$$

(bar) (bar)

Volumetric gas flow at average pressure (use ideal gas law)

$$F_{g,av} (m^3 / s) = F_{N,av} (mol / s) \times \frac{RT_{fermentor}}{p(N / m^2)}$$

$$R = 8.314 \text{ J/molK}$$

T_f = absolute fermentor temperature

$$F_{N,av} = (F_{N,in} + F_{N,out})/2 \text{ mol gas/second}$$

Volumetric gas flow at bottom pressure

$$F_{g,b} (m^3/s) = F_{N,bottom} (mol/s) \frac{RT_{fermentor}}{p_b (N / m^2)}$$

4.5 Heat removal

Stanton heat number:

$$St_{heat} = \frac{UA_c}{\rho_{cw} C_p F_{cw}} = \frac{\text{capacity for heat transfer through coil}}{\text{convective heat transport capacity in cooling water}}$$

- A_c = Cooling surface (m^2)
 F_{cw} = Cooling water flow (m^3/h)
 ρ_{cw} = Density of cooling water (kg/m^3)
 C_p = Heat capacity cooling water ($KJ/(kg^\circ K)$)
 U = Overall heat transfer coefficient ($kg/(m^2h^\circ K)$)

Cooling Loop:

$$F_{coolingloop} (m^3 / s) = \frac{R_Q (kJ / s)}{\rho c_p \Delta T (kJ / m^3)}$$

$$\rho c_p = 4180 \frac{kJ}{m^3 K} \text{ (heat capacity broth)}$$

ΔT = temperature shock in external cooling loop

c_p = heat capacity broth

Time period for cold shock

$$t_{coldshock}(s) = \frac{M \text{ tonne(or } m^3)}{F_{coolingloop} (m^3 / s)}$$

M = broth mass in the fermenter

Biological heat production

1 molO₂ \equiv 450 kJ

Heat of evaporation of water

43 kJ/mol water

4.6 Broth mixing and gradients

Dimensionless mixing equation (mixing time t_m in seconds, e in W/kg):

$$N_{mix} = \frac{t_m e^{\frac{1}{3}}}{D^{\frac{2}{3}}} = \frac{\alpha}{\beta^{\frac{4}{3}} \gamma^{\frac{1}{3}}} \left(\frac{L_s}{H} \right)^2 \left(\frac{H}{D} \right)^2$$

Power input per broth mass (W/kg)

$$e = P/M$$

Power input (Watt)

Gas $P = F_N RT \ln(p_b/p_t)$

F_n = Molar gas flow rate (mol/s)

R = 8.314 J/mol K (gas constant)

T = Absolute temperature (Kelvin; $K = 273.15 + T(^{\circ}C)$)

Stirrer $P = N_p \rho_L N^3 D_s^5$

N_p (gassed power number, Rushton) ≈ 3.0

ρ_L = broth density (kg/m^3)

N = rotational frequency stirrer (s^{-1})

D_s = stirrer diameter (m)

Circulation time

$$t_c = t_m/4$$

Broth circulation flow inside fermenter

$$F_{circ} (m^3/s) = \frac{M (m^3 \text{ broth})}{t_c}$$

Substrate concentration at the feedpoint

$$C_{s, \text{feedpoint}} (mol \text{ substrate}/m^3) = \frac{F_{N, \text{glucose}} (mol \text{ substrate} / s)}{F_{circ} (m^3 / s)} = \frac{F_{in} C_{s, in} (mol \text{ substrate} / s)}{F_{circ} (m^3 / s)}$$

4.7 Vertical gradients: t = top, b = bottom

Gradients in O₂-transfer rate

(see 4.4)

O₂-solubility at top and bottom

$$\text{Top} \quad c_{o,t}^* = \alpha_o Y_{o,\text{out}} p_t$$

$$\text{Bottom} \quad c_{o,b}^* = \alpha_o Y_{o,\text{in}} p_b$$

Average O₂- solubility

$$c_{o,\text{av}}^* = [c_{o,t}^* + c_{o,b}^*]/2$$

Bottom c_o (molO₂/m³ broth)

$$c_{o,b} = c_{o,b}^* - (|R_o|/V_L)/(k_L a)_b$$

Top c_o (molO₂/m³ broth)

$$c_{o,t} = c_{o,t}^* - (|R_o|/V_L)/(k_L a)_t$$

Top CO₂ (molCO₂/m³ broth)

$$c_{c,t}^* = \alpha_c Y_{c,\text{out}} p_t$$

$$c_{c,t} = c_{c,t}^* + (|R_c|/V_L)/(k_L a)_t$$

Bottom CO₂ (molCO₂/m³ broth)

$$c_{c,b} = c_{c,t} + \frac{1}{2} t_{\text{circ}} R_c/V_L$$

Week 5

5.1: Towards and integrated process

Equilibrium equation

$$y_1 = K * x_1$$

Mass balance example

$$Hx_0 + Ly_0 = Hx_1 + Ly_1$$

Separation factor

$$S = L/H * K$$

5.4: Separation principles

Extraction yield

$$\text{extraction yield (single stage)} = \frac{Vy_1}{Lx_0} = \frac{S}{1 + S}$$

$$\text{extraction yield (for } N \text{ stages)} = \frac{Vy_1}{Lx_0} = 1 - \frac{S - 1}{S^{N+1} - 1}$$

Week 6

6.2: Process economics

Net Present Value

$$NPV = \sum_{k=1}^n \frac{CF_k}{(1+i)^k}$$

Payback time

$$\text{Payback period} = \frac{CAPEX}{\text{annual net profit}}$$

Net Cash Flow

$$\text{Net Cash Flow} = \text{Revenues} - \text{OPEX} - \text{taxes}$$

Return on Investment

$$ROI = \frac{\text{net profit}}{CAPEX}$$

Purchase Equipment cost – scaling

$$\frac{\text{equipment cost}}{\text{reference equipment cost}} = \left(\frac{\text{size}}{\text{reference size}} \right)^i \quad (i \sim 0.6)$$

6.5: Technology improvement

Technology Impact (IPAT)

$$\text{Impact (on environment)} = \text{Population} * \text{Affluence} * \text{Technology}$$