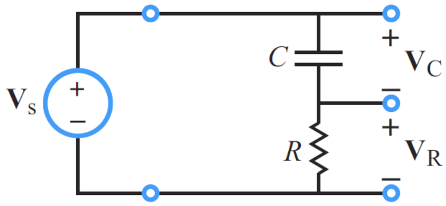
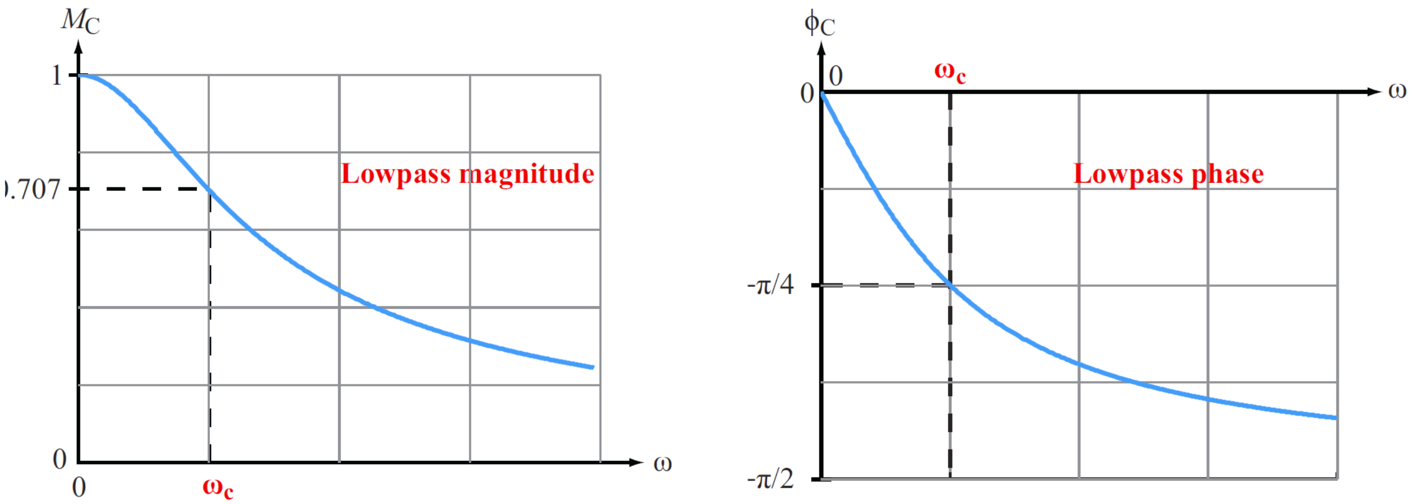
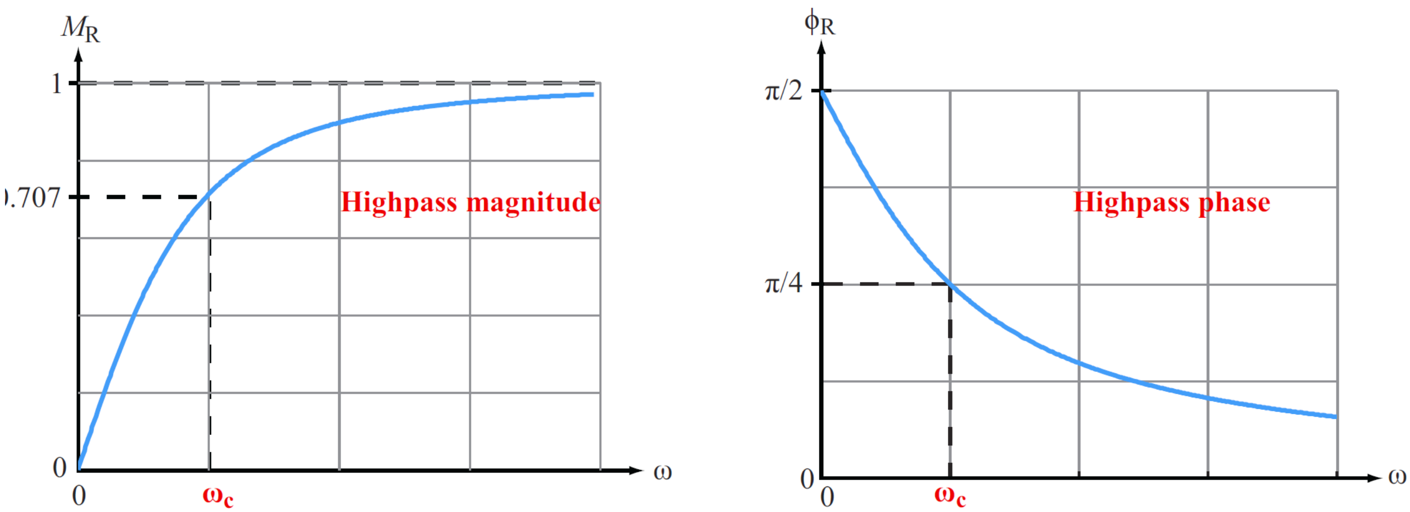
1^{st} order Low and High Pass RC Filter Example



(a) RC circuit



(b) Magnitude and phase angle of $\hat{\mathbf{H}}_C(w) = \mathbf{V}_C / \mathbf{V}_s$



(c) Magnitude and phase angle of $\hat{\mathbf{H}}_R(\omega) = \mathbf{V}_R / \mathbf{V}_s$

(a) An RC circuit; (b) if we measure the voltage across the capacitor (given the input V_S), we obtain a lowpass filter; (c) if we measure the voltage across the resistor (given the input V_S), we obtain a highpass filter. The expressions that generate the plots above are worked out on the next page.

The Lowpass Filter

Application of voltage division gives

$$\mathbf{V}_\mathrm{C} = \frac{\mathbf{V}_\mathrm{s}\mathbf{Z}_\mathrm{C}}{R+\mathbf{Z}_\mathrm{C}} = \frac{\mathbf{V}_\mathrm{s}/j\omega C}{R+\frac{1}{j\omega C}}.$$

The transfer function corresponding to V_C is

$$\mathbf{H}_\mathrm{C}(\omega) = \frac{\mathbf{V}_\mathrm{C}}{\mathbf{V}_\mathrm{s}} = \frac{1}{1+j\omega RC},$$

where we have multiplied the numerator and denominator of the top expression by jωC to simplify the form of the expression. In terms of its magnitude $M_C(\omega)$ and phase angle $\phi_C(\omega)$, the transfer function is given by

$$\mathbf{H}_\mathrm{C}(\omega) = M_\mathrm{C}(\omega)\, e^{j\phi_\mathrm{C}(\omega)}$$

with

$$M_\mathrm{C}(\omega) = |\mathbf{H}_\mathrm{C}(\omega)| = \frac{1}{\sqrt{1+\omega^2R^2C^2}}$$

and

$$\phi_\mathrm{C}(\omega) = -\tan^{-1}(\omega RC).$$

At dc, the capacitor acts like an open circuit—allowing no current to flow through the loop—with the obvious consequence that $V_C = V_s$. At very high values of ω, the capacitor acts like a short circuit, in which case the voltage across it is approximately zero.

The Highpass Filter

The output across R is

$$\mathbf{H}_\mathrm{R}(\omega) = \frac{\mathbf{V}_\mathrm{R}}{\mathbf{V}_\mathrm{s}} = \frac{j\omega RC}{1+j\omega RC}$$

The magnitude and phase angle of $H_R(\omega)$ are given by

$$M_\mathrm{R}(\omega) = |\mathbf{H}_\mathrm{R}(\omega)| = \frac{\omega RC}{\sqrt{1+\omega^2R^2C^2}}$$

And

$$\phi_\mathrm{R}(\omega) = \frac{\pi}{2} - \tan^{-1}(\omega RC)$$