Video 1.1

Vijay Kumar and Ani Hsieh

Robotics: Dynamics and Control

Vijay Kumar and Ani Hsieh
University of Pennsylvania

Why?

- Robots live in a physical world
- The physical world is governed by the laws of motion
- Fundamental understanding of dynamics of robots

The Goal

- Models of robots
  - Robot manipulators, ground robots, flying robots…
- Beyond geometric and kinematic models to dynamic models
- Use dynamic models for real world applications

Dynamics

Two sets of problems:

- Forward dynamics

  How do robots move when we apply forces or torques to the actuators, or currents/voltages to the motors?

- Inverse dynamics

  What forces or torques or currents or voltages to apply to achieve a desired output (force or moment or velocity or acceleration)?

Video 1.2
Vijay Kumar and Ani Hsieh

Dynamics and Control Introduction

Vijay Kumar and Ani Hsieh
University of Pennsylvania

What should you know?

- 3-D vectors, geometry
- Vector calculus, kinematics
- Rotation matrices
- Transformation matrices

What you will learn

- How to create dynamic models of robots?
- How to simulate robotic systems?
- How to control robotic systems?

Dynamics

- Particle dynamics
  - Kinematics
  - Kinetics
- Rigid body dynamics
  - Kinematics
  - Kinetics
- Application to chains of rigid bodies
  - Newton-Euler Equations of Motion
  - Lagrange's Equations of Motion

Simulation

- Forward Dynamics

  How does the robot move if you apply a set of forces or torques at the actuators

http://money.cnn.com/2015/04/07/technology/sawyer-robot-manufacturing-revolution/

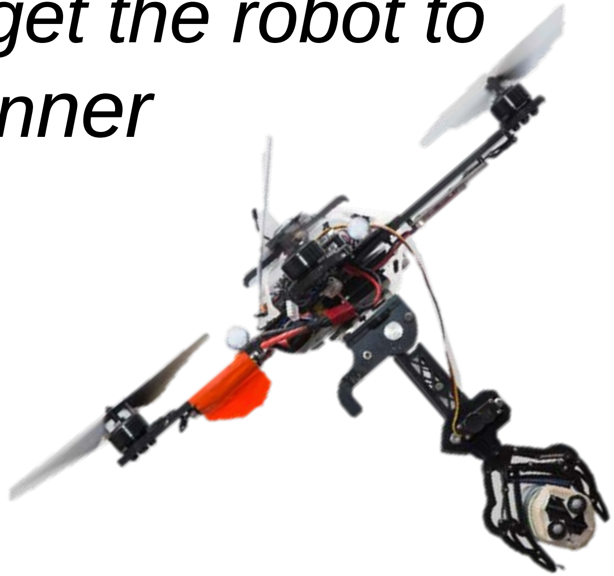
J. Thomas, G. Loianno, J. Polin, K. Sreenath, and V. Kumar, "Toward autonomous avian-inspired grasping for micro aerial vehicles," *Bioinspiration and Biomimetics*, vol. 9, no. 2, p. 025010, June 2014.

Control

- Inverse Dynamics

What forces or torques need to be applied by the actuators in order to get the robot to move or act in a desired manner

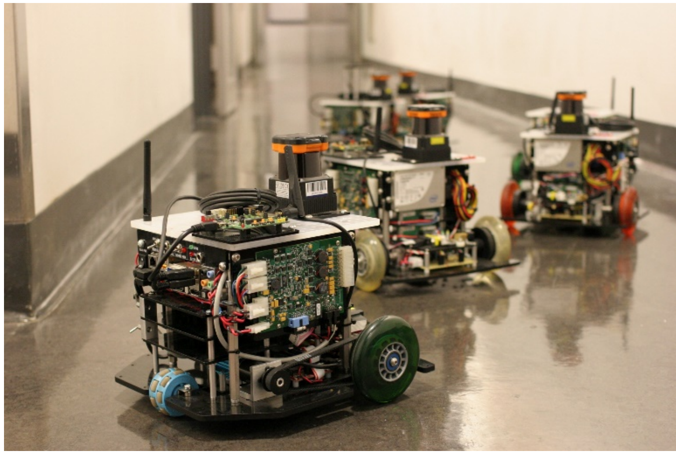
http://money.cnn.com/2015/04/07/technology/sawyer-robot-manufacturing-revolution/



J. Thomas, G. Loianno, J. Polin, K. Sreenath, and V. Kumar, "Toward autonomous avian-inspired grasping for micro aerial vehicles," *Bioinspiration and Biomimetics*, vol. 9, no. 2, p. 025010, June 2014.

Applications

- Robot manipulators
- Ground robots: wheeled
- Flying robots: quadrotors



Video 1.3

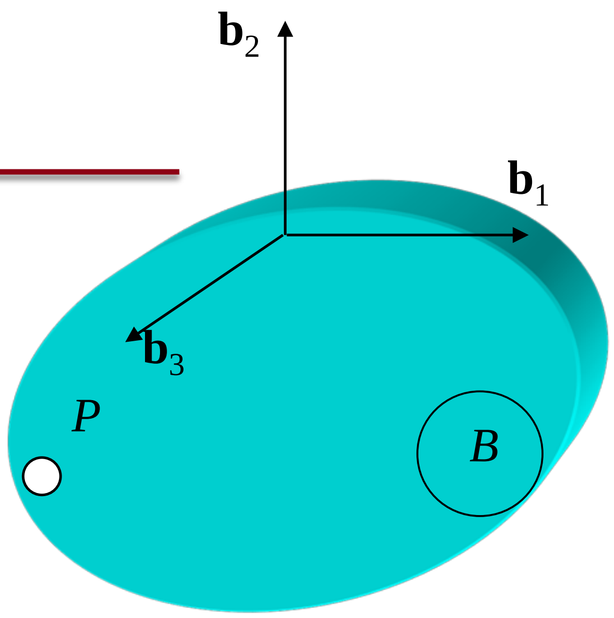
Vijay Kumar and Ani Hsieh

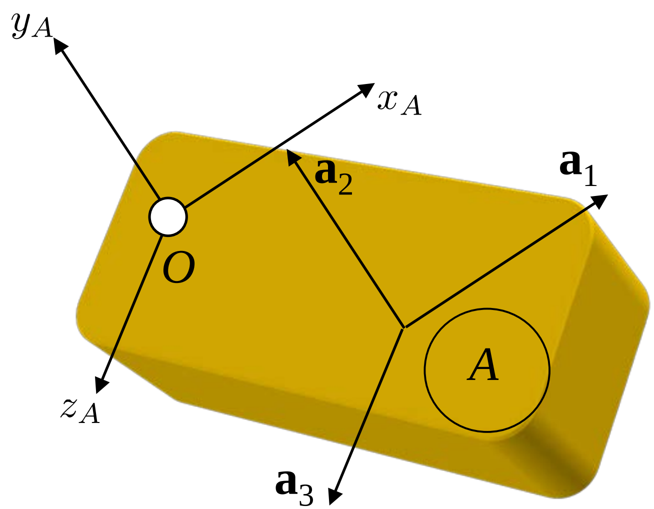
Dynamics and Control Review

Vijay Kumar and Ani Hsieh
University of Pennsylvania

Reference Frames

- Reference frame A
  - Origin O
  - Basis vectors $\{\mathbf{a}_1, \mathbf{a}_2, \mathbf{a}_3\}$
- Reference frame B
  - Origin P
  - Basis vectors $\{\mathbf{b}_1, \mathbf{b}_2, \mathbf{b}_3\}$



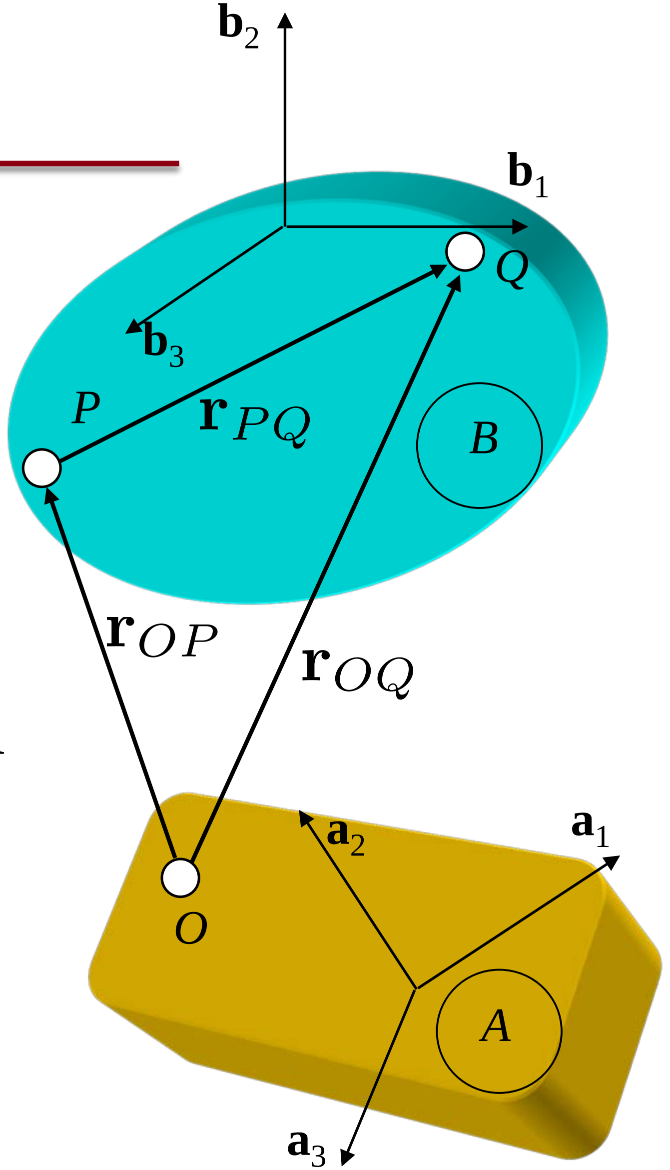


Position Vectors

- Reference frame A
  - Origin O
  - Basis vectors $\{\mathbf{a}_1, \mathbf{a}_2, \mathbf{a}_3\}$
- Position Vectors
  - Position vectors for P and Q in A

    $\mathbf{r}_{OP}$ $\mathbf{r}_{OQ}$
  - Position vector of Q in B

    $\mathbf{r}_{PQ}$



Position Vectors

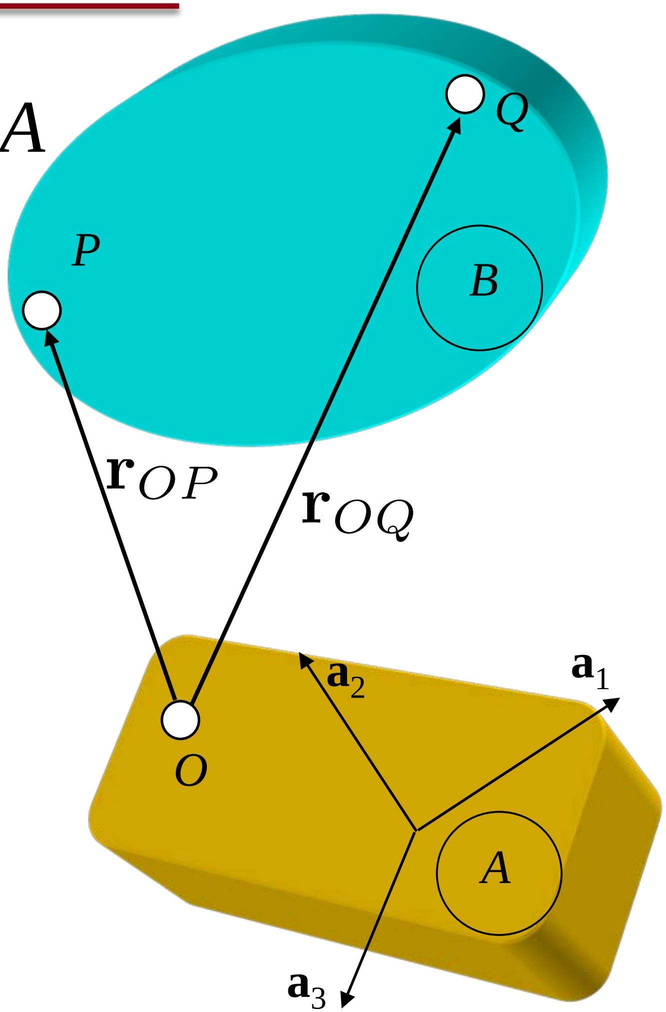
- Position vectors for P and Q in A

$$\mathbf{r}_{OP} = p_1\mathbf{a}_1 + p_2\mathbf{a}_2 + p_3\mathbf{a}_3$$

$$\begin{bmatrix} p_1 \\ p_2 \\ p_3 \end{bmatrix}$$

$$\mathbf{r}_{OQ} = q_1\mathbf{a}_1 + q_2\mathbf{a}_2 + q_3\mathbf{a}_3$$

$$\begin{bmatrix} q_1 \\ q_2 \\ q_3 \end{bmatrix}$$



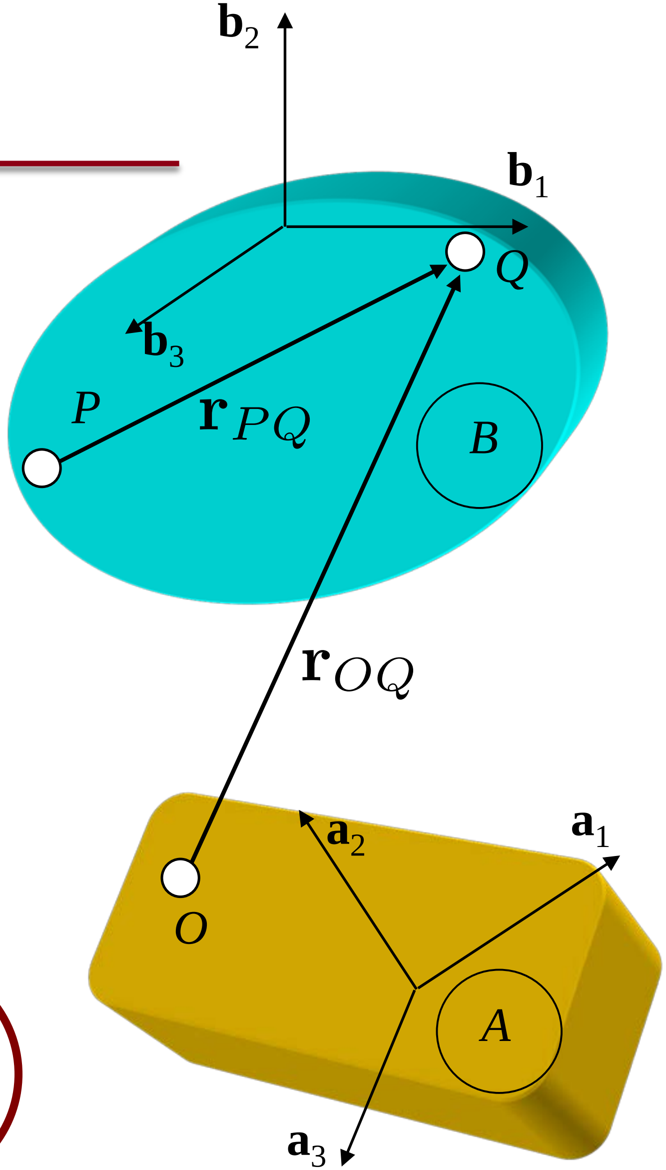
Transformations

- Reference frames $A,\ B$
  - Origins $O,\ P$
  - Basis vectors $\{\mathbf{a}_1, \mathbf{a}_2, \mathbf{a}_3\}$ $\{\mathbf{b}_1, \mathbf{b}_2, \mathbf{b}_3\}$
- Rigid Body Transformation
  - Position vector of Q in A

$$\mathbf{r}_{OQ} = q_1\mathbf{a}_1 + q_2\mathbf{a}_2 + q_3\mathbf{a}_3$$

  - Position vector Q in B

$$\mathbf{r}_{PQ} = q_1'\mathbf{b}_1 + q_2'\mathbf{b}_2 + q_3'\mathbf{b}_3$$



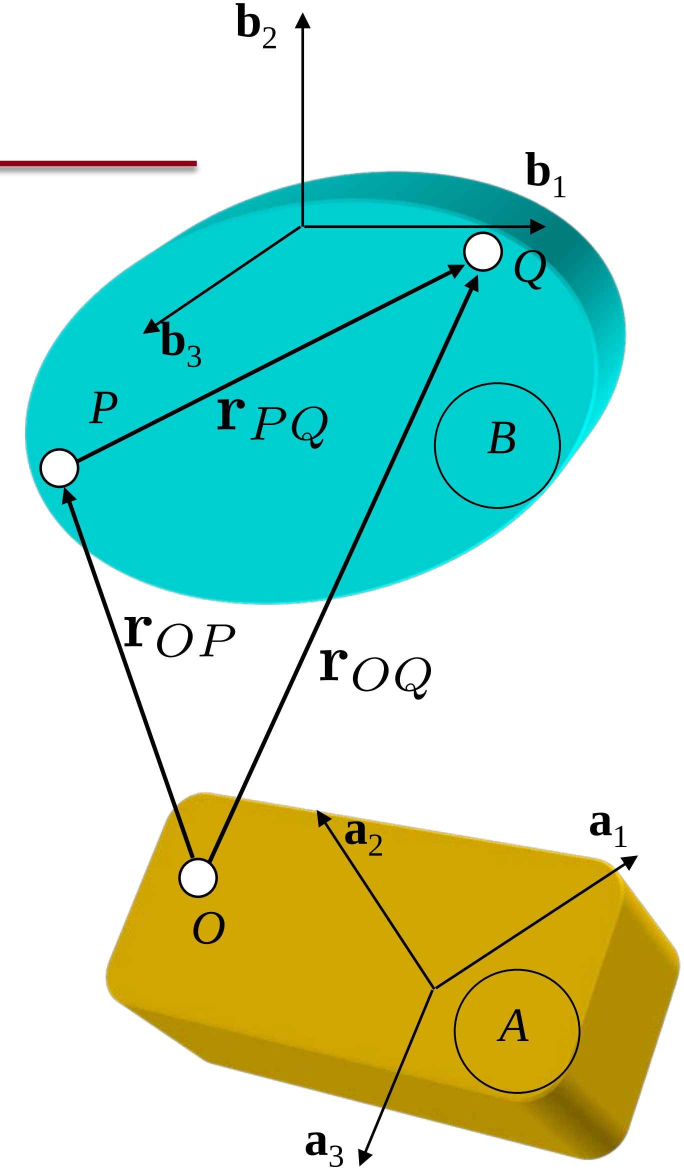
Transformations

$$\begin{bmatrix} q_1 \\ q_2 \\ q_3 \end{bmatrix} \Longleftrightarrow \begin{bmatrix} q'_1 \\ q'_2 \\ q'_3 \end{bmatrix}$$

$$\mathbf{r}_{OQ} = \mathbf{r}_{OP} + \mathbf{r}_{PQ}$$

$$\begin{aligned} q_1\mathbf{a}_1 + q_2\mathbf{a}_2 + q_3\mathbf{a}_3 &= p_1\mathbf{a}_1 + p_2\mathbf{a}_2 + p_3\mathbf{a}_3 \\ &\quad + q'_1\mathbf{b}_1 + q'_2\mathbf{b}_2 + q'_3\mathbf{b}_3 \end{aligned}$$

$$\begin{bmatrix} q_1 \\ q_2 \\ q_3 \end{bmatrix} = \begin{bmatrix} p_1 \\ p_2 \\ p_3 \end{bmatrix} + \begin{bmatrix} q'_1 \\ q'_2 \\ q'_3 \end{bmatrix}$$



Transformations

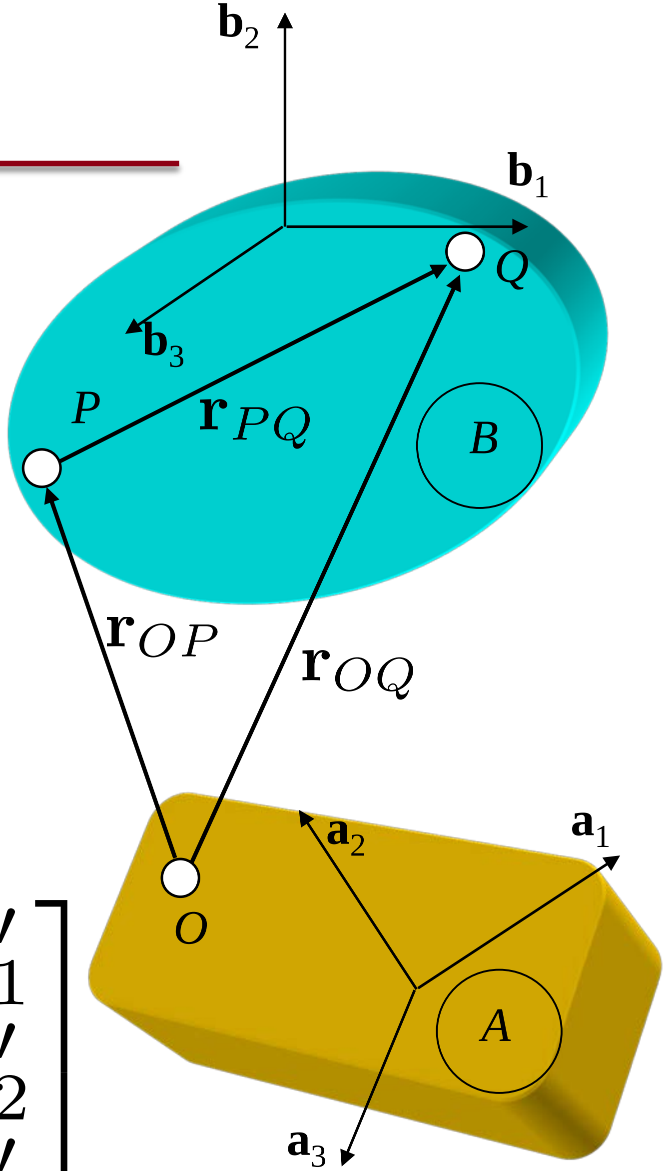
$$\begin{bmatrix} q_1 \\ q_2 \\ q_3 \end{bmatrix} \Longleftrightarrow \begin{bmatrix} q'_1 \\ q'_2 \\ q'_3 \end{bmatrix}$$

$$\mathbf{r}_{OQ} = \mathbf{r}_{OP} + \mathbf{r}_{PQ}$$

$$q_1\mathbf{a}_1 + q_2\mathbf{a}_2 + q_3\mathbf{a}_3$$
$$= p_1\mathbf{a}_1 + p_2\mathbf{a}_2 + p_3\mathbf{a}_3$$
$$+ q'_1\mathbf{b}_1 + q'_2\mathbf{b}_2 + q'_3\mathbf{b}_3$$

$$\begin{bmatrix} q_1 \\ q_2 \\ q_3 \end{bmatrix} = \begin{bmatrix} p_1 \\ p_2 \\ p_3 \end{bmatrix} + \mathbf{R}_{AB} \begin{bmatrix} q'_1 \\ q'_2 \\ q'_3 \end{bmatrix}$$

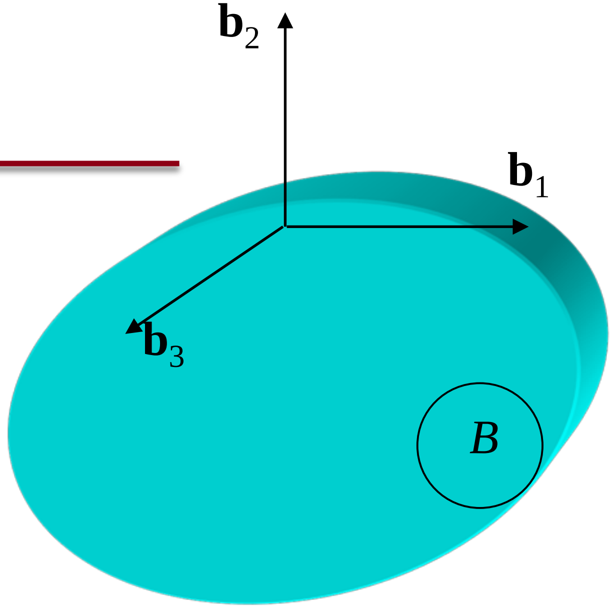
Rotation Matrix



Rotation Matrix

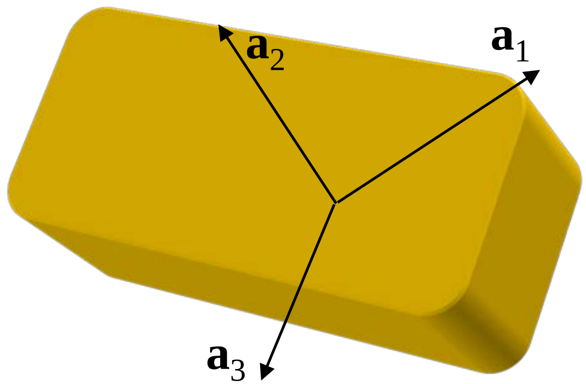
$$\mathbf{r}_{OQ} = \mathbf{r}_{OP} + \mathbf{r}_{PQ}$$

$$q_1\mathbf{a}_1 + q_2\mathbf{a}_2 + q_3\mathbf{a}_3$$
$$= p_1\mathbf{a}_1 + p_2\mathbf{a}_2 + p_3\mathbf{a}_3$$
$$+ q_1'\mathbf{b}_1 + q_2'\mathbf{b}_2 + q_3'\mathbf{b}_3$$



$$\begin{bmatrix} q_1 \\ q_2 \\ q_3 \end{bmatrix} = \begin{bmatrix} p_1 \\ p_2 \\ p_3 \end{bmatrix} + \mathbf{R}_{AB} \begin{bmatrix} q_1' \\ q_2' \\ q_3' \end{bmatrix}$$

$$R_{AB} = \begin{bmatrix} b_1 \cdot a_1 & b_2 \cdot a_1 & b_3 \cdot a_1 \\ b_1 \cdot a_2 & b_2 \cdot a_2 & b_3 \cdot a_2 \\ b_1 \cdot a_3 & b_2 \cdot a_3 & b_3 \cdot a_3 \end{bmatrix}$$



Homogeneous Transformation Matrix

$$\begin{bmatrix} q_1 \\ q_2 \\ q_3 \end{bmatrix} = \begin{bmatrix} p_1 \\ p_2 \\ p_3 \end{bmatrix} + \mathbf{R}_{AB} \begin{bmatrix} q_1' \\ q_2' \\ q_3' \end{bmatrix}$$

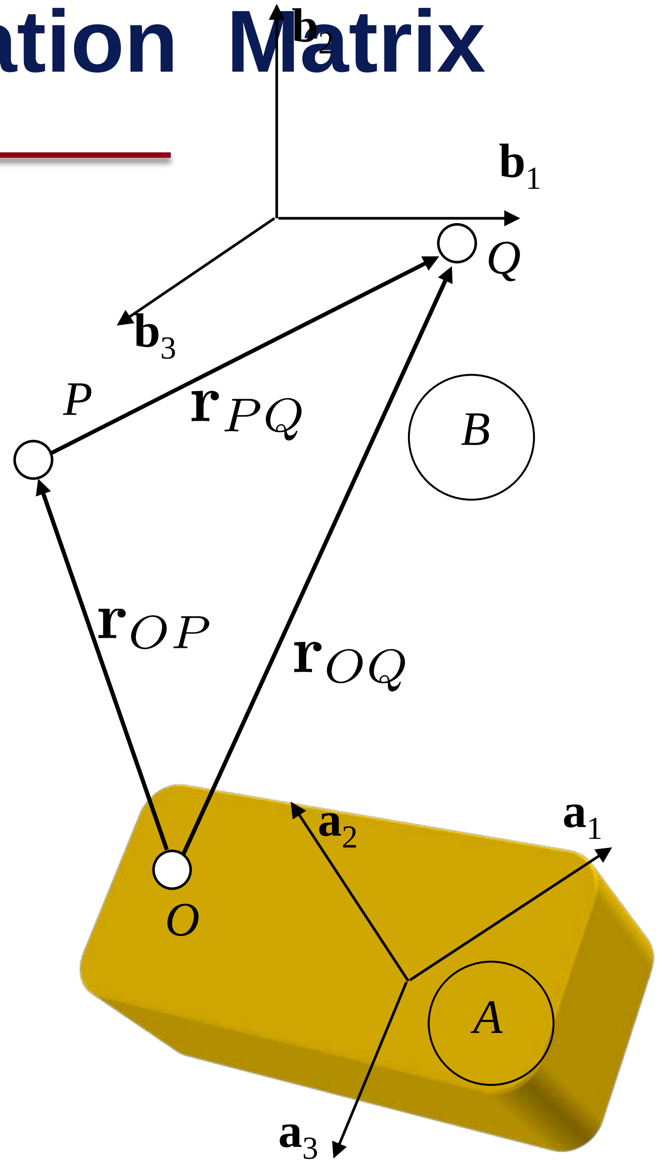
$$\begin{bmatrix} q_1 \\ q_2 \\ q_3 \\ 1 \end{bmatrix} = \begin{bmatrix} \mathbf{b}_1.\mathbf{a}_1 & \mathbf{b}_1.\mathbf{a}_2 & \mathbf{b}_1.\mathbf{a}_3 & p_1 \\ \mathbf{b}_2.\mathbf{a}_1 & \mathbf{b}_2.\mathbf{a}_2 & \mathbf{b}_2.\mathbf{a}_3 & p_2 \\ \mathbf{b}_3.\mathbf{a}_1 & \mathbf{b}_3.\mathbf{a}_2 & \mathbf{b}_3.\mathbf{a}_3 & p_3 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} q_1' \\ q_2' \\ q_3' \\ 1 \end{bmatrix}$$

Position of *Q* in *A*

Position of *Q* in *B*

$$\mathbf{T}_{AB}$$

4x4 homogeneous transformation matrix



ONLINE LEARNING

Video 1.4
Vijay Kumar and Ani Hsieh

Dynamics and Control
Velocity and Acceleration Analysis

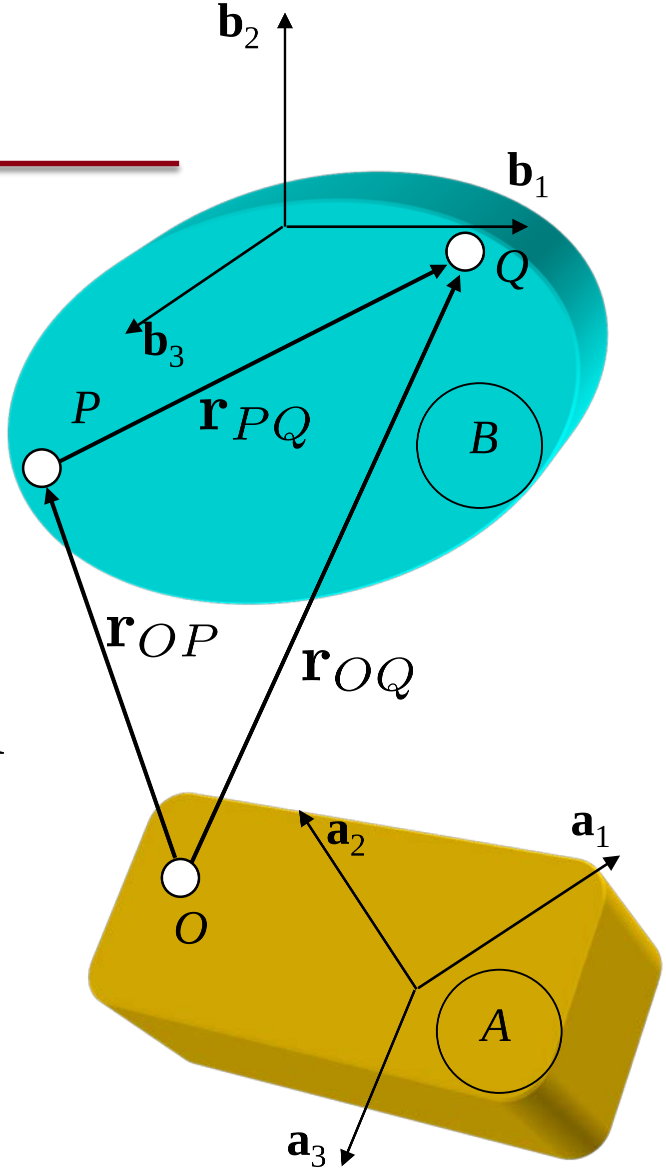
Vijay Kumar and Ani Hsieh
University of Pennsylvania

Position Vectors

- Reference frame A
  - Origin O
  - Basis vectors $\{\mathbf{a}_1, \mathbf{a}_2, \mathbf{a}_3\}$
- Position Vectors
  - Position vectors for P and Q in A

    $\mathbf{r}_{OP}$ $\mathbf{r}_{OQ}$
  - Position vector of Q in B

    $\mathbf{r}_{PQ}$



Velocity Vectors

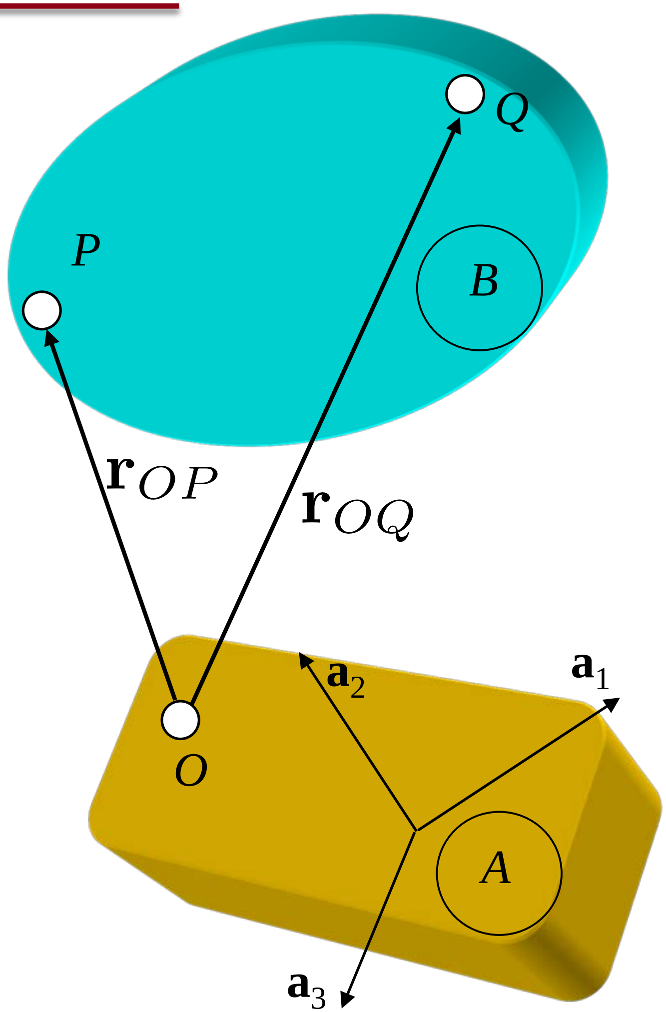
- Velocity of P and Q in A

$$\mathbf{v}_P = \dot{p}_1\mathbf{a}_1 + \dot{p}_2\mathbf{a}_2 + \dot{p}_3\mathbf{a}_3$$

$$\begin{bmatrix} \dot{p}_1 \\ \dot{p}_2 \\ \dot{p}_3 \end{bmatrix}$$

$$\mathbf{v}_Q = \dot{q}_1\mathbf{a}_1 + \dot{q}_2\mathbf{a}_2 + \dot{q}_3\mathbf{a}_3$$

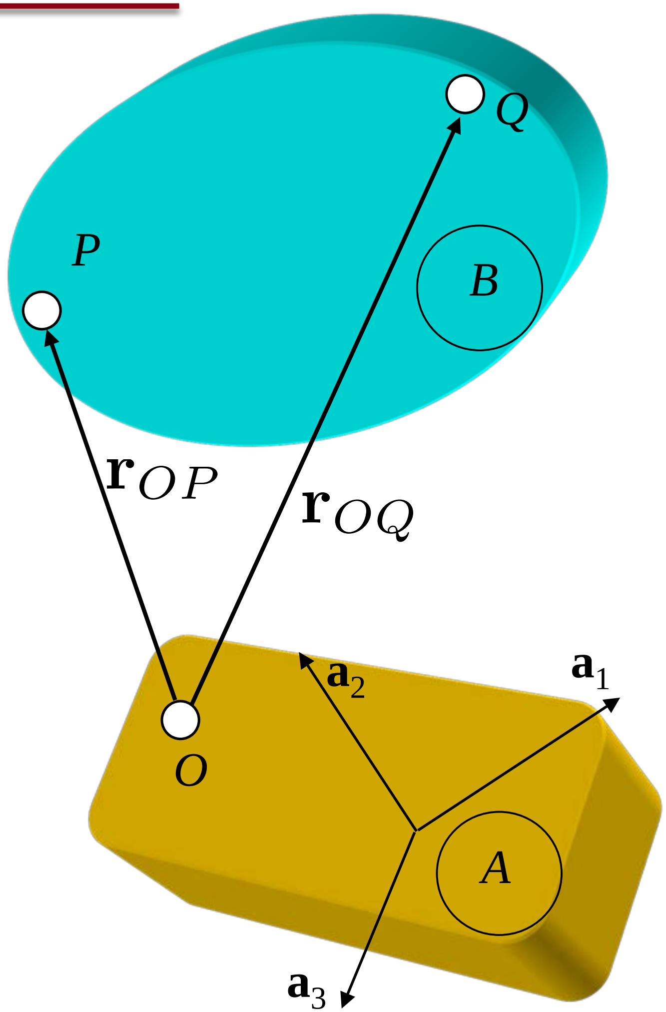
$$\begin{bmatrix} \dot{q}_1 \\ \dot{q}_2 \\ \dot{q}_3 \end{bmatrix}$$



Velocity Vectors

- Velocity of P and Q in B

Zero, since both points are fixed to B!



Velocity Vectors

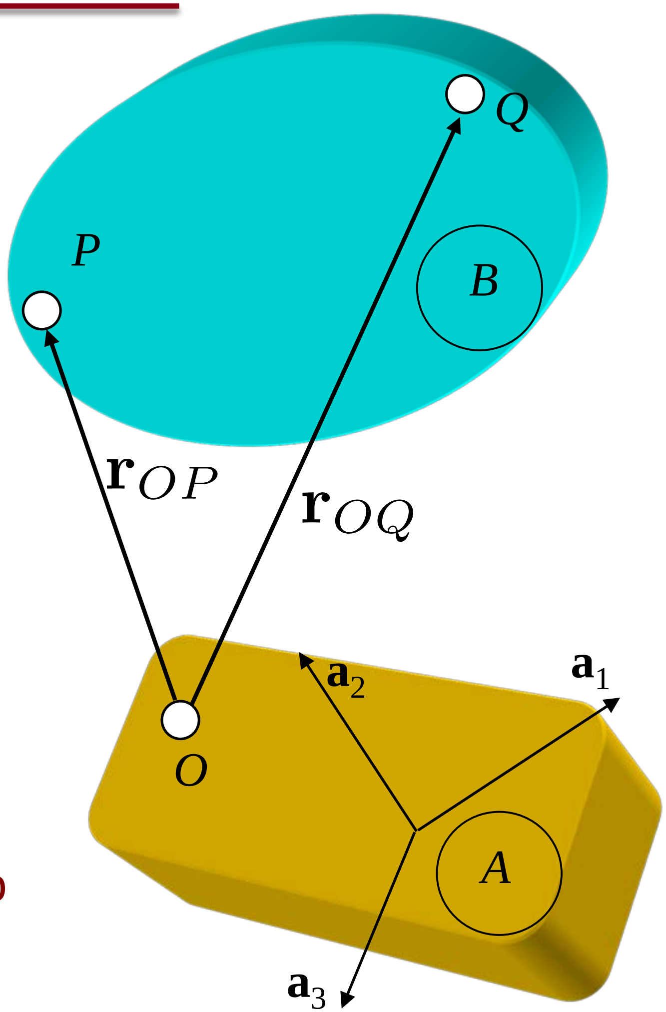
- Velocity of P and Q in A

$$\mathbf{v}_P = \dot{p}_1 \mathbf{a}_1 + \dot{p}_2 \mathbf{a}_2 + \dot{p}_3 \mathbf{a}_3$$

$$\begin{bmatrix} \dot{p}_1 \\ \dot{p}_2 \\ \dot{p}_3 \end{bmatrix}$$

$$\mathbf{v}_Q = \dot{q}_1 \mathbf{a}_1 + \dot{q}_2 \mathbf{a}_2 + \dot{q}_3 \mathbf{a}_3$$

$$\begin{bmatrix} \dot{q}_1 \\ \dot{q}_2 \\ \dot{q}_3 \end{bmatrix}$$



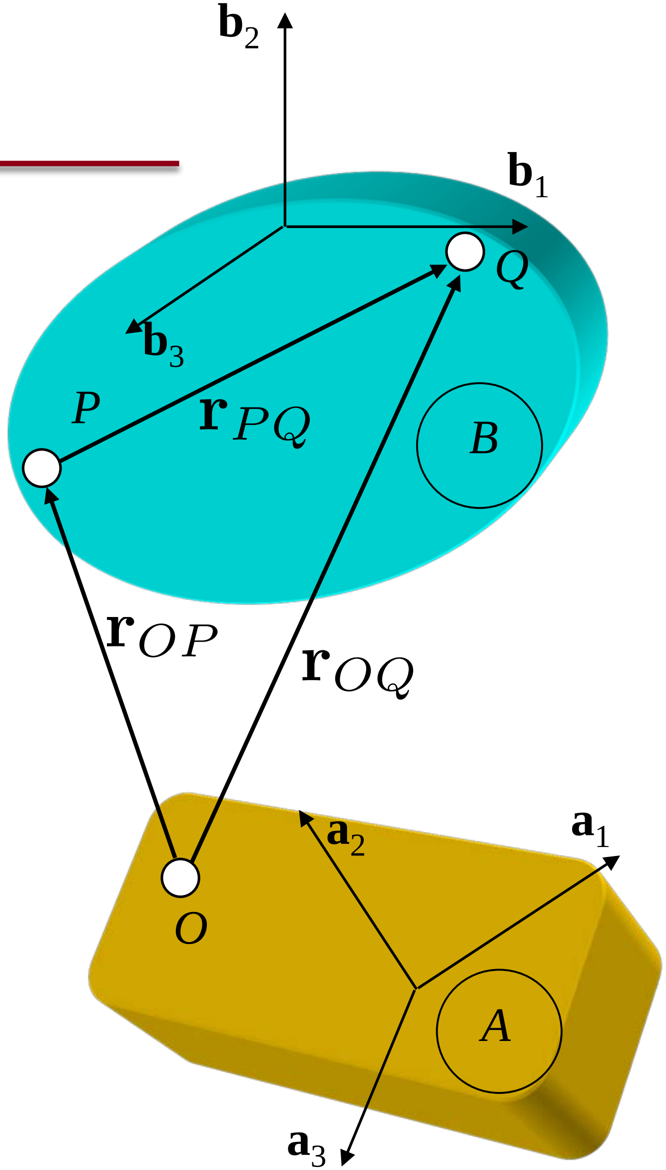
How to relate velocities of two points fixed to the same rigid body?

Recall ...

$$\mathbf{r}_{OQ} = \mathbf{r}_{OP} + \mathbf{r}_{PQ}$$

$$q_1\mathbf{a}_1 + q_2\mathbf{a}_2 + q_3\mathbf{a}_3$$
$$= p_1\mathbf{a}_1 + p_2\mathbf{a}_2 + p_3\mathbf{a}_3$$
$$+ q_1'\mathbf{b}_1 + q_2'\mathbf{b}_2 + q_3'\mathbf{b}_3$$

$$\begin{bmatrix} q_1 \\ q_2 \\ q_3 \end{bmatrix} = \begin{bmatrix} p_1 \\ p_2 \\ p_3 \end{bmatrix} + \mathbf{R}_{AB} \begin{bmatrix} q_1' \\ q_2' \\ q_3' \end{bmatrix}$$



Velocities of 2 points fixed to the same rigid body

$$\mathbf{r}_{OQ} = \mathbf{r}_{OP} + \mathbf{r}_{PQ}$$

$$\begin{bmatrix} q_1 \\ q_2 \\ q_3 \end{bmatrix} = \begin{bmatrix} p_1 \\ p_2 \\ p_3 \end{bmatrix} + \mathbf{R}_{AB} \begin{bmatrix} q_1' \\ q_2' \\ q_3' \end{bmatrix}$$

$$\mathbf{R}_{AB}^T \begin{bmatrix} q_1 - p_1 \\ q_2 - p_2 \\ q_3 - p_3 \end{bmatrix}$$

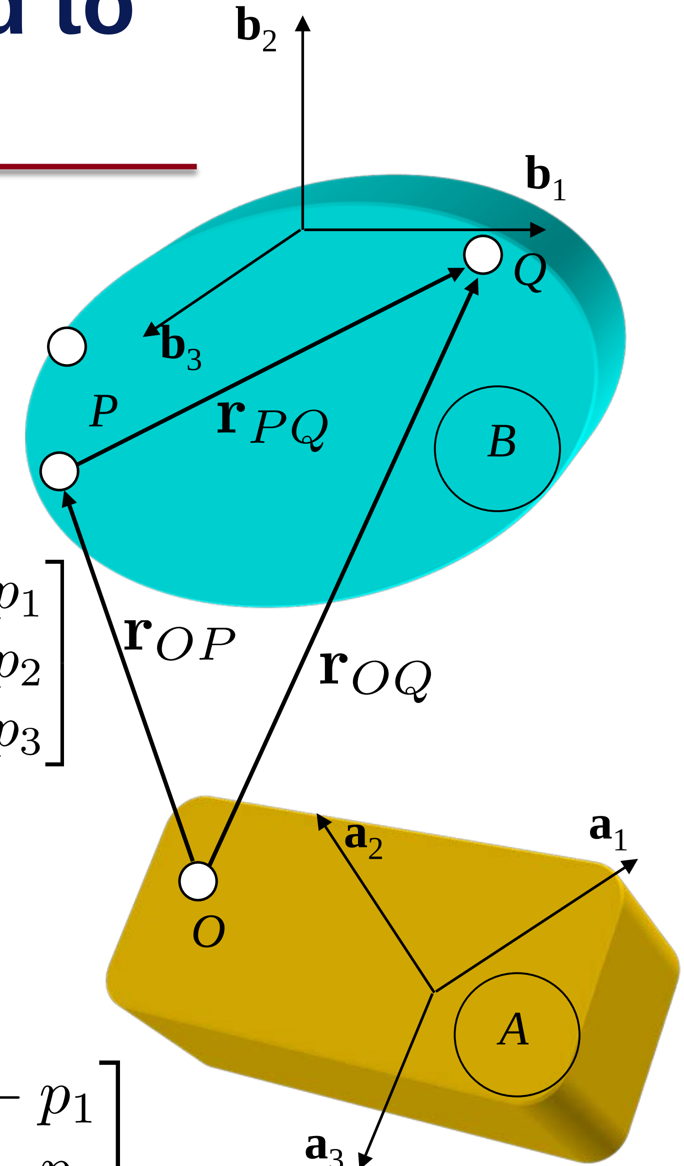
$$\frac{d}{dt}$$

$$\begin{bmatrix} \dot{q}_1 \\ \dot{q}_2 \\ \dot{q}_3 \end{bmatrix} = \begin{bmatrix} \dot{p}_1 \\ \dot{p}_2 \\ \dot{p}_3 \end{bmatrix} + \dot{\mathbf{R}}_{AB} \begin{bmatrix} q_1' \\ q_2' \\ q_3' \end{bmatrix}$$

$$\hat{\omega}_{AB}$$

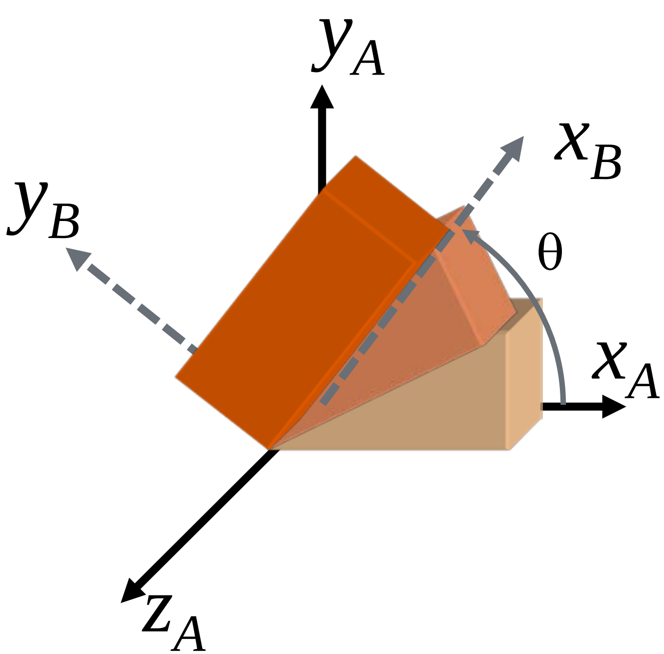
$$\begin{bmatrix} \dot{q}_1 \\ \dot{q}_2 \\ \dot{q}_3 \end{bmatrix} = \begin{bmatrix} \dot{p}_1 \\ \dot{p}_2 \\ \dot{p}_3 \end{bmatrix} + \dot{\mathbf{R}}_{AB} \mathbf{R}_{AB}^T \begin{bmatrix} q_1 - p_1 \\ q_2 - p_2 \\ q_3 - p_3 \end{bmatrix}$$

3x3 skew symmetric



Example: Rotation about a single axis

Rotation about the *z*-axis through θ



$$R_{AB} = \begin{bmatrix} \cos(\theta) & -\sin(\theta) & 0 \\ \sin(\theta) & \cos(\theta) & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\dot{R}_{AB} = \begin{bmatrix} -\sin(\theta) & -\cos(\theta) & 0 \\ \cos(\theta) & -\sin(\theta) & 0 \\ 0 & 0 & 0 \end{bmatrix} \dot{\theta}$$

$$R_{AB}^T = \begin{bmatrix} \cos(\theta) & \sin(\theta) & 0 \\ -\sin(\theta) & \cos(\theta) & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\hat{\omega}_{AB} = \begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \dot{\theta}$$

Velocities of 2 points fixed to the same rigid body

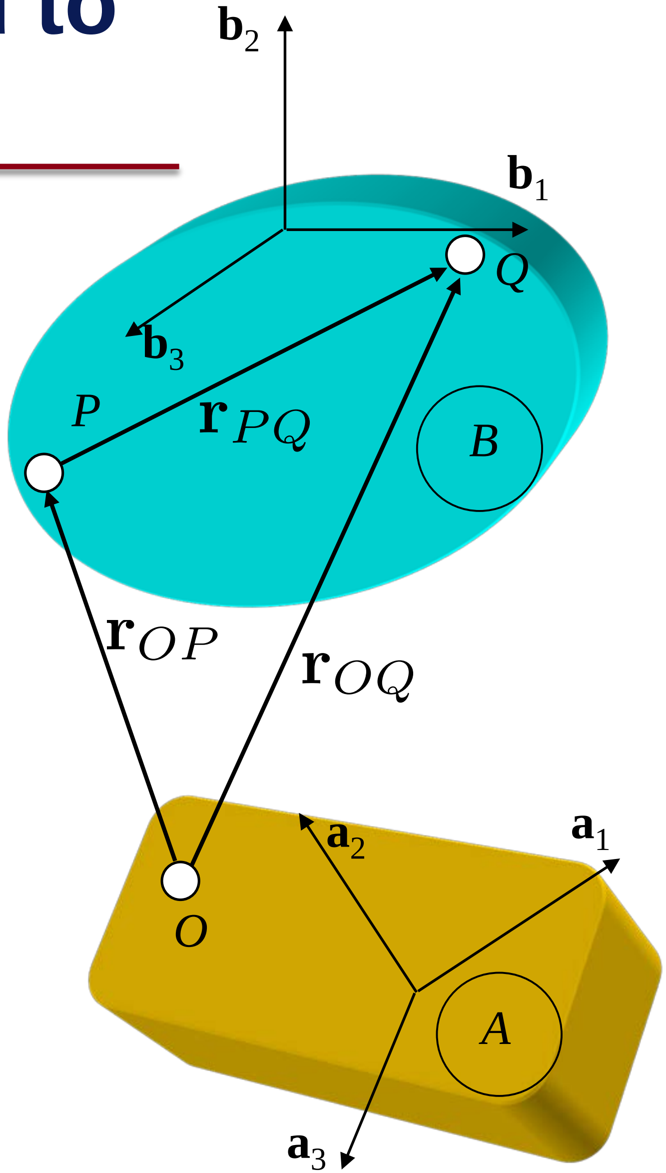
$$\mathbf{r}_{OQ} = \mathbf{r}_{OP} + \mathbf{r}_{PQ}$$

$$\begin{bmatrix} \dot{q}_1 \\ \dot{q}_2 \\ \dot{q}_3 \end{bmatrix} = \begin{bmatrix} \dot{p}_1 \\ \dot{p}_2 \\ \dot{p}_3 \end{bmatrix} + \underbrace{\dot{\mathbf{R}}_{AB}\mathbf{R}_{AB}^T}_{\hat{\omega}_{AB}} \begin{bmatrix} q_1 - p_1 \\ q_2 - p_2 \\ q_3 - p_3 \end{bmatrix}$$

Recall a 3x3 skew symmetric matrix encodes a cross product operation

$$\hat{\omega}_{AB} \begin{bmatrix} q_1 - p_1 \\ q_2 - p_2 \\ q_3 - p_3 \end{bmatrix} = \omega_{AB} \times \mathbf{r}_{PQ}$$

$$\mathbf{v}_Q = \mathbf{v}_P + \omega_{AB} \times \mathbf{r}_{PQ}$$



Acceleration Analysis

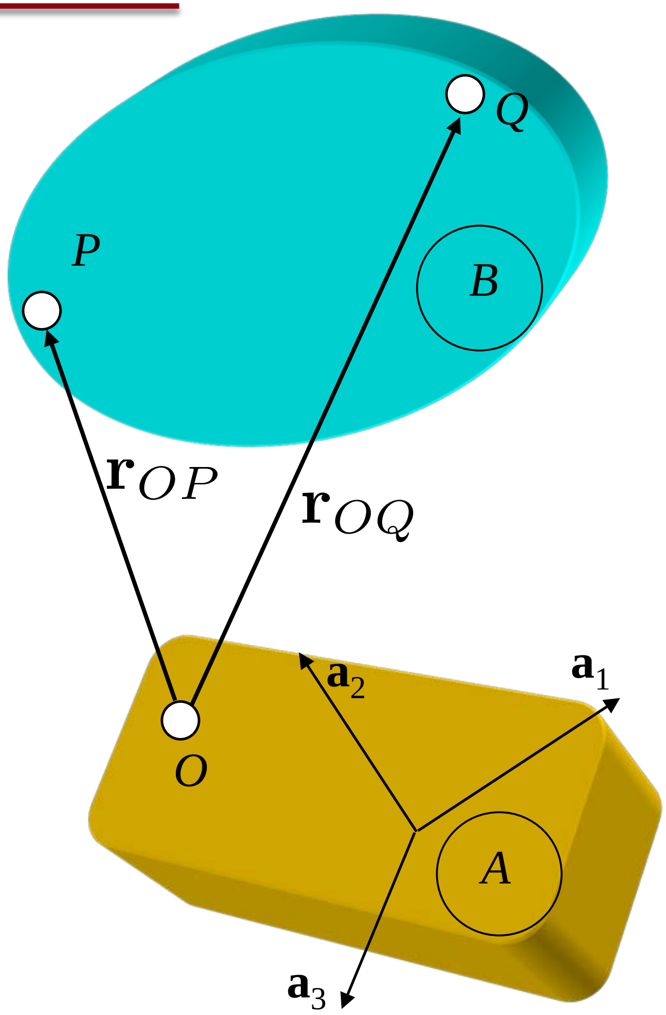
- Acceleration of P and Q in A

$$\mathbf{a}_P = \ddot{p}_1 \mathbf{a}_1 + \ddot{p}_2 \mathbf{a}_2 + \ddot{p}_3 \mathbf{a}_3$$

$$\begin{bmatrix} \ddot{p}_1 \\ \ddot{p}_2 \\ \ddot{p}_3 \end{bmatrix}$$

$$\mathbf{a}_Q = \ddot{q}_1 \mathbf{a}_1 + \ddot{q}_2 \mathbf{a}_2 + \ddot{q}_3 \mathbf{a}_3$$

$$\begin{bmatrix} \ddot{q}_1 \\ \ddot{q}_2 \\ \ddot{q}_3 \end{bmatrix}$$



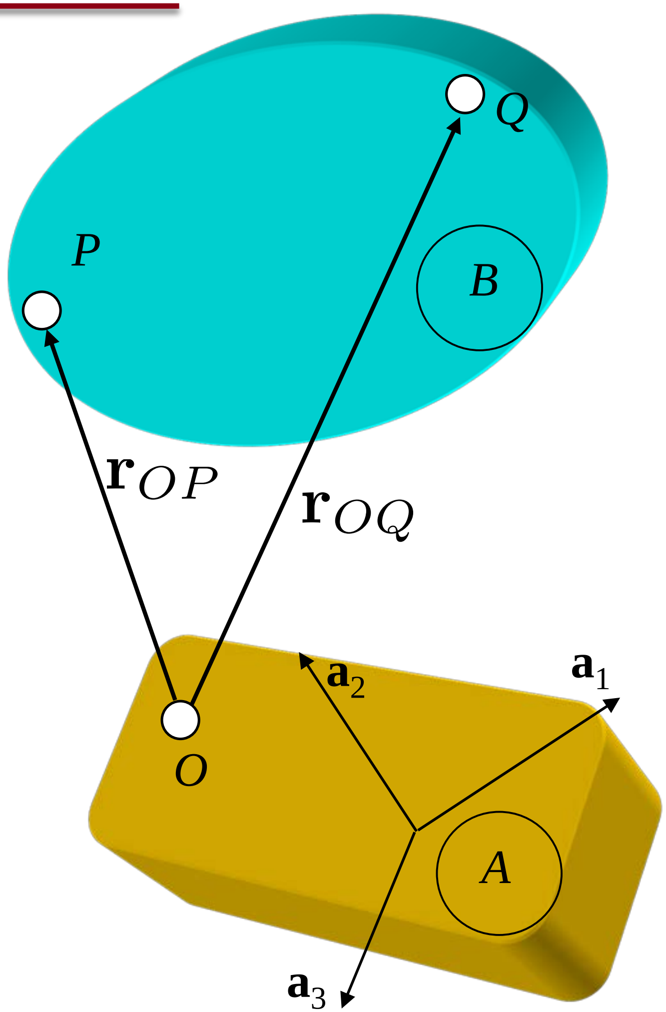
Two Approaches

- Lagrangian Mechanics

  need expressions of kinetic and potential energy, and external forces/moments

- Newtonian Mechanics

  need expressions for accelerations and external forces/moments



Video 1.5
Vijay Kumar and Ani Hsieh

Dynamics and Control
Velocity and Acceleration Analysis: Examples

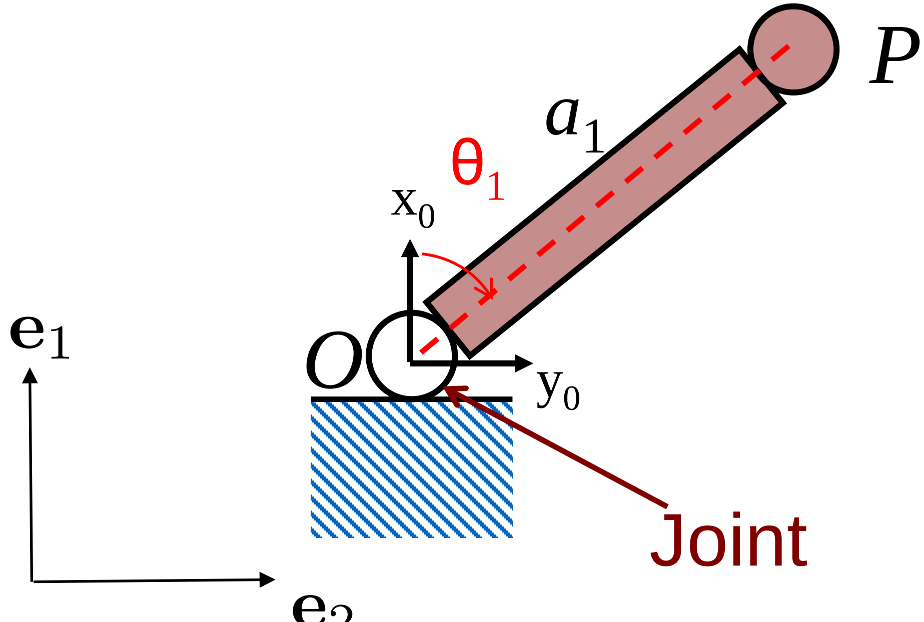
Vijay Kumar and Ani Hsieh
University of Pennsylvania

A one link manipulator

Inertial reference frame E

- Origin O
- Basis vectors $\{\mathbf{e}_1, \mathbf{e}_2, \mathbf{e}_3\}$

- P is fixed to link 1

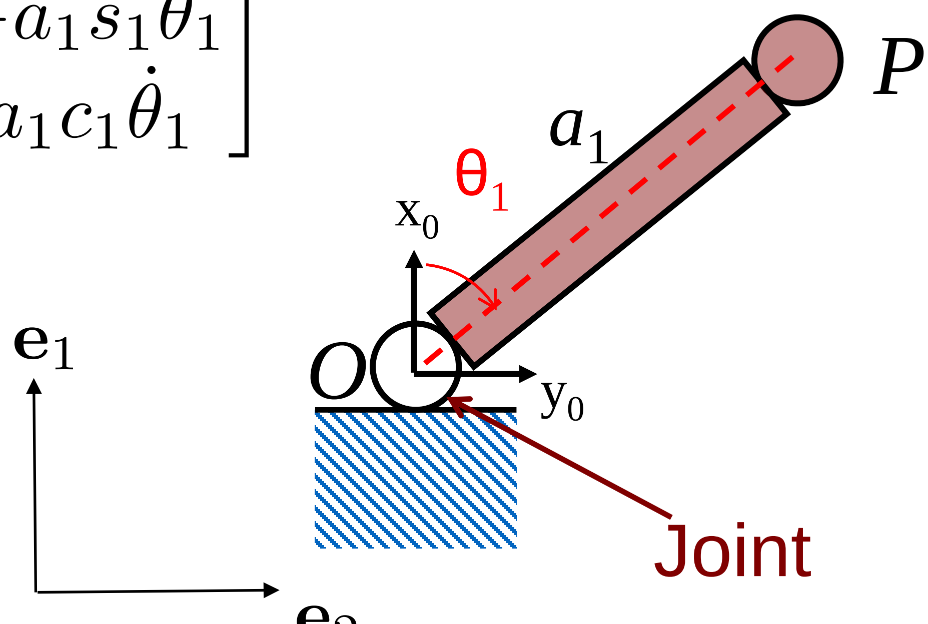


Position and Velocity Vectors

$$\mathbf{r}_{OP} = a_1 \cos\theta_1 \mathbf{e}_1 + a_1 \sin\theta_1 \mathbf{e}_2$$

$$\mathbf{v}_{OP} = -a_1 s_1 \dot{\theta}_1 \mathbf{e}_1 + a_1 c_1 \dot{\theta}_1 \mathbf{e}_2$$

$$\mathbf{v}_{OP} = \begin{bmatrix} -a_1 s_1 \dot{\theta}_1 \\ a_1 c_1 \dot{\theta}_1 \end{bmatrix}$$

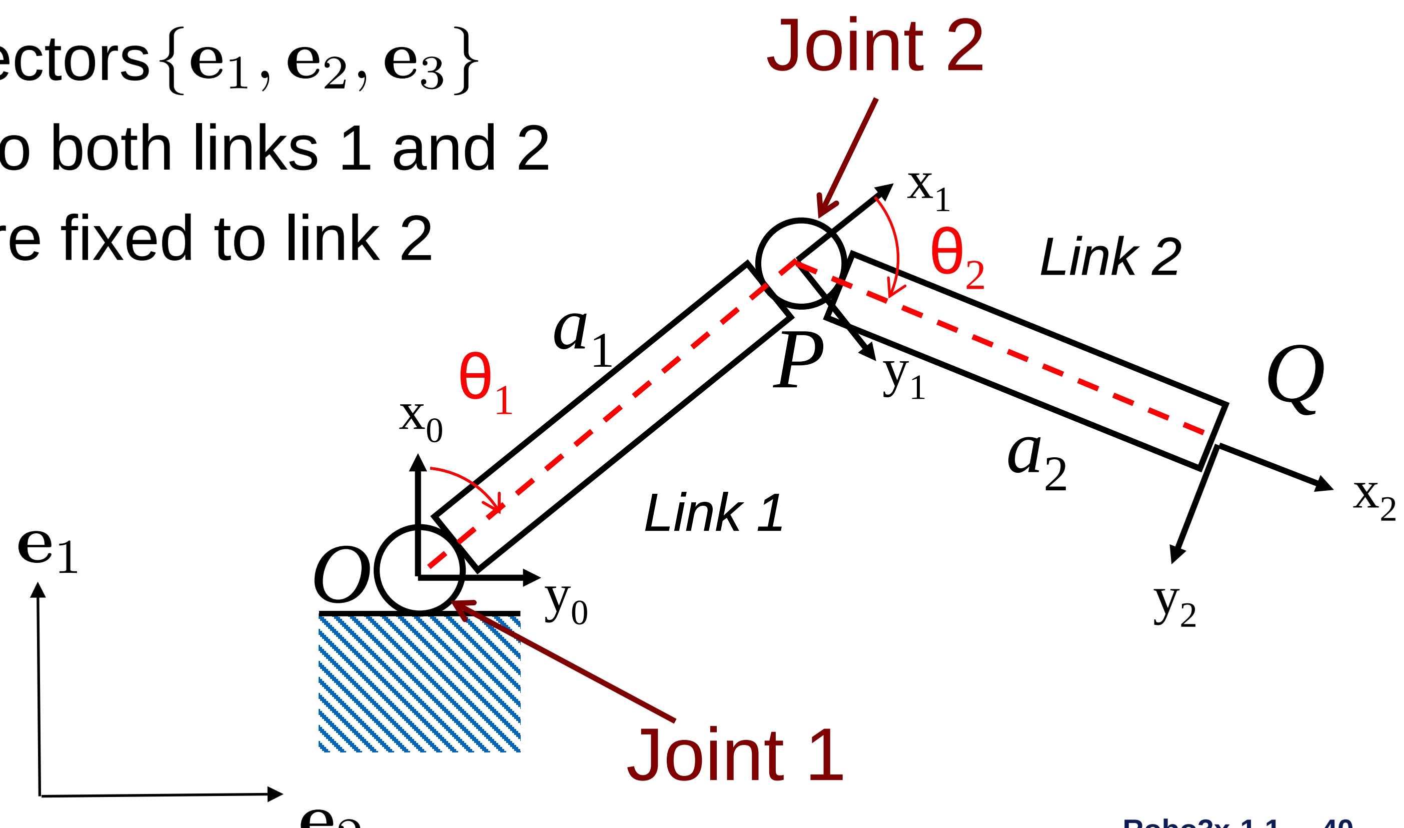


Two Link Manipulator

Inertial reference frame E

- Origin O
- Basis vectors $\{\mathbf{e}_1, \mathbf{e}_2, \mathbf{e}_3\}$

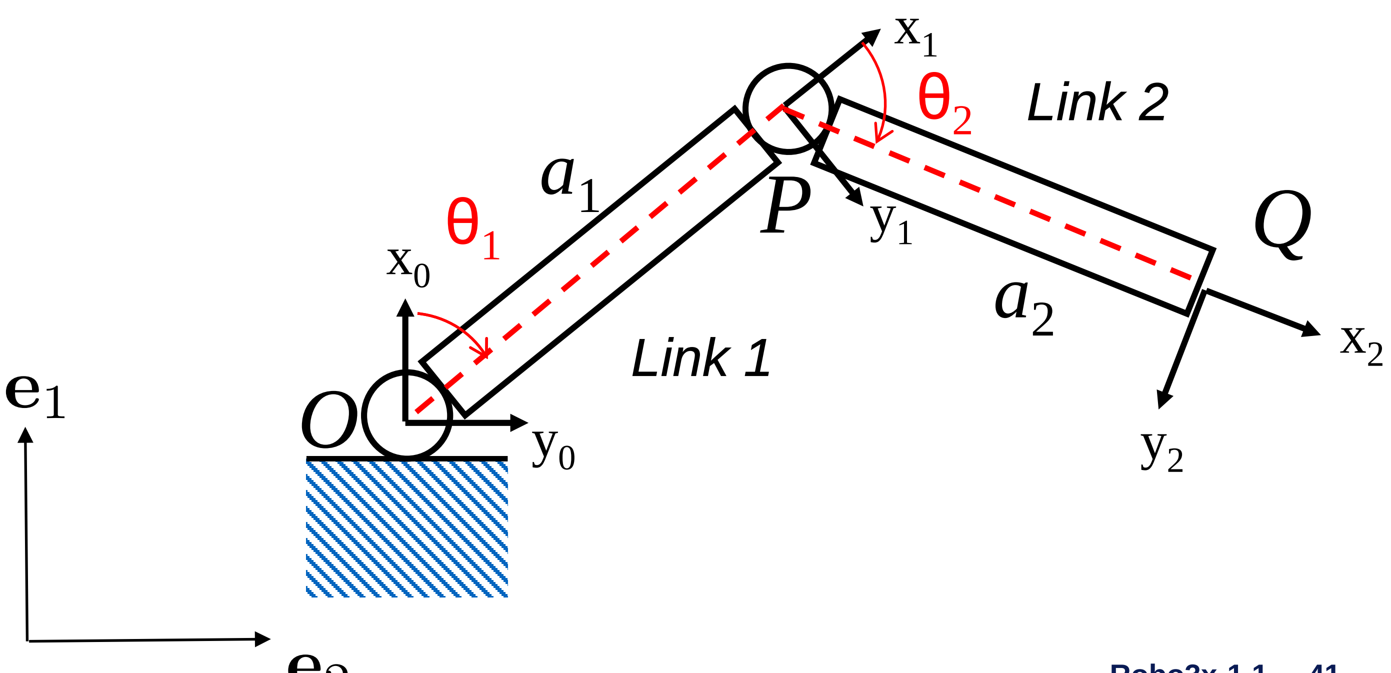
- P is fixed to both links 1 and 2
- P and Q are fixed to link 2



Position Vectors

$$\mathbf{r}_{OP} = a_1 \cos\theta_1 \mathbf{e}_1 + a_1 \sin\theta_1 \mathbf{e}_2$$

$$\mathbf{r}_{PQ} = a_2 \cos(\theta_1 + \theta_2)\mathbf{e}_1 + a_2 \sin(\theta_1 + \theta_2)\mathbf{e}_2$$



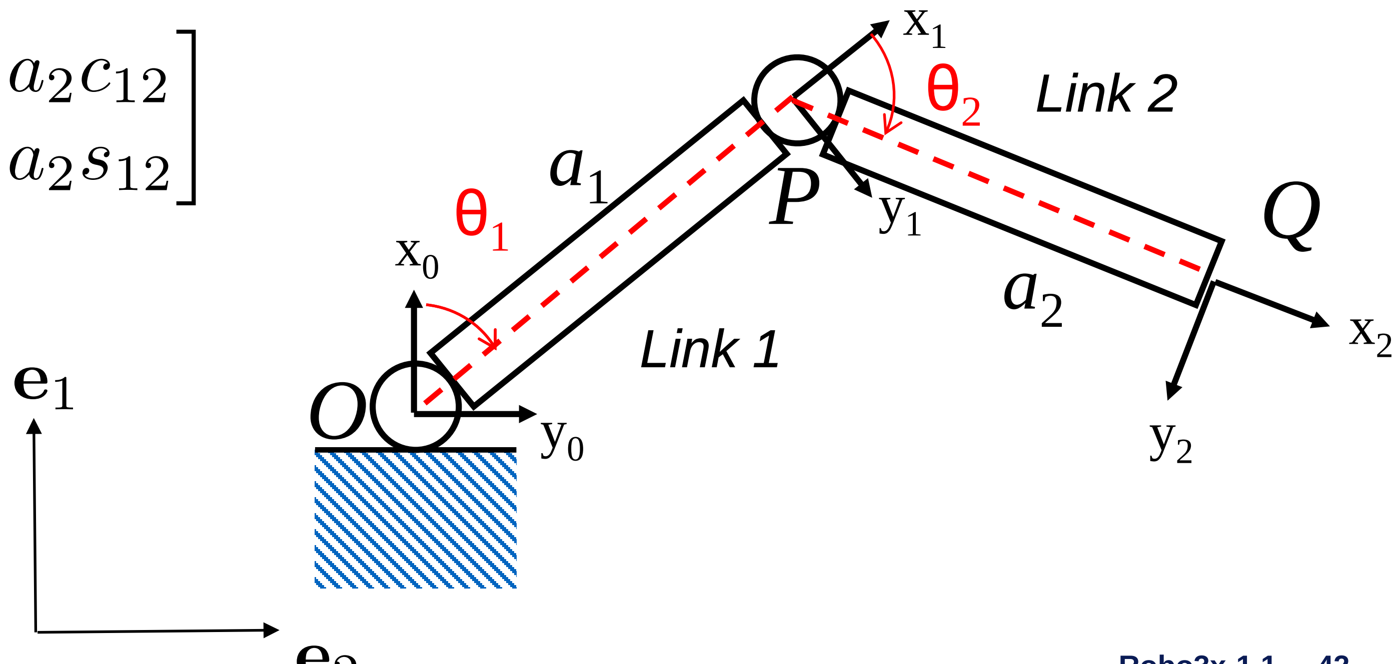
Position Vectors

$$\mathbf{r}_{OP} = a_1 \cos\theta_1 \mathbf{e}_1 + a_1 \sin\theta_1 \mathbf{e}_2$$

$$\mathbf{r}_{PQ} = a_2 \cos(\theta_1 + \theta_2)\mathbf{e}_1 + a_2 \sin(\theta_1 + \theta_2)\mathbf{e}_2$$

$$\mathbf{r}_{OQ} = (a_1 c_1 + a_2 c_{12})\mathbf{e}_1 + (a_1 s_1 + a_2 s_{12})\mathbf{e}_2$$

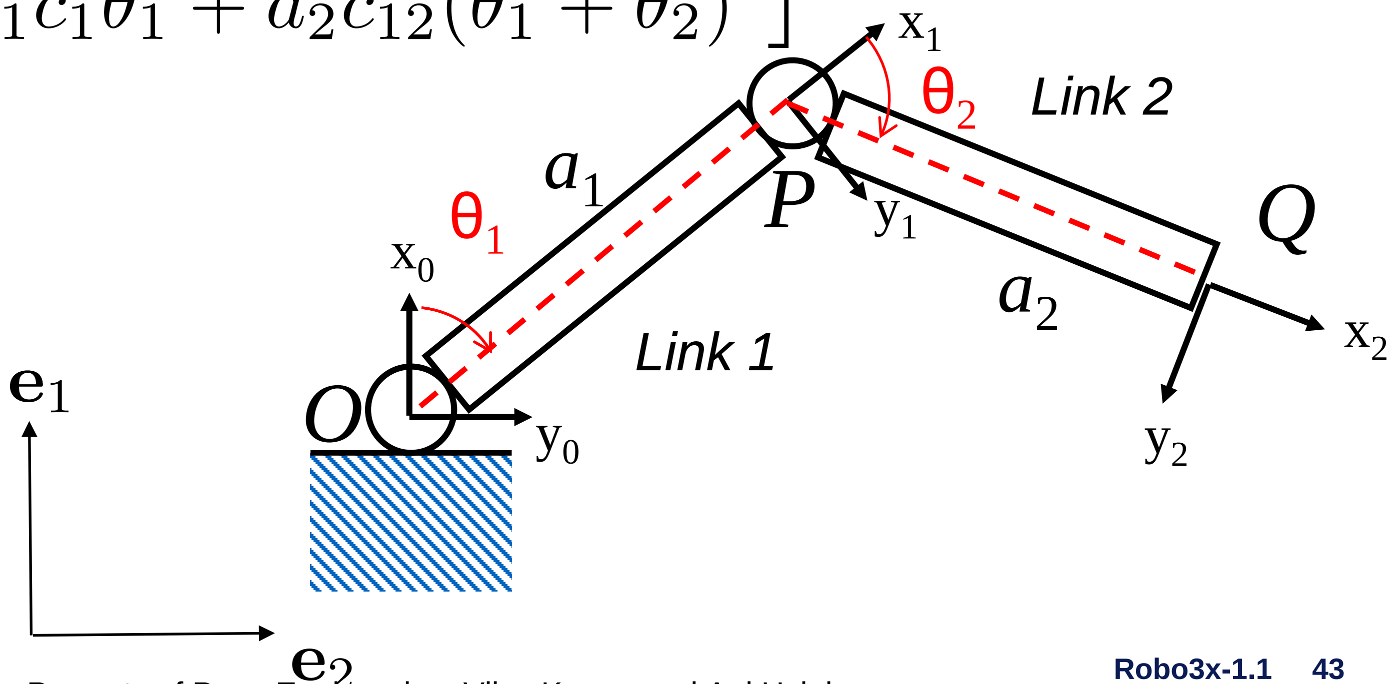
$$\begin{bmatrix} a_1 c_1 + a_2 c_{12} \\ a_1 s_1 + a_2 s_{12} \end{bmatrix}$$



Velocity of point Q in the inertial frame

$$\mathbf{r}_{OQ} = \begin{bmatrix} a_1 c_1 + a_2 c_{12} \\ a_1 s_1 + a_2 s_{12} \end{bmatrix}$$

$$\mathbf{v}_{OQ} = \begin{bmatrix} -a_1 s_1 \dot{\theta}_1 - a_2 s_{12}(\dot{\theta}_1 + \dot{\theta}_2) \\ a_1 c_1 \dot{\theta}_1 + a_2 c_{12}(\dot{\theta}_1 + \dot{\theta}_2) \end{bmatrix}$$



Velocity of point Q in the inertial frame – alternative approach

$$\mathbf{v}_Q = \mathbf{v}_P + \omega_{F_0 F_2} \times \mathbf{r}_{PQ}$$

$$(\dot{\theta}_1 + \dot{\theta}_2)\mathbf{e}_3$$

$$\mathbf{v}_P = \mathbf{v}_O + \omega_{F_0 F_1} \times \mathbf{r}_{OP}$$

$$\dot{\theta}_1 \mathbf{e}_3$$

$$\begin{bmatrix} -a_1 s_1 \dot{\theta}_1 - a_2 s_{12}(\dot{\theta}_1 + \dot{\theta}_2) \\ a_1 c_1 \dot{\theta}_1 + a_2 c_{12}(\dot{\theta}_1 + \dot{\theta}_2) \end{bmatrix}$$