ITMO UNIVERSITY

How to Win Coding Competitions: Secrets of Champions

Week 2: Computational complexity. Linear data structures
Lecture 3: Vector

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All other operations can be performed in linear time.

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- vector of 3 elements

3	6	2	

Extra space

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- vector of 5 elements

3	6	2	5	8			

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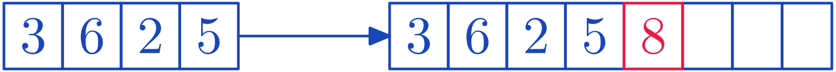
Let's allocate array of doubled size, copy all elements and insert new element.



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  - we'll call such operations *long* insert operations

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- total time of these insert operations is $O(1 + 2 + \ldots + 2^{\lfloor log_2 k \rfloor}) = O(2 \times 2^{\lfloor log_2 k \rfloor}) = O(k)$
- that means that *amortized* complexity of every insert operation is $O(1)$

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- let's decrease size of array if we have less then $\frac{1}{3}$ of elements in use
- it can be proved that even in this case *amortized* complexity of all operations is $O(1)$

Thank you
for your attention!