



ITMO UNIVERSITY

How to Win Coding Competitions: Secrets of Champions

Week 4: Algorithms on Graphs Lecture 5: Topological sort

Maxim Buzdalov
Saint Petersburg 2016

Assume you are solving a problem from a competition.
What should you do?

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What should you do?

Write the solution template

Read the statement

Invent the algorithm

Get the names of I/O files

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Submit the solution

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And in which order?

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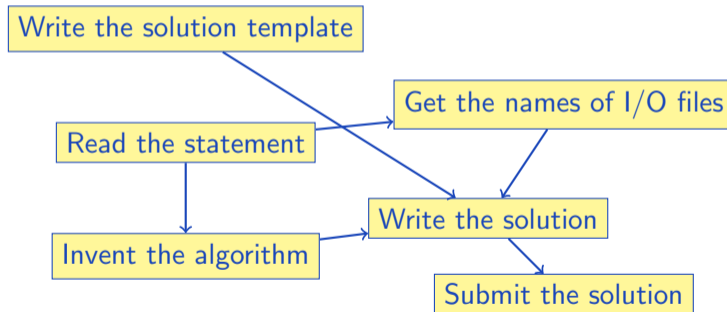
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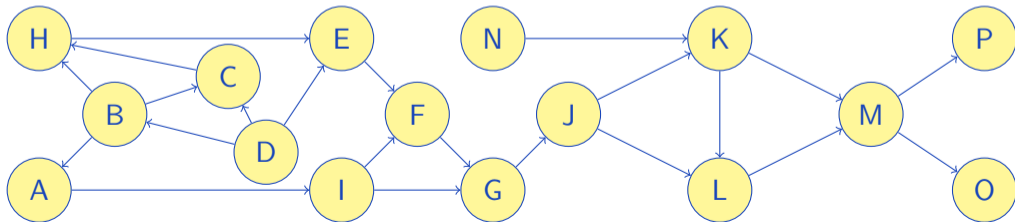
And in which order?



- For the directed **acyclic** graph $G = \langle V, E \rangle$,
a **topological sort** is an assignment $I : V \rightarrow [1; N]$ of indices to vertices, such that:
- ▶ for every two distinct vertices u, v it holds that $I(u) \neq I(v)$
 - ▶ for each edge $(u, v) \in E$ it holds that $I(u) < I(v)$

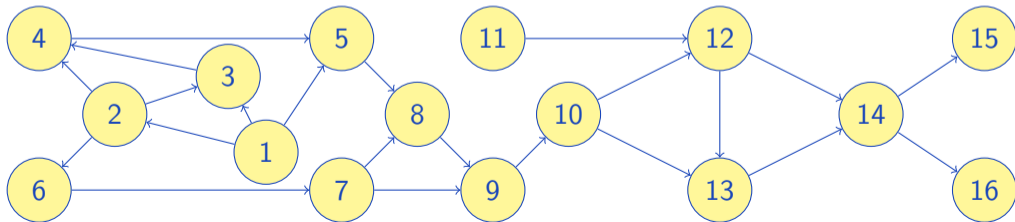
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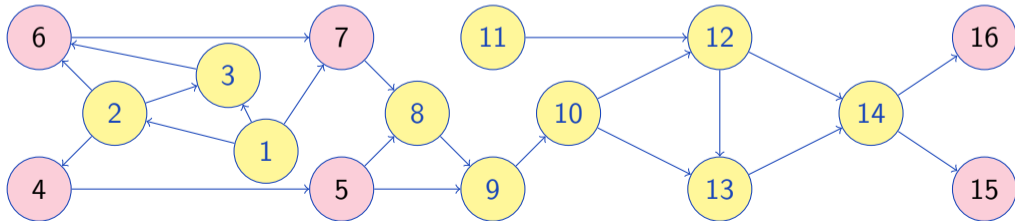
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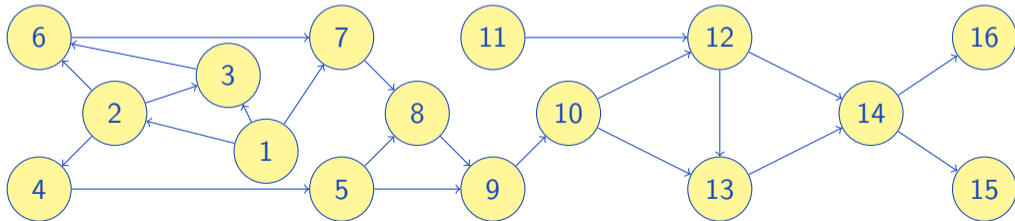


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Sometimes it is convenient to have an array $o[i]$ such that:

- ▶ $o[I(v)] = v$, or, just the same, $I(o[i]) = i$

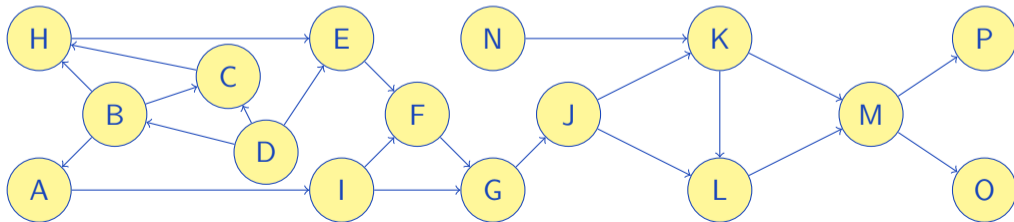


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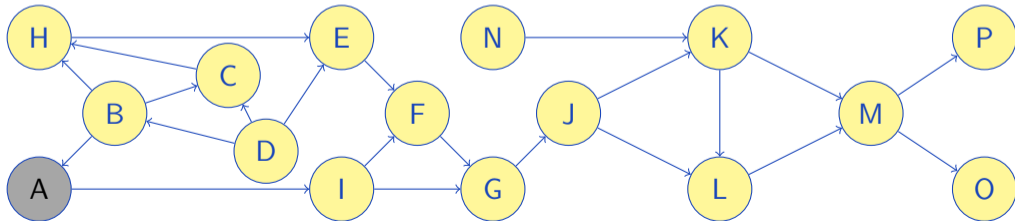
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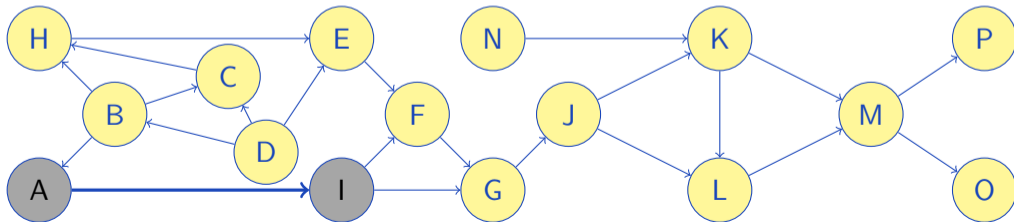
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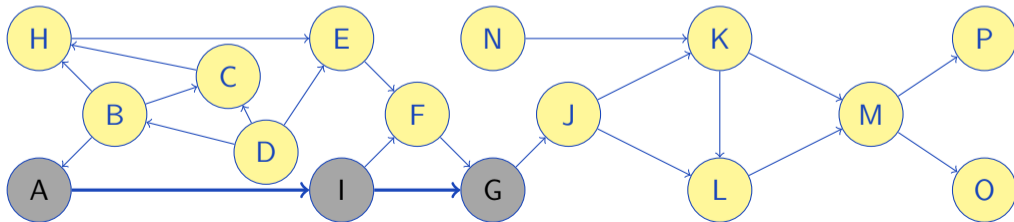
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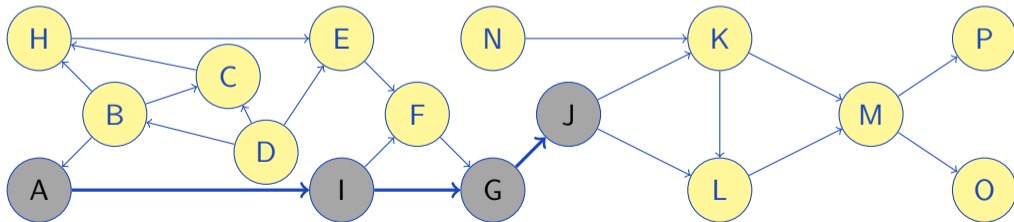
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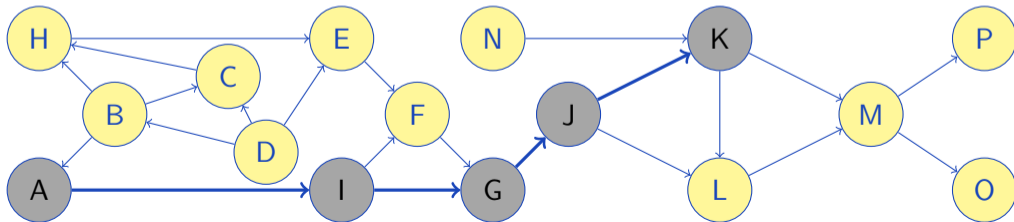
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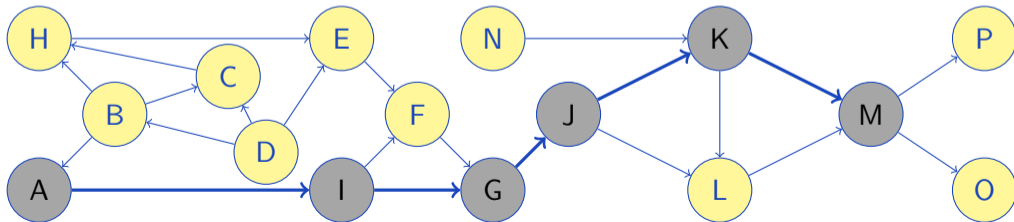
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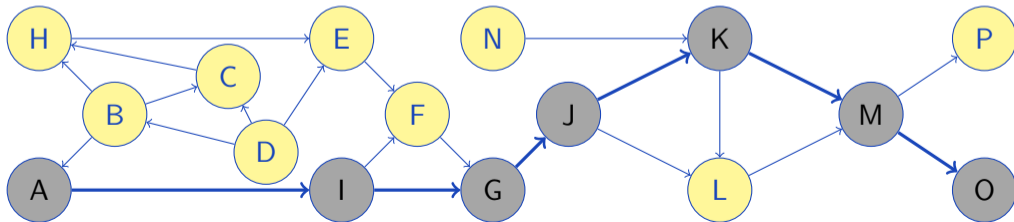
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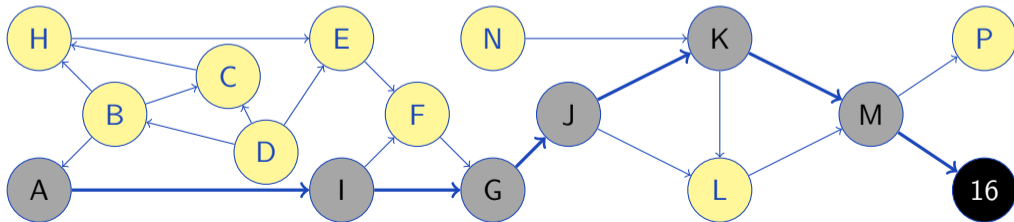
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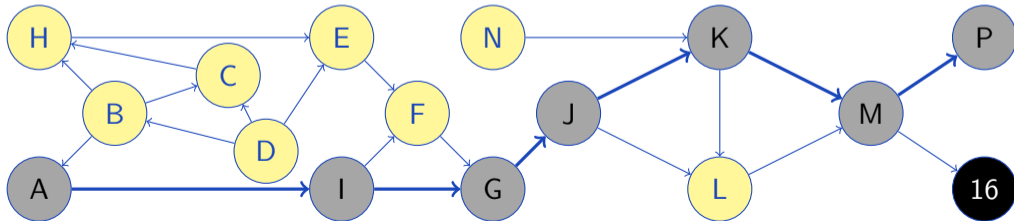
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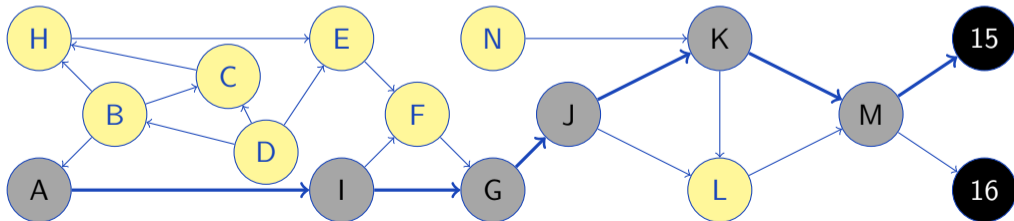
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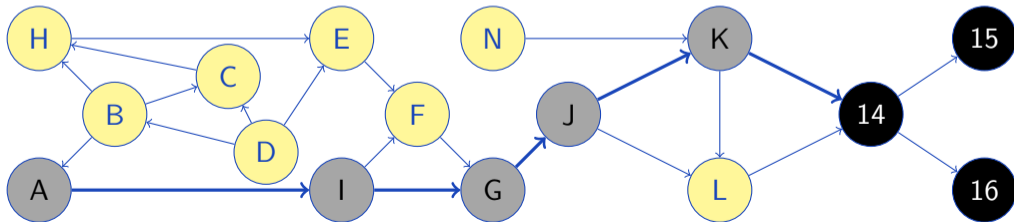
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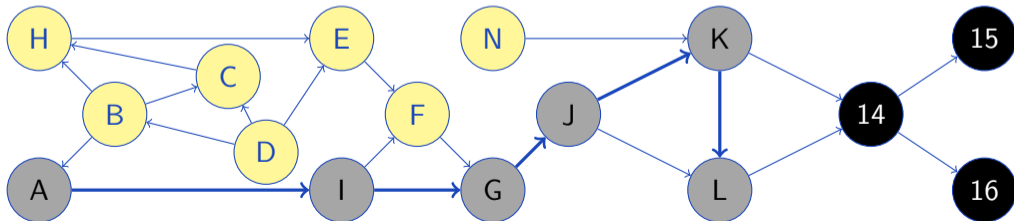
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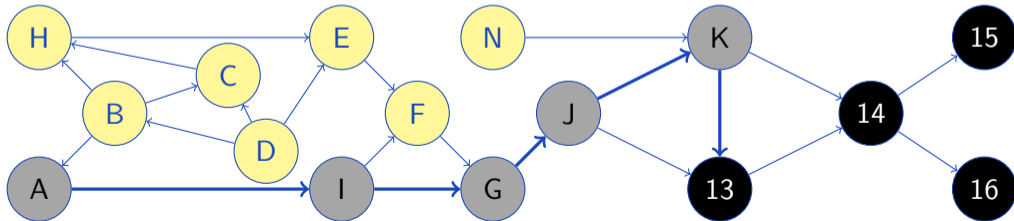
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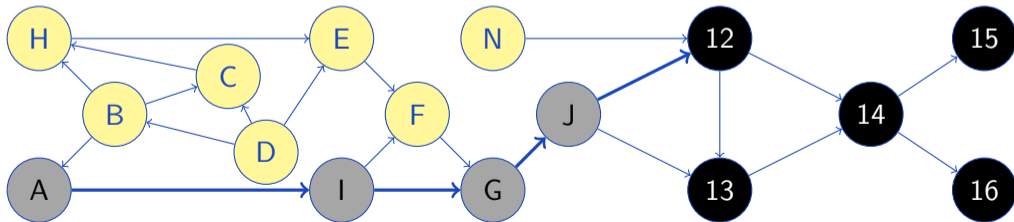
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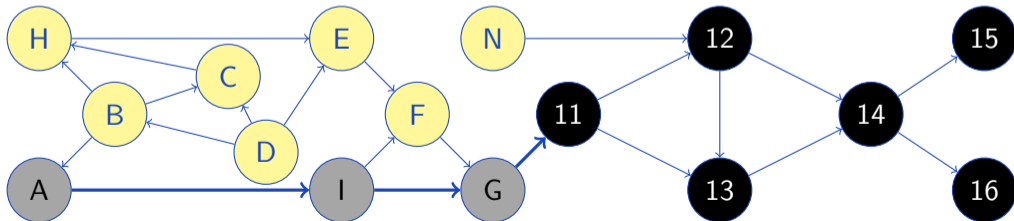
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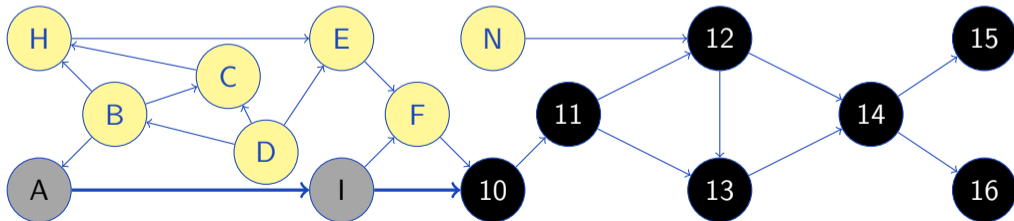
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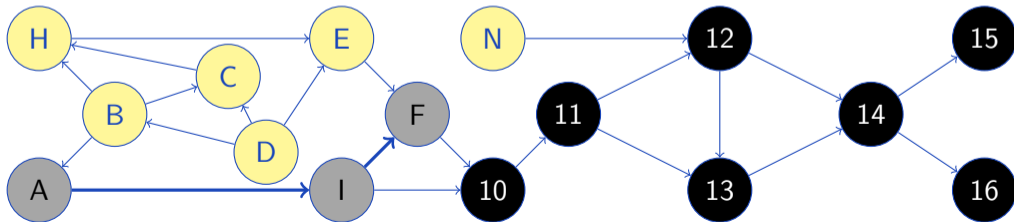
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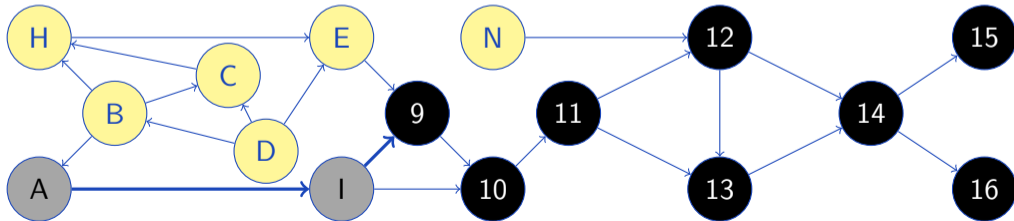


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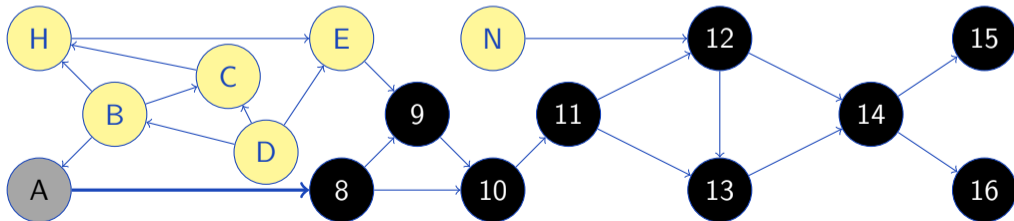
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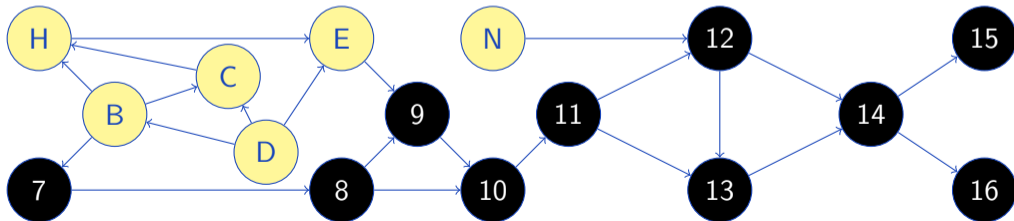
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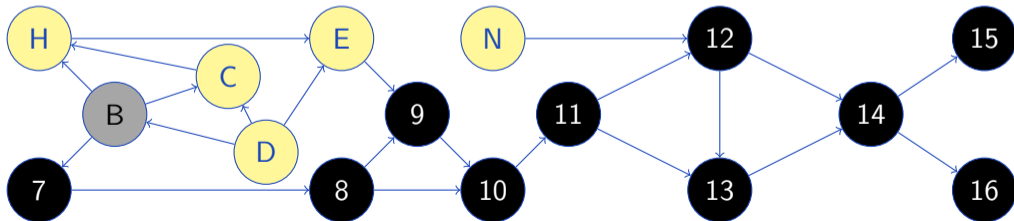
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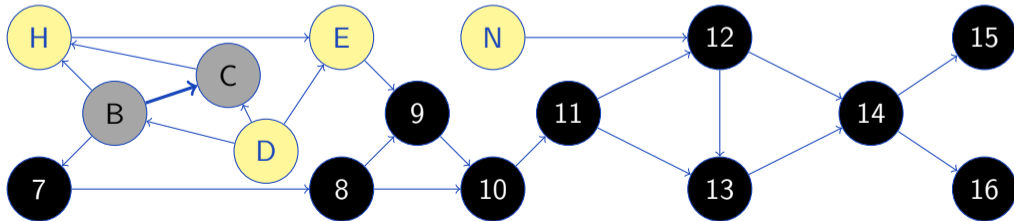
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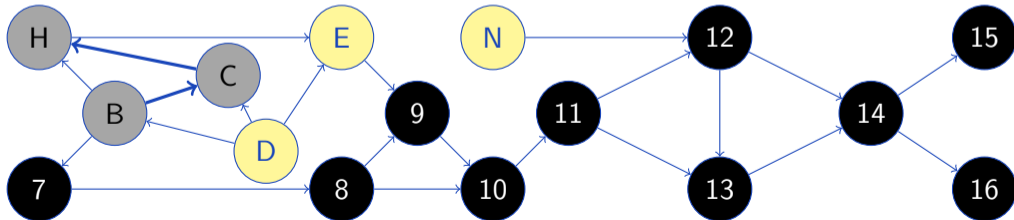
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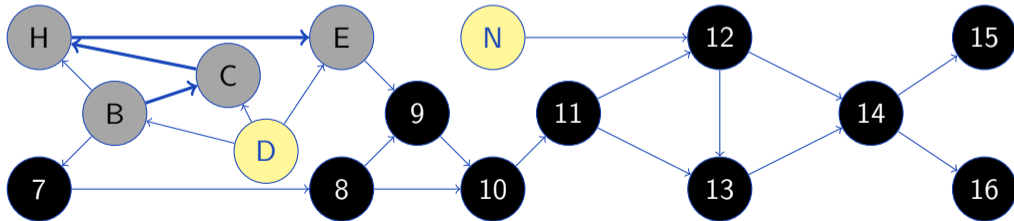
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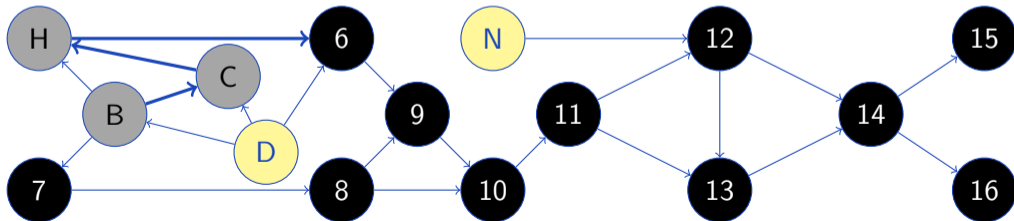


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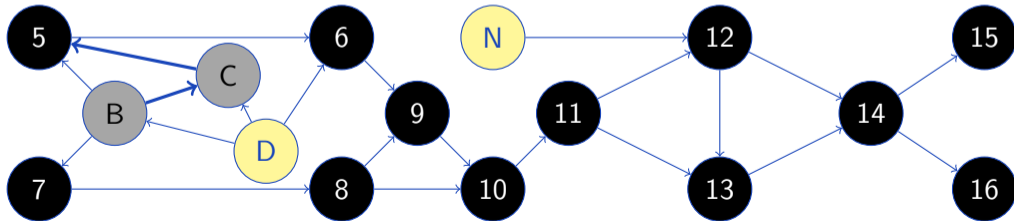
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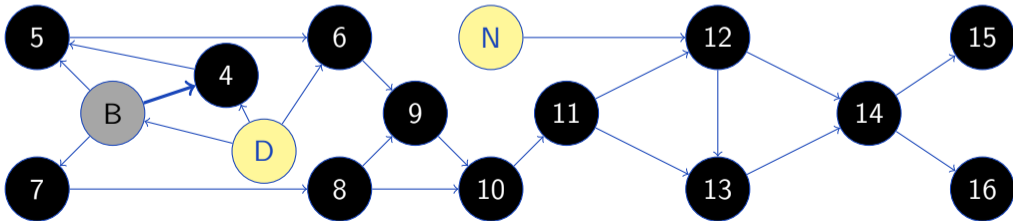


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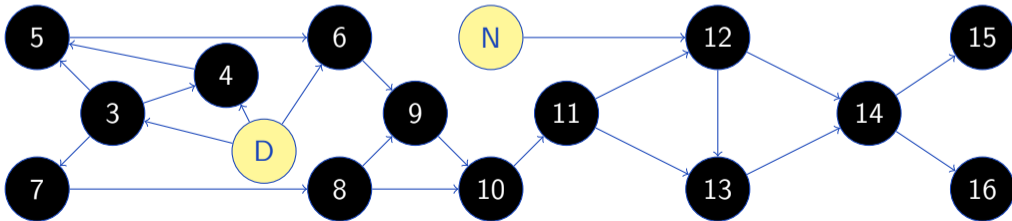
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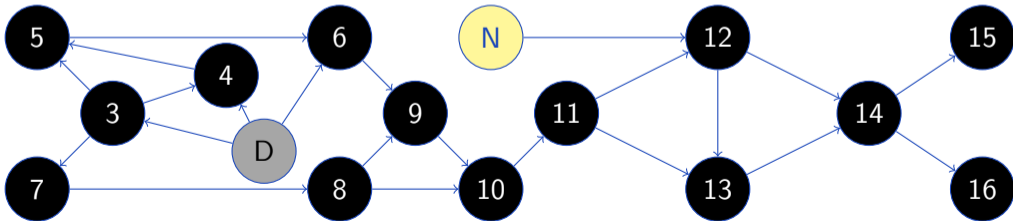
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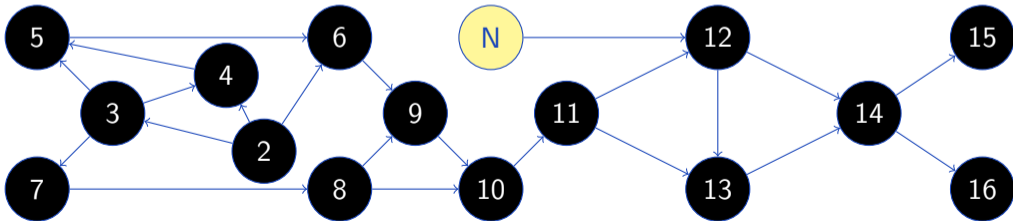
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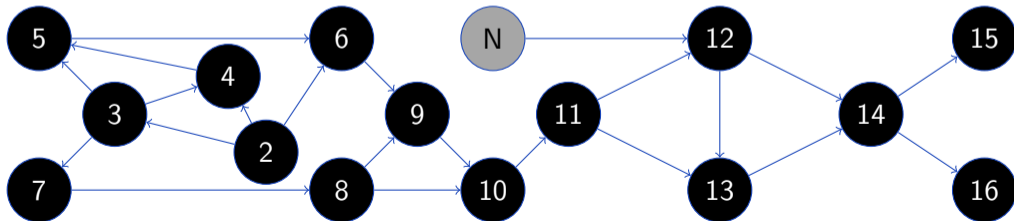
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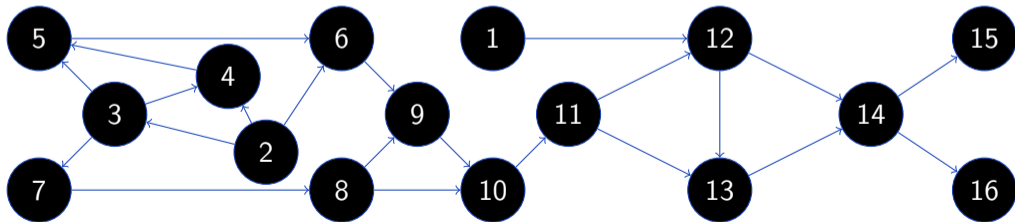
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 - ▶ So we may use topological sort to **find** the strong connectivity components

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procedure CONDENSATION( $V, E$ )  
  TOPOSORT( $V, E$ )  
   $A^R(v) = \{u \mid (v, u) \in E\}$   
   $C \leftarrow \{-1\}$   
   $K \leftarrow 1$   
  for  $i$  from 1 to  $|V|$  do  
    if  $C[o[i]] = -1$  then  
      DFS( $o[i], A^R, K$ )  
       $K \leftarrow K + 1$   
    end if  
  end for  
end procedure
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procedure DFS( $v, A, K$ )  
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- ▶ Second DFS traverses vertices in the order of topological sort
- ▶ ... and does it using reversed edges

