

CHAPTER

9**Exponential Models****Chapter Outline**

- 9.1** **EXPONENTIAL GROWTH**
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9.1 Exponential Growth

Learning Objectives

- Graph an exponential growth function.
- Compare graphs of exponential growth functions.
- Solve real-world problems involving exponential growth.

Introduction

Exponential functions are different than other functions you have seen before because now the variable appears as the exponent (or power) instead of the base. In this section, we will be working with functions where the base is a constant number and the exponent is the variable.

In general, the exponential function takes the form:

$$y = A \cdot b^x$$

where A is the **initial value**, and b is the **growth factor**, the amount that y gets multiplied by each time the value of x increases by 1. The growth factor can also be expressed as

$$b = 1 + r$$

where r is the rate of change. A population that grows by 5% each year would have a growth factor of

$$b = 1 + .05 = 1.05.$$

Example A

A colony of bacteria has a population of three thousand at noon on Sunday. During the next week, the colony's population doubles every day. What is the population of the bacteria colony at noon on Saturday? In this example, we want to describe something that doubles every time x increased by one. You could also think about this problem as bacteria having a rate of change of 100% every day, or $b = 1 + 1 = 2$.

The exponential function, therefore, is $y = 2^x$.

Let's make a table of values and calculate the population each day.

TABLE 9.1:

Day	0 (Sun)	1 (Mon)	2 (Tues)	3 (Wed)	4 (Thurs)	5 (Fri)	6 (Sat)
Population (in thousands)	3	6	12	24	48	96	192

To get the population of bacteria for the next day we simply multiply the current day's population by 2.

We start with a population of 3 (thousand):	$P = 3$
To find the population on Monday we double	$P = 3 \cdot 2$
The population on Tuesday will be double that on Monday	$P = 3 \cdot 2 \cdot 2$
The population on Wednesday will be double that on Tuesday	$P = 3 \cdot 2 \cdot 2 \cdot 2$
Thursday is double that on Wednesday	$P = 3 \cdot 2 \cdot 2 \cdot 2 \cdot 2$
Friday is double that on Thursday	$P = 3 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2$
Saturday is double that on Friday	$P = 3 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2$

You can see that this function describes a population that is multiplied by 2 each time a day passes.

If we define x as the number of days since Sunday at noon, then we can write the following.

$$P = 3 \cdot 2^x$$

This is a formula that we can use to calculate the population on any day.

For instance, the population on Saturday at noon will be $P = 3 \cdot 2^6 = 3 \cdot 64 = 192$ (thousand) bacteria.

We used $x = 6$, since Saturday at noon is six days after Sunday at noon.

Graphing Exponential Functions

Let's start this section by graphing some exponential functions. Since we don't yet know any special properties of exponential functions, we will graph using a table of values.

Example B

Graph the equation $y = 2^x$.

Solution

Let's begin by making a table of values that includes both negative and positive values of x .

To evaluate the positive values of x , we just plug into the function and evaluate.

$x = 1,$	$y = 2^1 = 2$
$x = 2,$	$y = 2^2 = 2 \cdot 2 = 4$
$x = 3,$	$y = 2^3 = 2 \cdot 2 \cdot 2 = 8$

TABLE 9.2:

x	y
-3	$\frac{1}{8}$
-2	$\frac{1}{4}$

TABLE 9.2: (continued)

x	y
-1	$\frac{1}{2}$
0	1
1	2
2	4
3	8

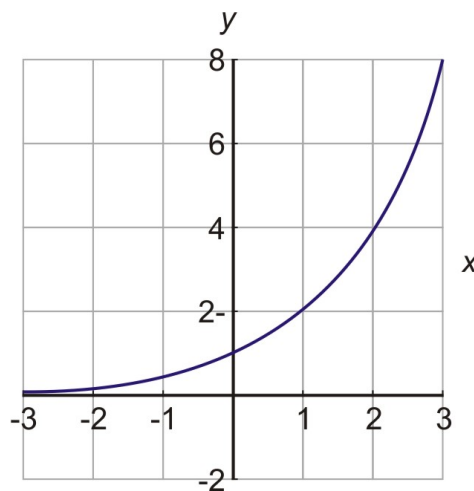
For $x = 0$, we must remember that a number to the power 0 is always 1.

$$x = 0, \quad y = 2^0 = 1$$

To evaluate the negative values of x , we must remember that x to a negative power means one over x to the same positive power.

$$\begin{aligned} x = -1, & \quad y = 2^{-1} = \frac{1}{2^1} = \frac{1}{2} \\ x = -2, & \quad y = 2^{-2} = \frac{1}{2^2} = \frac{1}{4} \\ x = -3, & \quad y = 2^{-3} = \frac{1}{2^3} = \frac{1}{8} \end{aligned}$$

When we plot the points on the coordinate axes we get the graph below. Exponentials always have this basic shape. That is, they start very small and then, once they start growing, they grow faster and faster, and soon they become extremely big!



You may have heard people say that something is growing **exponentially**. This implies that the growth is very quick. An exponential function actually starts slow, but then grows faster and faster all the time. Specifically, our function y above doubled each time we increased x by one.

This is the definition of exponential growth. There is a consistent fixed period during which the function will double or triple, or quadruple. The change is always a fixed proportion.

Compare Graphs of Exponential Growth Functions

Let's graph a few more exponential functions and see what happens as we change the constants A and b in the functions. The basic shape of the exponential function should stay the same, but the curve may become steeper or shallower depending on the constants we are using.

We mentioned that the general form of the exponential function is $y = A \cdot b^x$ where A is the initial amount and b is the factor that the amount gets multiplied by each time x is increased by one. Let's see what happens for different values of A .

Example C

Graph the exponential function $y = 3 \cdot 2^x$ and compare with the graph of $y = 2^x$.

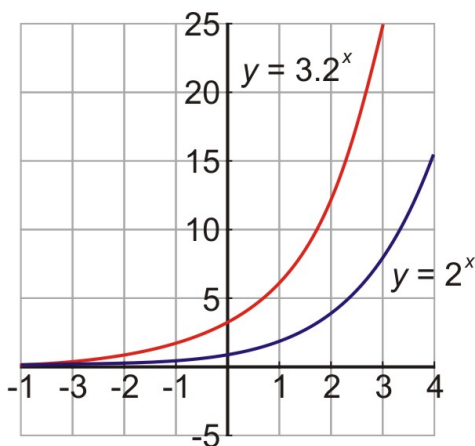
Solution

Let's make a table of values for $y = 3 \cdot 2^x$.

TABLE 9.3:

x	$y = 3 \cdot 2^x$
-2	$y = 3 \cdot 2^{-2} = 3 \cdot \frac{1}{2^2} = \frac{3}{4}$
-1	$y = 3 \cdot 2^{-1} = 3 \cdot \frac{1}{2^1} = \frac{3}{2}$
0	$y = 3 \cdot 2^0 = 3$
1	$y = 3 \cdot 2^1 = 6$
2	$y = 3 \cdot 2^2 = 3 \cdot 4 = 12$
3	$y = 3 \cdot 2^3 = 3 \cdot 8 = 24$

Now let's use this table to graph the function.



We can see that the function $y = 3 \cdot 2^x$ is bigger than function $y = 2^x$. In both functions, the value of y doubled every time x increases by one. However, $y = 3 \cdot 2^x$ “starts” with a value of 3, while $y = 2^x$ “starts” with a value of 1, so it makes sense that $y = 3 \cdot 2^x$ would be bigger as its values of y keep getting doubled.

You might think that if the initial value A is less than one, then the corresponding exponential function would be less than $y = 2^x$. This is indeed correct. Let's see how the graphs compare for $A = \frac{1}{3}$.

Example D

Graph the exponential function $y = \frac{1}{3} \cdot 2^x$ and compare with the graph of $y = 2^x$.

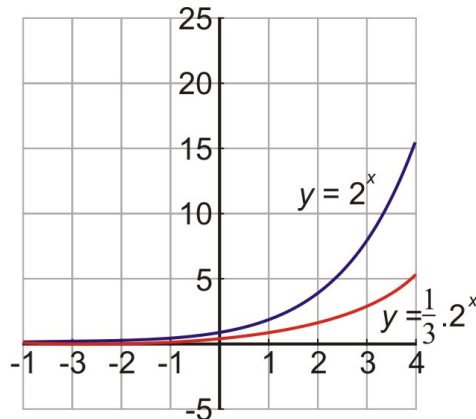
Solution

Let's make a table of values for $y = \frac{1}{3} \cdot 2^x$.

TABLE 9.4:

x	$y = \frac{1}{3} \cdot 2^x$
-2	$y = \frac{1}{3} \cdot 2^{-2} = \frac{1}{3} \cdot \frac{1}{2^2} = \frac{1}{12}$
-1	$y = \frac{1}{3} \cdot 2^{-1} = \frac{1}{3} \cdot \frac{1}{2^1} = \frac{1}{6}$
0	$y = \frac{1}{3} \cdot 2^0 = \frac{1}{3}$
1	$y = \frac{1}{3} \cdot 2^1 = \frac{2}{3}$
2	$y = \frac{1}{3} \cdot 2^2 = \frac{1}{3} \cdot 4 = \frac{4}{3}$
3	$y = \frac{1}{3} \cdot 2^3 = \frac{1}{3} \cdot 8 = \frac{8}{3}$

Now let's use this table to graph the function.



As expected, the exponential function $y = \frac{1}{3} \cdot 2^x$ is smaller than the exponential function $y = 2^x$.

Now, let's compare exponential functions whose bases are different.

The function $y = 2^x$ has a base of 2. That means that the value of y doubles every time x is increased by 1.

The function $y = 3^x$ has a base of 3. That means that the value of y triples every time x is increased by 1.

The function $y = 5^x$ has a base of 5. That means that the value of y gets multiplied by a factor of 5 every time x is increased by 1.

The function $y = 10^x$ has a base of 10. That means that the value of y gets multiplied by a factor of 10 every time x is increased by 1.

What do you think will happen as the base number is increased? Let's find out.

Example E

Now let's explore what happens when we change the value of b . Graph the following exponential functions of the same graph $y = 2^x, y = 3^x, y = 5^x, y = 10^x$.

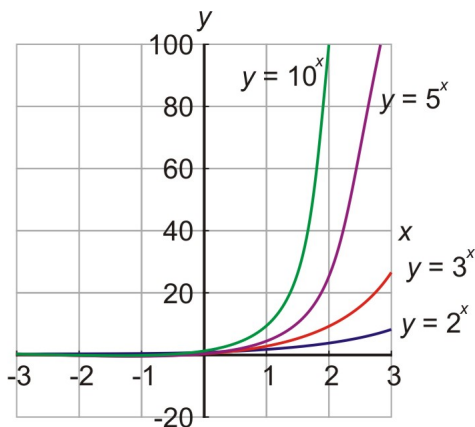
Solution

To graph these functions we should start by making a table of values for each of them.

TABLE 9.5:

x	$y = 2^x$	$y = 3^x$	$y = 5^x$	$y = 10^x$
-2	$\frac{1}{4}$	$\frac{1}{9}$	$\frac{1}{25}$	$\frac{1}{100}$
-1	$\frac{1}{2}$	$\frac{1}{3}$	$\frac{1}{5}$	$\frac{1}{10}$
0	1	1	1	1
1	2	3	5	10
2	4	9	25	100
3	8	27	125	1000

Now let's graph these functions.



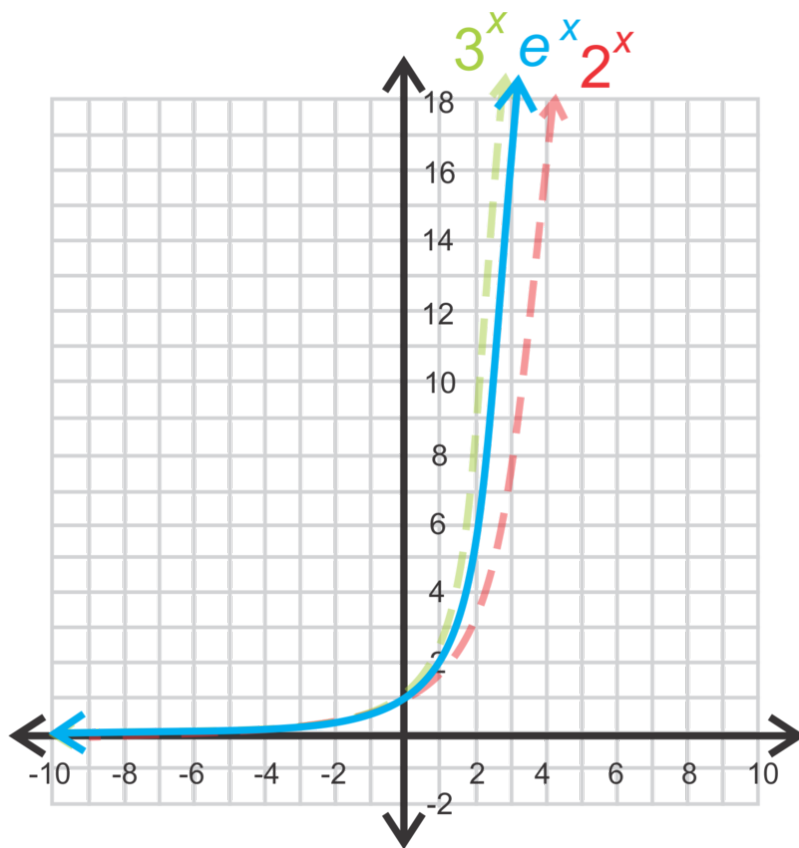
Notice that for $x = 0$ the values for all the functions are equal to 1. This means that the initial value of the functions is the same and equal to 1. Even though all the functions start at the same value, they increase at different rates. We can see that the bigger the base is the faster the values of y will increase. It makes sense that something that triples each time will increase faster than something that just doubles each time.

Growth with the natural base, e

Graph $y = e^x$. Identify the asymptote, y -intercept, domain and range.

Solution

As you would expect, the graph of e^x will curve between 2^x and 3^x .



The asymptote is $y = 0$ and the y -intercept is $(0, 1)$ because anything to the zero power is one. The domain is all real numbers and the range is all positive real numbers; $y > 0$.

Solve Real-World Problems Involving Exponential Growth

We will now examine some real-world problems where exponential growth occurs.

Example F

The population of a town is estimated to increase by 15% per year. The population today is 20 thousand. Make a graph of the population function and find out what the population will be ten years from now.

Solution

First, we need to write a function that describes the population of the town. The general form of an exponential function is.

$$y = A \cdot b^x$$

Define y as the population of the town.

Define x as the number of years from now.

A is the initial population, so $A = 20$ (thousand)

Finally, we must find what b (the growth factor) is. We are told that the population increases by 15% each year. This means that the value of b in our exponential equation would be

$$b = 1 + r = 1 + 0.15 = 1.15$$

Here is another way to reach the value of b : In order to get the total population for the following year, we must add the current population to the increase in population. In other words $A + 0.15A = 1.15A$. We see from this that the population must be multiplied by a factor of 1.15 each year.

This means that the base of the exponential is $b = 1.15$.

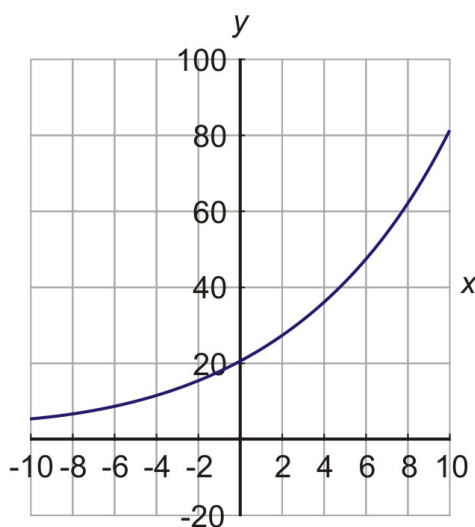
The formula that describes this problem is $y = 20 \cdot (1.15)^x$

Now let's make a table of values.

TABLE 9.6:

x	$y = 20 \cdot (1.15)^x$
-10	4.9
-5	9.9
0	20
5	40.2
10	80.9

Now let's graph the function.



Notice that we used negative values of x in our table of values. Does it make sense to think of negative time? In this case $x = -5$ represents what the population was five years ago, so it can be useful information. The question asked in the problem was “*What will be the population of the town ten years from now?*”

To find the population exactly, we use $x = 10$ in the formula. We found

$$y = 20 \cdot (1.15)^{10} = 80.911 \text{ thousands.}$$

Example G

Peter earned \$1500 last summer. If he deposited the money in a bank account that earns 5% interest compounded yearly, how much money will he have after five years?

Solution

This problem deals with interest which is compounded yearly. This means that each year the interest is calculated on the amount of money you have in the bank. That interest is added to the original amount and next year the interest is calculated on this new amount. In this way, you get paid interest on the interest.

Let's write a function that describes the amount of money in the bank. The general form of an exponential function is:

$$y = A \cdot b^x$$

Define y as the amount of money in the bank.

Define x as the number of years from now.

A is the initial amount, so $A = 1500$.

Now we must find what b is.

We are told that the interest is 5% each year.

Change percents into decimals 5% is equivalent to 0.05.

This means that the base of the exponential is $b = 1 + 0.05 = 1.05$. We see that Peter's money grew by 5% per year. Or, his amount of money was multiplied by 1.05 each year.

The formula that describes this problem is $y = 1500 \cdot (1.05)^x$

To find the total amount of money in the bank at the end of five years, we simply use $x = 5$ in our formula.

$$\text{Answer } y = 1500 \cdot (1.05)^5 = \$1914.42$$

Sample Problems with the Natural Log**Example H**

Gianna opens a savings account with \$1000 and it accrues interest *continuously* at a rate of 5%. What is the balance in the account after 6 years?

Solution

When solving a problem that involves continuous growth, you use the base e . In this example, the equation for continuous growth is $A = Pe^{rt}$, where A is the balance in the account, P is the amount put into the account when it was opened, r is the continuous rate of change, and t is the time the account was open. Therefore, the equation for this problem is $A = 1000e^{0.05(6)}$ and the account will have \$1349.86 in it.

Example I

The population of Springfield is growing exponentially. The growth can be modeled by the function $P = Ie^{0.055t}$, where P represents the projected population, I represents the current population of 100,000 in 2012 and t represents the number of years after 2012

- To the nearest person, what will the population be in 2022?
- In what year will the population double in size if this growth rate continues?

Example J

Naya invests \$7500 in an account which accrues interest continuously at a rate of 4.5%.

- Write an exponential growth function to model the value of her investment after t years.
- How much interest does Naya earn in the first six months to the nearest dollar?
- How much money, to the nearest dollar, is in the account after 8 years?

Review Questions

Graph the following exponential functions by making a table of values.

- $y = 3^x$
- $y = 5 \cdot 3^x$
- $y = 40 \cdot 4^x$
- $y = 3 \cdot 10^x$

Solve the following problems.

- A chain letter is sent out to 10 people telling everyone to make 10 copies of the letter and send each one to a new person. Assume that everyone who receives the letter sends it to ten new people and that it takes a week for each cycle. How many people receive the letter on the sixth week?
- Nadia received \$200 for her 10th birthday. If she saves it in a bank with a 7.5% interest compounded yearly, how much money will she have in the bank by her 21st birthday?

9.2 Exponential Decay

Learning Objectives

- Graph an exponential decay function.
- Compare graphs of exponential decay functions.
- Solve real-world problems involving exponential decay.

Introduction

In the last section, we looked at graphs of exponential functions. We saw that exponential functions describe a quantity that doubles, triples, quadruples, or simply gets multiplied by the same factor. All the functions we looked at in the last section were exponentially increasing functions. They started small and then became large very fast. In this section, we are going to look at exponentially decreasing functions. An example of such a function is a quantity that gets decreased by one half each time. Let's look at a specific example.

For her fifth birthday, Nadia's grandmother gave her a full bag of candy. Nadia counted her candy and found out that there were 160 pieces in the bag. As you might suspect Nadia loves candy so she ate half the candy on the first day. Her mother told her that if she eats it at that rate it will be all gone the next day and she will not have anymore until her next birthday. Nadia devised a clever plan. She will always eat half of the candy that is left in the bag each day. She thinks that she will get candy every day and her candy will never run out. How much candy does Nadia have at the end of the week? Would the candy really last forever?

Let's make a table of values for this problem.

Day	0	1	2	3	4	5	6	7
No. of Candies	160	80	40	20	10	5	2.5	1.25

You can see that if Nadia eats half the candies each day, then by the end of the week she only has 1.25 candies left in her bag.

Let's write an equation for this exponential function.

Nadia started with 160 pieces.

$$y = 160$$

After the first she has $\frac{1}{2}$ of that amount.

$$y = 160 \cdot \frac{1}{2}$$

After the second day she has $\frac{1}{2}$ of the last amount.

$$y = 160 \cdot \frac{1}{2} \cdot \frac{1}{2}$$

You see that in order to get the amount of candy left at the end of each day we keep multiplying by $\frac{1}{2}$. Another way of thinking about this is that the rate of change is - 50%, or -.5. This means $b = 1 + r = 1 - .5 = .5$

We can write the exponential function as

$$y = 160 \cdot \frac{1}{2}^x$$

Notice that this is the same general form as the exponential functions in the last section.

$$y = A \cdot b^x$$

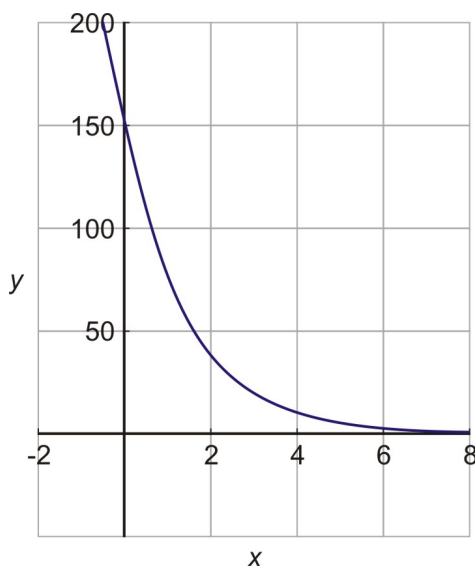
Here $A = 160$ is the initial amount and $b = \frac{1}{2}$ is the factor that the quantity gets multiplied by each time. The difference is that now b is a fraction that is less than one, instead of a number that is greater than one.

This is a good rule to remember for exponential functions.

If b is greater than one, then the exponential function increased, but

If b is less than one (but still positive), then the exponential function decreased

Let's now graph the candy problem function. The resulting graph is shown below.



So, will Nadia's candy last forever? We saw that by the end of the week she has 1.25 candies left so there does not seem to be much hope for that. But if you look at the graph you will see that the graph never really gets to zero.

Theoretically there will always be some candy left, but she will be eating very tiny fractions of a candy every day after the first week!

This is a fundamental feature of an exponential decay function. Its value get smaller and smaller and approaches zero but it never quite gets there. In mathematics we say that the function **asymptotes** to the value zero. This means that it approaches that value closer and closer without ever actually getting there.

Graph an Exponential Decay Function

The graph of an exponential decay function will always take the same basic shape as the one in the previous figure. Let's graph another example by making a table of values.

Example A

Graph the exponential function $y = 5 \cdot \left(\frac{1}{2}\right)^x$

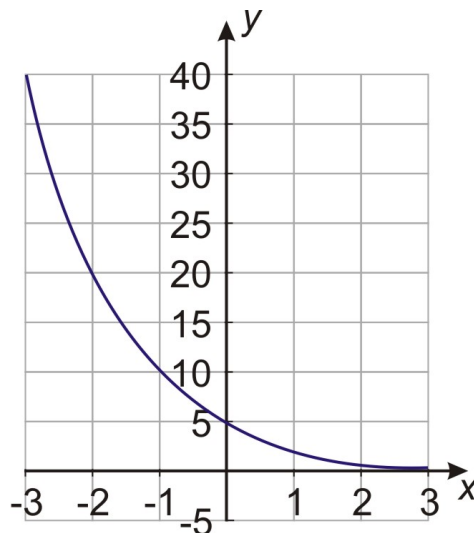
Solution

Let's start by making a table of values.

TABLE 9.7:

x	$y = 5 \cdot \left(\frac{1}{2}\right)^x$
-3	$y = 5 \cdot \left(\frac{1}{2}\right)^{-3} = 5 \cdot 2^3 = 40$
-2	$y = 5 \cdot \left(\frac{1}{2}\right)^{-2} = 5 \cdot 2^2 = 20$
-1	$y = 5 \cdot \left(\frac{1}{2}\right)^{-1} = 5 \cdot 2^1 = 10$
0	$y = 5 \cdot \left(\frac{1}{2}\right)^0 = 5 \cdot 1 = 5$
1	$y = 5 \cdot \left(\frac{1}{2}\right)^1 = \frac{5}{2}$
2	$y = 5 \cdot \left(\frac{1}{2}\right)^2 = \frac{5}{4}$

Now let's graph the function.



Remember that a fraction to a negative power is equivalent to its reciprocal to the same positive power.

We said that an exponential decay function has the same general form as an exponentially increasing function, but that the base b is a positive number less than one. When b can be written as a fraction, we can use the *Property of Negative Exponents* that we discussed in Section 8.3 to write the function in a different form.

For instance, $y = 5 \cdot \left(\frac{1}{2}\right)^x$ is equivalent to $5 \cdot 2^{-x}$.

These two forms are both commonly used so it is important to know that they are equivalent.

Example B

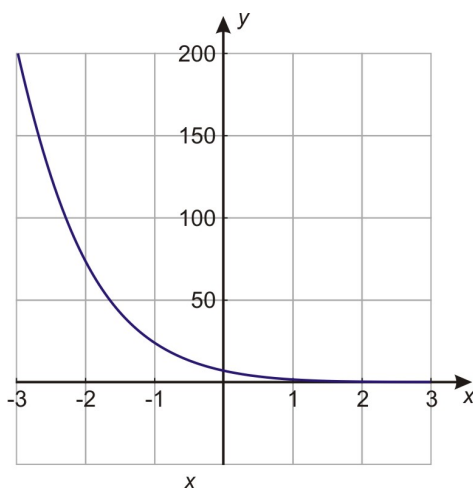
Graph the exponential function $y = 8 \cdot 3^{-x}$.

Solution

Here is our table of values and the graph of the function.

TABLE 9.8:

x	$y = 8 \cdot 3^{-x}$
-3	$y = 8 \cdot 3^{-(-3)} = 8 \cdot 3^3 = 216$
-2	$y = 8 \cdot 3^{-(-2)} = 8 \cdot 3^2 = 72$
-1	$y = 8 \cdot 3^{-(-1)} = 8 \cdot 3^1 = 24$
0	$y = 8 \cdot 3^0 = 8$
1	$y = 8 \cdot 3^{-1} = \frac{8}{3}$
2	$y = 8 \cdot 3^{-2} = \frac{8}{9}$



Compare Graphs of Exponential Decay Functions

You might have noticed that an exponentially decaying function is very similar to an exponentially increasing function. The two types of functions behave similarly, but they are backwards from each other.

The increasing function starts very small and increases very quickly and ends up very, very big. While the decreasing function starts very big and decreases very quickly to soon become very, very small. Let's graph two such functions together on the same graph and compare them.

Example C

Graph the functions $y = 4^x$ and $y = 4^{-x}$ on the same coordinate axes.

Solution

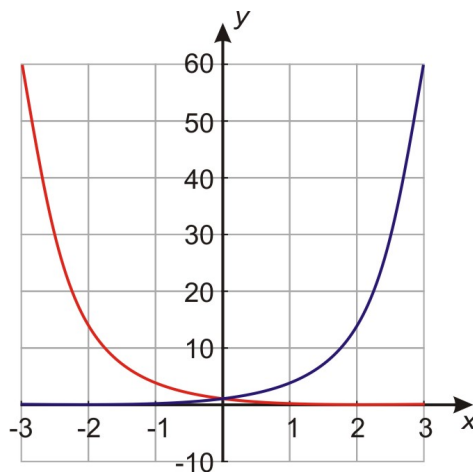
Here is the table of values and the graph of the two functions.

Looking at the values in the table we see that the two functions are “backwards” of each other in the sense that the values for the two functions are reciprocals.

TABLE 9.9:

x	$y = 4^x$	$y = 4^{-x}$
-3	$y = 4^{-3} = \frac{1}{64}$	$y = 4^{-(-3)} = 64$
-2	$y = 4^{-2} = \frac{1}{16}$	$y = 4^{-(-2)} = 16$
-1	$y = 4^{-1} = \frac{1}{4}$	$y = 4^{-(-1)} = 4$
0	$y = 4^0 = 1$	$y = 4^0 = 1$
1	$y = 4^1 = 4$	$y = 4^{-1} = \frac{1}{4}$
2	$y = 4^2 = 16$	$y = 4^{-2} = \frac{1}{16}$
3	$y = 4^3 = 64$	$y = 4^{-3} = \frac{1}{64}$

Here is the graph of the two functions. Notice that the two functions are mirror images of each other if the mirror is placed vertically on the y -axis.

**Solve Real-World Problems Involving Exponential Decay**

Exponential decay problems appear in several application problems. Some examples of these are **half-life problems**, and **depreciation problems**. Let's solve an example of each of these problems.

Example D: Half-Life

A radioactive substance has a half-life of one week. In other words, at the end of every week the level of radioactivity is half of its value at the beginning of the week. The initial level of radioactivity is 20 counts per second.

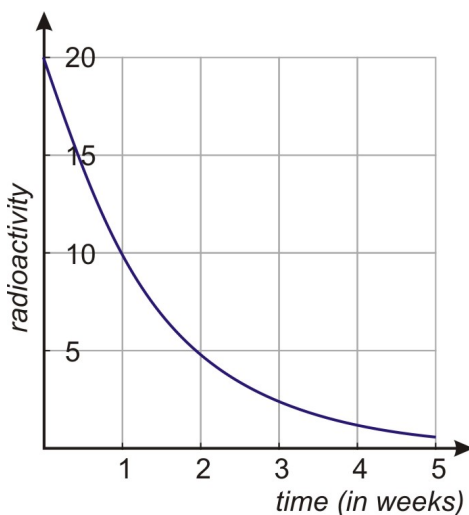
- Draw the graph of the amount of radioactivity against time in weeks.
- Find the formula that gives the radioactivity in terms of time.
- Find the radioactivity left after three weeks

Solution

Let's start by making a table of values and then draw the graph.

TABLE 9.10:

time	radioactivity
0	20
1	10
2	5
3	2.5
4	1.25
5	0.625



- a.
- b. Exponential decay fits the general formula $y = A \cdot b^x$

In this case:

y is the amount of radioactivity

x is the time in weeks

$A = 20$ is the starting amount

$b = \frac{1}{2}$ since the substance loses half its value each week

The formula for this problem is: $y = 20 \cdot \left(\frac{1}{2}\right)^x$ or $y = 20 \cdot 2^{-x}$.

- c. Finally, to find out how much radioactivity is left after three weeks, we use $x = 3$ in the formula we just found.

$$y = 20 \cdot \left(\frac{1}{2}\right)^3 = \frac{20}{8} = 2.5$$

Example E: Depreciation

The cost of a new car is \$32,000. It depreciates at a rate of 15% per year. This means that it loses 15% of each value each year.

- Draw the graph of the car's value against time in year.
- Find the formula that gives the value of the car in terms of time.
- Find the value of the car when it is four years old.

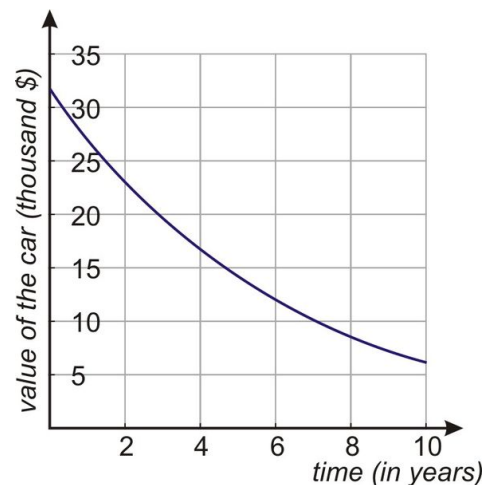
Solution

Let's start by making a table of values. To fill in the values we start with 32,000 at time $t = 0$. Then we multiply the value of the car by 85% for each passing year ($b = 1 + 4 = 1 - 0.15 = .85$). Remember that 85% means that we multiply by the decimal 0.85.

TABLE 9.11:

Time	Value(Thousands)
0	32
1	27.2
2	23.1
3	19.7
4	16.7
5	14.2

Now draw the graph



-
- Let's start with the general formula

$$y = A \cdot b^x$$

In this case:

y is the value of the car

x is the time in years

$A = 32$ is the starting amount in thousands

$b = 0.85$ since we multiply the amount by this factor to get the value of the car next year

The formula for this problem is $y = 32 \cdot (0.85)^x$.

- Finally, to find the value of the car when it is four years old, we use $x = 4$ in the formula we just found. $y = 32 \cdot (0.85)^4 = 16.7$ thousand dollars or \$16,704 if we don't round.

Review Questions

Graph the following exponential decay functions.

1. $y = \frac{1}{5}^x$
2. $y = 4 \cdot \left(\frac{2}{3}\right)^x$
3. $y = 3^{-x}$
4. $y = \frac{3}{4} \cdot 6^{-x}$

Solve the following application problems.

1. The cost of a new ATV (all-terrain vehicle) is \$7200. It depreciates at 18% per year. Draw the graph of the vehicle's value against time in years. Find the formula that gives the value of the ATV in terms of time. Find the value of the ATV when it is ten year old.
2. A person is infected by a certain bacterial infection. When he goes to the doctor the population of bacteria is 2 million. The doctor prescribes an antibiotic that reduces the bacteria population to $\frac{1}{4}$ of its size each day.
 - a. Draw the graph of the size of the bacteria population against time in days.
 - b. Find the formula that gives the size of the bacteria population in term of time.
 - c. Find the size of the bacteria population ten days after the drug was first taken.
 - d. Find the size of the bacteria population after 2 weeks (14 days)

9.3 Revisiting Rate of Change

Learning Objectives

- Differentiate between linear and exponential functions by their rate of change.

The Rumor

Two girls in a small town once shared a secret, just between the two of them. They couldn't stand it though, and each of them told three friends. Of course, their friends couldn't keep secrets, either, and each of them told three of their friends. Those friends told three friends, and those friends told three friends, and so on... and pretty soon the whole town knew the secret. There was nobody else to tell!

These girls experienced the startling effects of an exponential function.

If you start with the two girls who each told three friends, you can see that they told six people or $2 \cdot 3$.

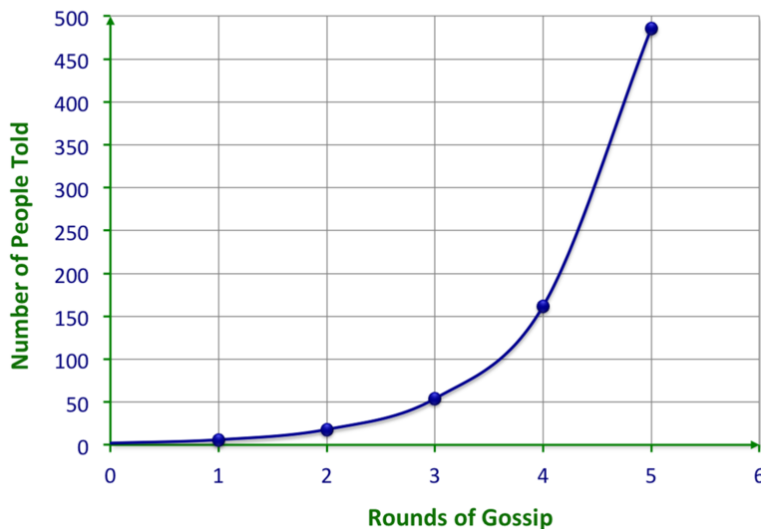
Those six people each told three others, so that $6 \cdot 3$ or $2 \cdot 3 \cdot 3$ —they told 18 people.

Those 18 people each told 3, so that now is $18 \cdot 3$ or $2 \cdot 3 \cdot 3 \cdot 3$ or 54 people.

As we did with linear functions, we could make a table of values and calculate the number of people told after each round of gossip.

x rounds of gossip	0	1	2	3	4	5
y people told	2	6	18	54	162	486

The Gossip Mill



This is clearly not a linear (constant) rate of change. But it does have a characteristic rate of change that identifies it as an exponential function, as we'll learn below.

Linear Rate of Change

One method for identifying functions is to look at the rate of change in the dependent variable. If the **difference** between values of the dependent variable is constant each time we change the independent variable by the same amount, then the function is **linear**.

Example A: Linear Rate of Change

Determine if the function represented by the following table of values is linear.

x	y	difference of y-values
-2	-4	} $-1 + 4 = 3$
-1	-1	
0	2	} $2 + 1 = 3$
1	5	
2	8	} $5 - 2 = 3$

If we take the difference between consecutive y-values, we see that each time the x-value increases by one, the y-value always increases by 3.

NOTE: Be sure when using this approach to make sure that the difference between consecutive x-values is constant.

In mathematical notation, we can write the linear property as follows:

If $\frac{y_2 - y_1}{x_2 - x_1}$ is always the same for values of the dependent and independent variables, then the points are on a line. Notice that the expression we wrote is the definition of the slope of a line.

Exponential Rate of Change

There is also a specific rate of change pattern that will help you identify exponential functions. If the **ratio** between values of the dependent variable is constant each time we change the independent variable by the same amount, then the function is **exponential**.

Example B: Exponential Rate of Change

Determine if the function represented by the following table of values is exponential. Note that the independent variable (x) is changing by the same amount.

x	y	ratio of y -values
0	4	} $\frac{12}{4} = 3$
1	12	
2	36	} $\frac{36}{12} = 3$
3	108	
4	324	} $\frac{108}{36} = 3$

a.

If we take the ratio of consecutive y -values, we see that each time the x -value increases by one, the y -value is multiplied by 3.

Since the ratio is always the same, **the function is exponential.**

x	y	ratio of y -values
0	240	} $\frac{120}{240} = \frac{1}{2}$
1	120	
2	60	} $\frac{60}{120} = \frac{1}{2}$
3	30	
4	15	} $\frac{30}{60} = \frac{1}{2}$

b.

If we take the ratio of consecutive y -values, we see that each time the x -value increases by one, the y -value is multiplied by $\frac{1}{2}$.

Since the ratio is always the same, **the function is exponential.**

Write Equations for Functions

Once we identify which type of function fits the given values, we can write an equation for the function by starting with the general form for that type of function.

Example C

Determine what type of function represents the values in the following table.

TABLE 9.12:

x	y

TABLE 9.12: (continued)

x	y
0	3
1	1
2	-3
3	-7
4	-11

Solution

Let's first check the difference of consecutive values of y . (NOTE: the consecutive values of x are changing by the same amount each time).

x	y	difference of y -values
0	5	} $1 - 5 = -4$
1	1	
2	-3	} $-3 - 1 = -4$
3	-7	
4	-11	} $-7 + 3 = -4$

If we take the difference between consecutive y -values, we see that each time the x -value increases by one, the y -value always decreases by 4. Since the difference is always the same, **the function is linear**.

To find the equation for the function that represents these values, we start with the general form of a linear function.

$$y = mx + b$$

Here m is the slope of the line and is defined as the quantity by which y increases every time the value of x increases by one. The constant b is the value of the function when $x = 0$. Therefore, the function is

$$y = -4x + 5$$

Example D

Determine what type of function represents the values in the following table.

TABLE 9.13:

x	y
0	400

TABLE 9.13: (continued)

x	y
1	100
2	25
3	6.25
4	1.5625

Solution

Let's check the ratio of consecutive values of y .

x	y	ratio of y -values
0	400	} $\frac{100}{400} = \frac{1}{4}$
1	100	
2	25	} $\frac{25}{100} = \frac{1}{4}$
3	6.25	
4	1.5625	} $\frac{6.25}{25} = \frac{1}{4}$

If we take the ratio of consecutive y -values, we see that each time the x -value increases by one, the y -value is multiplied by $\frac{1}{4}$.

Since the ratio is always the same, **the function is exponential**.

To find the equation for the function that represents these values, we start with the general form of an exponential function, as we will see in this section:

$$y = A \cdot b^x$$

b is the ratio between the values of y each time that x is increased by one. The constant A is the value of the function when $x = 0$. Therefore, our answer is

$$y = 400 \left(\frac{1}{4} \right)^x$$

Review Questions

1. Determine whether the data in the following tables can be represented by a linear function.

TABLE 9.14:

x	y
-4	10
-3	7
-2	4
-1	1
0	-2
1	-5

TABLE 9.15:

x	y
-2	4
-1	3
0	2
1	3
2	6
3	11

TABLE 9.16:

x	y
0	50
1	75
2	100
3	125
4	150
5	175

2. Determine whether the data in the following tables can be represented by an exponential function.

TABLE 9.17:

x	y
0	200
1	300
2	1800
3	8300
4	25800
5	62700

TABLE 9.18:

x	y
0	120
1	180
2	270

TABLE 9.18: (continued)

x	y
3	405
4	607.5
5	911.25

TABLE 9.19:

x	y
0	4000
1	2400
2	1440
3	864
4	518.4
5	311.04

3. Determine what type of function represents the values in the following table and find the equation of the function.

TABLE 9.20:

x	y
0	400
1	500
2	625
3	781.25
4	976.5625

TABLE 9.21:

x	y
-9	-3
-7	-2
-5	-1
-3	0
-1	1
1	2

TABLE 9.22:

x	y
-3	14
-2	4
-1	-2
0	-4
1	-2
2	4

TABLE 9.22: (continued)

x	y
3	14

4. The following table shows the rate of pregnancies (per 1000) for US women aged 15 to 19. (source: US Census Bureau). Make a scatterplot with the rate of pregnancies as the dependent variable and the number of years since 1990 as the independent variable. Find which curve fits this data the best and predict the rate of teen pregnancies in the year 2010.

TABLE 9.23:

Year	Rate of Pregnancy (per 1000)
1990	116.9
1991	115.3
1992	111.0
1993	108.0
1994	104.6
1995	99.6
1996	95.6
1997	91.4
1998	88.7
1999	85.7
2000	83.6
2001	79.5
2002	75.4

9.4 A Quick Review of Logarithms

Learning Objectives

- Recall the relationship between logs and exponents.
- Differentiate between common log (base 10) and natural log (base e)
- Solve for an unknown exponent using logs.

What is a log?

When working with exponential functions, you will often be asked to solve an equation using logs. In general, to solve an equation means to find the value(s) of the variable that makes the equation a true statement. For example, if you were asked to solve the equation $\log_2 x = 5$ for x , how would you do that?

First, we have to think about what “log” *means*. A **logarithm** (or “log” for short) *is* an exponent.

$$\log_b n = a \text{ and } b^a = n$$

\downarrow $n = \text{power (result obtained by raising } b \text{ to the power of } a)$
 \uparrow $b = \text{base}$ \uparrow $a = \text{exponent}$

As an example, consider the equation $\log_2 x = 5$.

The equation $\log_2 x = 5$ means that $2^5 = x$.

So the solution to the equation is $x = 2^5 = 32$.

Relationship between Logs and Exponents

Every exponential expression can be written in logarithmic form. For example, the equation $x = 2^y$ can also be written as $y = \log_2 x$. Notice that the exponential form of an expression emphasizes the power, while the logarithmic form emphasizes the exponent.

Example A

Rewrite each exponential expression as a log expression.

- $3^4 = 81$
- $b^{4x} = 52$

Solution

- a. In order to rewrite an expression, you must identify its base, its exponent, and its power. The 3 is the base, so it is placed as the subscript in the log expression. The 81 is the power, and so it is placed after the “log”. Thus we have: $3^4 = 81$ is the same as $\log_3 81 = 4$.

To read this expression, we say “the logarithm base 3 of 81 equals 4.” This is equivalent to saying “3 to the 4th power equals 81.”

- b. The b is the base, and the expression $4x$ is the exponent, so we have: $\log_b 52 = 4x$. We say, “log base b of 52, equals $4x$.”

Example B

Evaluate each log.

- $\log 1$
- $\log 10$
- $\log \sqrt{10}$

Solution

Remember that $\log x$ (with no base specified) commonly refers to $\log_{10} x$:

- $\log 1 = 0$ because $10^0 = 1$.
- $\log 10 = 1$ because $10^1 = 10$
- $\log \sqrt{10} = \frac{1}{2}$ because $\sqrt{10} = 10^{1/2}$

Example C

Evaluate the function $f(x) = \log_2 x$ for the values:

- $x = 2$
- $x = 1$
- $x = -2$

Solution

- a. If $x = 2$, we have:

$$f(x) = \log_2 x$$

$$f(2) = \log_2 2$$

To determine the value of $\log_2 2$, you can ask yourself: “2 to what power equals 2?” Answering this question is often easy if you consider the exponential form: $2^? = 2$

The missing exponent is 1. So we have $f(2) = \log_2 2 = 1$

- b. If $x = 1$, we have:

$$f(x) = \log_2 x$$

$$f(1) = \log_2 1$$

As we did in (a), we can consider the exponential form: $2^? = 1$. The missing exponent is 0. So we have $f(1) = \log_2 1 = 0$.

c. If $x = -2$, we have:

$$f(x) = \log_2 x$$

$$f(-2) = \log_2 -2$$

Again, consider the exponential form: $2^? = -2$. There is no such exponent. Therefore $f(-2) = \log_2 -2$ does not exist.

Common vs. Natural Log

Although a log function can have any positive number as a base, there are really only two bases that are commonly used in the real world. Both may be written without a base noted, like: $\log x$, so you may need to use the context to decide which is appropriate.

The **common log** is a log with base 10. It is used to define pH, earthquake magnitude, and sound decibel levels, among many many other common real-world values.

The **natural log**, sometimes written $\ln(x)$, is a log with base e . The **number e** is approximately 2.71828 and is used in any number of calculations involving continuous growth in chemistry, physics, biology, finance, etc.. The number e is called the **natural number** (or base), or the **Euler number**, after its discoverer, Leonhard Euler.

Vocabulary

Argument: The expression “inside” a logarithmic expression. The argument represents the “power” in the exponential relationship.

Exponential functions are functions with the input variable (the x term) in the exponent.

Logarithmic functions are the inverse of exponential functions. Recall: $\log_b n = a$ is equivalent to $b^a = n$.

log: The shorthand term for ‘the logarithm of’, as in: “ $\log_b n$ ” = “the logarithm, base ‘b’, of ‘n’”.

Natural Number (Euler Number): The number e , such that as $n \rightarrow \infty$, $(1 + \frac{1}{n})^n \rightarrow e$. $e \approx 2.71828$.

Guided Practice

Solve the following exponential equations.

- $4^{x-8} = 16$

- $2(7)^{3x+1} = 48$

- $\frac{2}{3} \cdot 5^{x+2} + 9 = 21$

- $8^{2x-3} - 4 = 5$.

Solutions

- Change 16 to 4^2 and set the exponents equal to each other.

$$4^{x-8} = 16$$

$$4^{x-8} = 4^2$$

$$x - 8 = 2$$

$$x = 10$$

2. Divide both sides by 2 and then take the log of both sides. Here we choose to use natural log (ln).

$$2(7)^{3x+1} = 48$$

$$7^{3x+1} = 24$$

$$\ln 7^{3x+1} = \ln 24$$

$$(3x + 1)\ln 7 = \ln 24$$

$$3x + 1 = \frac{\ln 24}{\ln 7}$$

$$3x = -1 + \frac{\ln 24}{\ln 7}$$

$$x = -\frac{1}{3} + \frac{\ln 24}{3\ln 7} \approx 0.211$$

3. Subtract 9 from both sides and multiply both sides by $\frac{3}{2}$. Then, take the log of both sides.

$$\frac{2}{3} \cdot 5^{x+2} + 9 = 21$$

$$\frac{2}{3} \cdot 5^{x+2} = 12$$

$$5^{x+2} = 18$$

$$(x + 2)\log 5 = \log 18$$

$$x = \frac{\log 18}{\log 5} - 2 \approx -0.204$$

4. Add 4 to both sides and then take the log of both sides.

$$8^{2x-3} - 4 = 5$$

$$8^{2x-3} = 9$$

$$\log 8^{2x-3} = \log 9$$

$$(2x - 3)\log 8 = \log 9$$

$$2x - 3 = \frac{\log 9}{\log 8}$$

$$2x = 3 + \frac{\log 9}{\log 8}$$

$$x = \frac{3}{2} + \frac{\log 9}{2\log 8} \approx 2.028$$

Notice that in these problems, we did not find the numeric value of any of the logs until the very end. This will reduce rounding errors and ensure that we have the most accurate answer.

More Practice

Use logarithms and a calculator to solve the following equations for x . Round answers to three decimal places.

1. $5^x = 65$
2. $7^x = 75$
3. $2^x = 90$
4. $3^{x-2} = 43$
5. $6^{x+1} + 3 = 13$
6. $6(11^{3x-2}) = 216$
7. $8 + 13^{2x-5} = 35$
8. $\frac{1}{2} \cdot 7^{x-3} - 5 = 14$

9.5 Using Exponential Growth & Decay Models

Learning Objectives

- Use different exponential functions in real-life situations.
- Solve for the exponent in an exponential function using logs.

Review of Growth and Decay Models

When a real-life quantity increases by a percentage over a period of time, the final amount can be modeled by the equation: $A = P(1 + r)^t$, where A is the final amount, P is the initial amount, r is the rate (or percentage), and t is the time (in years). $1 + r$ is known as the **growth factor**. Note that the growth factor is equivalent to b in the formulas introduced previously.

Conversely, a real-life quantity can decrease by a percentage over a period of time. The final amount can be modeled by the same equation, but recall that the rate of change will be negative, so the value of b will be smaller than one.

Examples of Exponential Growth

Example A

The population of Coleman, Texas grows at a 2% rate annually. If the population in 2000 was 5981, what was the population in 2010? Round up to the nearest person.

Solution

First, set up an equation using the growth factor. $r = 0.02$, $t = 10$, and $P = 5981$.

$$A = P(1 + r)^t$$

$$\begin{aligned} A &= 5981(1 + 0.02)^{10} \\ &= 5981(1.02)^{10} \\ &= 7291 \text{ people} \end{aligned}$$

Example B

You deposit \$1000 into a savings account that pays 2.5% annual interest. Find the balance after 3 years if the interest rate is compounded a) annually, b) monthly, c) daily.

Solution

For part a, we will use $A = 1000(1.025)^3 = 1076.89$, as we would expect from Example A.

But, to determine the amount if it is compounded in amounts other than yearly, we need to alter the equation. For compound interest, the equation is $A = P \left(1 + \frac{r}{n}\right)^{nt}$, where n is the number of times the interest is compounded within a year. For part b, $n = 12$ because there are 12 months in one year.

$$\begin{aligned} A &= 1000 \left(1 + \frac{0.025}{12}\right)^{12 \cdot 3} \\ &= 1000(1.002)^{36} \\ &= 1077.80 \end{aligned}$$

In part c, $n = 365$.

$$\begin{aligned} A &= 1000 \left(1 + \frac{0.025}{365}\right)^{365 \cdot 3} \\ &= 1000(1.000068)^{1095} \\ &= 1077.88 \end{aligned}$$

Finding the Value of x in an Exponential Growth Model

The previous examples provided us with a way to find the exponential equation from the information given. In a previous chapter, we were told we had a town with an initial population of 20,000 with an estimated growth of 15% per year. With that information, we were able to answer a question like, "How big will the population be in 10 years?" But what if we wanted to ask the question: "When will the population reach 100 million?" We could use the graph of our exponential function and guess at the value of the x -axis when the population is equal to 100 million. But that would only be a guess. To find the actual value of x in our exponential equation, we must use our knowledge of logs or natural logs.

Here was our equation for the population growth in this town:

$$f(x) = 20 \cdot (1.15)^x$$

We want to know when the population will reach 100 million:

$$100 = 20 \cdot 1.15^x$$

Let's solve for x :

$$\frac{100}{20} = 1.15^x$$

$$5 = 1.15^x$$

Now we can use logs:

$$\log(5) = \log(1.15)^x$$

$$\log(5) = x \cdot \log(1.15)$$

$$\frac{\log(5)}{\log(1.15)} = x$$

$$11.52 = x$$

So, when will the population reach 100 million? In approximately 11 and a half years – or 11.52 years to be exact. Let's hope there is enough space in town.

If you prefer, you can also use natural logs to find the same value. The rules of natural log are the same as logs. Using natural logs has the added benefit of being able to handle the number e .

To use natural logs, we would start the same:

$$100 = 20 \cdot 1.15^x$$

$$\frac{100}{20} = 1.15^x$$

$$5 = 1.15^x$$

Now, we use the natural log: $\ln(5) = \ln(1.15)^x$

$$\frac{\ln(5)}{\ln(1.15)} = x$$

$$11.52 = x$$

Examples of Exponential Decay

Example C

You buy a new car for \$35,000. If the value of the car decreases by 12% each year, what will the value of the car be in 5 years?

Solution

This is a decay function because the value *decreases*.

$$\begin{aligned} A &= 35000(1 - 0.12)^5 \\ &= 35000(0.88)^5 \\ &= 18470.62 \end{aligned}$$

The car would be worth \$18,470.62 after five years.

Example D

The half-life of an isotope of barium is about 10 years. The half-life of a substance is the amount of time it takes for half of that substance to decay. If a nuclear scientist starts with 200 grams of barium, how many grams will remain after 100 years?

This is an example of exponential decay. Half-life refers to a 50% decay, so $b = 1 - 0.05 = 0.05$. Our starting value is 200 grams, and we know that it takes 10 years for half of the isotope to decay. Therefore, the equation should read:

$$\begin{aligned} A &= 200 \cdot \frac{1}{2}^{10} \\ A &= 200 \cdot \frac{1}{1024} = 0.195 \end{aligned}$$

Therefore, 0.195 grams of the barium still remain 100 years later.

Finding the Value of x in an Exponential Decay Model

The previous examples provided us with a way to find the exponential decay equation from the information given. In a previous chapter, we had a car with an initial value of \$32,000 that depreciated at 15% per year. We were able

to answer a question like, "How much will my car be worth when it is four years old?" But what if we wanted to ask, "When will my car be worth only \$10,000 dollars?" We could use a graph of our exponential function and guess at the value on the x-axis when the value of my car is \$10,000. But that would only be a guess. To find the actual value of x in our exponential equation, we must use our knowledge of logs or natural logs.

Here's is our equation for the car: $f(x) = 32 \cdot 0.85^x$

We want to know when the car will be worth \$10,000:

$$10 = 32 \cdot 0.85^x$$

Let's solve for x :

$$\frac{10}{32} = 0.85^x$$

$$0.3125 = 0.85^x$$

$$\ln(0.3125) = \ln(0.85)^x$$

$$\ln(0.3125) = x \cdot \ln(0.85)$$

$$\frac{\ln(0.3125)}{\ln(0.85)} = x$$

$$7.16 = x$$

So when will our car be worth \$10,000? In approximately 7 years – or 7.16 years to be exact.

Extension: Transformations to Achieve Linearity

We can transform an exponential relationship between X and Y into a linear relationship. We commonly use transformations in everyday life. For example, the Richter scale, which measures earthquake intensity, is an example of making transformations of non-linear data.

Consider the following exponential relationship, and take the log of both sides as shown:

$$\begin{aligned} y &= ab^x \\ \log y &= \log(ab^x) \\ \log y &= \log a + \log b^x \\ \log y &= \log a + x \log b \end{aligned}$$

In this example, a and b are real numbers (constants), so this is now a linear relationship between the variables x and $\log y$.

Thus, you can find a least squares line for these variables.

Let's take a look at an example to help clarify this concept. Say that we were interested in making a case for investing and examining how much return on investment one would get on \$100 over time. Let's assume that we invested \$100 in the year 1900 and that this money accrued 5% interest every year. The table below details how much we would have each decade:

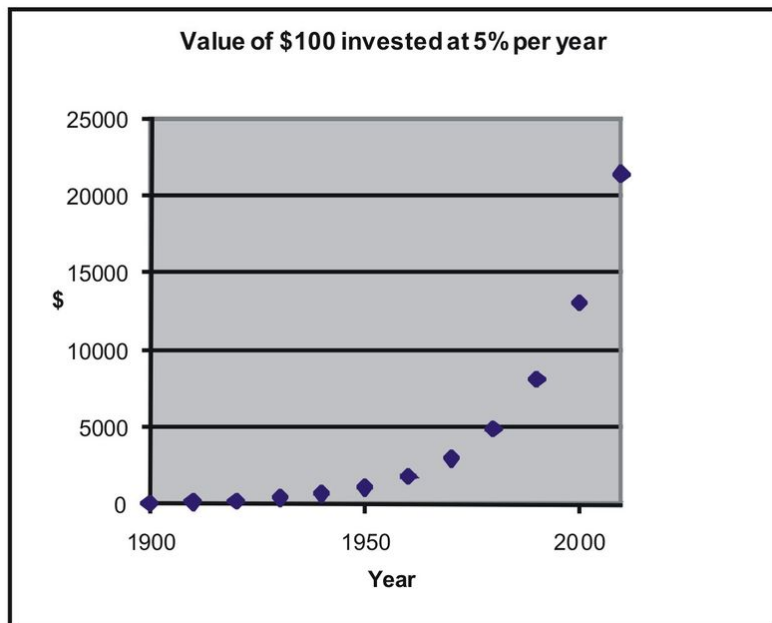
TABLE 9.24: Table of account growth assuming \$100 invested in 1900 at 5% annual growth.

Year	Investment with 5% Each Year
1900	100
1910	163
1920	265
1930	432
1940	704
1950	1147
1960	1868

TABLE 9.24: (continued)

Year	Investment with 5% Each Year
1970	3043
1980	4956
1990	8073
2000	13150
2010	21420

If we graphed these data points, we would see that we have an exponential growth curve.

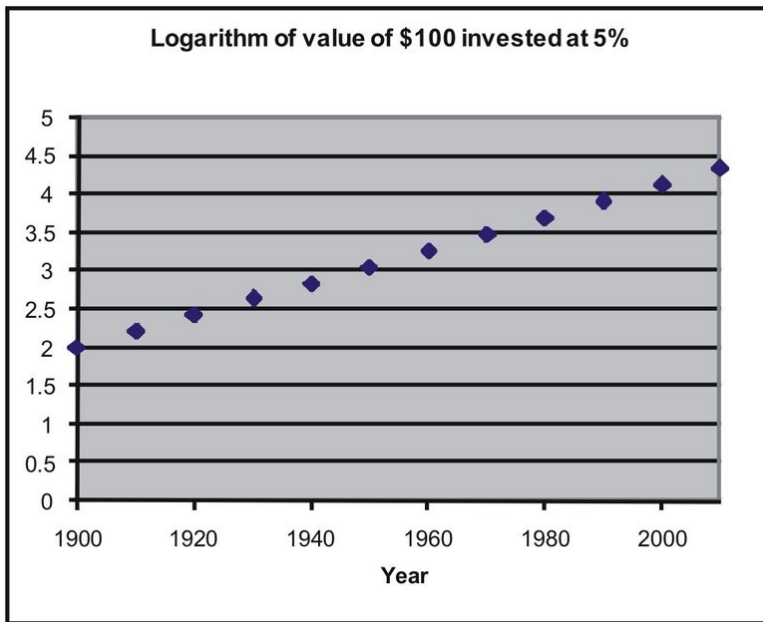


Say that we wanted to fit a linear regression line to these data. First, we would transform these data using logarithmic transformations as follows:

TABLE 9.25: Account growth data and values after a logarithmic transformation.

Year	Investment with 5% Each Year	Log of amount
1900	100	2
1910	163	2.211893
1920	265	2.423786
1930	432	2.635679
1940	704	2.847572
1950	1147	3.059465
1960	1868	3.271358
1970	3043	3.483251
1980	4956	3.695144
1990	8073	3.907037
2000	13150	4.118930
2010	21420	4.330823

If we plotted these transformed data points, we would see that we have a linear relationship as shown below:



We can now perform a linear regression on (year, log of amount), and we will find the following relationship:

$$Y = 0.021X - 38.2$$

in which:

X is representing year

Y is representing log of amount.

These transformed models are trickier to interpret, but they are often useful for modeling.

Vocabulary

Growth Factor: The amount, $(1 + r)$, an exponential function grows by. Populations and interest commonly use growth factors.

Decay Factor: The amount, $(1 - r)$, an exponential function decreases by. Populations, depreciated values, and radioactivity commonly use decay factors.

Guided Practice

- Tommy bought a truck 7 years ago that is now worth \$12,348. If the value of his truck decreased 14% each year, how much did he buy it for? Round to the nearest dollar.
- The Wetakayomoola credit card company charges an Annual Percentage Rate (APR) of 21.99%, compounded monthly. If you have a balance of \$2000 on the card, what would the balance be after 4 years (assuming you do not make any payments)? If you pay \$200 a month to the card, how long would it take you to pay it off? You may need to make a table to help you with the second question.
- As the altitude increases, the atmospheric pressure (the pressure of the air around you) decreases. For every 1000 feet up, the atmospheric pressure decreases about 4%. The atmospheric pressure at sea level is 101.3. If you are on top of Hevenly Mountain at Lake Tahoe (elevation about 10,000 feet) what is the atmospheric pressure?

Solutions

1. Tommy needs to use the formula $A = P(1 + r)^t$ and solve for P . (remember r is negative).

$$\begin{aligned} 12348 &= P(1 - 0.14)^7 \\ 12348 &= P(0.86)^7 && \text{Tommy's truck was originally } \$35,490. \\ \frac{12348}{(0.86)^7} &= P \approx 35490 \end{aligned}$$

2. You need to use the formula $A = P\left(1 + \frac{r}{n}\right)^{nt}$, where $n = 12$ because the interest is compounded monthly.

$$\begin{aligned} A &= 2000 \left(1 + \frac{0.2199}{12}\right)^{12 \cdot 4} \\ &= 2000(1.018325)^{48} \\ &= 4781.65 \end{aligned}$$

To determine how long it will take you to pay off the balance, you need to find how much interest is compounded in one month, subtract \$200, and repeat. A table might be helpful. For each month after the first, we will use the equation, $B = R\left(1 + \frac{0.2199}{12}\right)^{12 \cdot \left(\frac{1}{12}\right)} = R(1.018325)$, where B is the current balance and R is the remaining balance from the previous month. For example, in month 2, the balance (including interest) would be $B = 1800\left(1 + \frac{0.2199}{12}\right)^{12 \cdot \left(\frac{1}{12}\right)} = 1800 \cdot 1.018325 = 1832.99$.

TABLE 9.26:

Month	1	2	3	4	5	6	7	8	9	10	11
Balance	2000	1832.99	1662.91	1489.72	1313.35	930.09	790.87	640.06	476.69	299.73	108.03
Payment	200	200.00	200.00	200.00	200.00	200.00	200.00	200.00	200.00	200.00	108.03
Remainder	\$1800	1632.99	1462.91	1289.72	913.35	730.09	590.87	440.06	276.69	99.73	0

It is going to take you 11 months to pay off the balance and you are going to pay 108.03 in interest, making your total payment \$2108.03.

3. The equation will be $A = 101,325(1 - 0.04)^{100} = 1709.39$. The decay factor is only raised to the power of 100 because for every 1000 feet the pressure decreased. Therefore, $10,000 \div 1000 = 100$. Atmospheric pressure is what you don't feel when you are at a higher altitude and can make you feel light-headed.

More Practice

Use an exponential growth or exponential decay function to model the following scenarios and answer the questions.

- Sonya's salary increases at a rate of 4% per year. Her starting salary is \$45,000. What is her annual salary, to the nearest \$100, after 8 years of service?
- The value of Sam's car depreciates at a rate of 8% per year. The initial value was \$22,000. What will his car be worth after 12 years to the nearest dollar?
- Rebecca is training for a marathon. Her weekly long run is currently 5 miles. If she increase her mileage each week by 10%, will she complete a 20 mile training run within 15 weeks?

4. An investment grows at a rate of 6% per year. How much, to the nearest \$100, should Noel invest if he wants to have \$100,000 at the end of 20 years?
5. Charlie purchases a 7 year old used *RV* for \$54,000. If the rate of depreciation was 13% per year during those 7 years, how much was the *RV* worth when it was new? Give your answer to the nearest one thousand dollars.
6. The value of homes in a neighborhood increase in value an average of 3% per year. What will a home purchased for \$180,000 be worth in 25 years to the nearest one thousand dollars?
7. The population of a community is decreasing at a rate of 2% per year. The current population is 152,000. How many people lived in the town 5 years ago?
8. The value of a particular piece of land worth \$40,000 is increasing at a rate of 1.5% per year. Assuming the rate of appreciation continues, how long will the owner need to wait to sell the land if he hopes to get \$50,000 for it? Give your answer to the nearest year.