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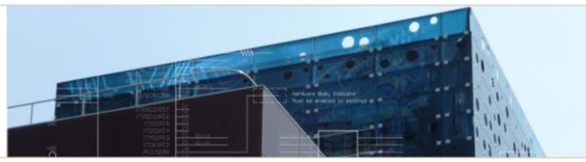
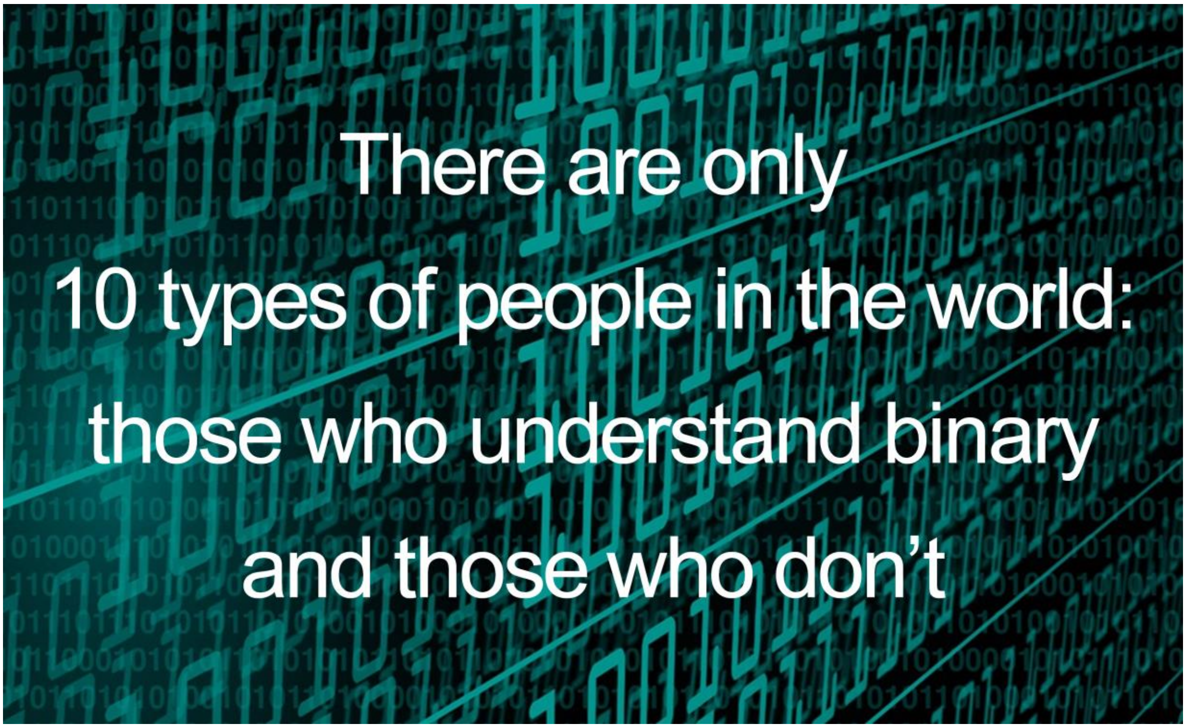


Data storage and representation

The Binary System

In this unit we'll learn the basics about the binary system.



Let's begin with a typical geek joke: there are 10 types of people: those who understand binary and those who don't. Do you get it? If you don't, don't worry. I'll try to explain it.

decimal	binary
0	0
1	1

$2^1 = 2$ possibilities

with one bit, we can just represent two possibilities: zero and one.

decimal	binary
0	00
1	01
2	10
3	11

2^2 = 4 possibilities

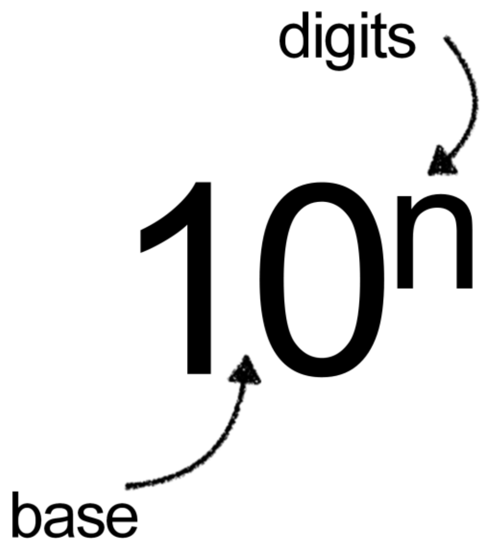
using two bits, and with all possible combinations, we can represent 4 situations. That's two raised to two.

decimal	binary
0	000
1	001
2	010
3	011
4	100
5	101
6	110
7	111

$2^3 = 8$ possibilities

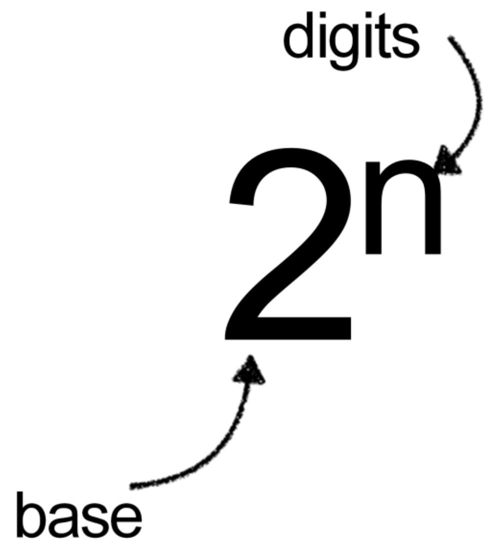
and we can continue with 3 bits, resulting 8 possibilities, from zero zero zero to one one one.

#digits	#combinations
0	1
1	10 (0-9)
2	100 (0-99)
3	1000 (0-999)



In the decimal system we know how to calculate that: the number of different possibilities that we can count is the numeric base (10) raised to the number of digits that we use. For example, with 3 digits we can count from zero to nine hundred and ninety nine. That's ten raised to 3 numbers.

#digits	#combinations
0	1
1	2
2	4
3	8
4	16
5	32
6	64
7	128
8	256
9	512
10	1024



The equivalent in binary is that we have two (our numeric base) raised to the digits that we use. In the table on the left you'll find the first 10 powers of two. Are they familiar? Exactly, the memory, or the capacity of a usb pen is measured with these figures.

10^3	10^2	10^1	10^0
2	1	9	3

2 x 1000 + 1 x 100 + 9 x 10 + 3 x 1 =2193

How can we convert a binary number into a decimal one? It’s simple. Think about the meaning of a decimal number. The figures represent the units, ten, hundred, and so on. So we just multiply the figure for the correspondent power of ten and sum the result

10^3	10^2	10^1	10^0
2	1	9	3

$$2 \times 1000 + 1 \times 100 + 9 \times 10 + 3 \times 1 = 2193$$

2^3	2^2	2^1	2^0
1	1	0	1

$$1 \times 8 + 1 \times 4 + 0 \times 2 + 1 \times 1 = 13$$

The binary system is exactly the same: we have to multiply each figure by the power of two that corresponds to its position and sum the result. In this case, eight plus four plus one, that is, thirteen.

represent 167 in binary

2^7	2^6	2^5	2^4	2^3	2^2	2^1	2^0
128	64	32	16	8	4	2	1
1							

137-128 = 39

The oposite operation is simple too. To represent a number in binary, lets say one hundred and sixty seven, we have to locate the next smaller power of two: one hundred and twenty-eight. We write down a one in this position, we subtract the value to the original number and continue doing the same. Now, we have to represent thirty-nine

represent 167 in binary

2^7	2^6	2^5	2^4	2^3	2^2	2^1	2^0
128	64	32	16	8	4	2	1
1							
1	0	1					

137-128 = 39

39-32 = 7

We fill with zeros the positions not used. We do the same: mark thirty-two, the difference is seven, and we continue...

represent 167 in binary

2^7	2^6	2^5	2^4	2^3	2^2	2^1	2^0	
128	64	32	16	8	4	2	1	
1								137-128 = 39
1	0	1						39-32 = 7
1	0	1	0	0	1			7 - 4 = 3

four

represent 167 in binary

2^7	2^6	2^5	2^4	2^3	2^2	2^1	2^0	
128	**64**	**32**	**16**	**8**	**4**	**2**	**1**	
1								137-128 = 39
1	0	1						39-32 = 7
1	0	1	0	0	1			7 - 4 = 3
1	0	1	0	0	1	1		3 - 2 = 1

Two

represent 167 in binary

2^7	2^6	2^5	2^4	2^3	2^2	2^1	2^0	
128	64	32	16	8	4	2	1	
1								137-128 = 39
1	0	1						39-32 = 7
1	0	1	0	0	1			7 - 4 = 3
1	0	1	0	0	1	1		3 - 2 = 1
1	0	1	0	0	1	1	1	1 - 1 = 0

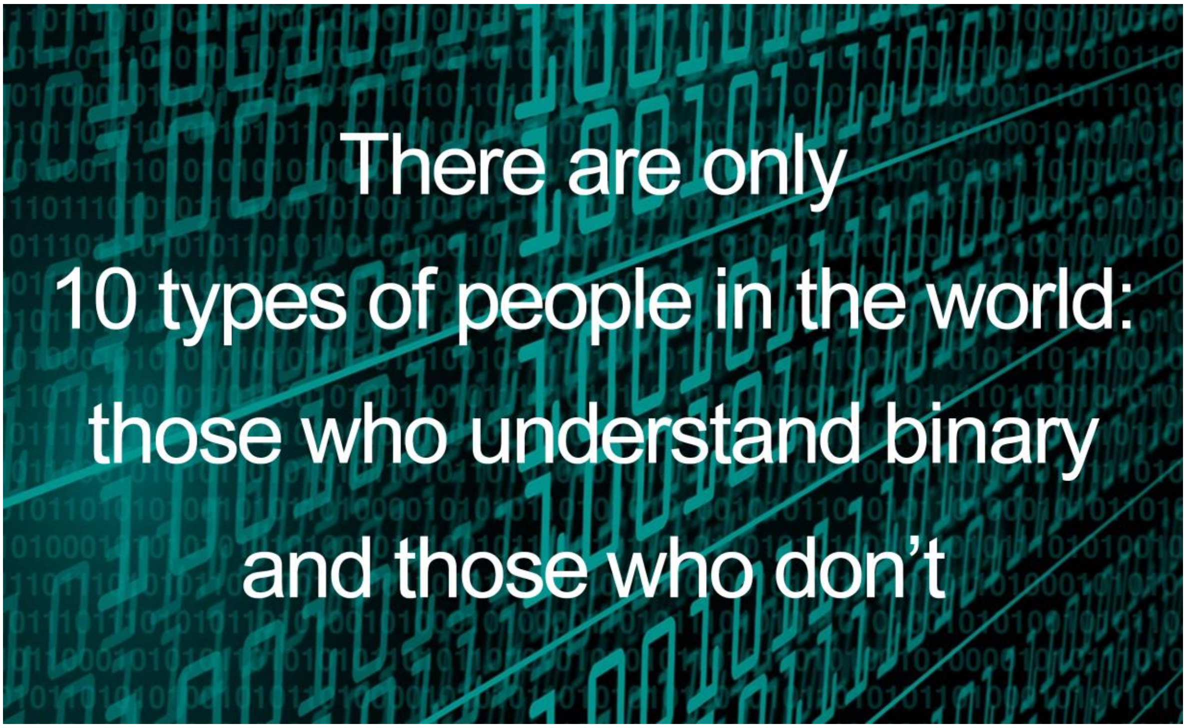
and one

represent 167 in binary

2^7	2^6	2^5	2^4	2^3	2^2	2^1	2^0	
128	**64**	**32**	**16**	**8**	**4**	**2**	**1**	
1								137-128 = 39
1	0	1						39-32 = 7
1	0	1	0	0	1			7 - 4 = 3
1	0	1	0	0	1	1		3 - 2 = 1
1	0	1	0	0	1	1	1	1 - 1 = 0

1 x 128 + 1 x 32 + 1 x 4+ 1 x 2+1 x 1

So this is the representation of one hundred and sixty seven. You can check that it's correct.



And maybe you can now get the joke: there are only TWO types of people in the world.

decinal	binary	octal
0	000 000	0
1	000 001	1
2	000 010	2
3	000 011	3
4	000 100	4
5	000 101	5
6	000 110	6
7	000 111	7
8	001 000	10
9	001 001	11
10	001 010	12

3-bit combination

The binary system is very useful to convert to other numeric system useful in computing. For example, the octal system, that uses 8 digits, has a direct translation if we take the bits in groups of three.

unix file permission

owner group others

rwx rwx rwx

r: read
w: write
x: execute

symbolic		numeric
rwxrwxrwx	111 111 111	777
rw-r—r--	110 100 100	644

Octal representation is used to represent the file permissions in Unix, were we have three permissions: read, write and execution. That can be codified by an octal number.

decimal	Binary	hexadecimal
0	0000 0000	0
1	0000 0001	1
2	0000 0010	2
3	0000 0011	3
4	0000 0100	4
5	0000 0101	5
6	0000 0110	6
7	0000 0111	7
8	0000 1000	8
9	0000 1001	9
10	0000 1010	A
11	0000 1011	B
12	0000 1100	C
13	0000 1101	D
14	0000 1110	E
15	0000 1111	F

4-bit combination

Or the hexadecimal system, that uses base sixteen, is equivalent to take the bits in groups of four.

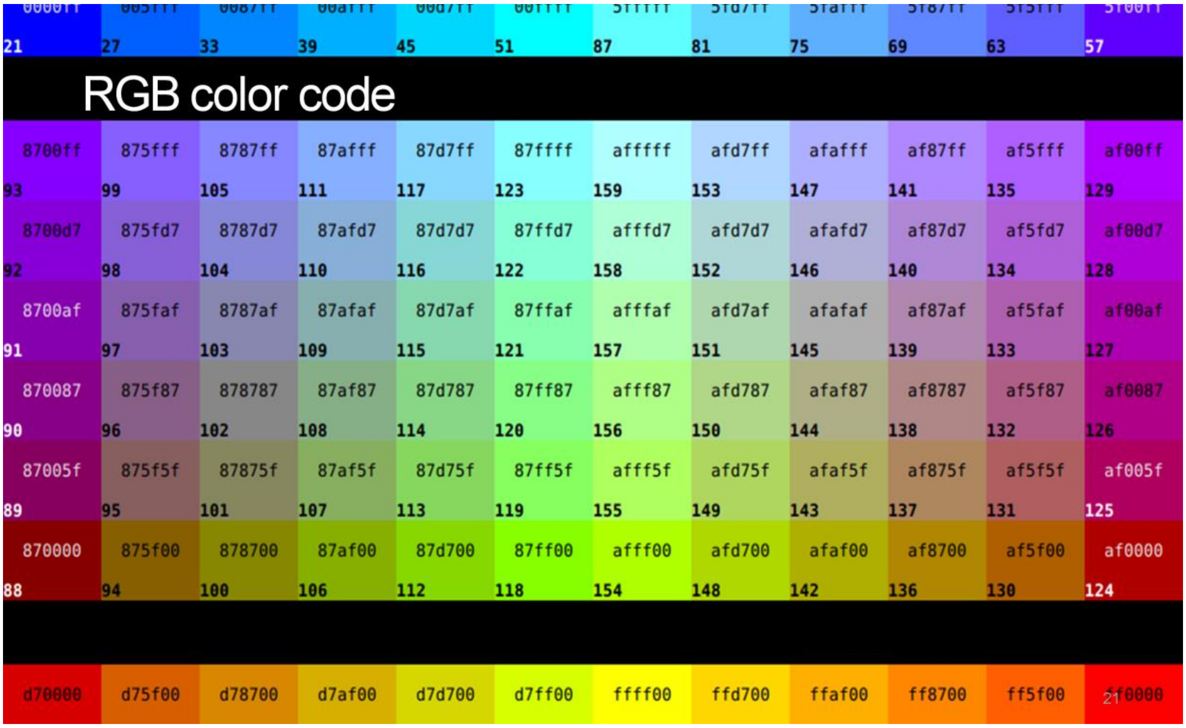
MAC address

identifier for network controllers
48 bits = 6 bytes = 6 groups of 2 hexadecimal numbers

01:23:45:67:89:ab

0000 0001 1010 1011

Hexadecimal codes are used for long chains of bits, such as the MAC address of a network interface card, formed by forty-eight bits...



or the representation of colours in RGB, using two hexadecimal figures for each component: red, green and blue

Attribution

The sources of some of these figures are :

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