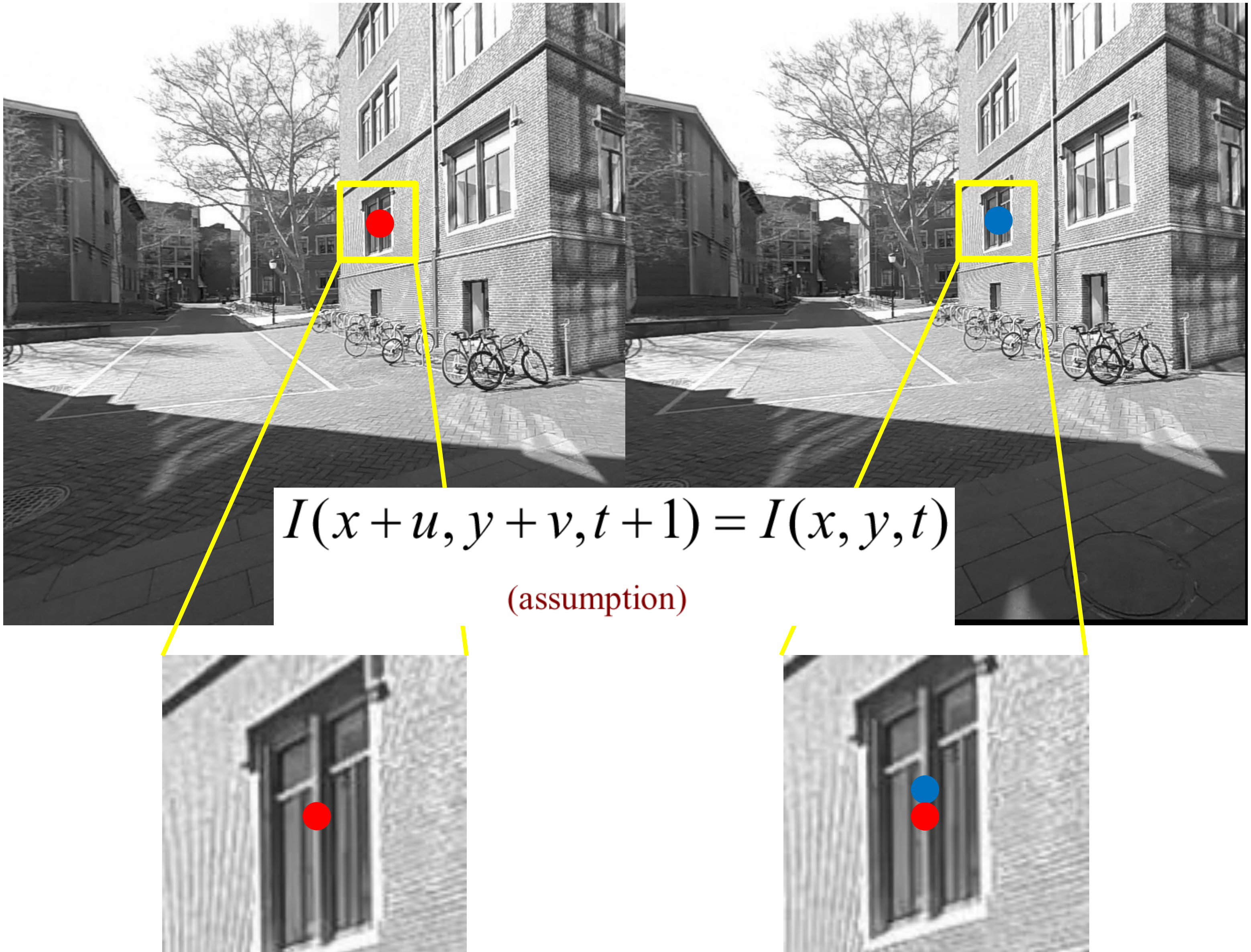
Penn Engineering

ONLINE LEARNING

Video 6.1

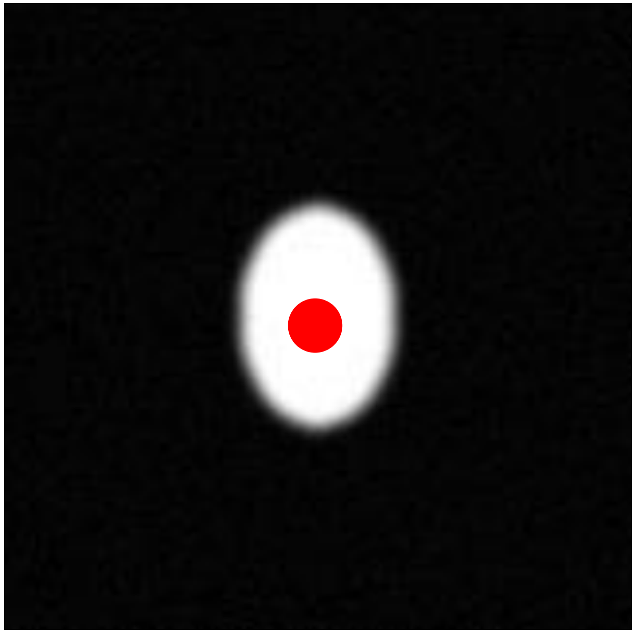
Jianbo Shi

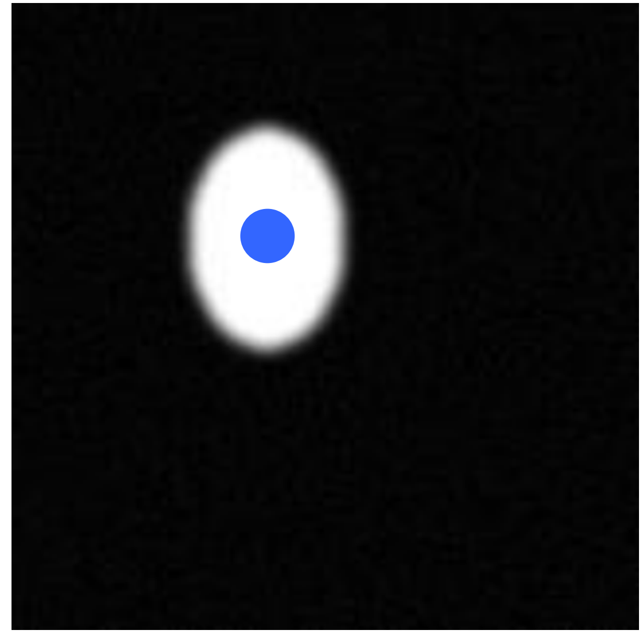
Optical Flow: 2D point correspondences



$$I(x+u, y+v, t+1) = I(x, y, t)$$

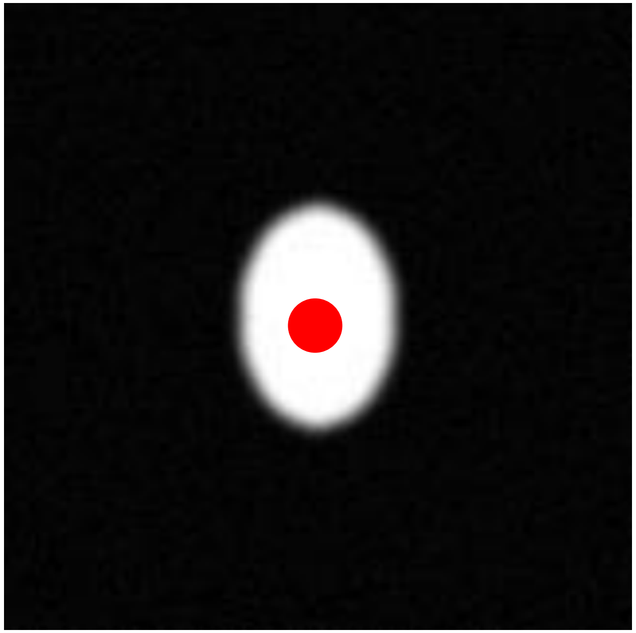
(assumption)

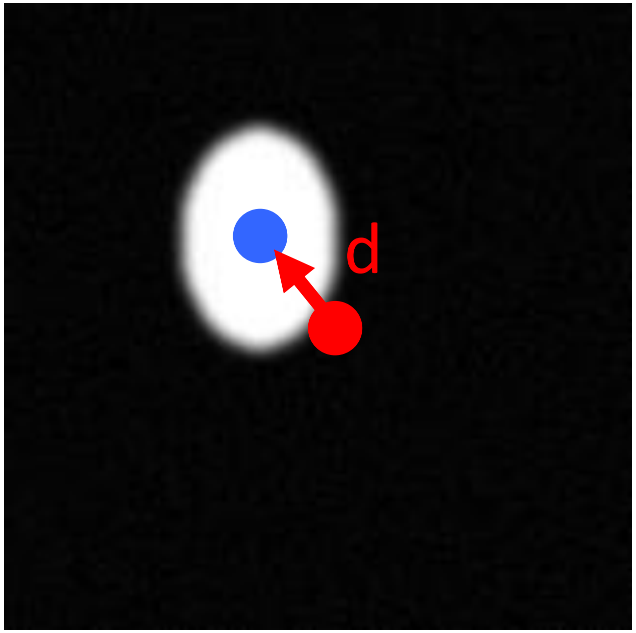




$\mathbf{I}(\mathbf{x})$ $\quad$ $\mathbf{J}(\mathbf{x})$

$t = 0$ $\quad$ $t = 1$

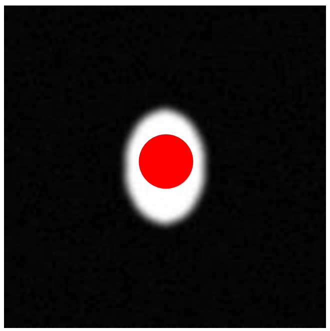




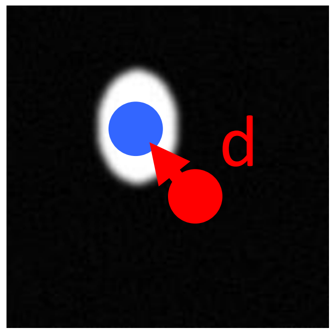
$$\mathbf{I}(\mathbf{x}) \qquad\qquad \mathbf{J}(\mathbf{x})$$

$$I(\mathrm{x}) = J(\mathrm{x} + \mathrm{d})$$

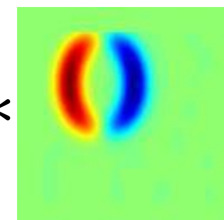
$\mathbf{I}(\mathbf{x})$ $\qquad\qquad$ $\mathbf{J}(\mathbf{x})$



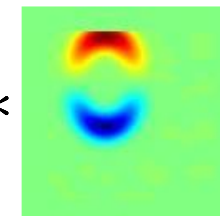
$=$



$+ \; \mathbf{d_x} *$



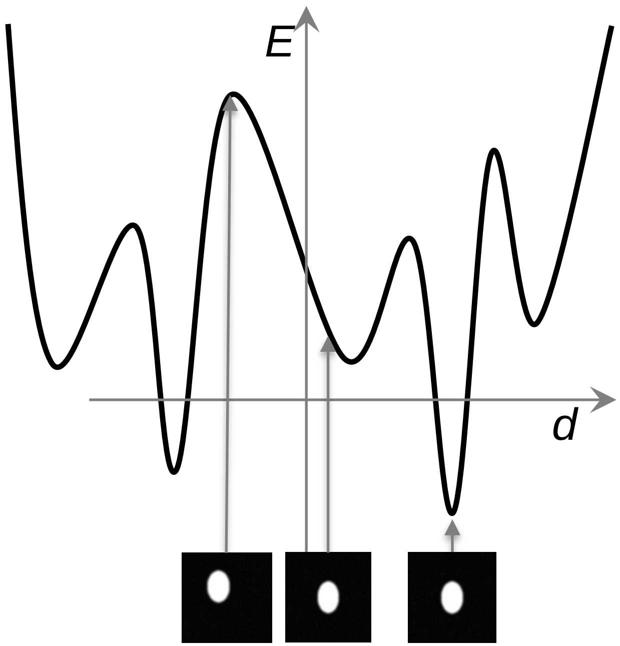
$+ \; \mathbf{d_y} *$

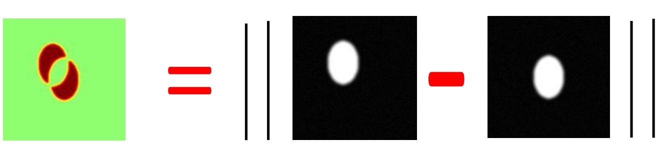


$$I(\mathrm{x}) = J(\mathrm{x} + \mathrm{d})$$

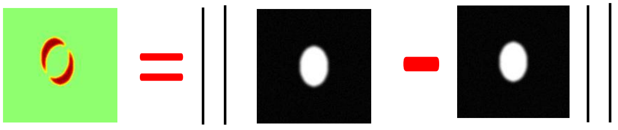
Correspondence cost

$$\min_{\mathbf{d}} E = \left\| J(\mathrm{x}+\mathrm{d}) - I(\mathrm{x}) \right\|^2$$





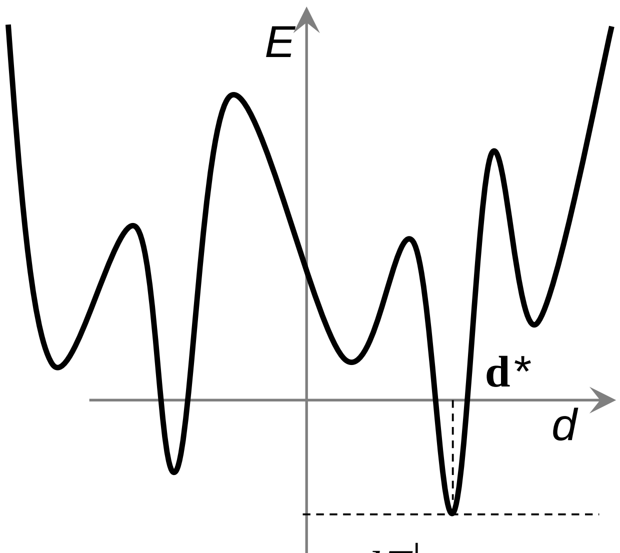
E(d=(0,0))



E(d=(-7,-9))

Step 1: $\left.\frac{dE}{d\mathbf{d}}\right|_{\mathbf{d}=\mathbf{d}^*} = 0$

$$E(\mathrm{d}) = \left\| J(\mathrm{x}+\mathrm{d}) - I(\mathrm{x}) \right\|^2$$

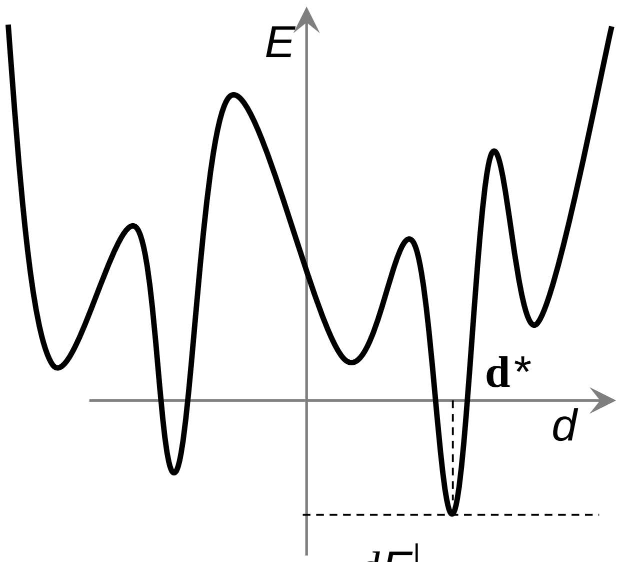


$$\left.\frac{dE}{d\mathbf{d}}\right|_{\mathbf{d}=\mathbf{d}^*} = 0$$

$$E(\mathrm{d}) = \left(J(\mathrm{x}+\mathrm{d}) - I(\mathrm{x}) \right)^{\mathrm{T}} \left(J(\mathrm{x}+\mathrm{d}) - I(\mathrm{x}) \right)$$

Step 1: $\left.\frac{dE}{d\mathbf{d}}\right|_{\mathbf{d}=\mathbf{d}^*} = 0$

$$E(\mathrm{d}) = \left\| J(\mathrm{x}+\mathrm{d}) - I(\mathrm{x}) \right\|^2$$



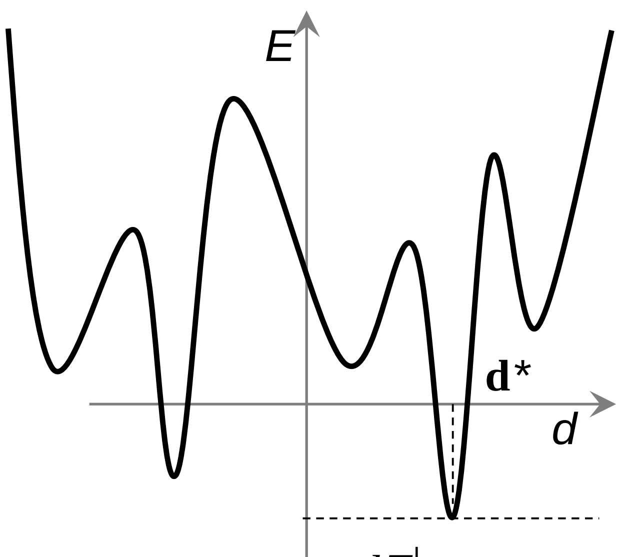
$$\left.\frac{dE}{d\mathbf{d}}\right|_{\mathbf{d}=\mathbf{d}^*} = 0$$

$$E(\mathrm{d}) = \left(J(\mathrm{x}+\mathrm{d}) - I(\mathrm{x}) \right)^{\mathrm{T}} \left(J(\mathrm{x}+\mathrm{d}) - I(\mathrm{x}) \right)$$

$$\frac{\partial E}{\partial \mathrm{d}} = 2 \frac{\partial J(\mathrm{x}+\mathrm{d})}{\partial \mathrm{d}}^{\mathrm{T}} \left(J(\mathrm{x}+\mathrm{d}) - I(\mathrm{x}) \right)$$

Step 1: $\left.\frac{dE}{d\mathbf{d}}\right|_{\mathbf{d}=\mathbf{d}^*} = 0$

$$E(\mathrm{d}) = \left\| J(\mathrm{x}+\mathrm{d}) - I(\mathrm{x}) \right\|^2$$



$$\left.\frac{dE}{d\mathbf{d}}\right|_{\mathbf{d}=\mathbf{d}^*} = 0$$

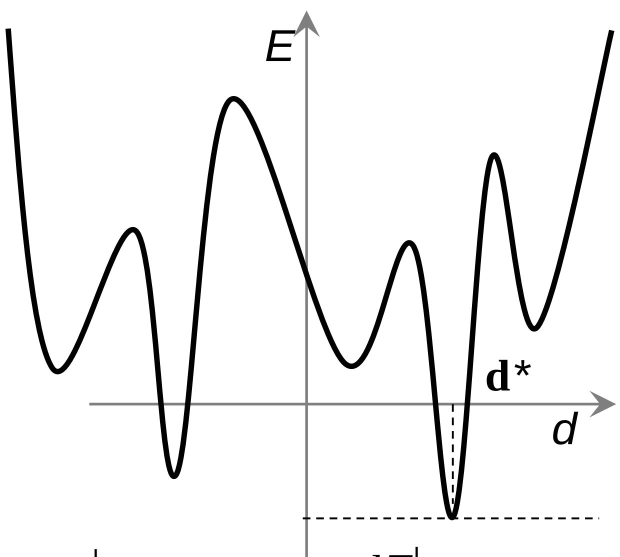
$$E(\mathrm{d}) = \left(J(\mathrm{x}+\mathrm{d}) - I(\mathrm{x}) \right)^{\mathrm{T}} \left(J(\mathrm{x}+\mathrm{d}) - I(\mathrm{x}) \right)$$

$$\frac{\partial E}{\partial \mathrm{d}} = 2 \frac{\partial J(\mathrm{x}+\mathrm{d})^{\mathrm{T}}}{\partial \mathrm{d}} \left(J(\mathrm{x}+\mathrm{d}) - I(\mathrm{x}) \right)$$

$$\frac{\partial E}{\partial \mathrm{d}} = 2 \frac{\partial J(\mathrm{x})^{\mathrm{T}}}{\partial \mathrm{x}} \left(J(\mathrm{x}+\mathrm{d}) - I(\mathrm{x}) \right)$$

Step 1: $\left.\frac{dE}{d\mathbf{d}}\right|_{\mathbf{d}=\mathbf{d}^*} = 0$

$$E(\mathrm{d}) = \left\| J(\mathrm{x}+\mathrm{d}) - I(\mathrm{x}) \right\|^2$$



$\left.\frac{dE}{d\mathbf{d}}\right|_{\mathbf{d}=\mathbf{d}^\#} = 0$ $\left.\frac{dE}{d\mathbf{d}}\right|_{\mathbf{d}=\mathbf{d}^*} = 0$

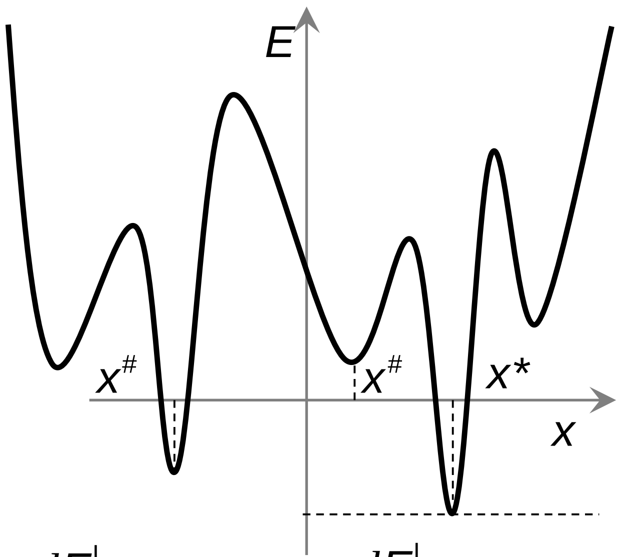
$$\frac{\partial E}{\partial \mathrm{d}} = 2\frac{\partial J(\mathrm{x})}{\partial \mathrm{x}}^{\mathrm{T}} \left(J(\mathrm{x}+\mathrm{d}) - I(\mathrm{x}) \right)$$

where $\frac{\partial J(\mathrm{x})}{\partial \mathrm{x}} = \left[\frac{\partial J(x,y)}{\partial x}, \frac{\partial J(x,y)}{\partial y} \right]$: Image Gradient

Step 1: $\left.\frac{dE}{d\mathbf{d}}\right|_{\mathbf{d}=\mathbf{d}^*} = 0$

$$E(\mathrm{d}) = \left\| J(\mathrm{x}+\mathrm{d}) - I(\mathrm{x}) \right\|^2$$

$$\frac{\partial E}{\partial \mathrm{d}} = 2\frac{\partial J(\mathrm{x})}{\partial \mathrm{x}}^{\mathrm{T}}\left(J(\mathrm{x}+\mathrm{d}) - I(\mathrm{x})\right)$$



$\left.\frac{dE}{dx}\right|_{x=x^{\#}} = 0$ $\left.\frac{dE}{dx}\right|_{x=x^{*}} = 0$

$$\frac{\partial E}{\partial \mathrm{d}} = 2\frac{\partial J(\mathrm{x})}{\partial \mathrm{x}}^{\mathrm{T}}\left(J(\mathrm{x}+\mathrm{d}) - I(\mathrm{x})\right) = 0$$

Find **d** such that the above equation is satis

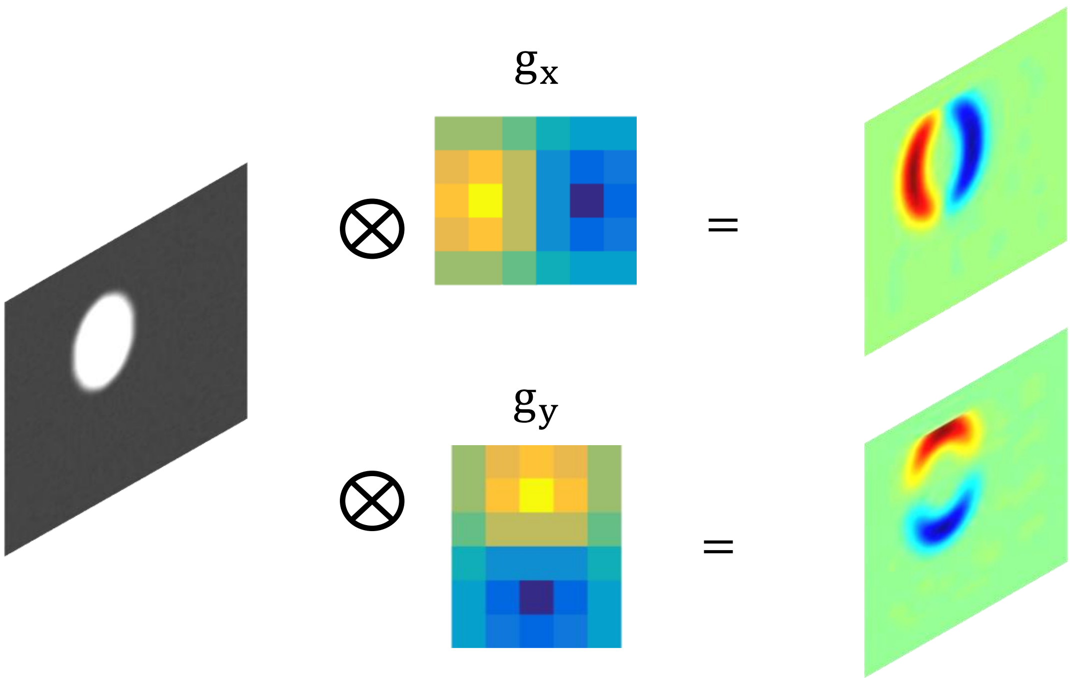
Step 1: $\left.\frac{dE}{d\mathbf{d}}\right|_{\mathbf{d}=\mathbf{d}^*} = 0$

$$\frac{\partial E}{\partial \mathrm{d}} = 2\frac{\partial J(\mathrm{x})}{\partial \mathrm{x}}^{T}\left(J(\mathrm{x}+\mathrm{d}) - I(\mathrm{x})\right) = 0$$

Find **d** such that the above equation is satis

$$\frac{\delta \mathbf{J}(\mathbf{x})}{\delta x} =$$

$$\frac{\delta \mathbf{J}(\mathbf{x})}{\delta y} =$$

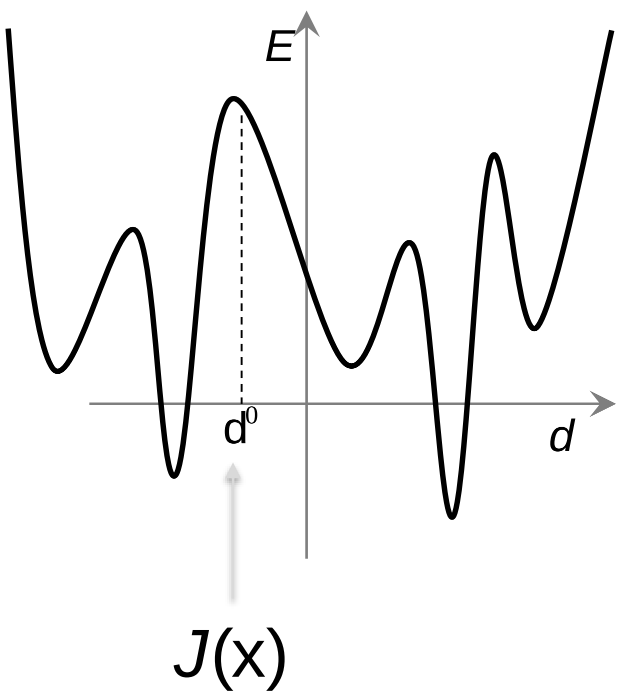


Video 6.2
Jianbo Shi

Nonlinear System

Find **d** such that the above equation is satisfied

$$\frac{\partial E}{\partial \mathrm{d}} = 2\frac{\partial J(\mathrm{x})^{T}}{\partial \mathrm{x}}\left(J(\mathrm{x}+\mathrm{d}) - I(\mathrm{x})\right) = 0$$



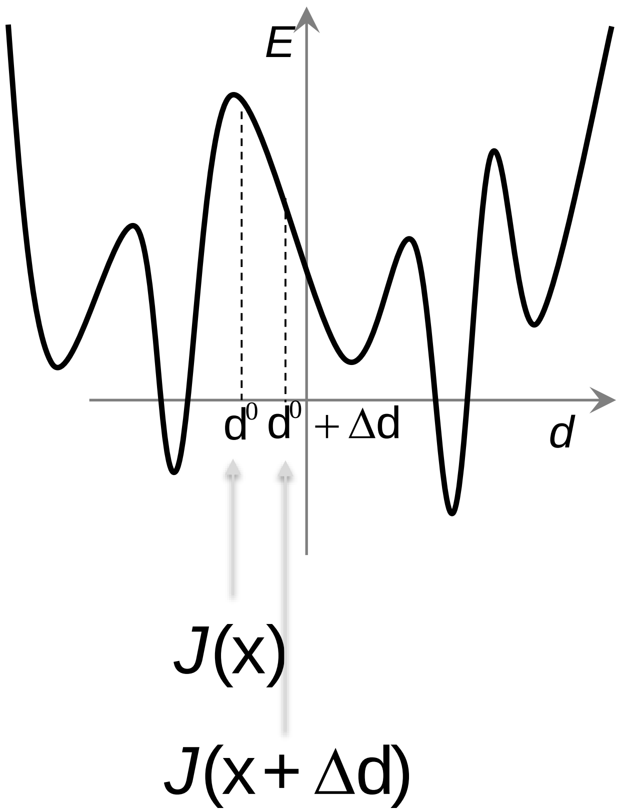
Idea: how to predict an image when it is shifted by

This is a nonlinear process, easy to carry out by image warping, but not easy to write down as an equation.

Nonlinear System

Find **d** such that the above equation is satisfied

$$\frac{\partial E}{\partial \mathrm{d}} = 2\frac{\partial J(\mathrm{x})^T}{\partial \mathrm{x}}\left(J(\mathrm{x}+\mathrm{d}) - I(\mathrm{x})\right) = 0$$

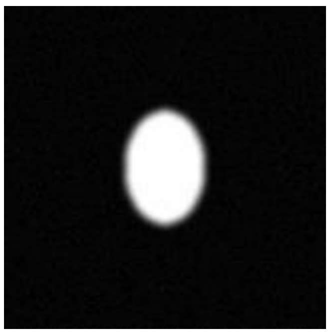
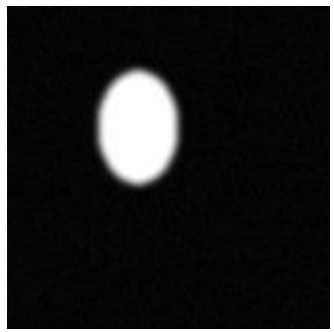
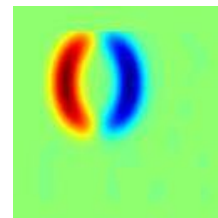
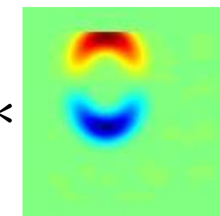


Idea: how to predict an image when it is shifted by $\Delta\mathrm{d}$

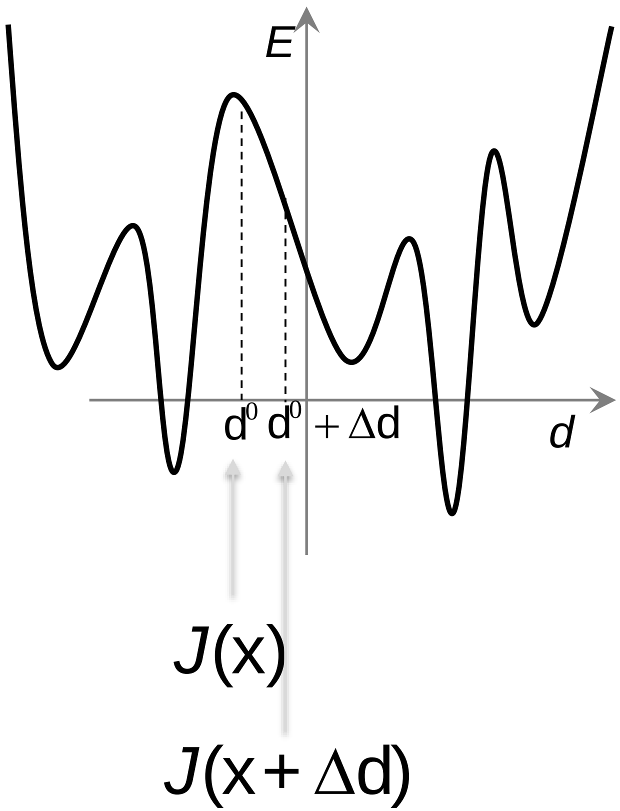
Taylor expansion:

$$J(\mathrm{x}+\Delta\mathrm{d}) = J(\mathrm{x}) + \frac{\partial J(\mathrm{x})}{\partial \mathrm{x}}\Delta\mathrm{d} + \mathrm{H.O.T.}$$

Step 2: Taylor expansion $J(\mathrm{x}+\Delta \mathrm{d}) = J(\mathrm{x}) + \frac{\partial J(\mathrm{x})}{\partial \mathrm{x}}\Delta \mathrm{d} + \mathrm{H.O.T.}$

 =  + $\mathbf{d_x} *$  + $\mathbf{d_y} *$ 

Step 2: Taylor expansion



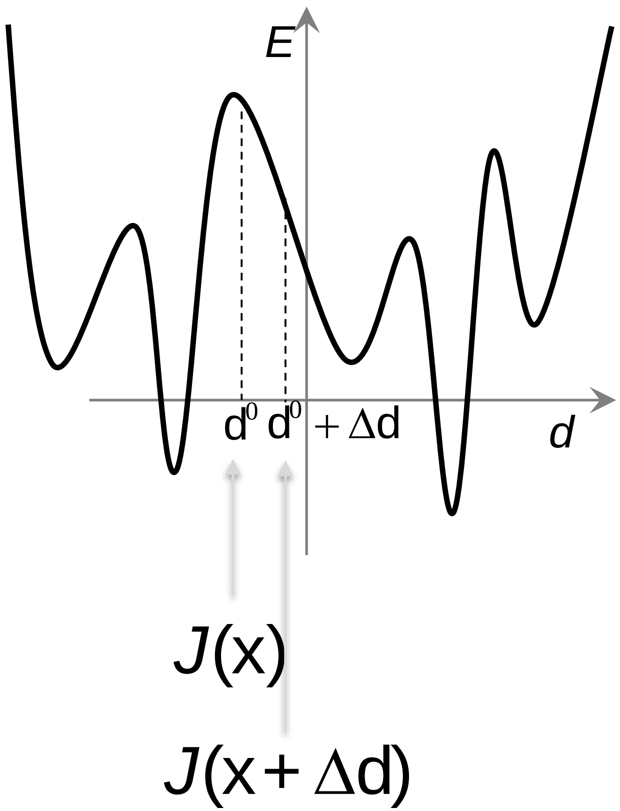
Find **d** such that the above equation is satisfied

$$\frac{\partial E}{\partial d} = 2\frac{\partial J(x)}{\partial x}^{T}\left(J(x+d) - I(x)\right) = 0$$

$$J(x+\Delta d) = J(x) + \frac{\partial J(x)}{\partial x}\Delta d + \text{H.O.T.}$$

$$\rightarrow \frac{\partial J(x)}{\partial x}^{T}\left(\frac{\partial J(x)}{\partial x}\Delta d\right) = \frac{\partial J(x)}{\partial x}^{T}\left(I(x) - J(x)\right)$$

Step 2: Taylor expansion



Find **d** such that the above equation is satisfie

$$\frac{\partial E}{\partial \mathrm{d}} = 2\frac{\partial J(\mathrm{x})}{\partial \mathrm{x}}^{\mathrm{T}}\left(J(\mathrm{x}+\mathrm{d}) - I(\mathrm{x})\right) = 0$$

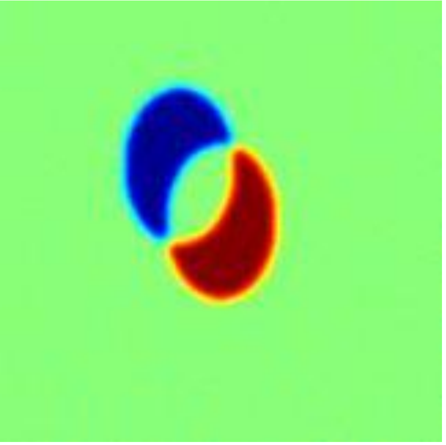
$$J(\mathrm{x}+\Delta\mathrm{d}) = J(\mathrm{x}) + \frac{\partial J(\mathrm{x})}{\partial \mathrm{x}}\Delta\mathrm{d} + \mathrm{H.O.T.}$$

$$\rightarrow \frac{\partial J(\mathrm{x})}{\partial \mathrm{x}}^{\mathrm{T}}\left(\frac{\partial J(\mathrm{x})}{\partial \mathrm{x}}\Delta\mathrm{d}\right) = \frac{\partial J(\mathrm{x})}{\partial \mathrm{x}}^{\mathrm{T}}\left(I(\mathrm{x}) - J(\mathrm{x})\right)$$

$$\rightarrow \left(\frac{\partial J(\mathrm{x})}{\partial \mathrm{x}}^{\mathrm{T}}\frac{\partial J(\mathrm{x})}{\partial \mathrm{x}}\right)\Delta\mathrm{d} = \frac{\partial J(\mathrm{x})}{\partial \mathrm{x}}^{\mathrm{T}}\left(I(\mathrm{x}) - J(\mathrm{x})\right)$$

$$\left(\frac{\partial J(x)^T}{\partial x}\frac{\partial J(x)}{\partial x}\right)\boxed{\Delta d} = \frac{\partial J(x)^T}{\partial x}\left(I(x) - J(x)\right)$$

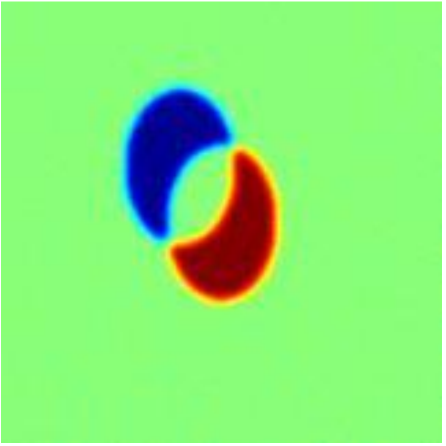
2D unknowns flow vector per pixel, 2 equations



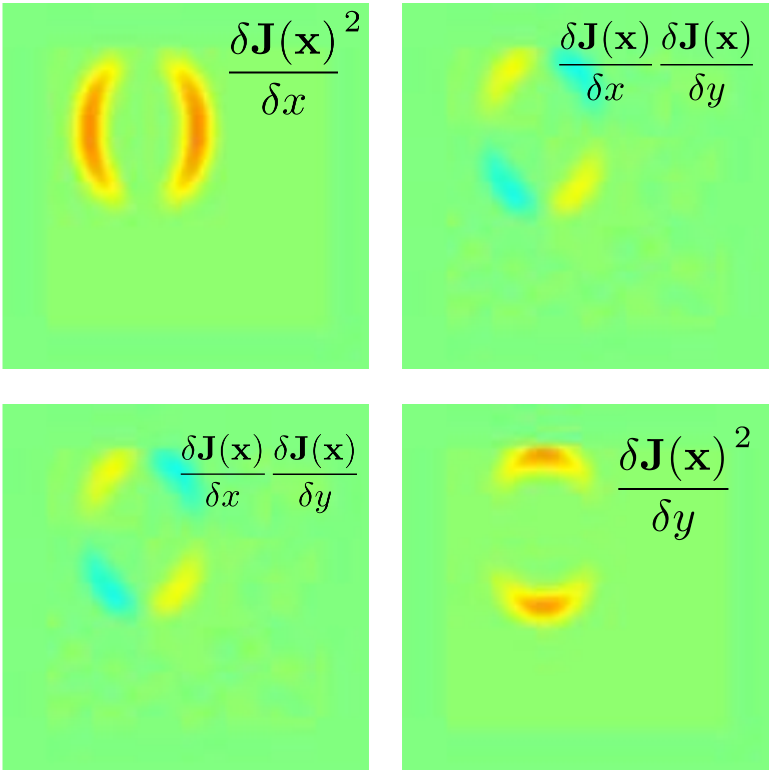
$$\left(\frac{\partial J(x)^T}{\partial x}\frac{\partial J(x)}{\partial x}\right)\Delta d = \frac{\partial J(x)^T}{\partial x}\left(I(x) - J(x)\right)$$

2D unknowns flow vector per pixel, 2 equations

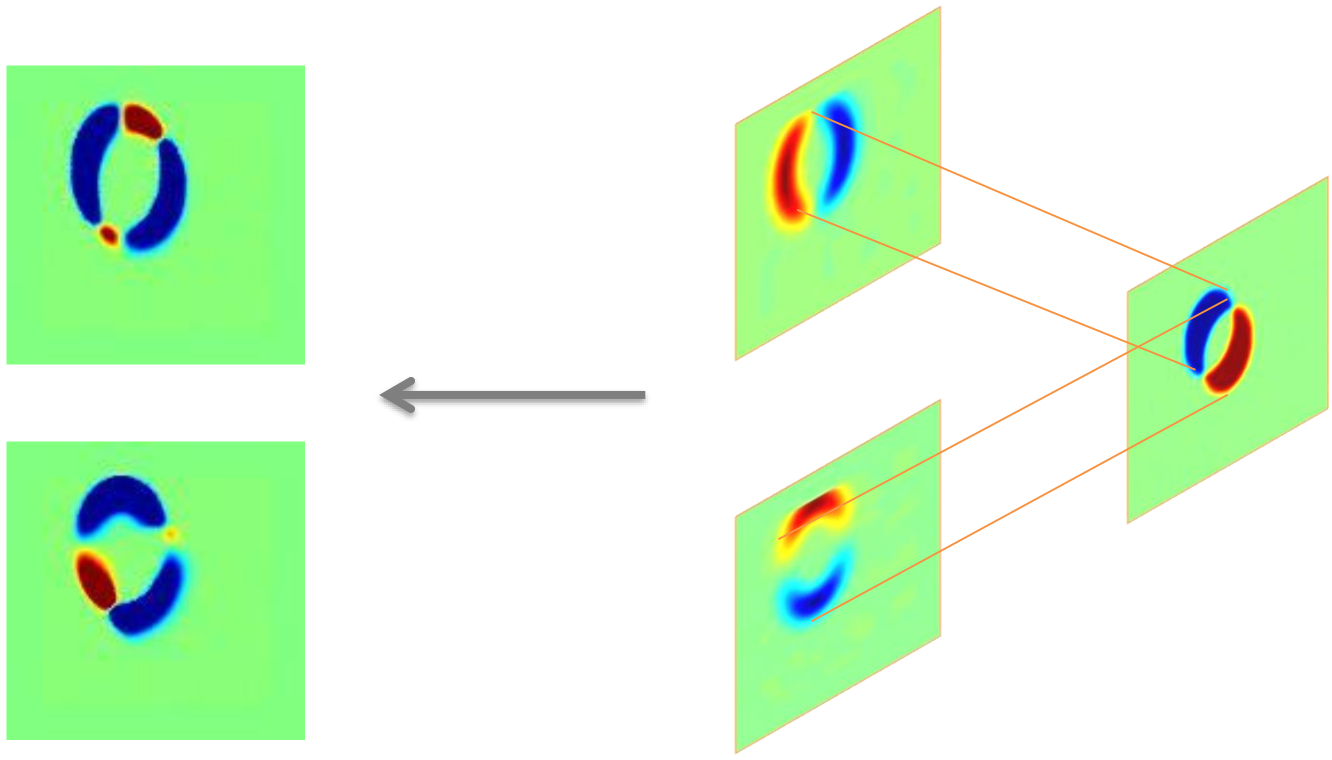
Also known as second moment matrix



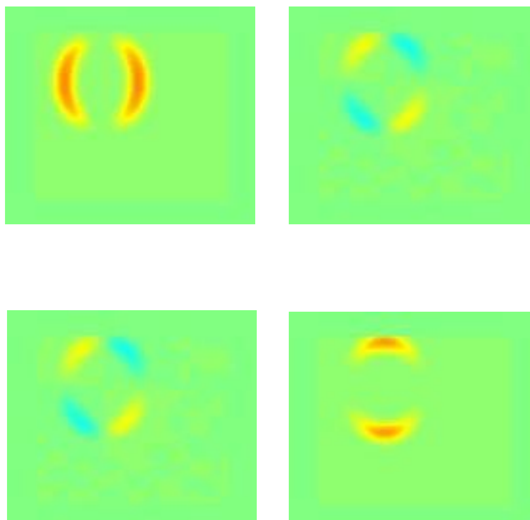
$$\left(\frac{\partial J(x)}{\partial x}^{T} \frac{\partial J(x)}{\partial x} \right)$$



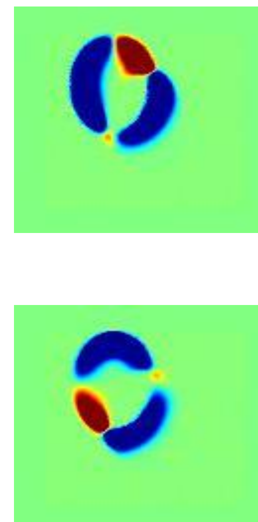
$$\frac{\partial J(\mathrm{x})^T}{\partial \mathrm{x}}\left(I(\mathrm{x})-J(\mathrm{x})\right)$$



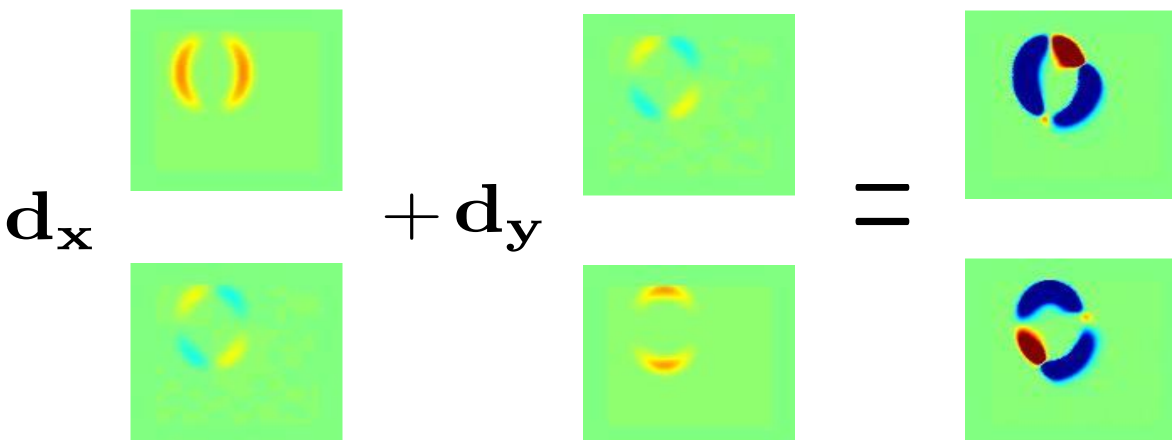
$$\left(\frac{\partial J(x)^T}{\partial x}\frac{\partial J(x)}{\partial x}\right)\Delta d = \frac{\partial J(x)^T}{\partial x}\left(I(x)-J(x)\right)$$



$$\begin{matrix}\mathbf{d_x} \\ \mathbf{d_y}\end{matrix} =$$

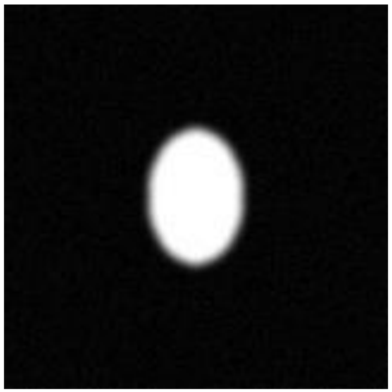


$$\left(\frac{\partial J(x)^T}{\partial x}\frac{\partial J(x)}{\partial x}\right)\Delta d = \frac{\partial J(x)^T}{\partial x}\big(I(x) - J(x)\big)$$

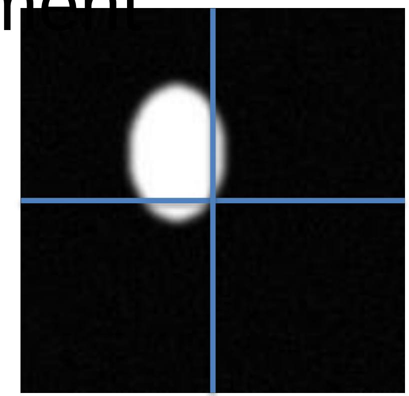


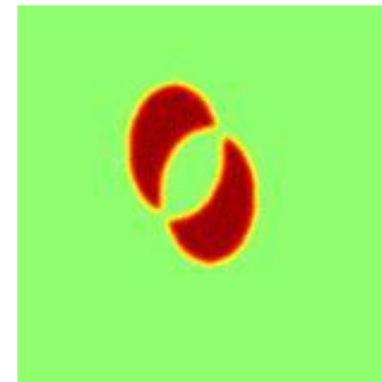
$\mathbf{I}(\mathbf{x})$ $\mathbf{J}(\mathbf{x})$ Error

Solve for displacement

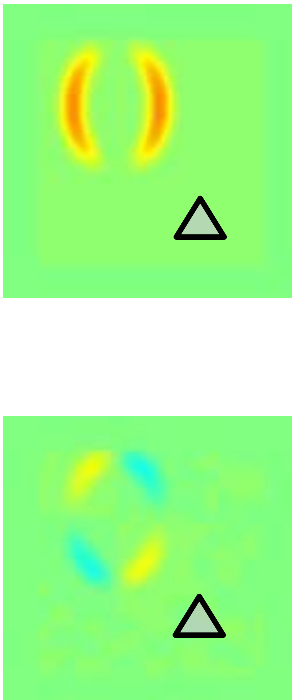


$$\mathbf{d} = (-7, -9)$$





$\mathbf{d_x}$



$+\mathbf{d_y}$

$=$

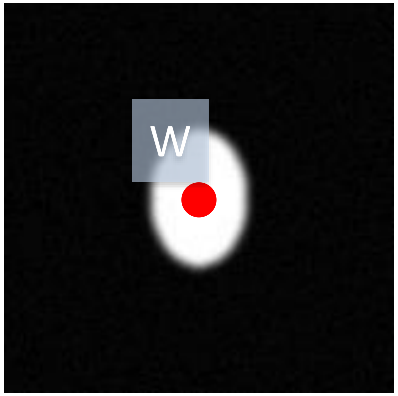
$$\left(\frac{\partial J(\mathrm{x})}{\partial \mathrm{x}}^{\mathsf{T}} \frac{\partial J(\mathrm{x})}{\partial \mathrm{x}}\right)\Delta \mathrm{d} = \frac{\partial J(\mathrm{x})}{\partial \mathrm{x}}^{\mathsf{T}}\big(I(\mathrm{x}) - J(\mathrm{x})\big)$$

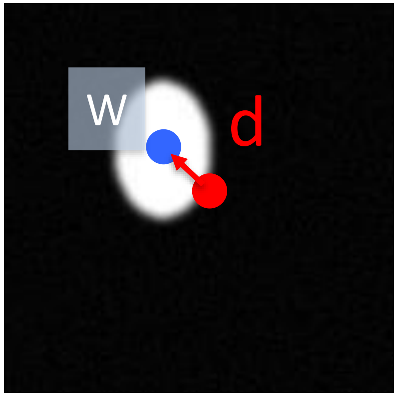
$$\mathbf{d_x}\begin{bmatrix} 0 \\ \triangle \\ 0 \\ \triangle \end{bmatrix} + \mathbf{d_y}\begin{bmatrix} 0 \\ \triangle \\ 0 \\ \triangle \end{bmatrix} = \begin{bmatrix} 0 \\ \triangle \\ 0 \\ \triangle \end{bmatrix}$$

Cannot solve for the displacement

$$\min_{\mathbf{d}} E = \sum_{x \in W} \left\| J(x + d) - I(x) \right\|^2$$

Pooling over a window

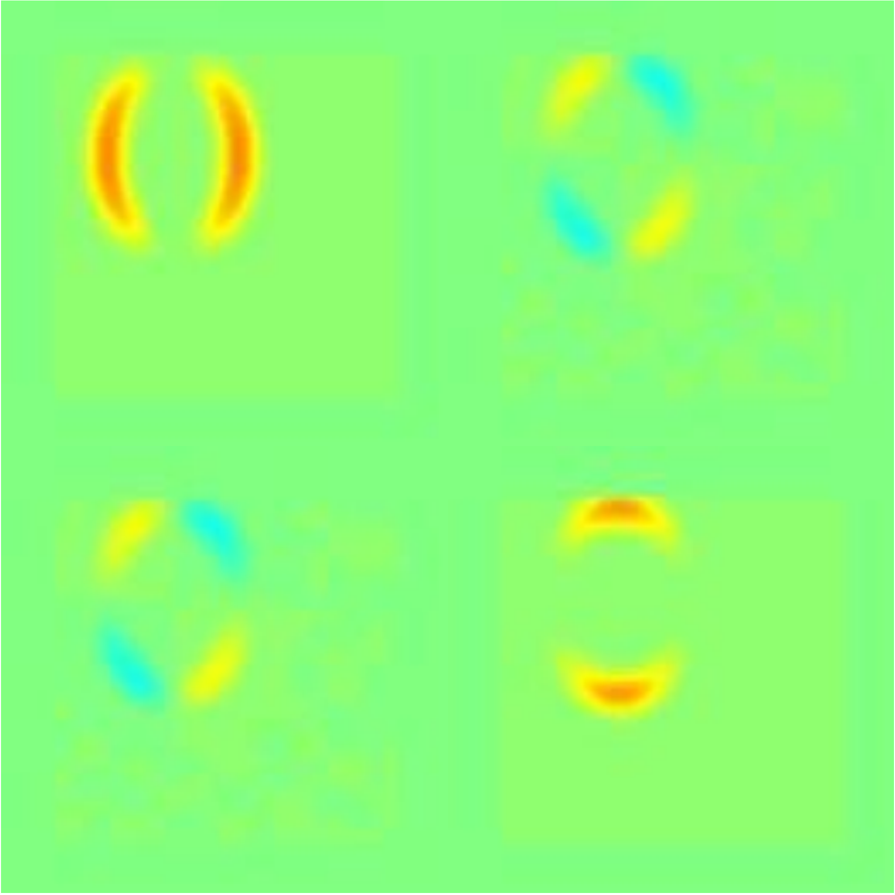




$$\min_{\mathbf{d}} E = \sum_{\mathrm{x} \in W} \left\| J(\mathrm{x}+\mathrm{d}) - I(\mathrm{x}) \right\|^2$$

$$\sum_{\mathrm{x} \in W} \left(\frac{\partial J(\mathrm{x})^{\mathrm{T}}}{\partial \mathrm{x}} \frac{\partial J(\mathrm{x})}{\partial \mathrm{x}} \right) \Delta \mathrm{d} = \sum_{\mathrm{x} \in W} \frac{\partial J(\mathrm{x})^{\mathrm{T}}}{\partial \mathrm{x}} \left(I(\mathrm{x}) - J(\mathrm{x}) \right)$$

Summing over pixels



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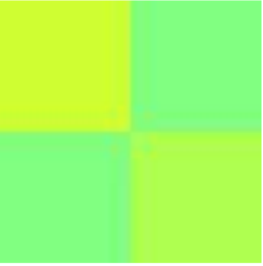
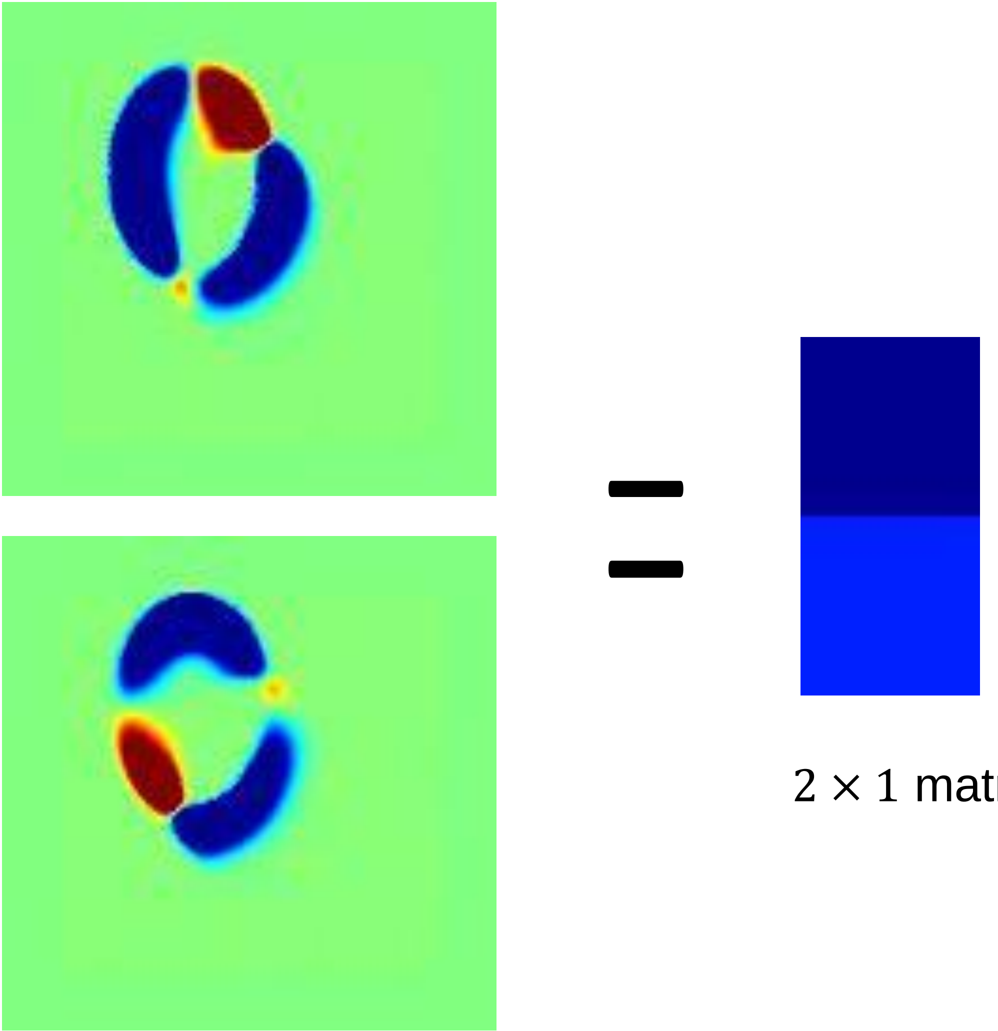


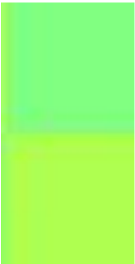
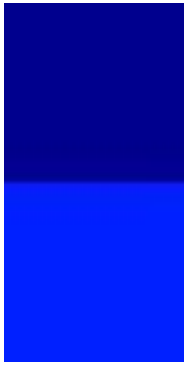
2×2 matrix

Summing over pixels



2×1 matrix

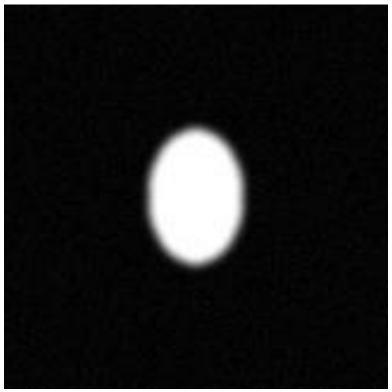
$$\sum_{x\in W}\left(\frac{\partial J(x)^{T}}{\partial x}\frac{\partial J(x)}{\partial x}\right)\Delta d=\sum_{x\in W}\frac{\partial J(x)^{T}}{\partial x}\big(I(x)-J(x)\big)$$

$\mathbf{d_x}$ $+\,\mathbf{d_y}$  $=$ 

Video 6.3
Jianbo Shi

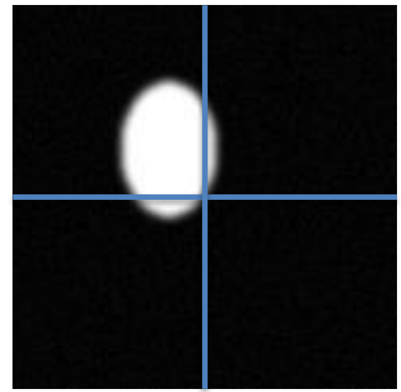
Step 3: Solve for displacement, warp image, and iter

$\mathbf{I}(\mathbf{x})$

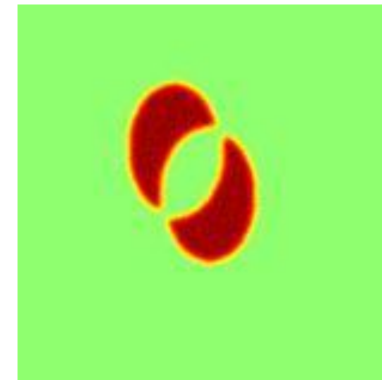


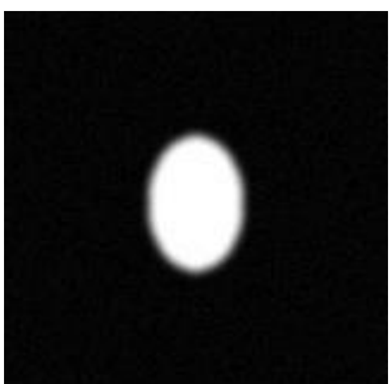
$\mathbf{d} = (-7, -9)$

$\mathbf{J}(\mathbf{x})$



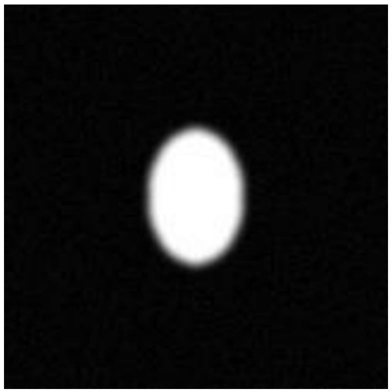
Error



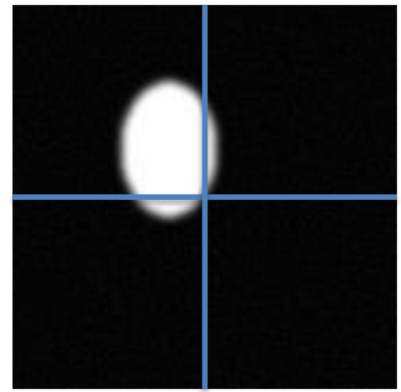


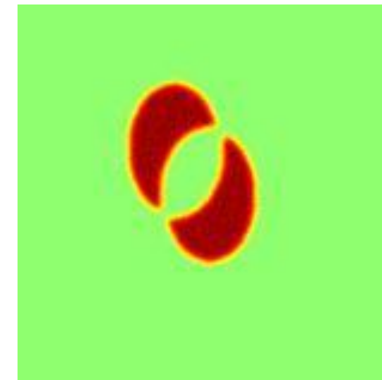
Step 3: Solve for displacement, warp image, and iter

$\mathbf{I}(\mathbf{x})$ $\mathbf{J}(\mathbf{x})$ Error

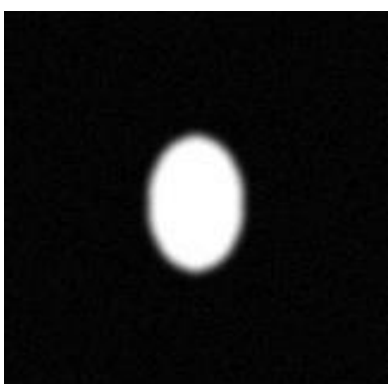


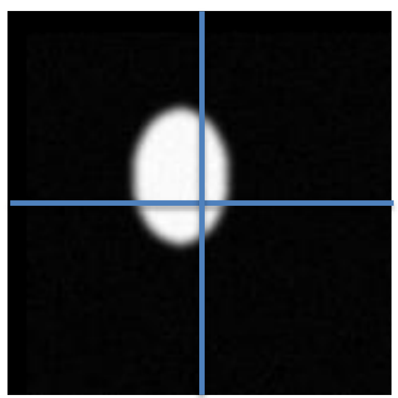
$$\mathbf{d} = (-\mathbf{7}, -\mathbf{9})$$

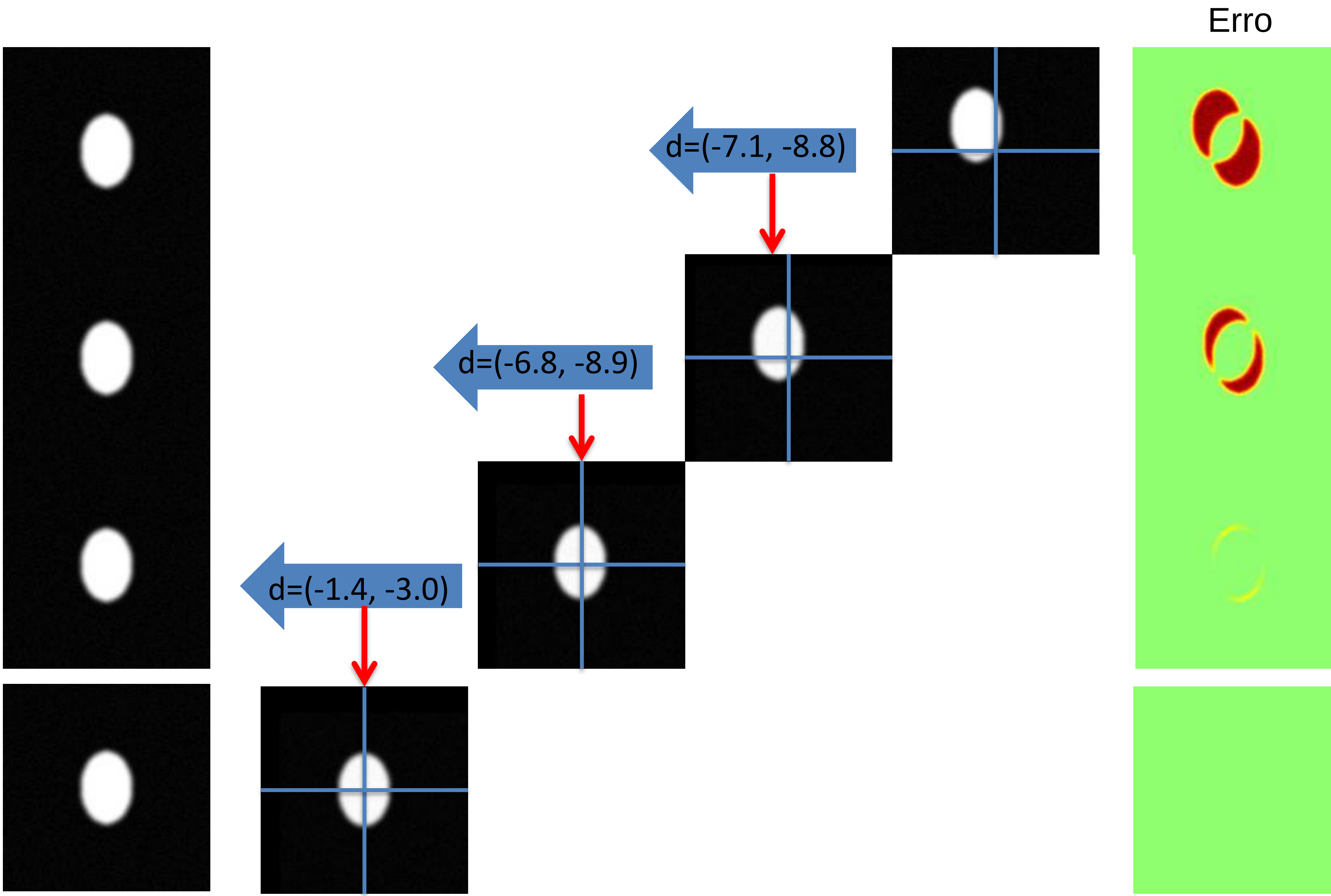


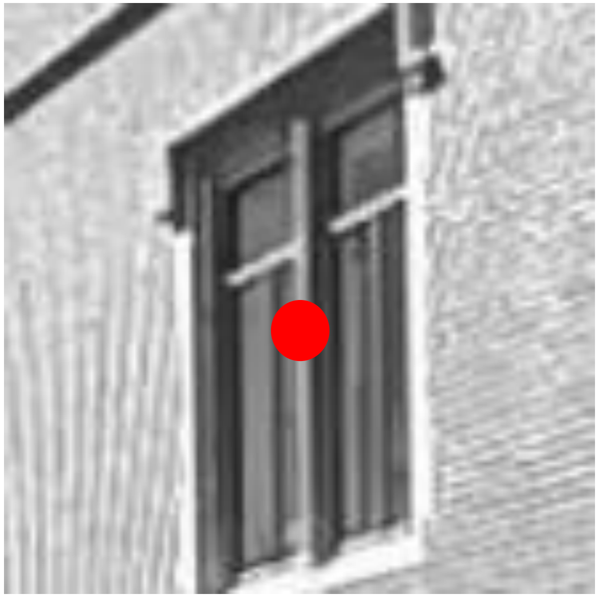


$$\mathbf{J}^{t=1}(\mathbf{x}) = \mathbf{J}(\mathbf{x} + \mathbf{d})$$






Erro
d=(-7.1, -8.8)
d=(-6.8, -8.9)
d=(-1.4, -3.0)



$$\mathbf{I}(\mathbf{x})$$

$$t = 0$$

$$\mathbf{J}(\mathbf{x})$$

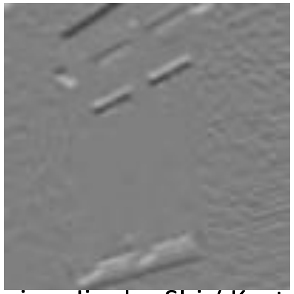
$$t = 1$$

$$\left(\frac{\partial J(\mathrm{x})^{T}}{\partial \mathrm{x}}\frac{\partial J(\mathrm{x})}{\partial \mathrm{x}}\right)\Delta \mathrm{d} = \frac{\partial J(\mathrm{x})^{T}}{\partial \mathrm{x}}\left(I(\mathrm{x}) - J(\mathrm{x})\right)$$

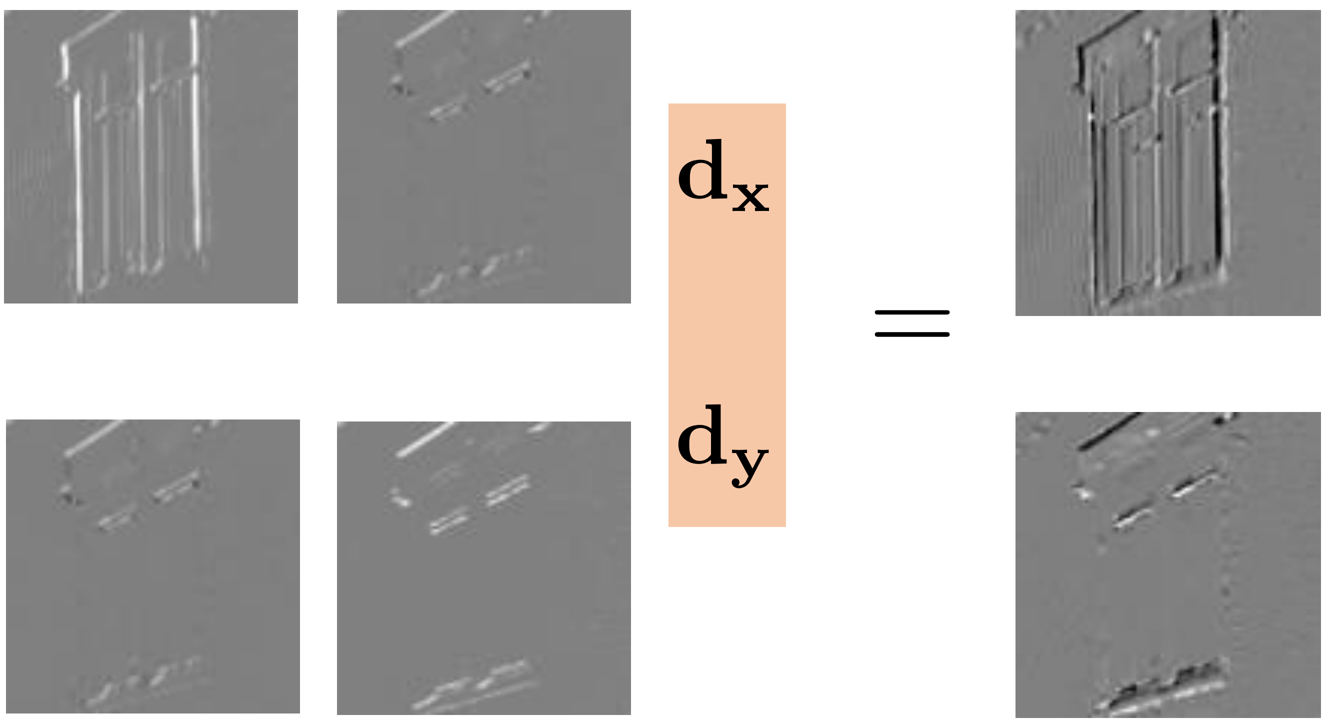
$$\left(\frac{\partial J(x)^T}{\partial x}\frac{\partial J(x)}{\partial x}\right)\Delta d = \frac{\partial J(x)^T}{\partial x}\left(I(x) - J(x)\right)$$

$$\frac{\delta \mathbf{J}(\mathbf{x})}{\delta x} =$$

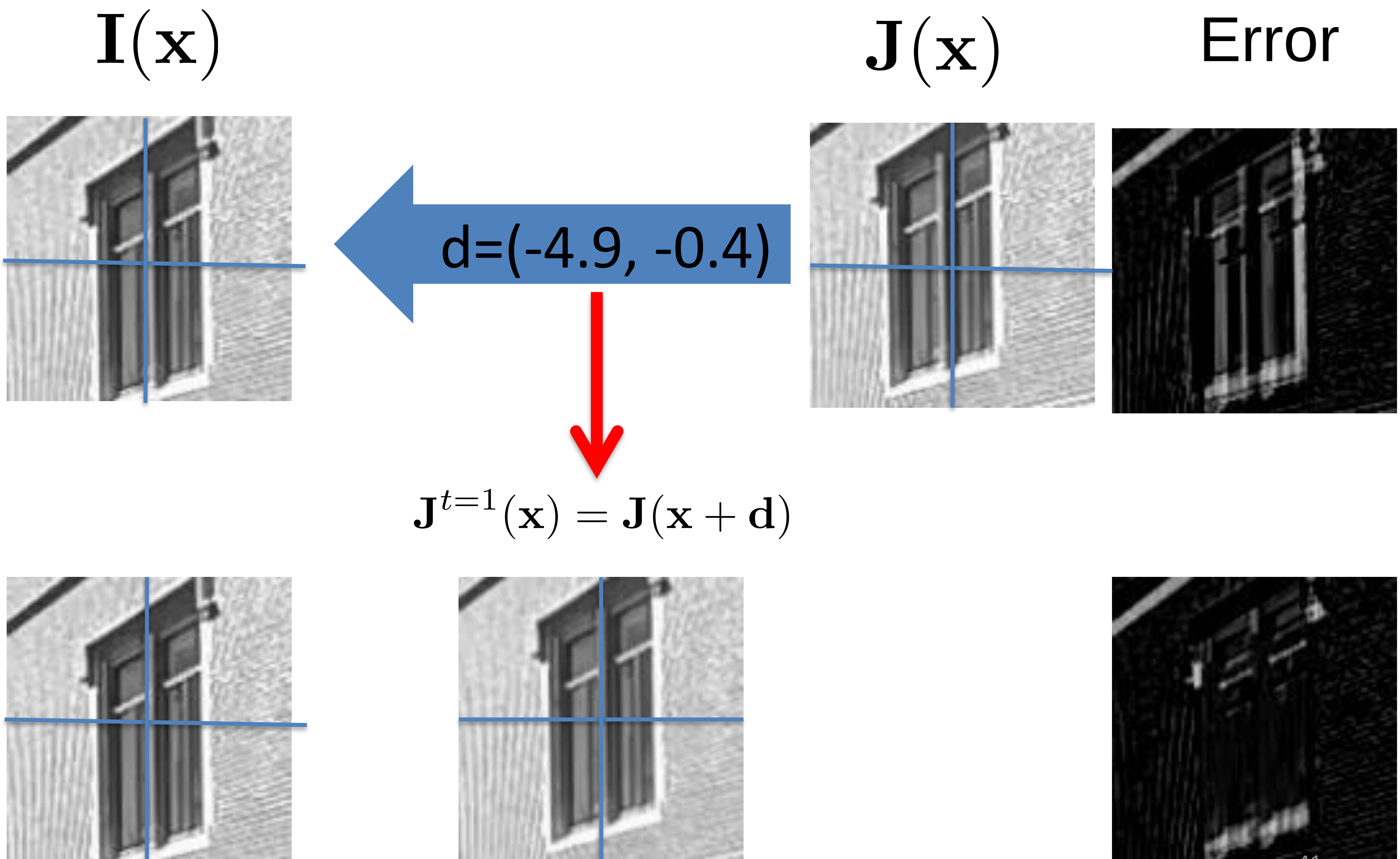
$$\frac{\delta \mathbf{J}(\mathbf{x})}{\delta y} =$$

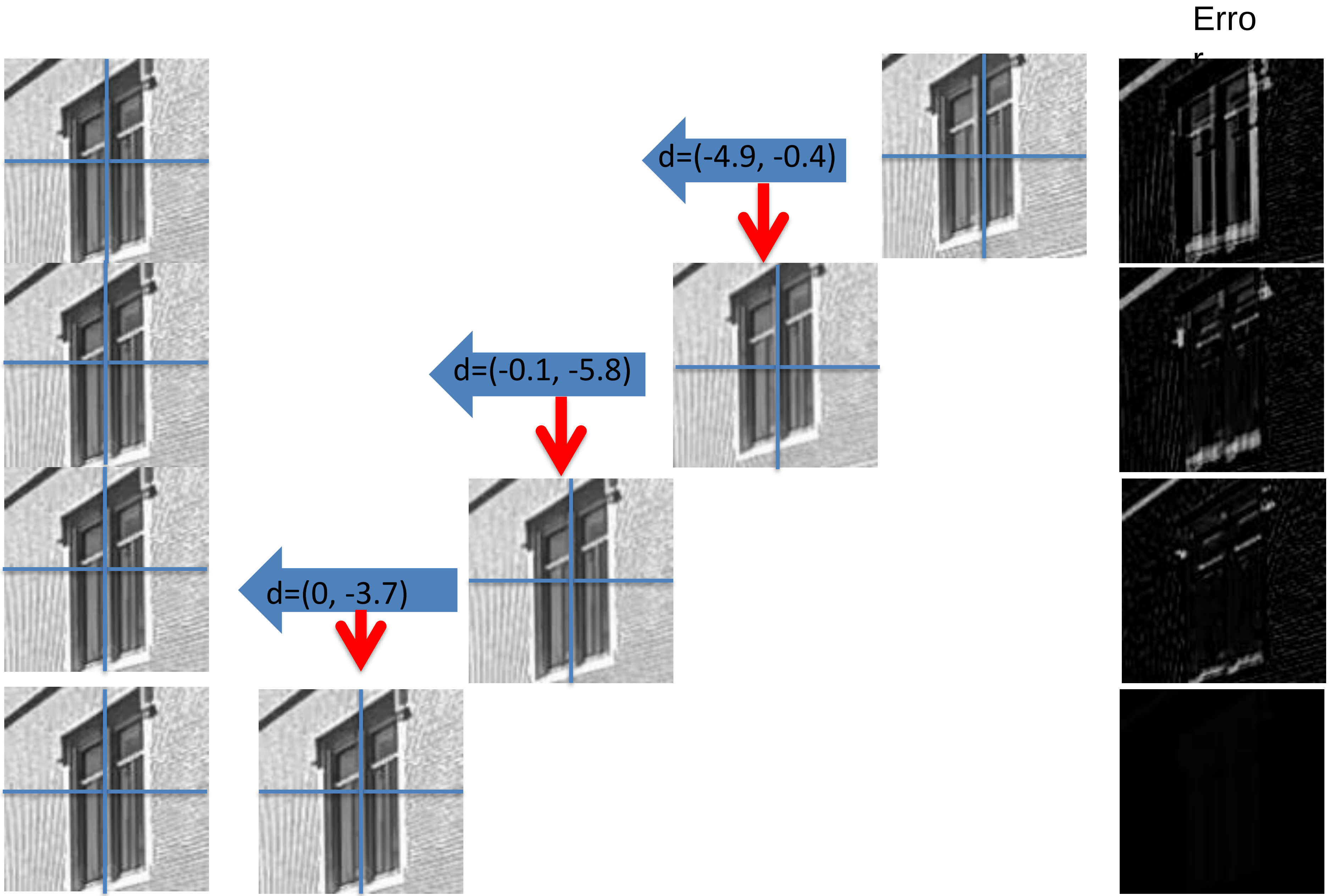


$$\left(\frac{\partial J(x)^T}{\partial x}\frac{\partial J(x)}{\partial x}\right)\Delta d = \frac{\partial J(x)^T}{\partial x}\big(I(x)-J(x)\big)$$



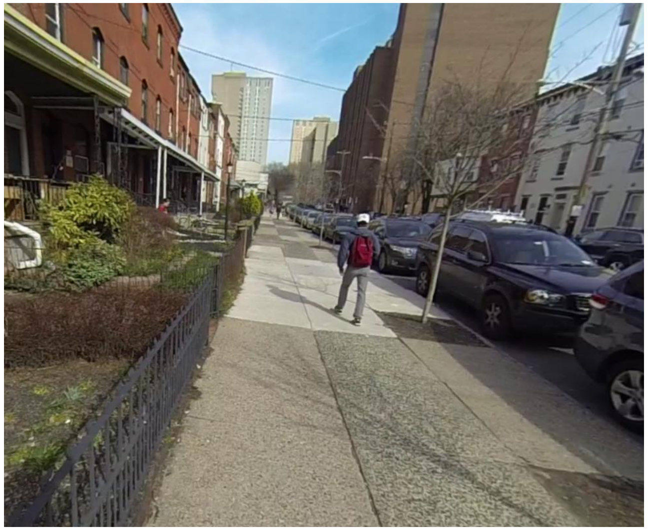
Step 3: Solve for displacement, warp image, an

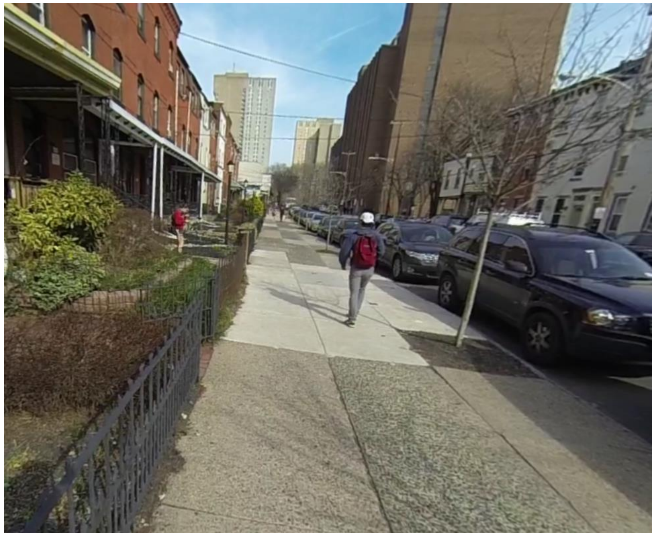


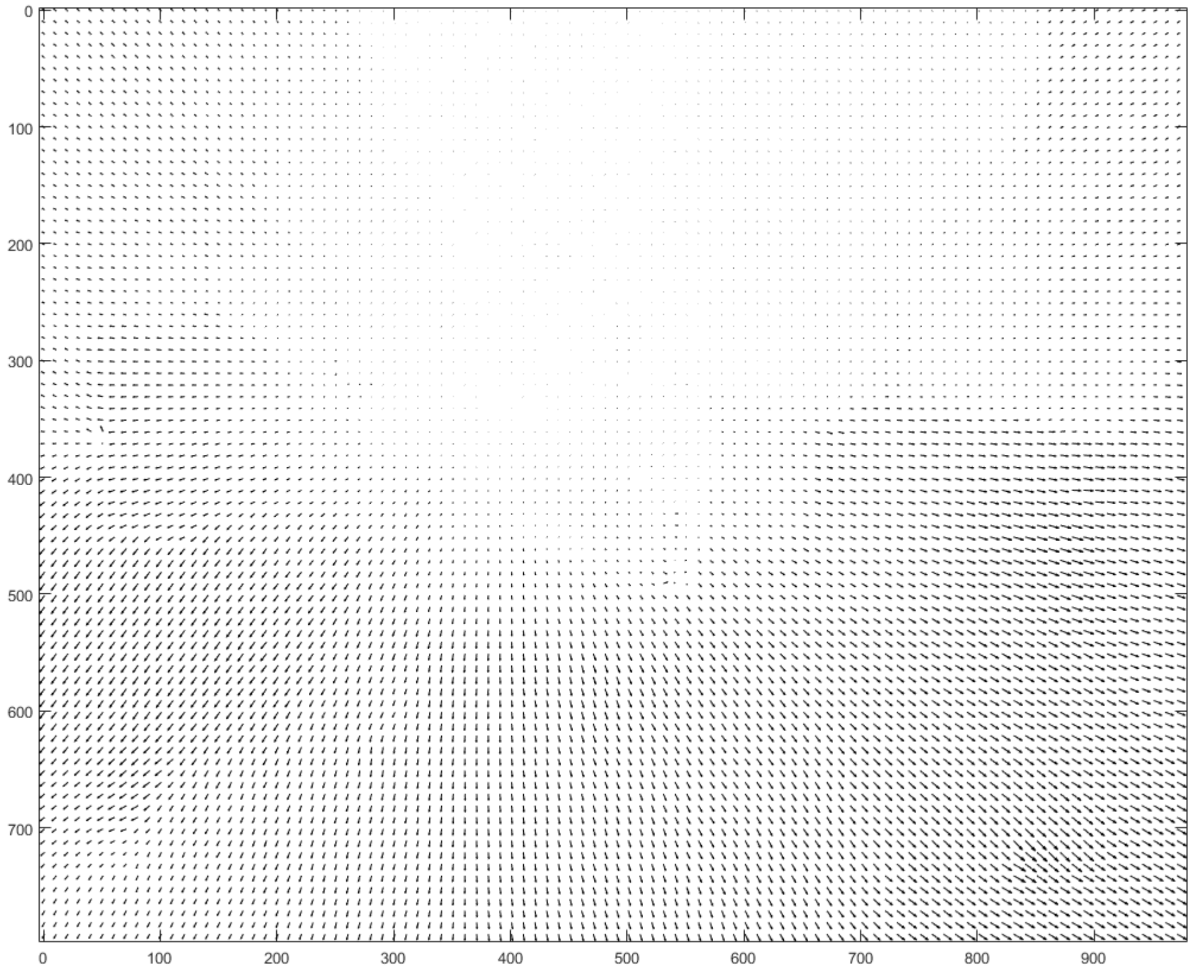

Erro
r
d=(-4.9, -0.4)
d=(-0.1, -5.8)
d=(0, -3.7)

Video 6.4
Jianbo Shi

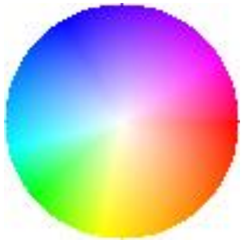
Dense optical flow encodes object motion

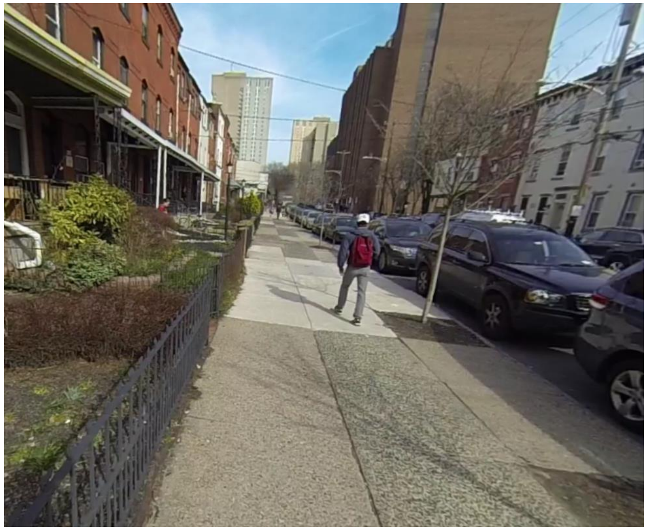


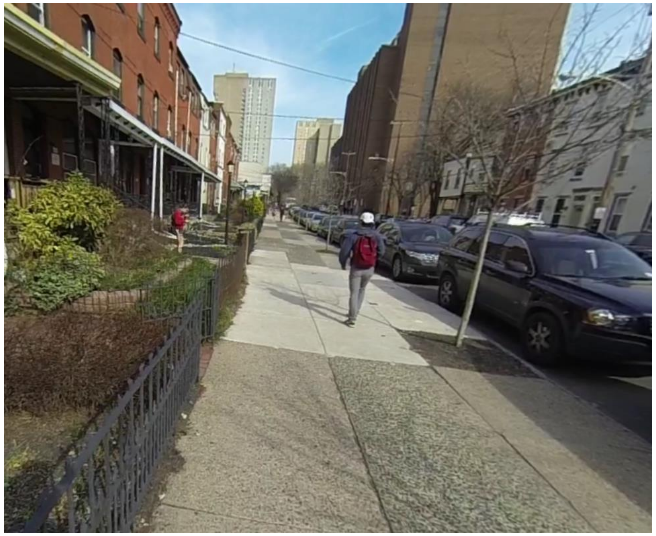




Dense optical flow encodes object motion



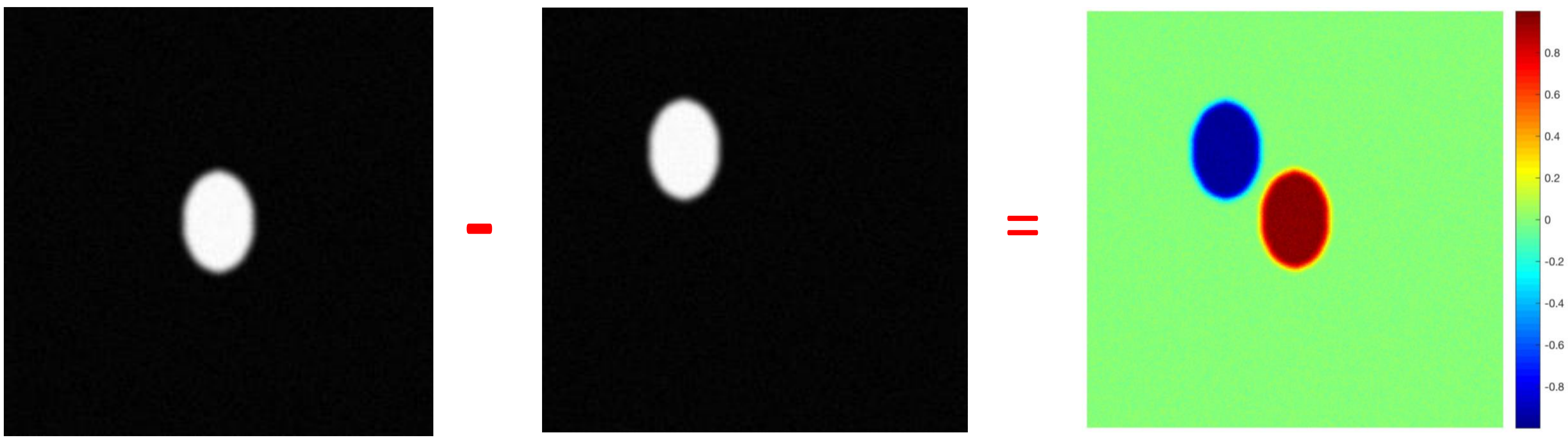




A Failed Case: fast movement

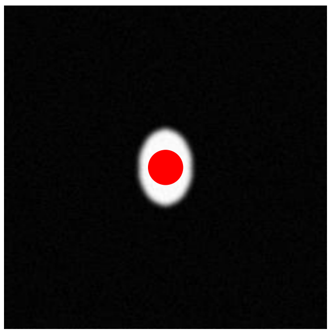
$$\left(\frac{\partial J(\mathrm{x})}{\partial \mathrm{x}}^{T}\frac{\partial J(\mathrm{x})}{\partial \mathrm{x}}\right)\Delta \mathrm{d} = \frac{\partial J(\mathrm{x})}{\partial \mathrm{x}}^{T}\left(I(\mathrm{x}) - J(\mathrm{x})\right)$$

A Failed Case: fast movement



$I(\mathrm{x}) = J(\mathrm{x} + \mathrm{d})$ $J(\mathrm{x})$

$\mathbf{I}(\mathbf{x})$ $\mathbf{J}(\mathbf{x})$



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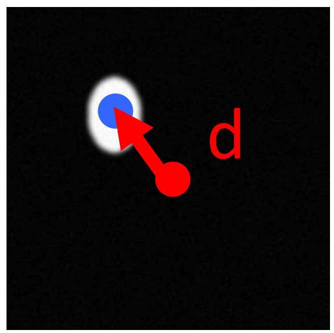
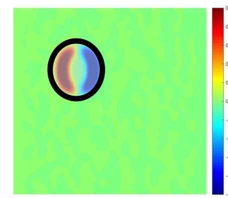
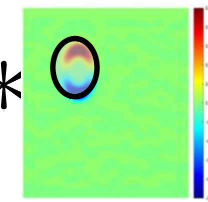


$+\,\mathbf{d_x} *$

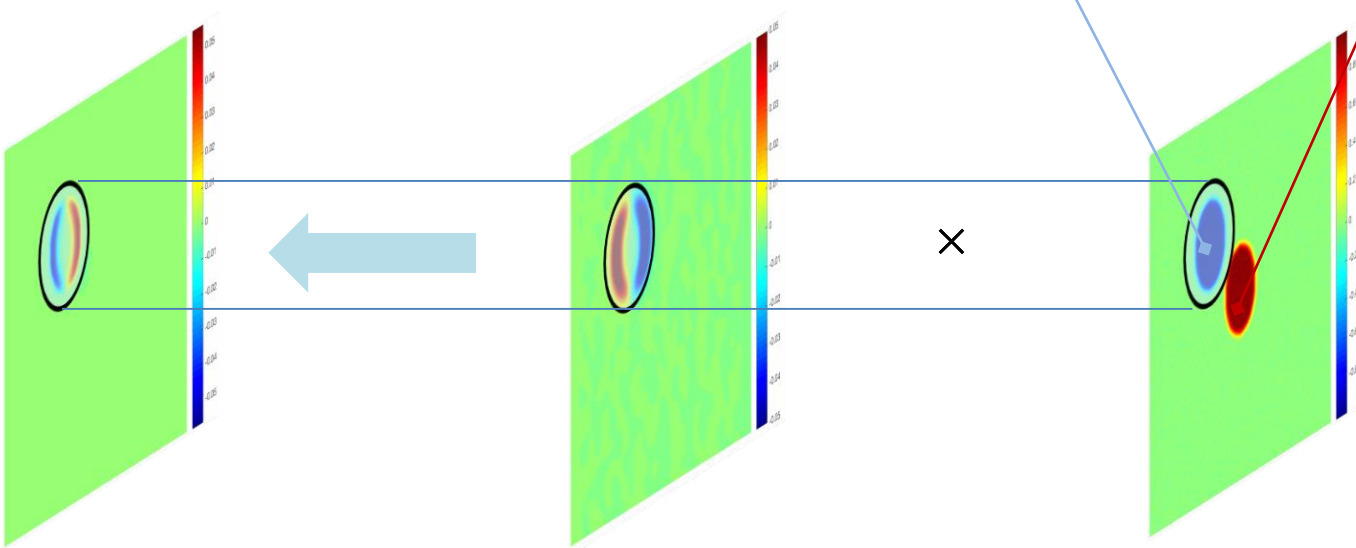


$+\,\mathbf{d_y} *$



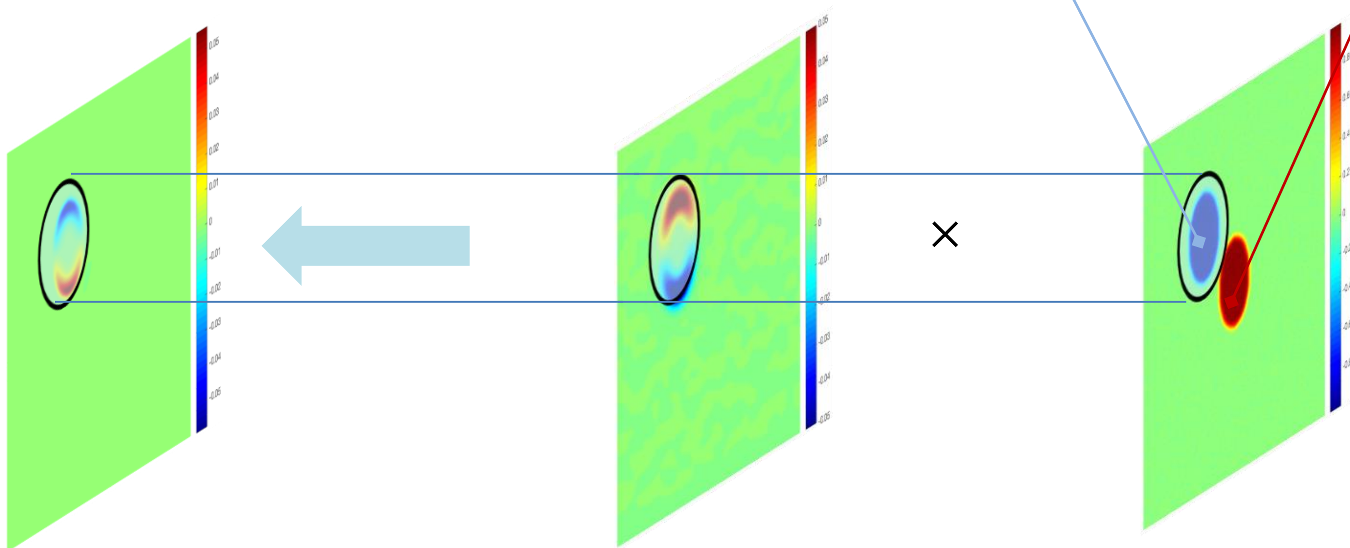
$$I(\mathrm{x}) = J(\mathrm{x}+\mathrm{d})$$

$$\frac{\partial J(x)^T}{\partial x}\left(I(x) - J(x)\right) \qquad \frac{\partial J(x)}{\partial x} \qquad -J(x) \qquad I(x)$$



The influence of I(x) is not incorporated!

$$\frac{\partial J(\mathrm{x})^{T}}{\partial \mathrm{x}}\left(I(\mathrm{x})-J(\mathrm{x})\right) \qquad \frac{\partial J(\mathrm{x})}{\partial y} \qquad -J(\mathrm{x}) \qquad I(\mathrm{x})$$



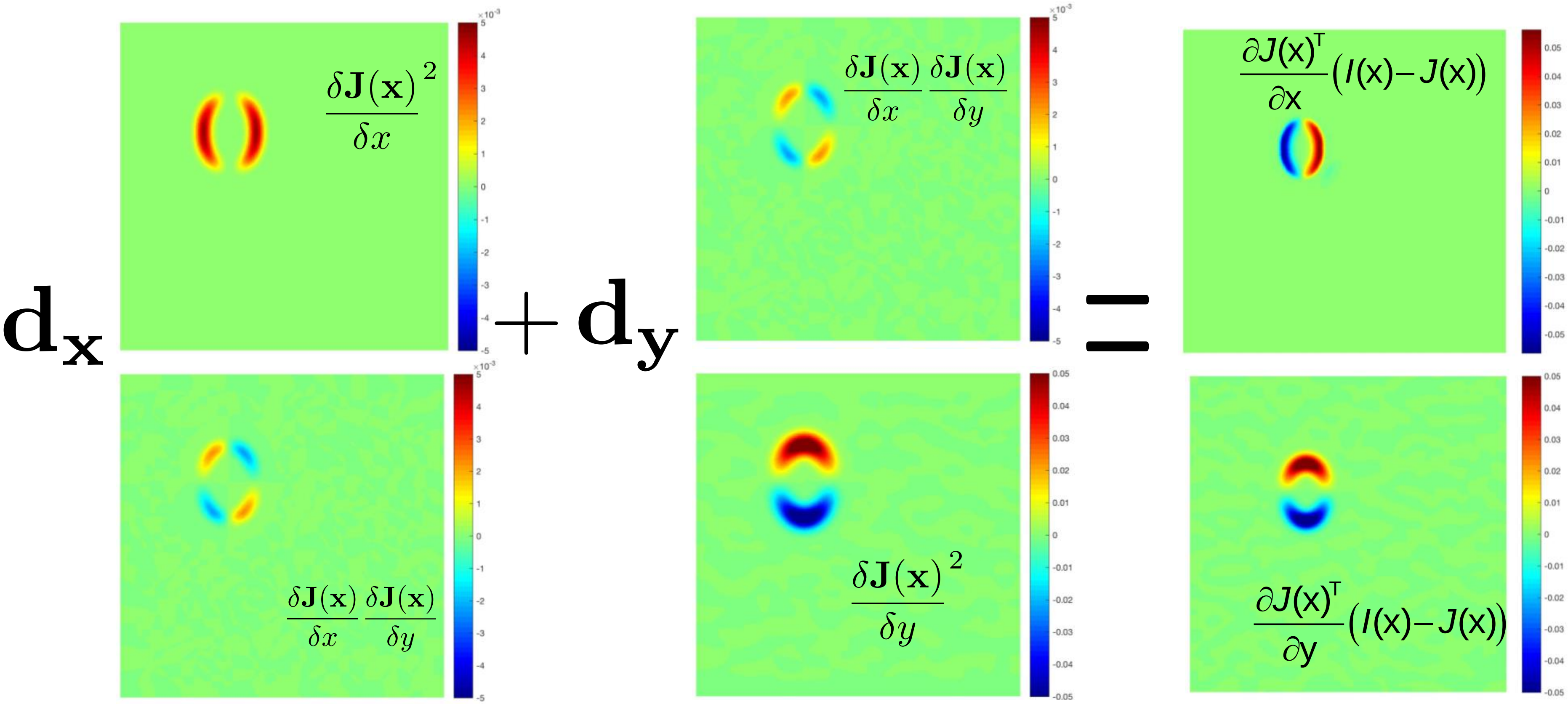
The influence of I(x) is not incorporated!

$$\left(\frac{\partial J(\mathrm{x})}{\partial \mathrm{x}}^{\mathrm{T}}\frac{\partial J(\mathrm{x})}{\partial \mathrm{x}}\right)\Delta \mathrm{d}=\frac{\partial J(\mathrm{x})}{\partial \mathrm{x}}^{\mathrm{T}}\left(I(\mathrm{x})-J(\mathrm{x})\right)$$

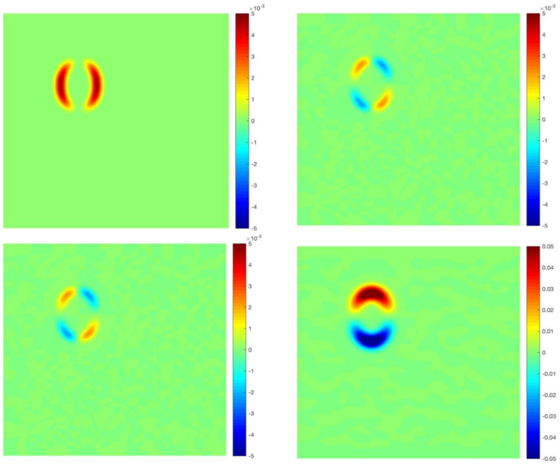
The influence of I(x) is Not included!

Guess what's the corresponding displacement?

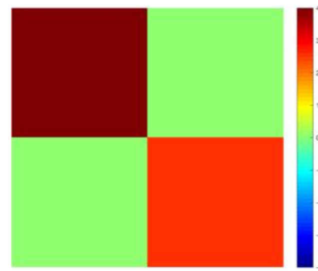
$$\left(\frac{\partial J(x)^T}{\partial x}\frac{\partial J(x)}{\partial x}\right)\Delta d = \frac{\partial J(x)^T}{\partial x}\big(I(x) - J(x)\big)$$



Summing over pixels



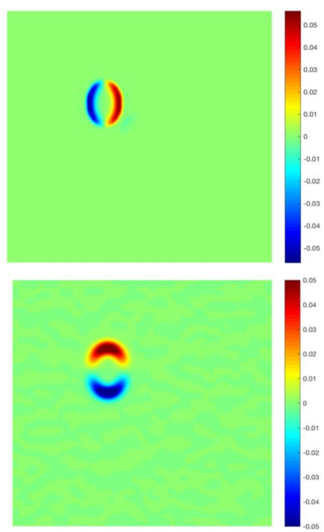
=



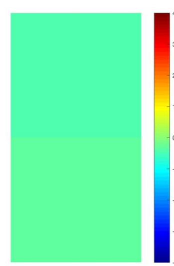
2×2
matrix

$$\frac{\partial J(x)^T}{\partial x} \frac{\partial J(x)}{\partial x}$$

Summing over pixels



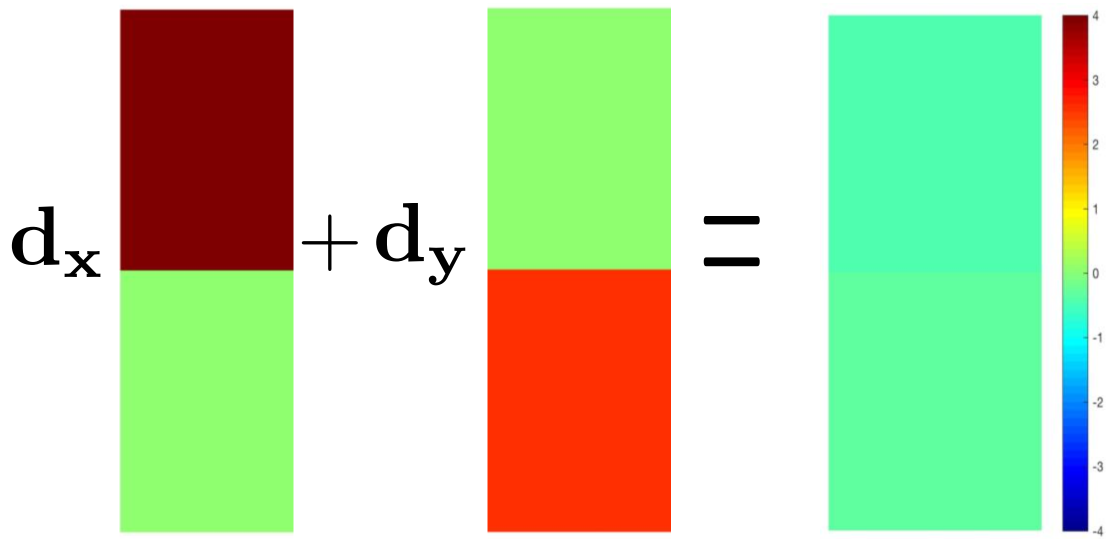
=



2×1
matri
x

$$\frac{\partial J(x)^T}{\partial x}\left(I(x) - J(x)\right)$$

$$\boxed{\left(\frac{\partial J(\mathrm{x})^{T}}{\partial \mathrm{x}}\frac{\partial J(\mathrm{x})}{\partial \mathrm{x}}\right)}\Delta \mathrm{d}=\boxed{\frac{\partial J(\mathrm{x})^{T}}{\partial \mathrm{x}}\left(I(\mathrm{x})-J(\mathrm{x})\right)}$$



[−0.011, 0.09]

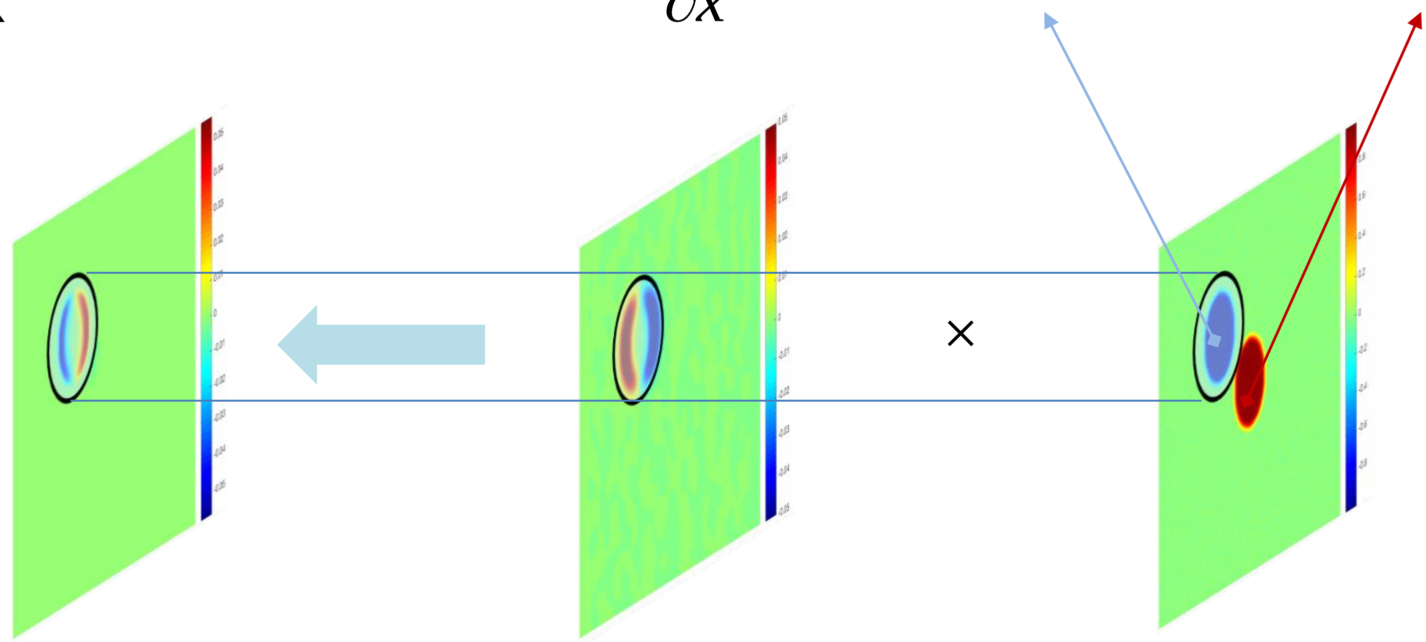
Almost zero motion, why?

$$\frac{\partial J(x)^T}{\partial x}\left(I(x) - J(x)\right)$$

$$\frac{\partial J(x)}{\partial x}$$

$$-J(x)$$

$$I(x)$$

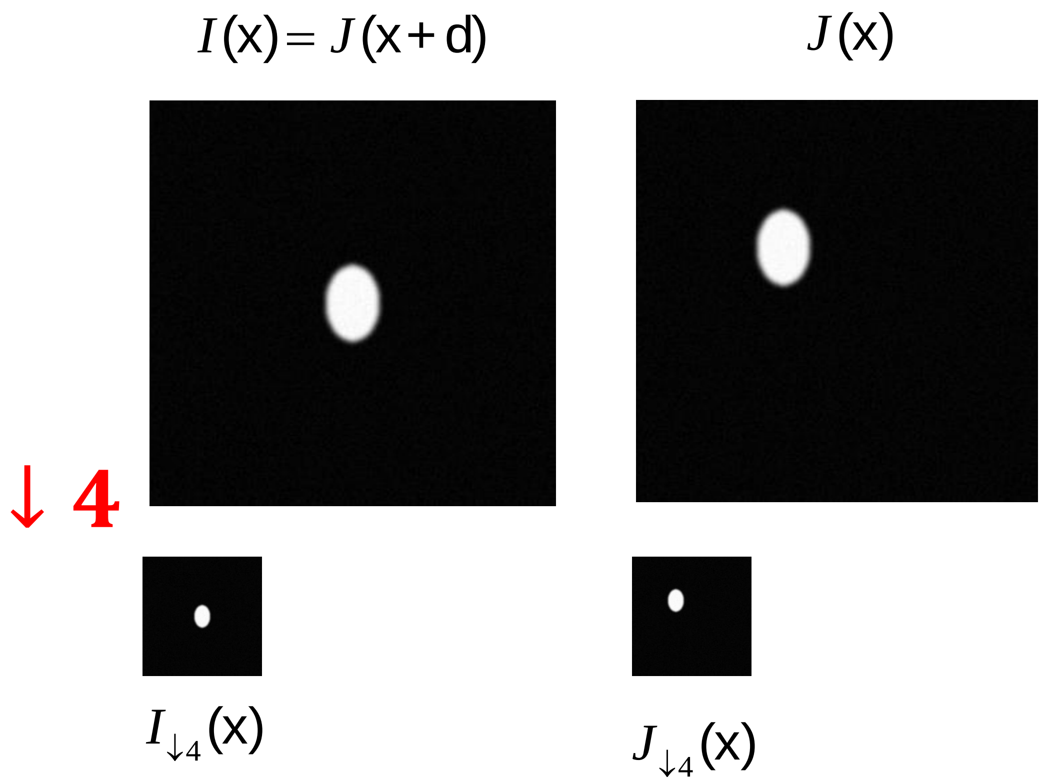


The influence of I(x) is not incorporated!

Video 6.5
Jianbo Shi

Solution 1: multi scale optical flow

Solution 2: increase the kernel size of gradient operator



Solution 1:
multi scale optical flow

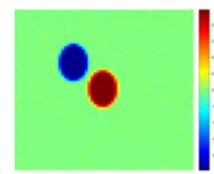
$$\left(\frac{\partial J_{\downarrow 4}(\mathrm{x})^{\mathrm{T}}}{\partial \mathrm{x}}\frac{\partial J_{\downarrow 4}(\mathrm{x})}{\partial \mathrm{x}}\right)\Delta \mathrm{d} = \frac{\partial J_{\downarrow 4}(\mathrm{x})^{\mathrm{T}}}{\partial \mathrm{x}}\left(I_{\downarrow 4}(\mathrm{x}) - J_{\downarrow 4}(\mathrm{x})\right)$$

$I_{\downarrow 4}(\mathrm{x})$

-

$J_{\downarrow 4}(\mathrm{x})$

=



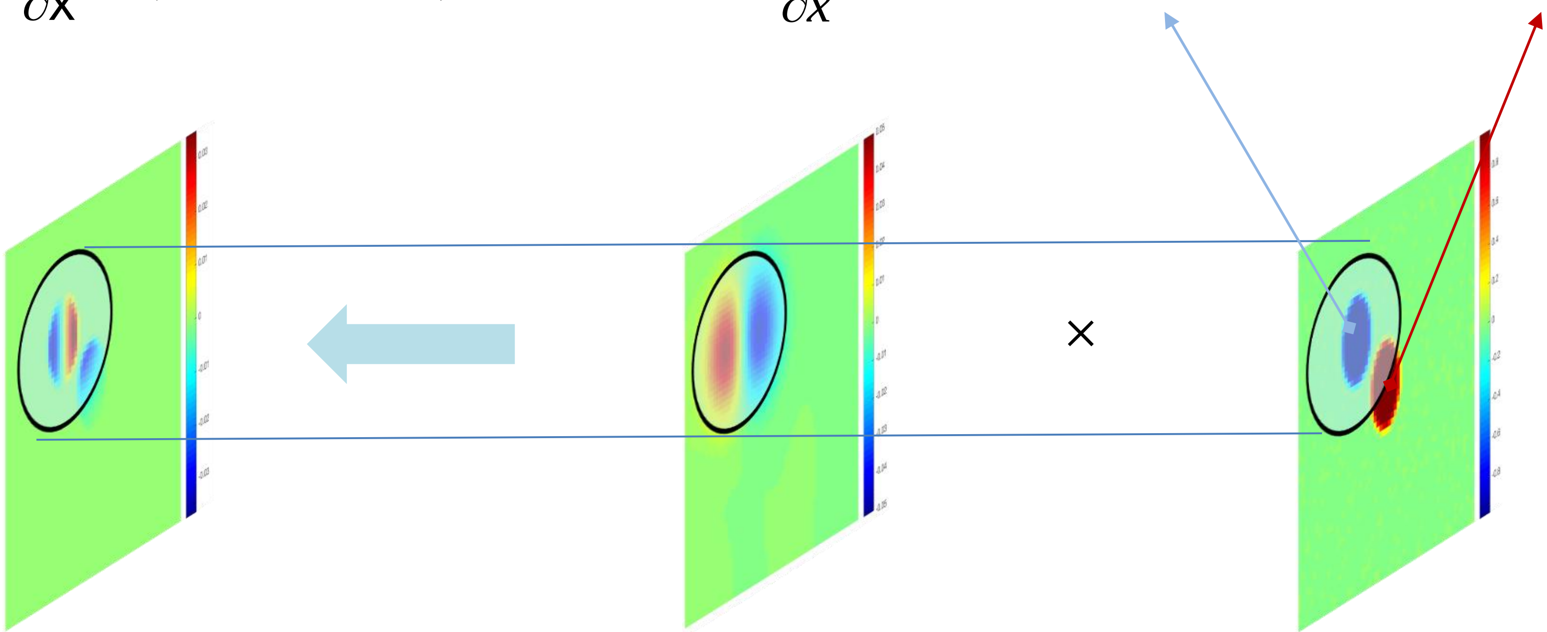
Start from a coarser resolution image

$$\frac{\partial J_{\downarrow 4}(\mathrm{x})^{\mathrm{T}}}{\partial \mathrm{x}}\left(I_{\downarrow 4}(\mathrm{x}) - J_{\downarrow 4}(\mathrm{x})\right)$$

$$\frac{\partial J_{\downarrow 4}(\mathrm{x})}{\partial x}$$

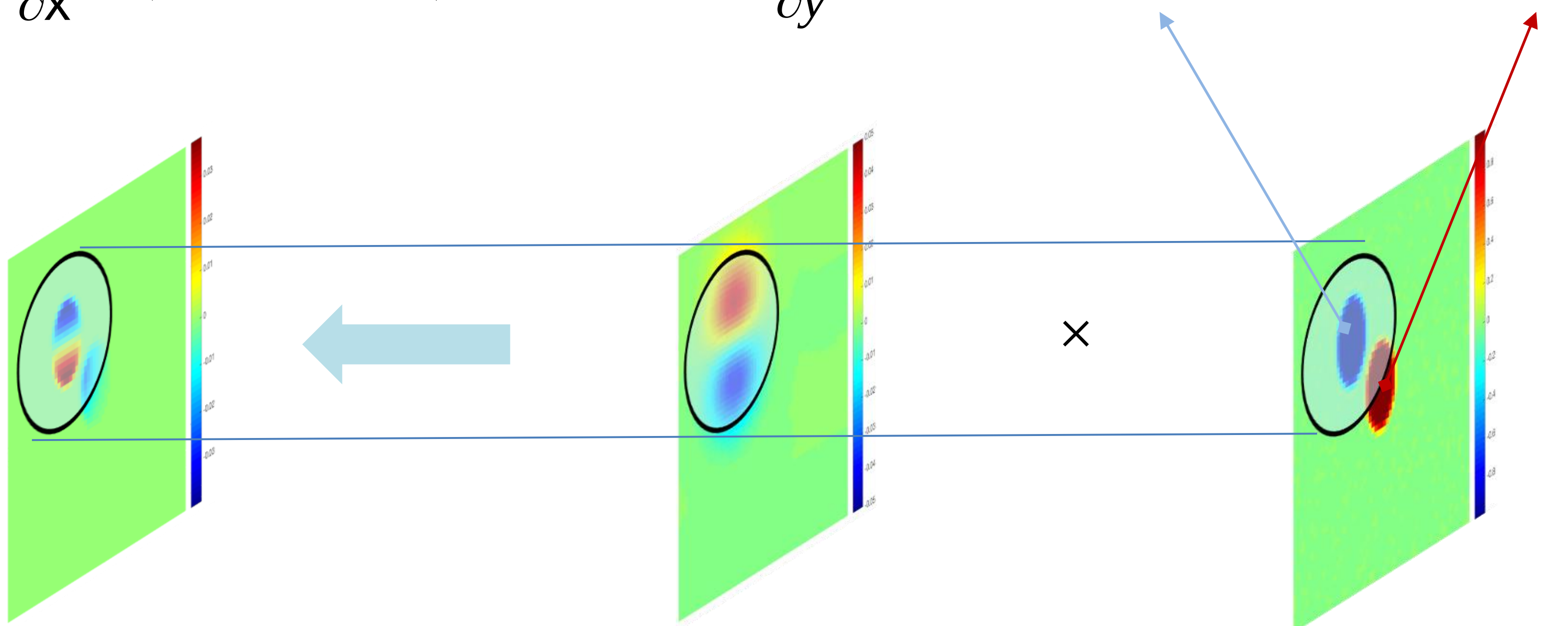
$$-J_{\downarrow 4}(\mathrm{x})$$

$$I_{\downarrow 4}(\mathrm{x})$$



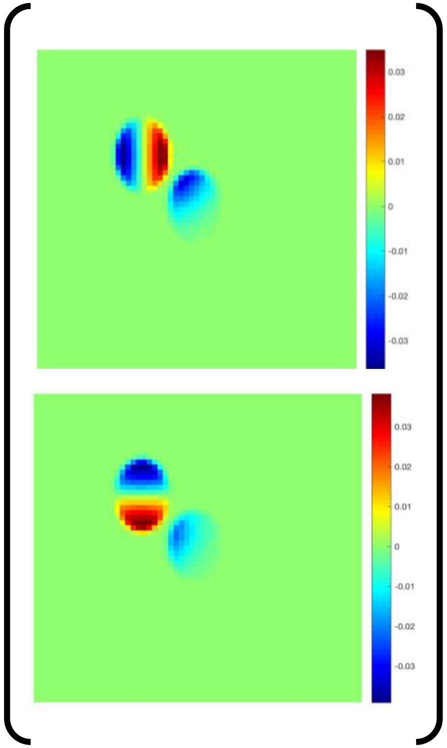
The influence of I(x) is incorporated!

$$\frac{\partial J_{\downarrow 4}(\mathrm{x})^{\mathrm{T}}}{\partial \mathrm{x}}\left(I_{\downarrow 4}(\mathrm{x})-J_{\downarrow 4}(\mathrm{x})\right) \qquad \frac{\partial J_{\downarrow 4}(\mathrm{x})}{\partial y} \qquad -J_{\downarrow 4}(\mathrm{x}) \qquad I_{\downarrow 4}(\mathrm{x})$$

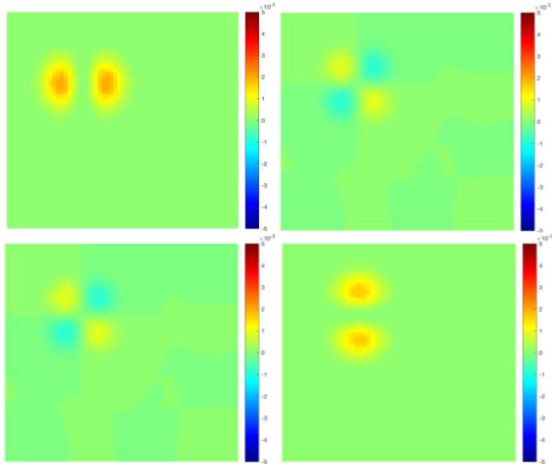


The influence of I(x) is incorporated!

$$\left(\frac{\partial J_{\downarrow 4}(\mathrm{x})}{\partial \mathrm{x}}^{\mathrm{T}}\frac{\partial J_{\downarrow 4}(\mathrm{x})}{\partial \mathrm{x}}\right)\Delta \mathrm{d}=\boxed{\frac{\partial J_{\downarrow 4}(\mathrm{x})}{\partial \mathrm{x}}^{\mathrm{T}}\left(I_{\downarrow 4}(\mathrm{x})-J_{\downarrow 4}(\mathrm{x})\right)}$$



Summing over pixels

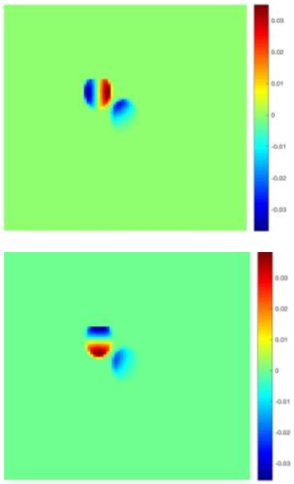


=

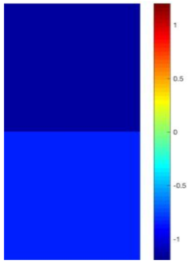
2×2
matrix

$$\frac{\partial J_{\downarrow 4}(\mathrm{x})^T}{\partial \mathrm{x}} \frac{\partial J_{\downarrow 4}(\mathrm{x})}{\partial \mathrm{x}}$$

Summing over pixels



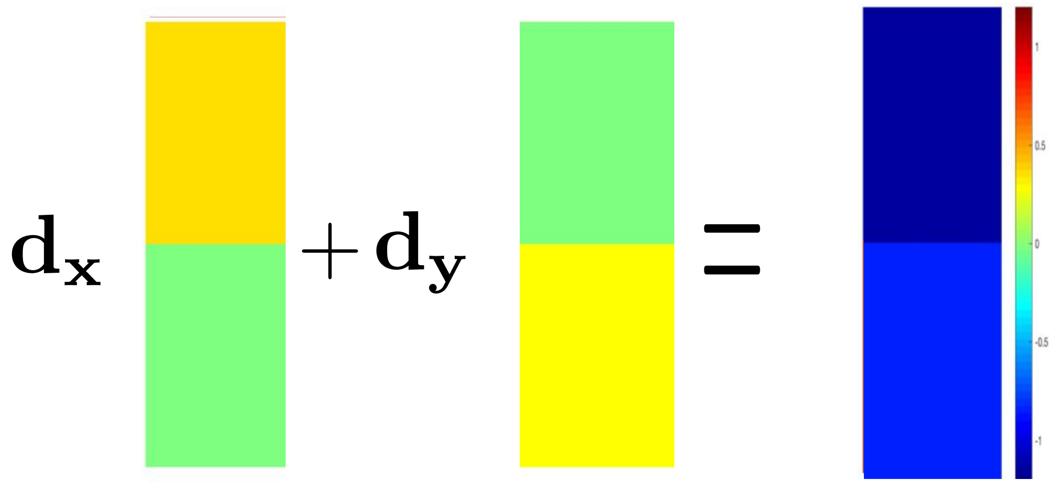
=



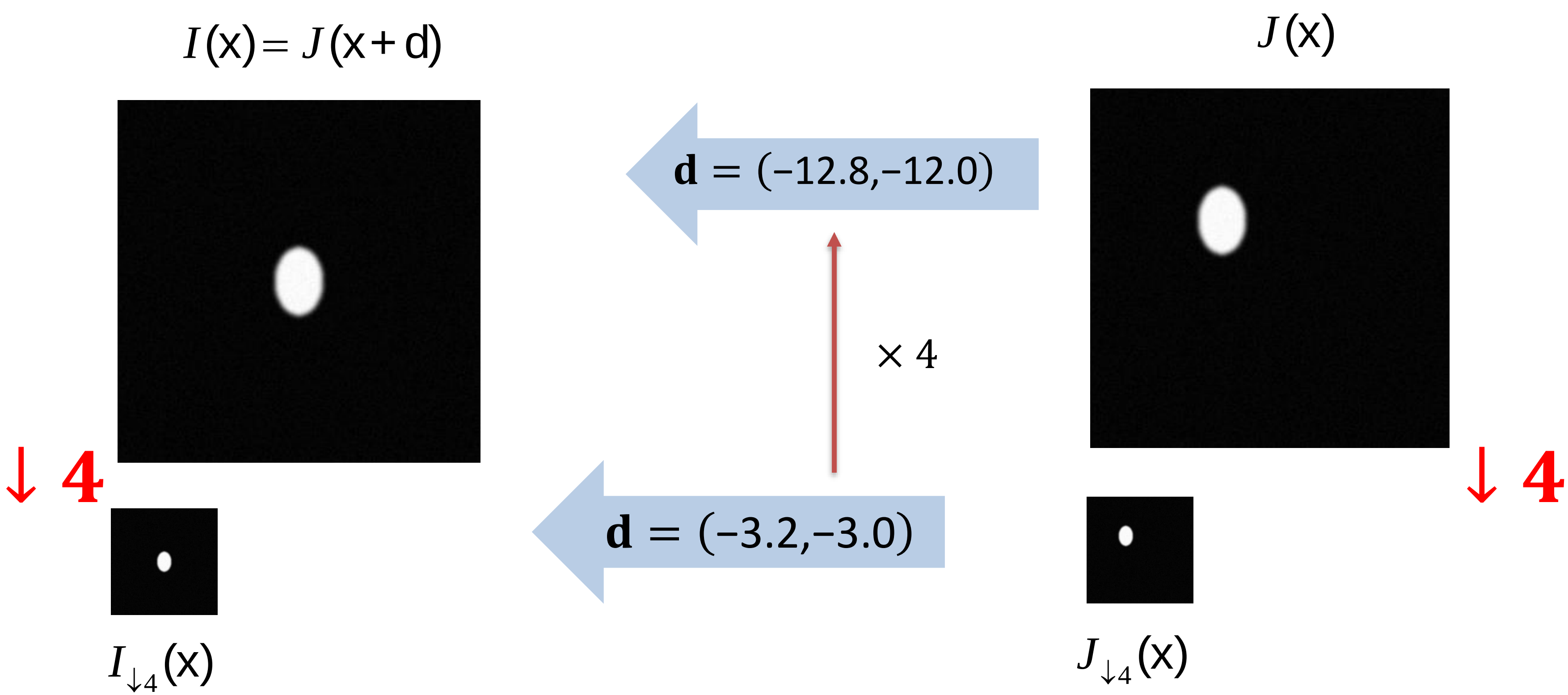
2×1
matrix

$$\frac{\partial J_{\downarrow 4}(\mathrm{x})^T}{\partial \mathrm{x}} \left(I_{\downarrow 4}(\mathrm{x}) - J_{\downarrow 4}(\mathrm{x}) \right)$$

$$\left(\frac{\partial J_{\downarrow 4}(x)^T}{\partial x}\frac{\partial J_{\downarrow 4}(x)}{\partial x}\right)\Delta d = \frac{\partial J_{\downarrow 4}(x)^T}{\partial x}\left(I_{\downarrow 4}(x) - J_{\downarrow 4}(x)\right)$$

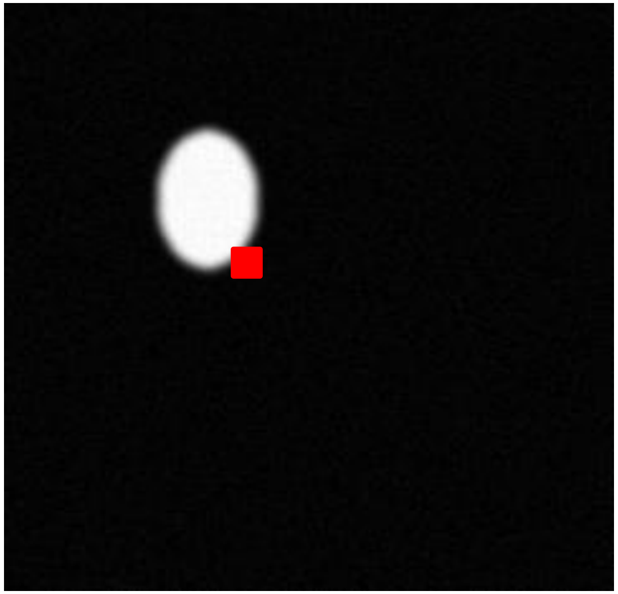


[−3.3 −3.0]

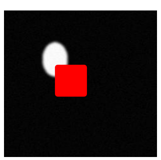


$I(\mathrm{x}) = J(\mathrm{x}+\mathrm{d})$
$J(\mathrm{x})$
$\mathbf{d} = (-12.8, -12.0)$
$\times 4$
$\downarrow 4$
$\downarrow 4$
$\mathbf{d} = (-3.2, -3.0)$
$I_{\downarrow 4}(\mathrm{x})$
$J_{\downarrow 4}(\mathrm{x})$

$J(\mathrm{x})$

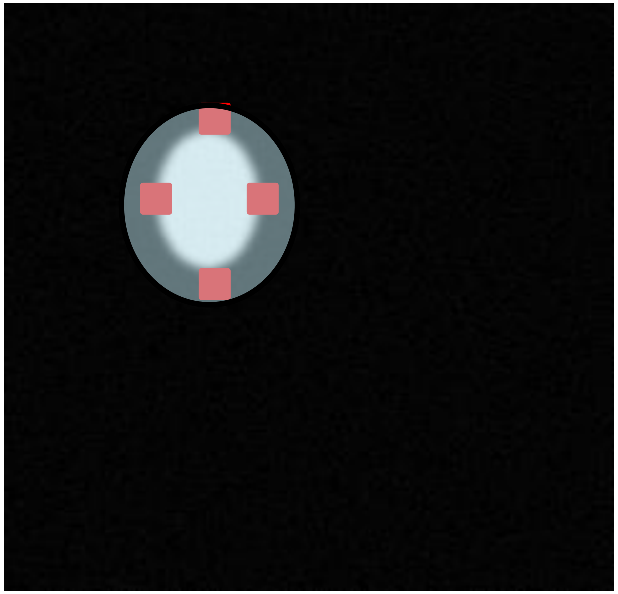


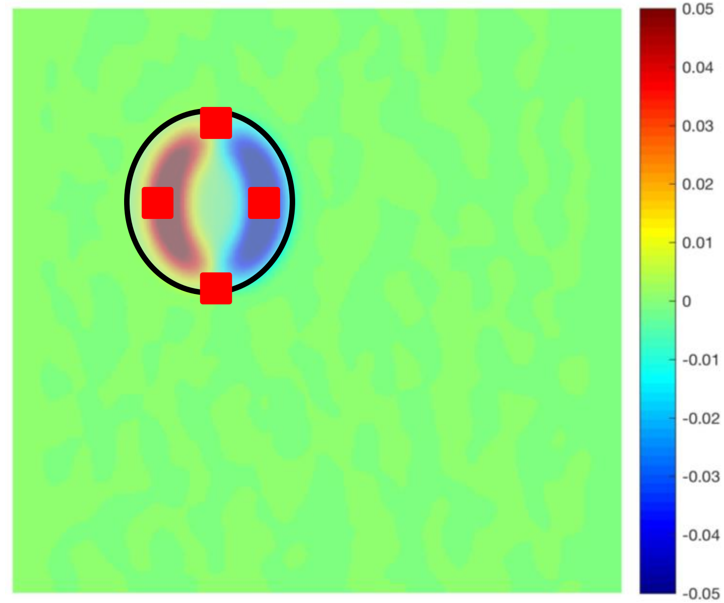
$\downarrow 4$

 $\otimes$ 

$J_{\downarrow 4}(\mathrm{x})$

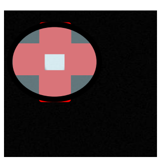
$J(\mathrm{x})$

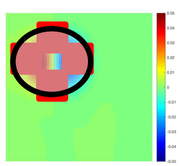




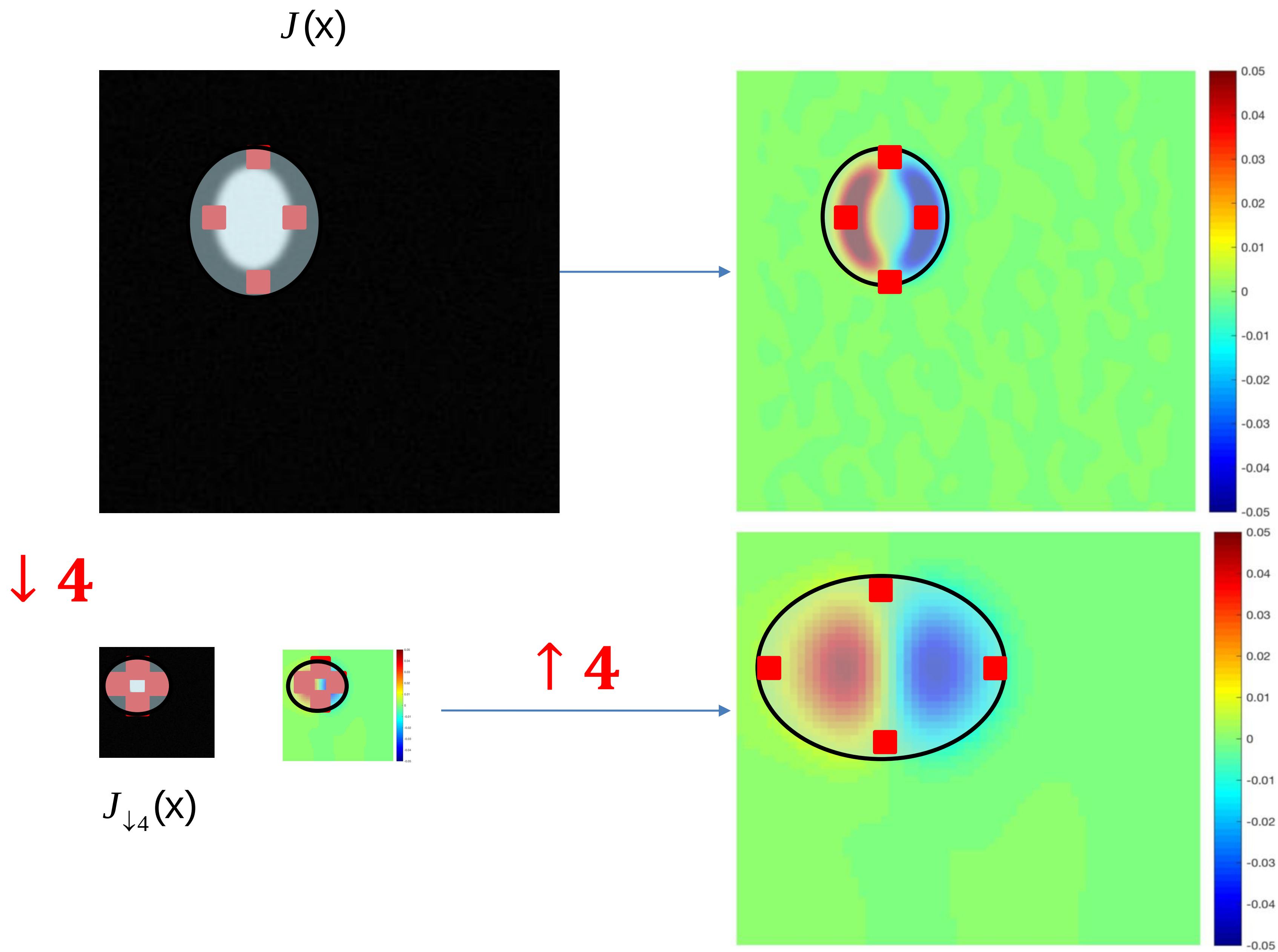
$\downarrow 4$

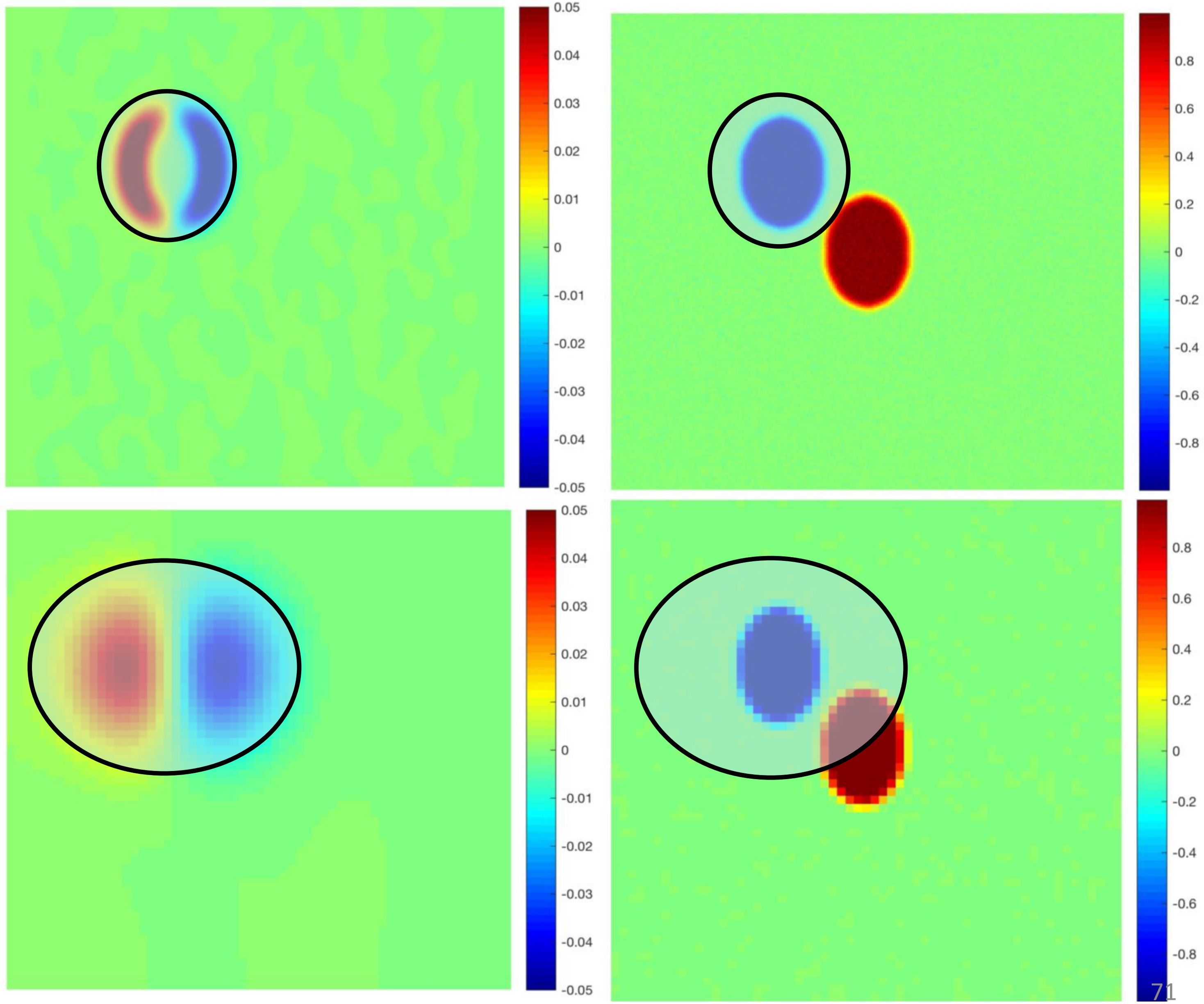
$1/4$





$J_{\downarrow 4}(\mathrm{x})$


$J(x)$
0.05
0.04
0.03
0.02
0.01
0
-0.01
-0.02
-0.03
-0.04
-0.05
$\downarrow 4$
$\uparrow 4$
$J_{\downarrow 4}(x)$


0.05
0.04
0.03
0.02
0.01
0
-0.01
-0.02
-0.03
-0.04
-0.05
0.8
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
-0.8

Down sampling image is equivalent to increase the kernel size.

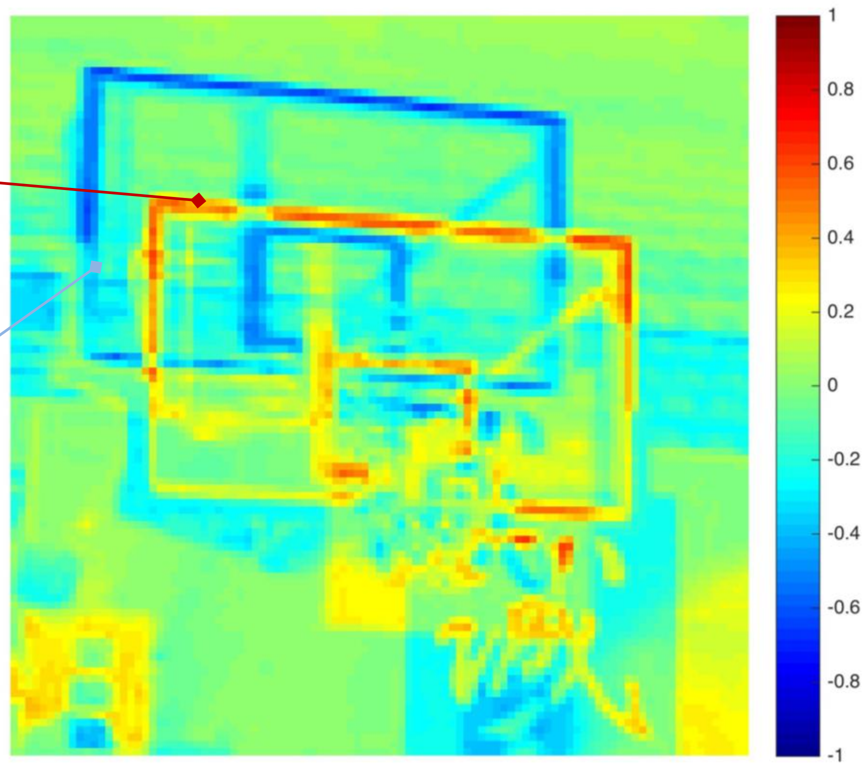
$$\mathbf{I}(\mathbf{x}) \qquad \mathbf{J}(\mathbf{x})$$

$$t = 0 \qquad t = 1$$

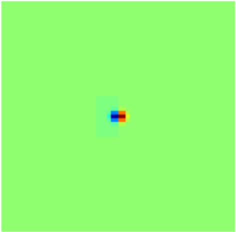
$$\left(\frac{\partial J(x)^T}{\partial x}\frac{\partial J(x)}{\partial x}\right)\Delta d = \frac{\partial J(x)^T}{\partial x}\boxed{\left(I(x) - J(x)\right)}$$

$\mathbf{I}(\mathbf{x})$

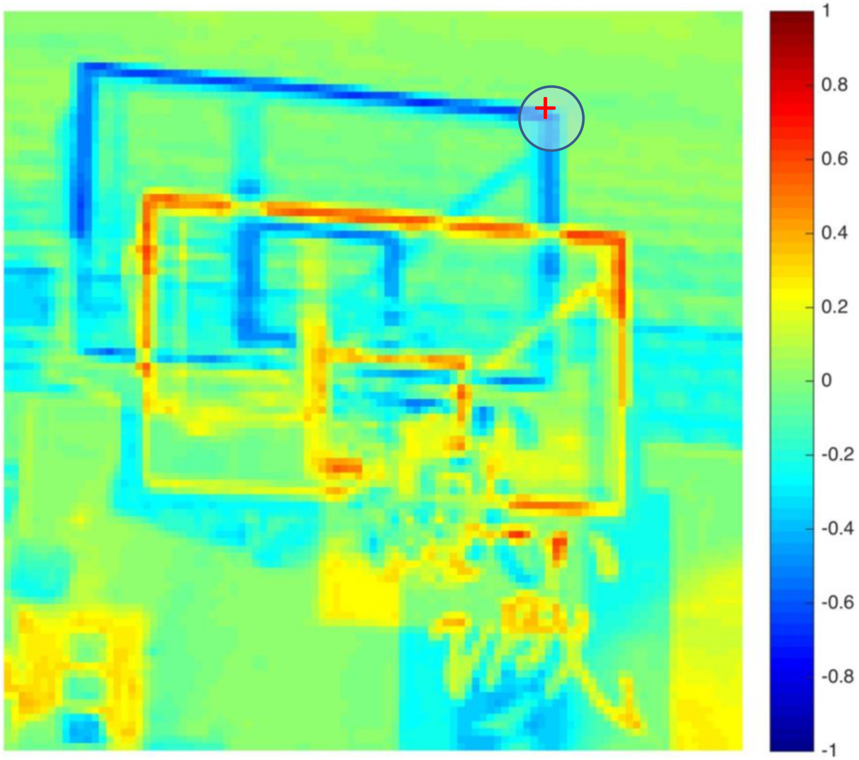
$-\mathbf{J}(\mathbf{x})$



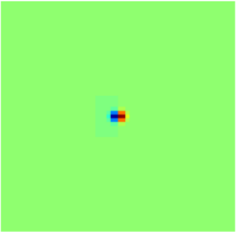
Solution 2: increase the kernel size of gradient operator

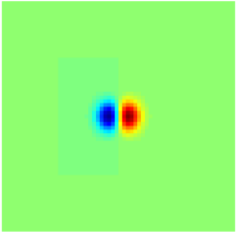


Kernel size

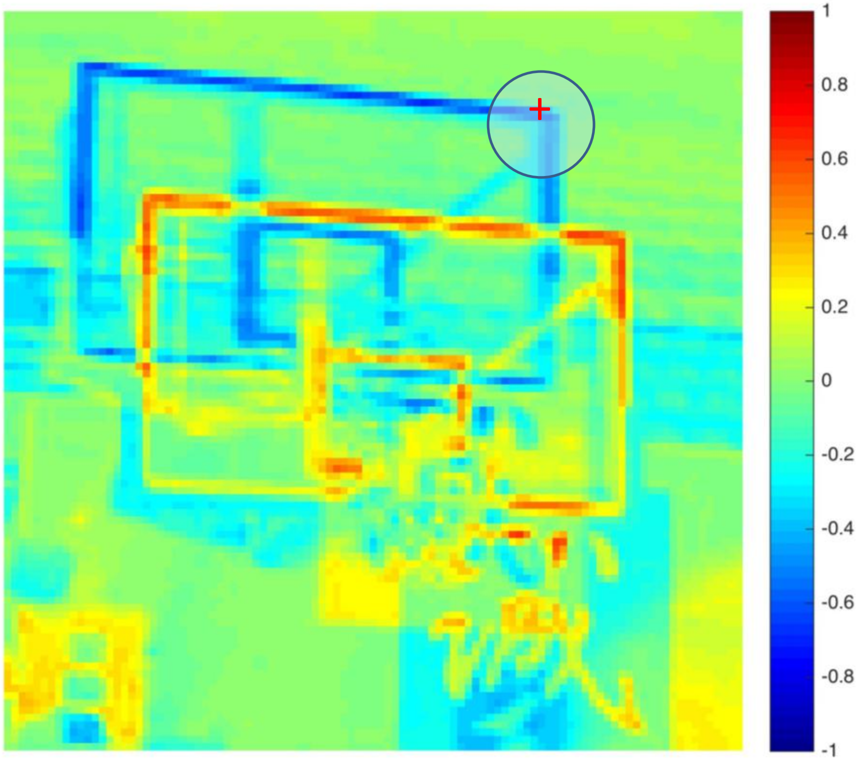


Solution 2: increase the kernel size of gradient operator

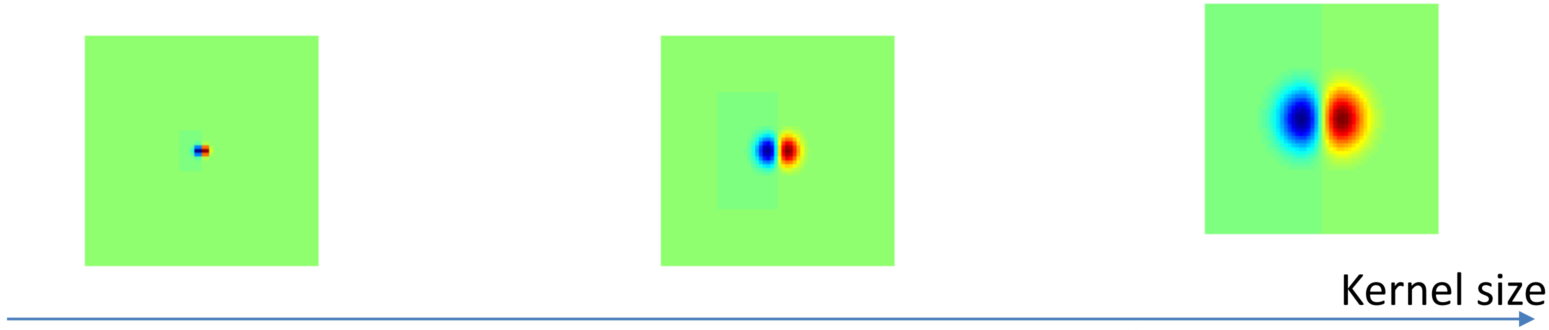


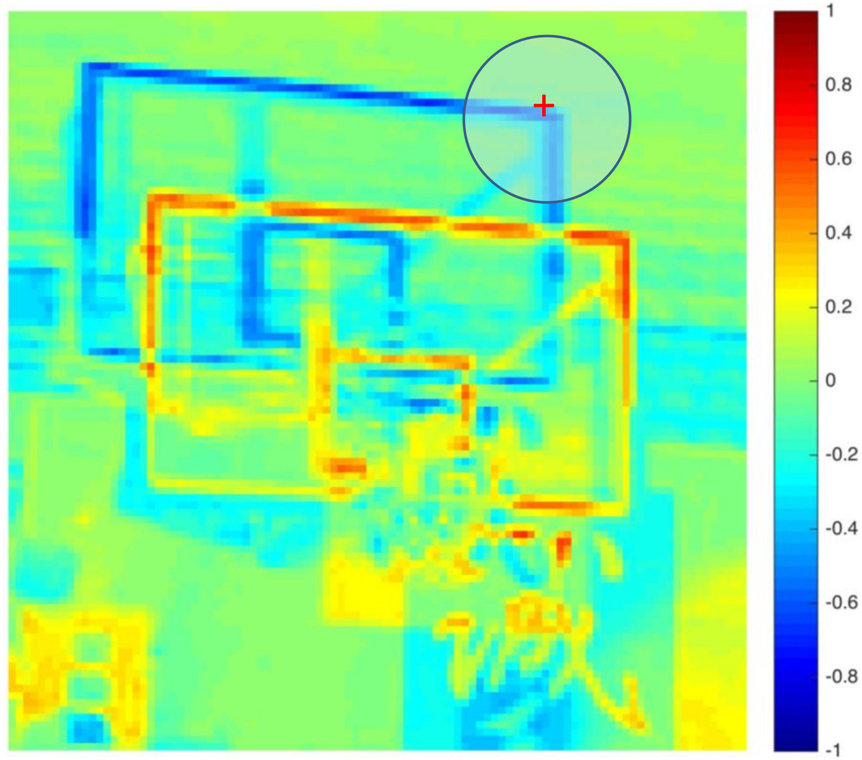


Kernel size

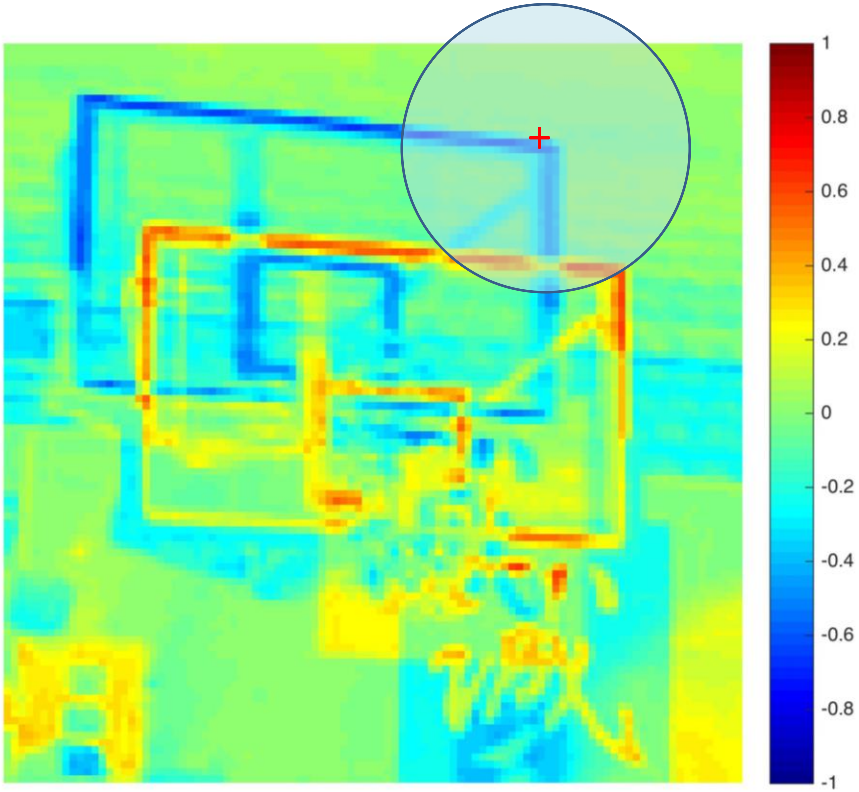


Solution 2: increase the kernel size of gradient operator

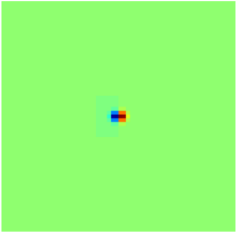




Solution 2: increase the kernel size of gradient operator



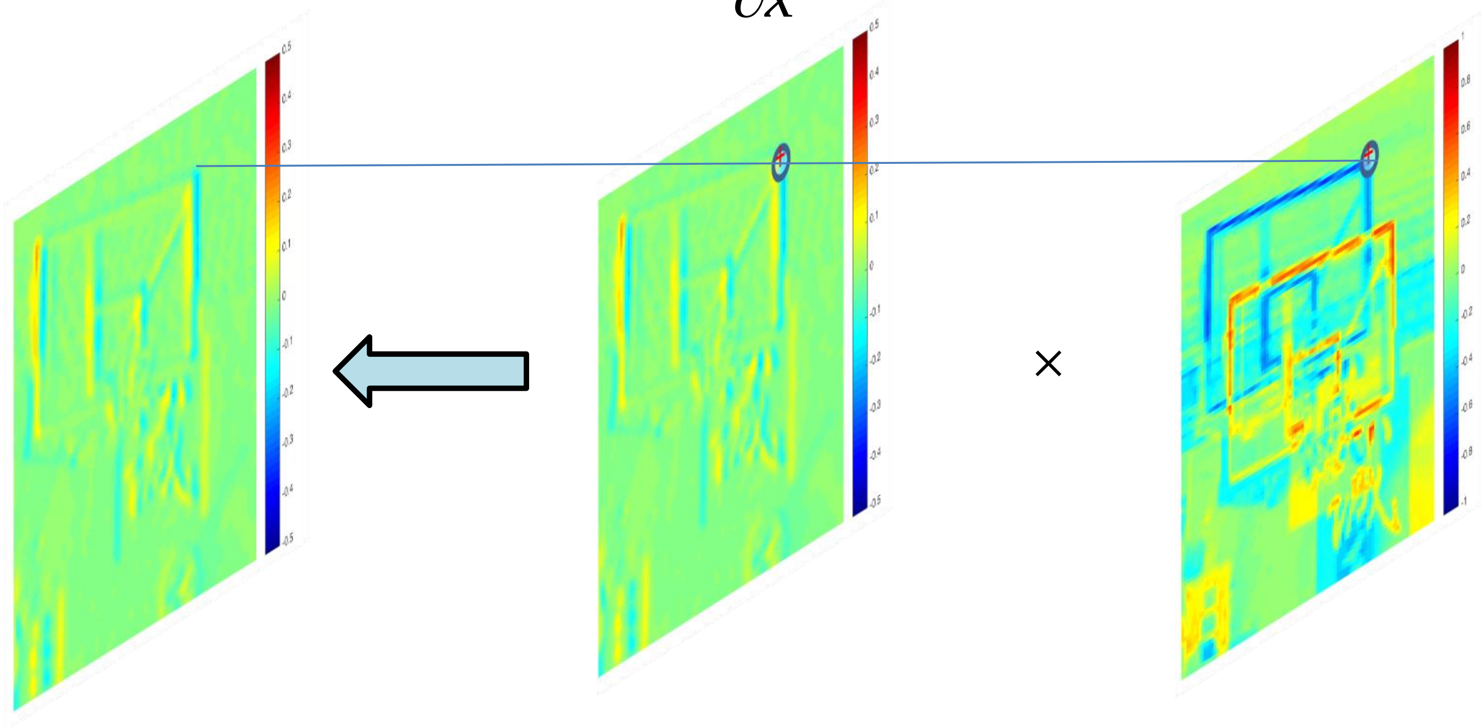
small kernel



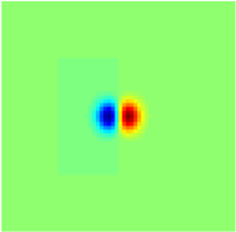
$$\frac{\partial J(x)^T}{\partial x}\big(I(x)-J(x)\big)$$

$$\frac{\partial J\ (x)}{\partial x}$$

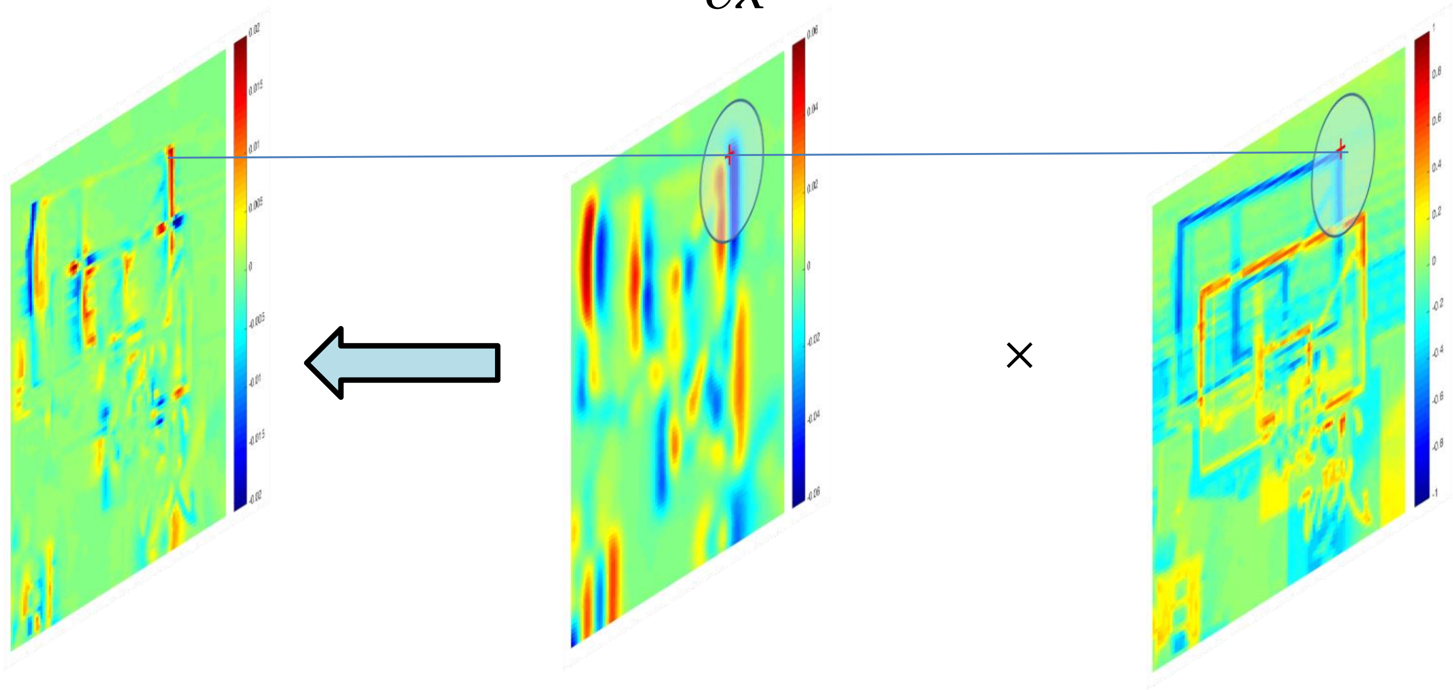
$$I(x)-J(x)$$



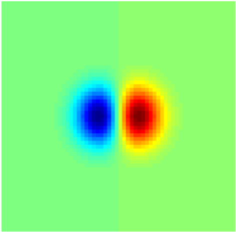
Median kernel



$$\frac{\partial J(\mathrm{x})^T}{\partial \mathrm{x}}\left(I(\mathrm{x})-J(\mathrm{x})\right) \qquad \frac{\partial J(\mathrm{x})}{\partial x} \qquad I(\mathrm{x})-J(\mathrm{x})$$



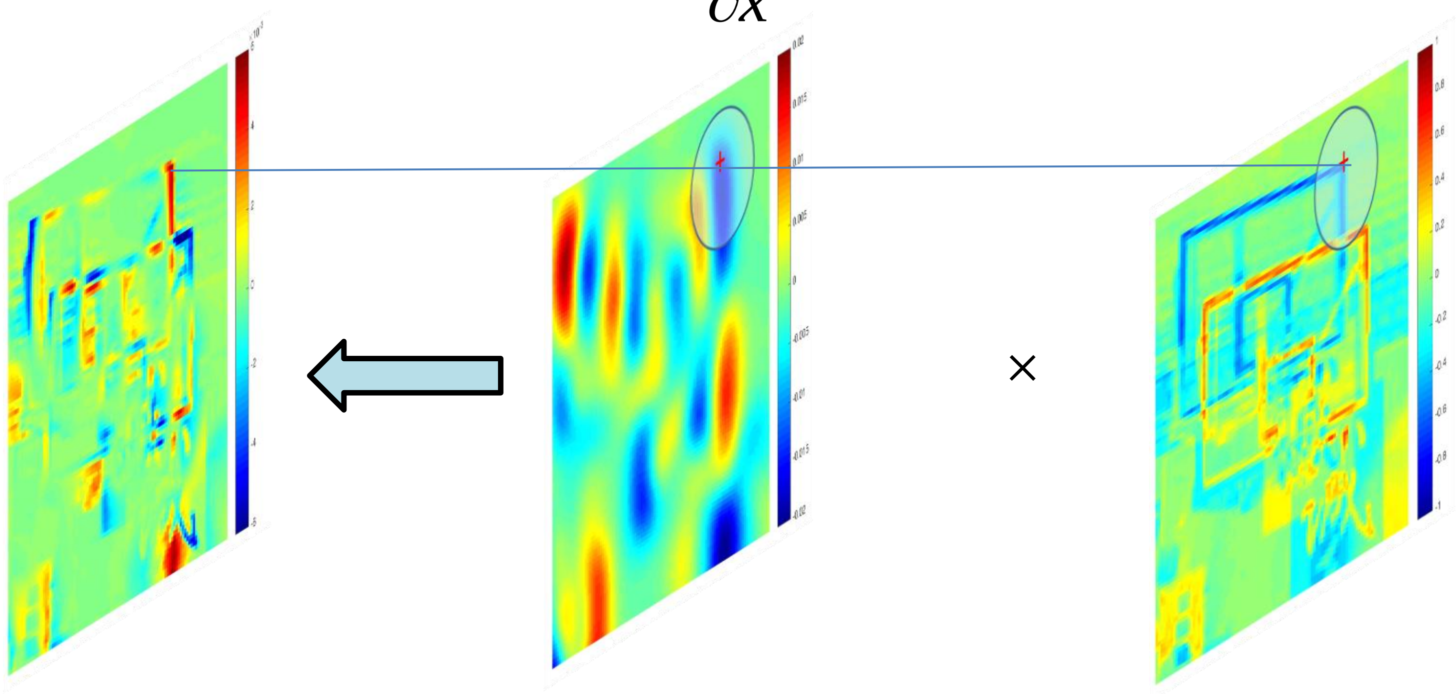
Large kernel



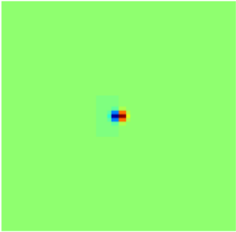
$$\frac{\partial J(\mathrm{x})^T}{\partial \mathrm{x}}\left(I(\mathrm{x})-J(\mathrm{x})\right)$$

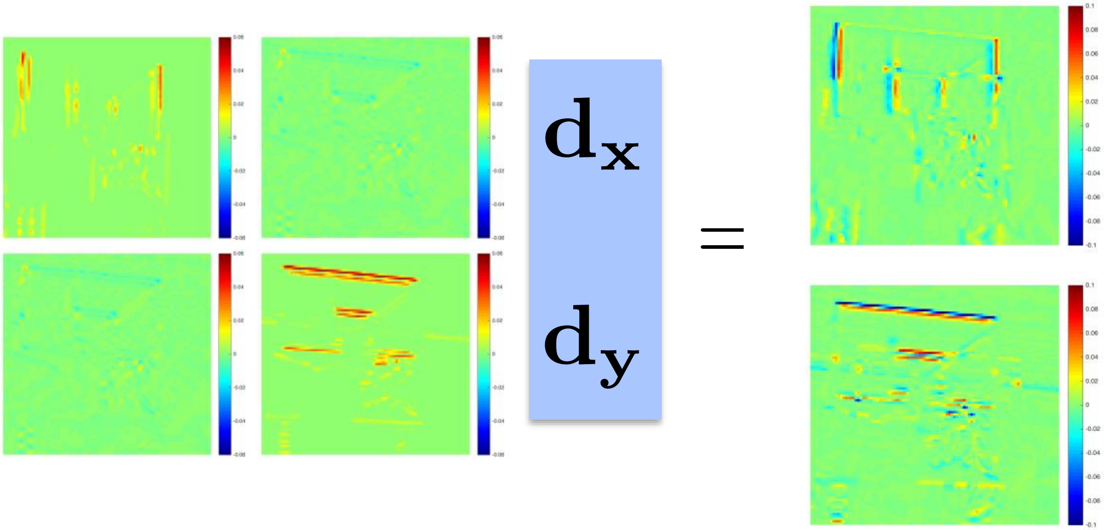
$$\frac{\partial J\ (\mathrm{x})}{\partial x}$$

$$I(\mathrm{x})-J(\mathrm{x})$$



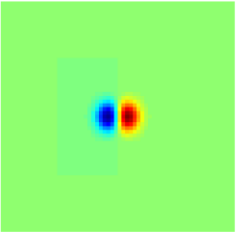
$$\left(\frac{\partial J(x)^T}{\partial x}\frac{\partial J(x)}{\partial x}\right)\Delta d = \frac{\partial J(x)^T}{\partial x}\big(I(x) - J(x)\big)$$

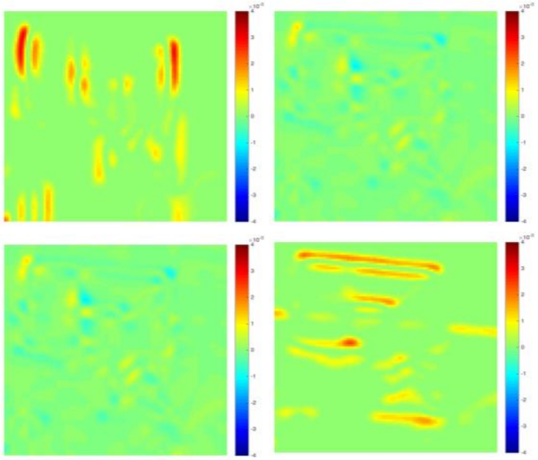




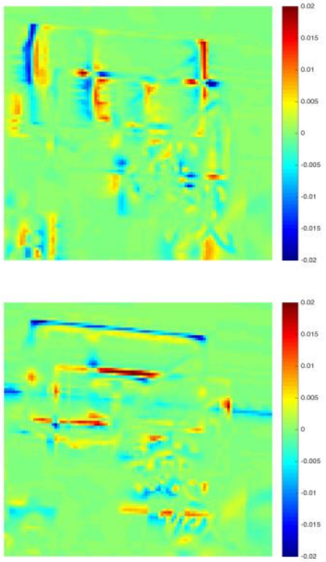
d=(-0.6, 1.1)

$$\left(\frac{\partial J(x)^T}{\partial x}\frac{\partial J(x)}{\partial x}\right)\Delta d = \frac{\partial J(x)^T}{\partial x}\big(I(x)-J(x)\big)$$



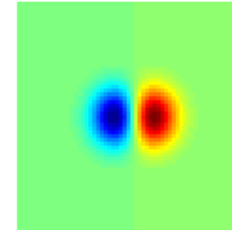


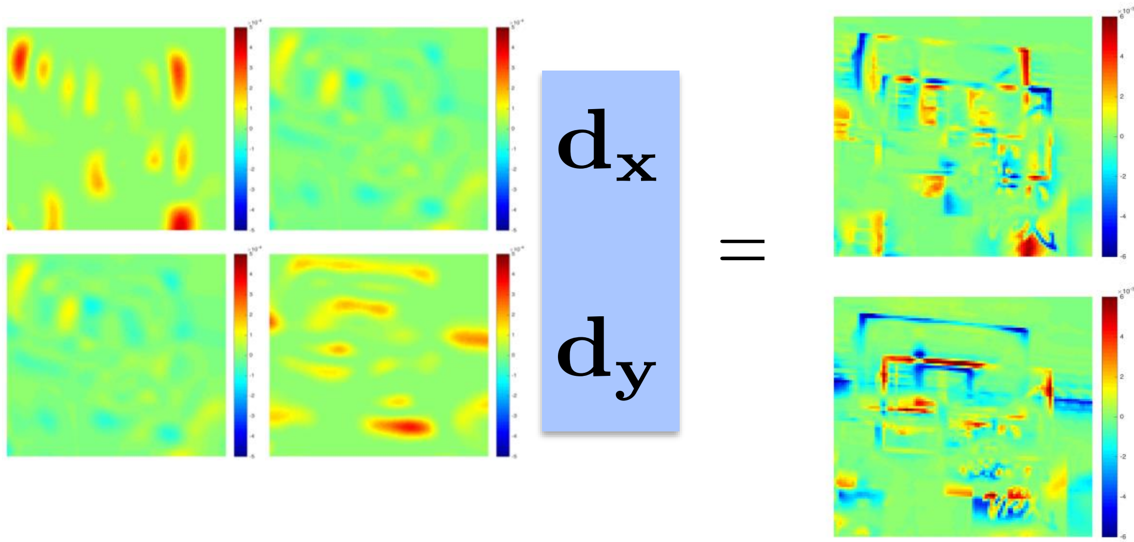
$$\begin{bmatrix} \mathbf{d_x} \\ \mathbf{d_y} \end{bmatrix} =$$



d=(-2.9, -3.0)

$$\left(\frac{\partial J(\mathrm{x})^{T}}{\partial \mathrm{x}}\frac{\partial J(\mathrm{x})}{\partial \mathrm{x}}\right)\Delta \mathrm{d} = \frac{\partial J(\mathrm{x})^{T}}{\partial \mathrm{x}}\left(I(\mathrm{x}) - J(\mathrm{x})\right)$$





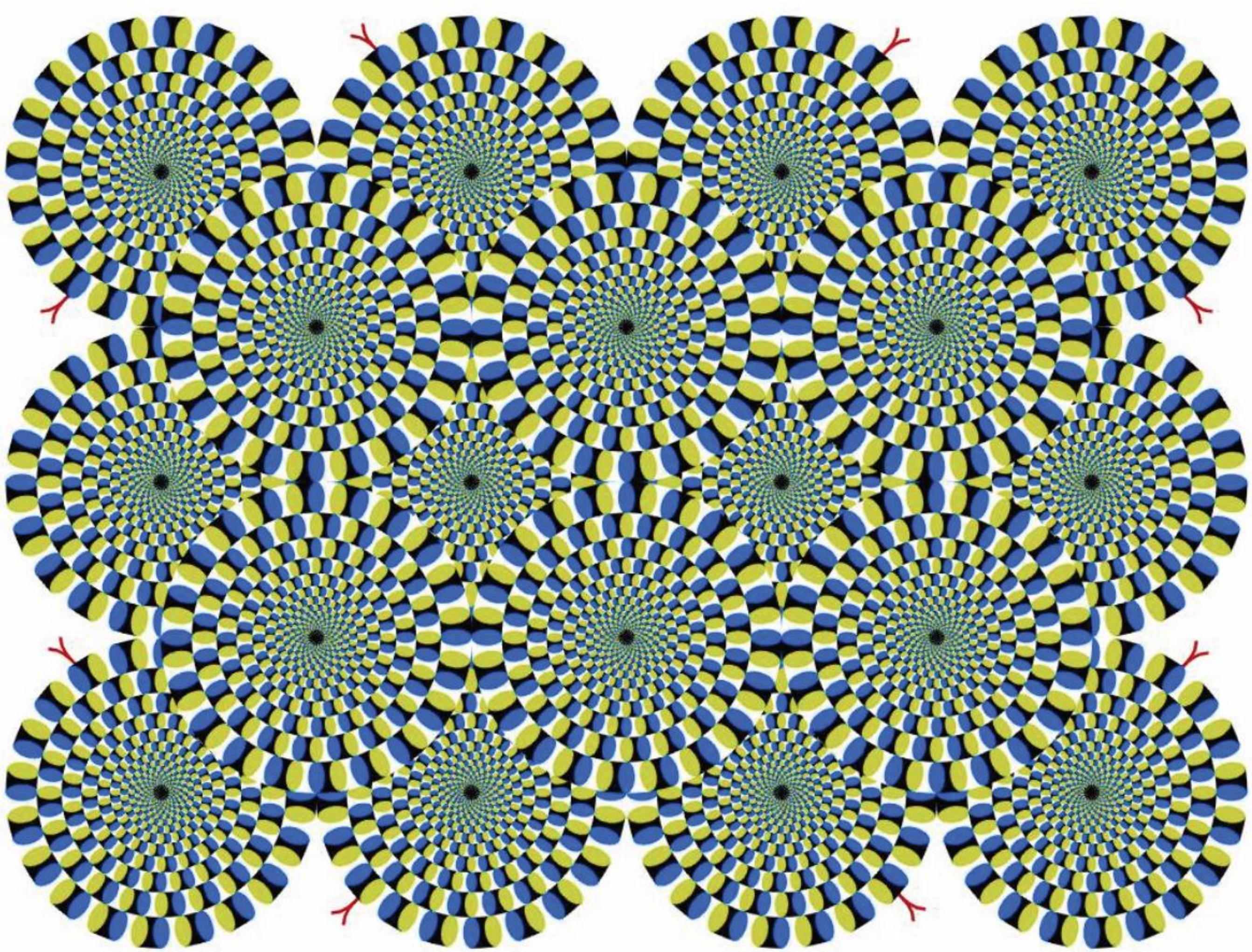
d=(-8.3, 19.0)

Penn Engineering

ONLINE LEARNING

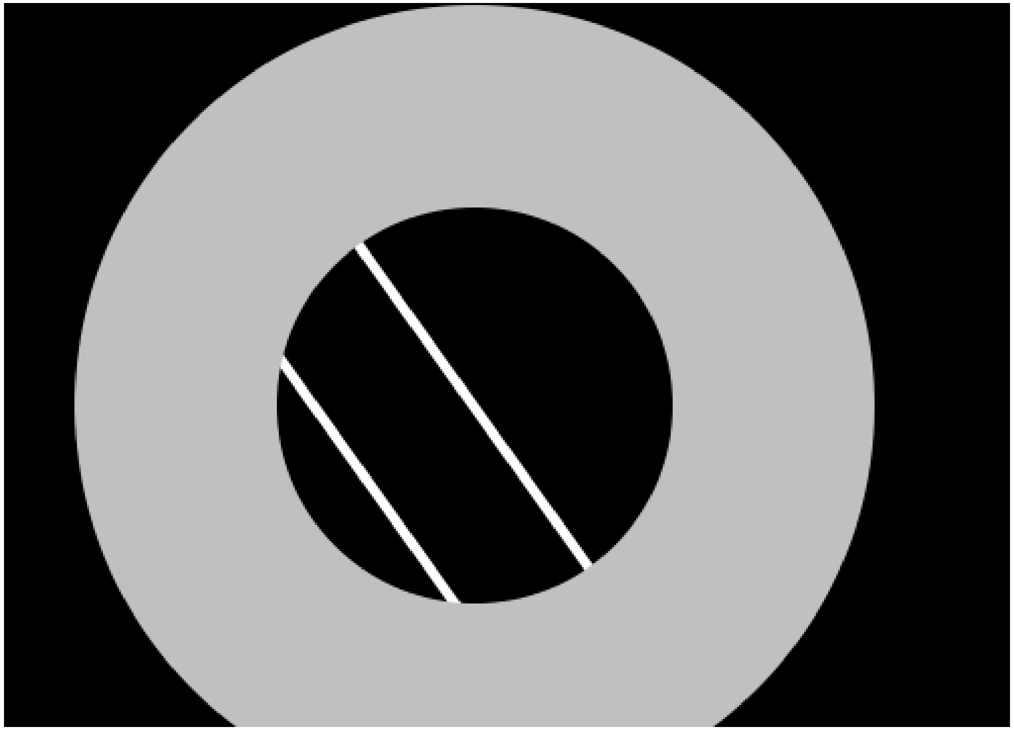
Video 6.6
Kostas Daniilidis

A robot is always in motion



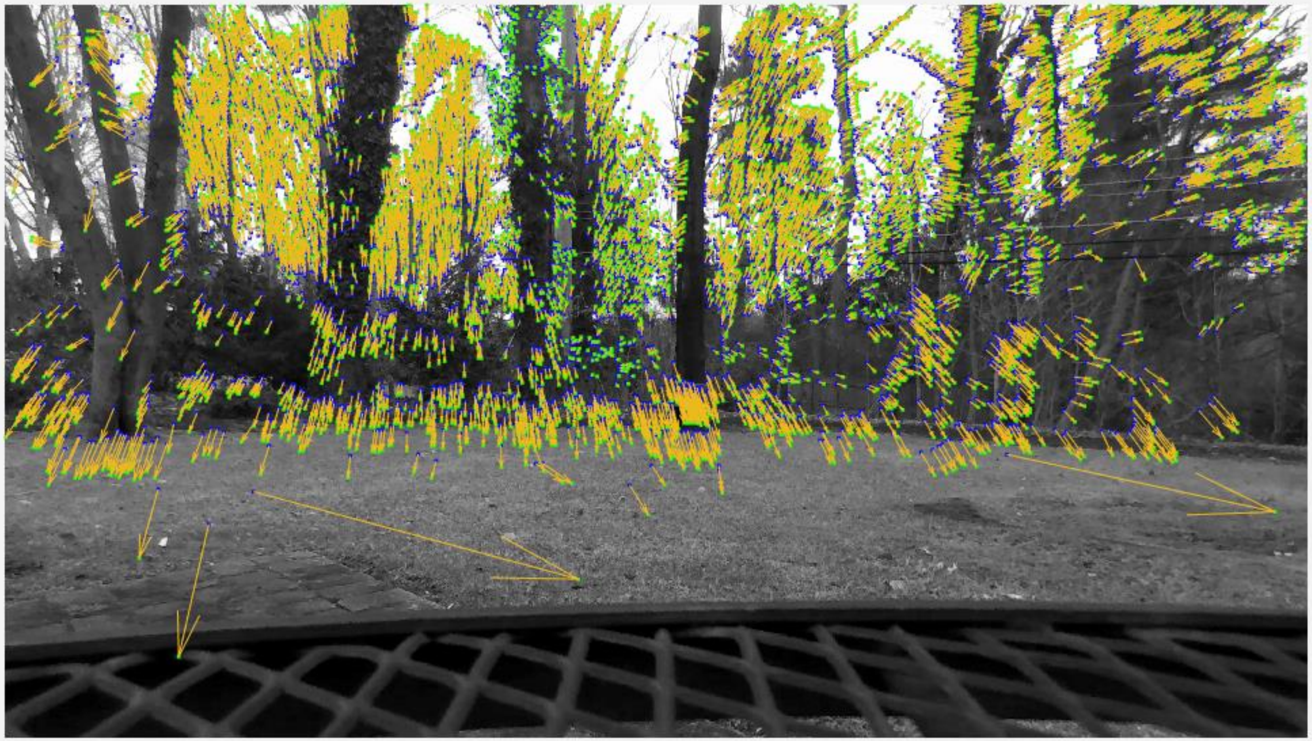
Ouchi

Aperture Problem

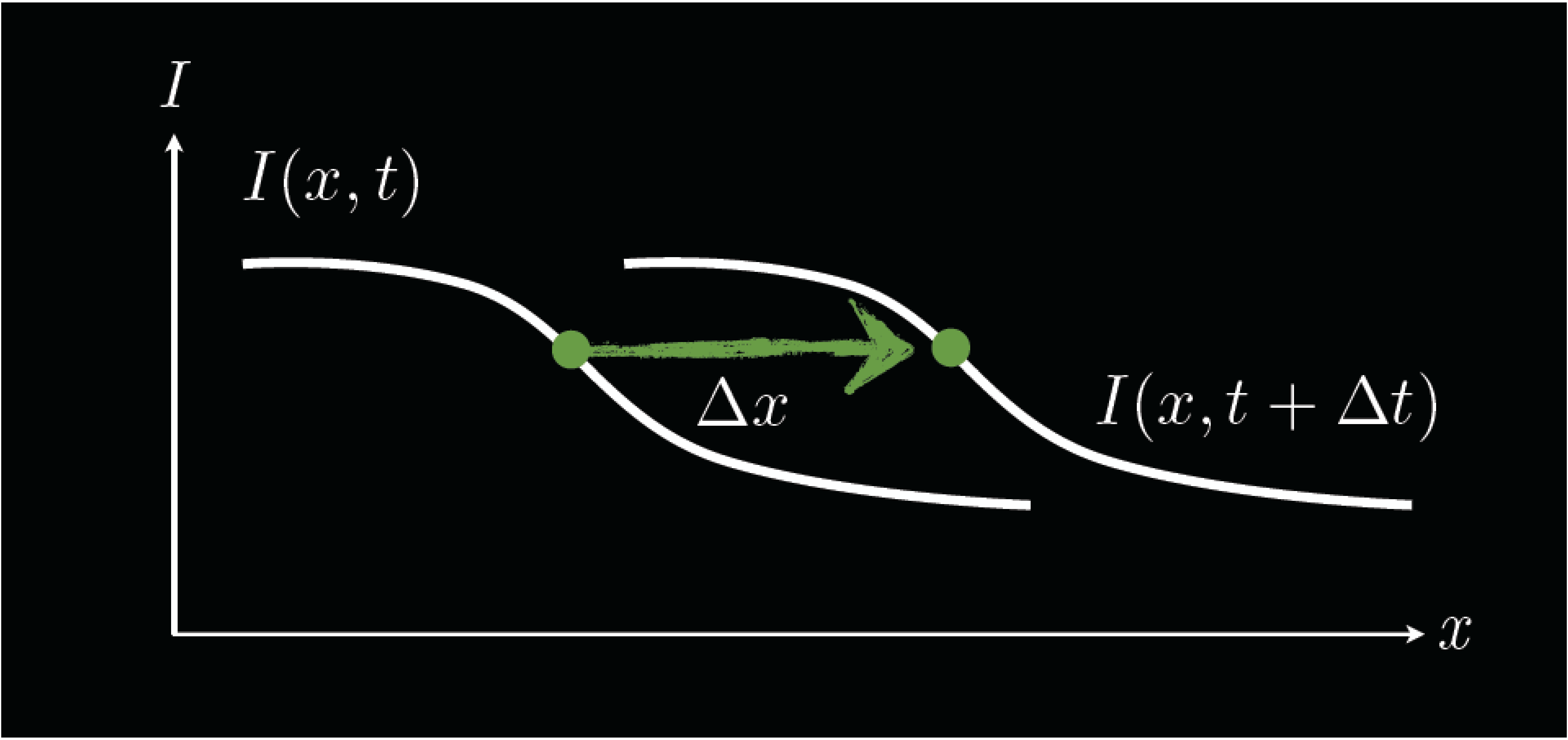


Barber Pole Illusion

Optical Flow



Brightness Constancy



$$I(x, y, t) = I(x + u\Delta t, y + v\Delta t, t + \Delta t)$$

assuming small duration

Assume just an xt-slice through the video

.. Like a wave!

$$f(x,t) = \cos(\omega_0(x - ut))$$

$$f_0(x) = \cos(\omega_0 x)$$

The Fourier transform of f is the following:

$$F(\omega_x, \omega_t) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f_0(x - ut)e^{-j\omega_x x + \omega_t t}dxdt$$

.. In the Fourier Domain

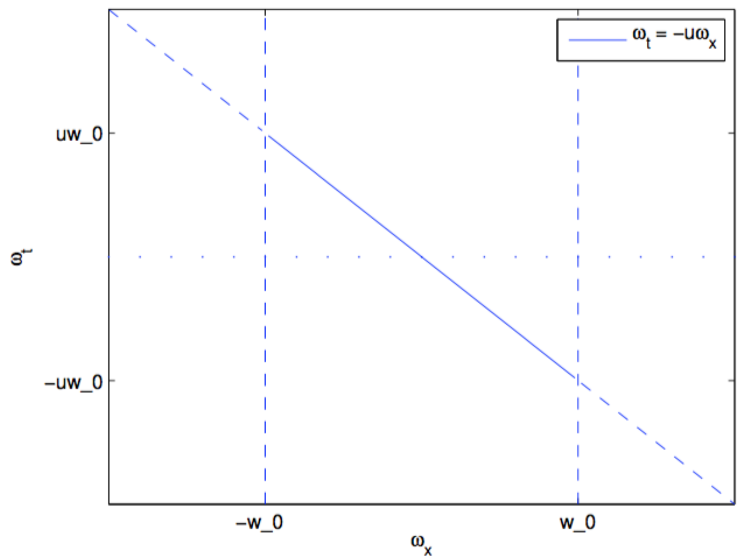
$$F(\omega_x, \omega_t) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f_0(x')e^{-j\omega_x(x'+ut)+\omega_t t}dx'dt$$
$$= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f_0(x')e^{-j\omega_x x'}e^{-j(\omega_x u+\omega_t)t}dx'dt$$
$$= F_0(\omega_x)\delta(\omega_x u + \omega_t)$$

Motion spectrum is

Taking the original spectrum, rotating, and stretching

- In our initial example $F_0(\omega_x) = \frac{1}{2}(\delta(\omega_x + \omega_0) + \delta(\omega_x - \omega_0))$: the $F_0(\omega_x, \omega_t)$ is made of two diracs located at $(\omega_0, -u\omega_0)$ and $(-\omega_0, +u\omega_0)$, which are at distance $\omega_0\sqrt{1+u^2}$ of the origin.

Fourier of a wave



Temporal aliasing

Temporal Aliasing