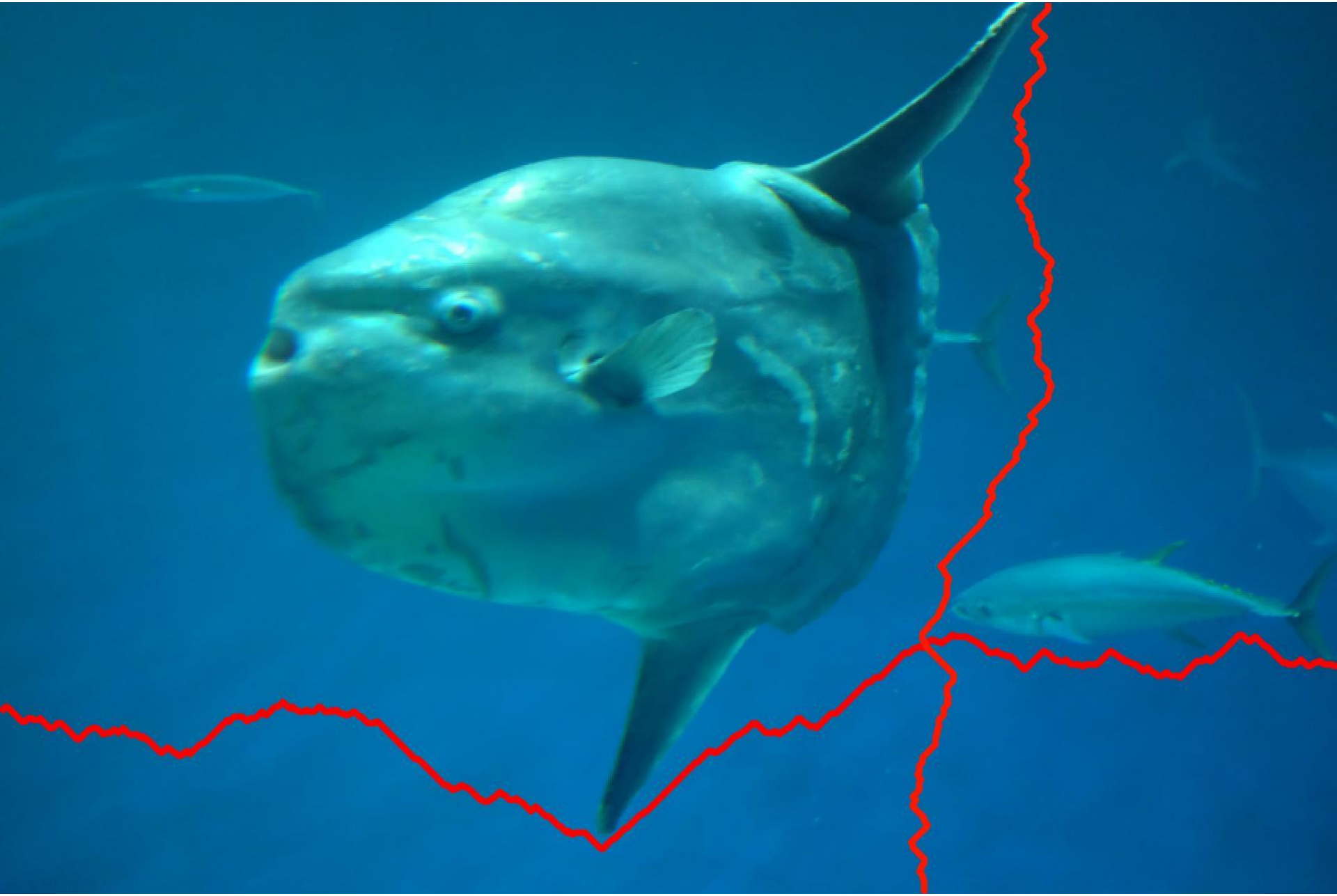
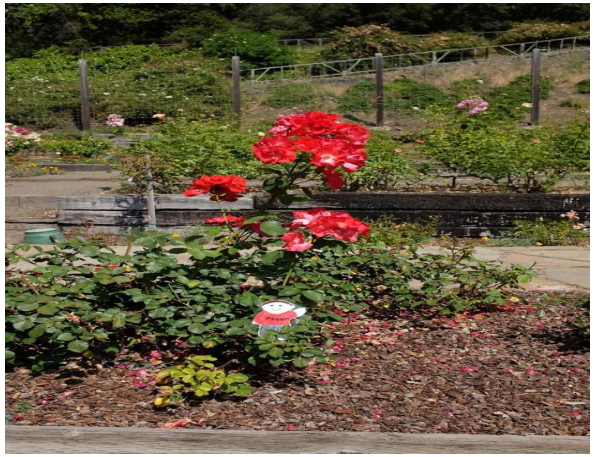
Video 9.1
Jianbo Shi



$820 \times 546 \times 3$

$420 \times 546 \times 3$



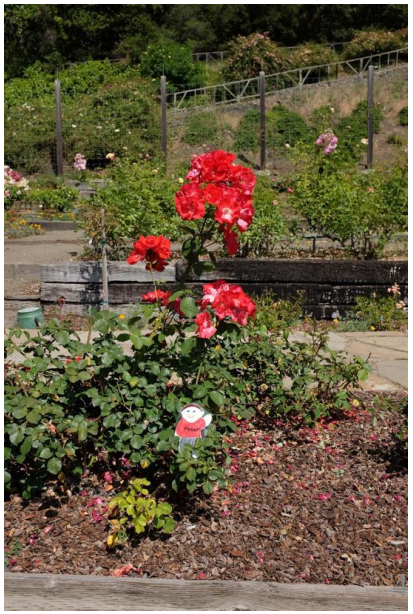
(a)

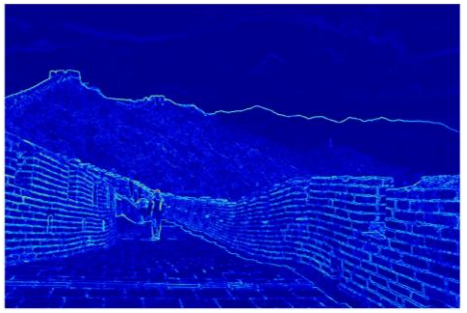
(b)

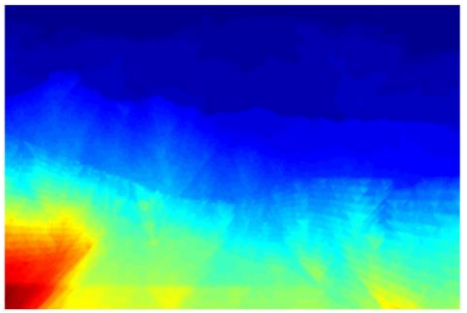
(c)

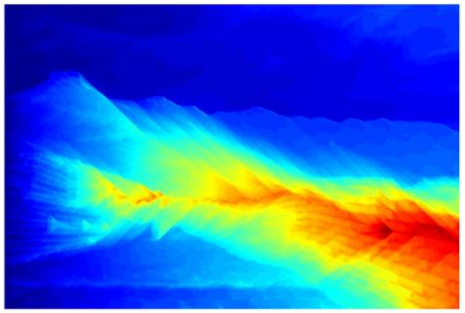
Guess

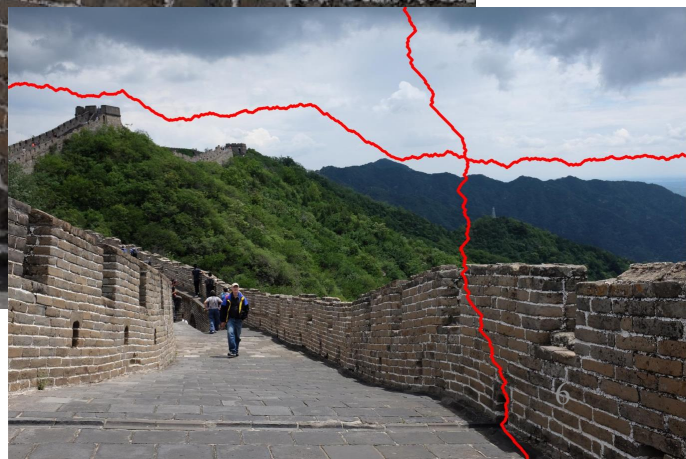
- We use “crop”, “scaling” and “carving” for resizing the given image
- Guess which one is for carving?



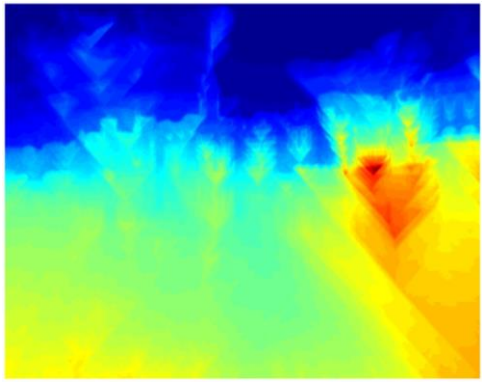


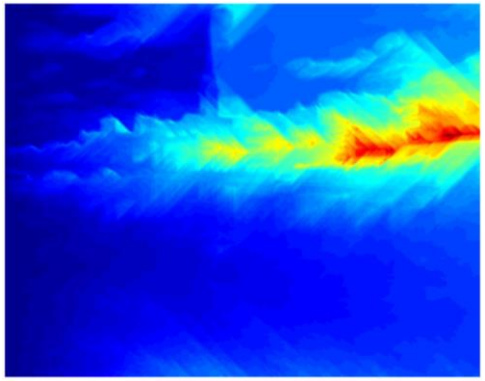


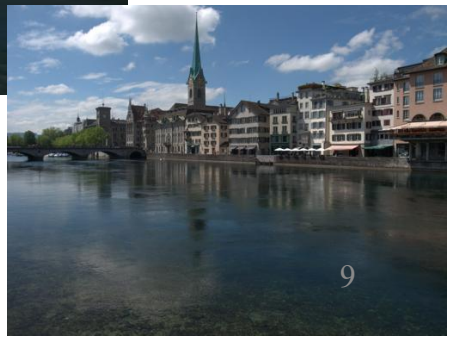




- Guess which one is for carving?

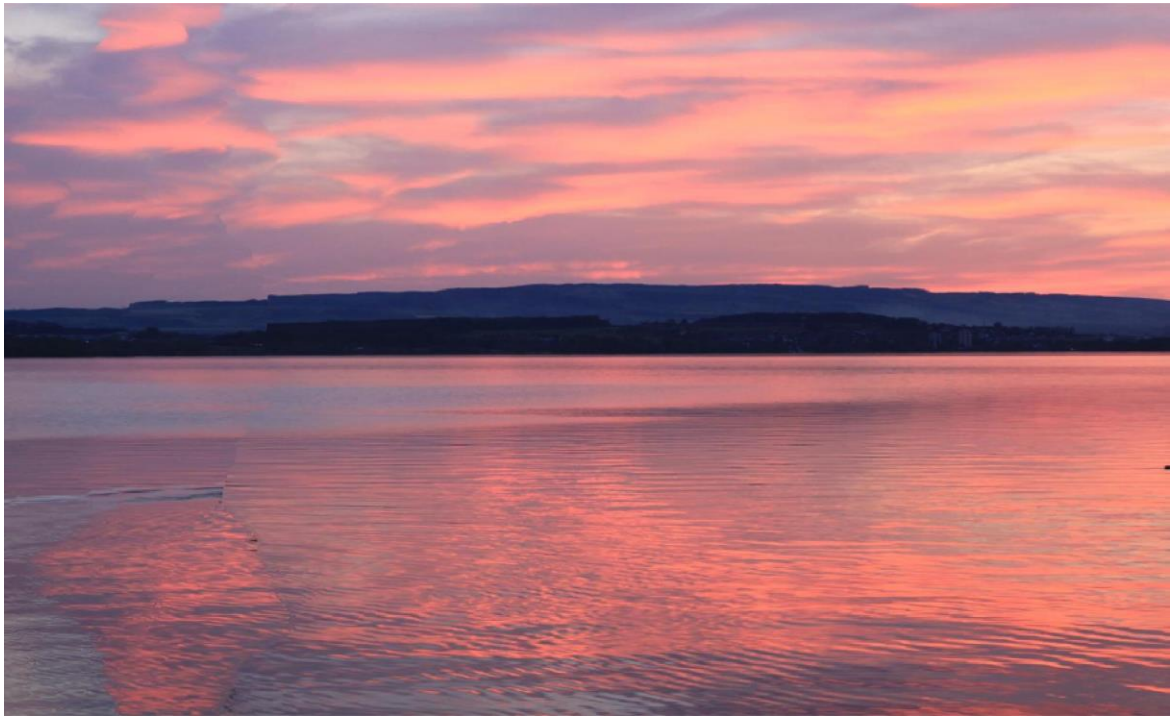


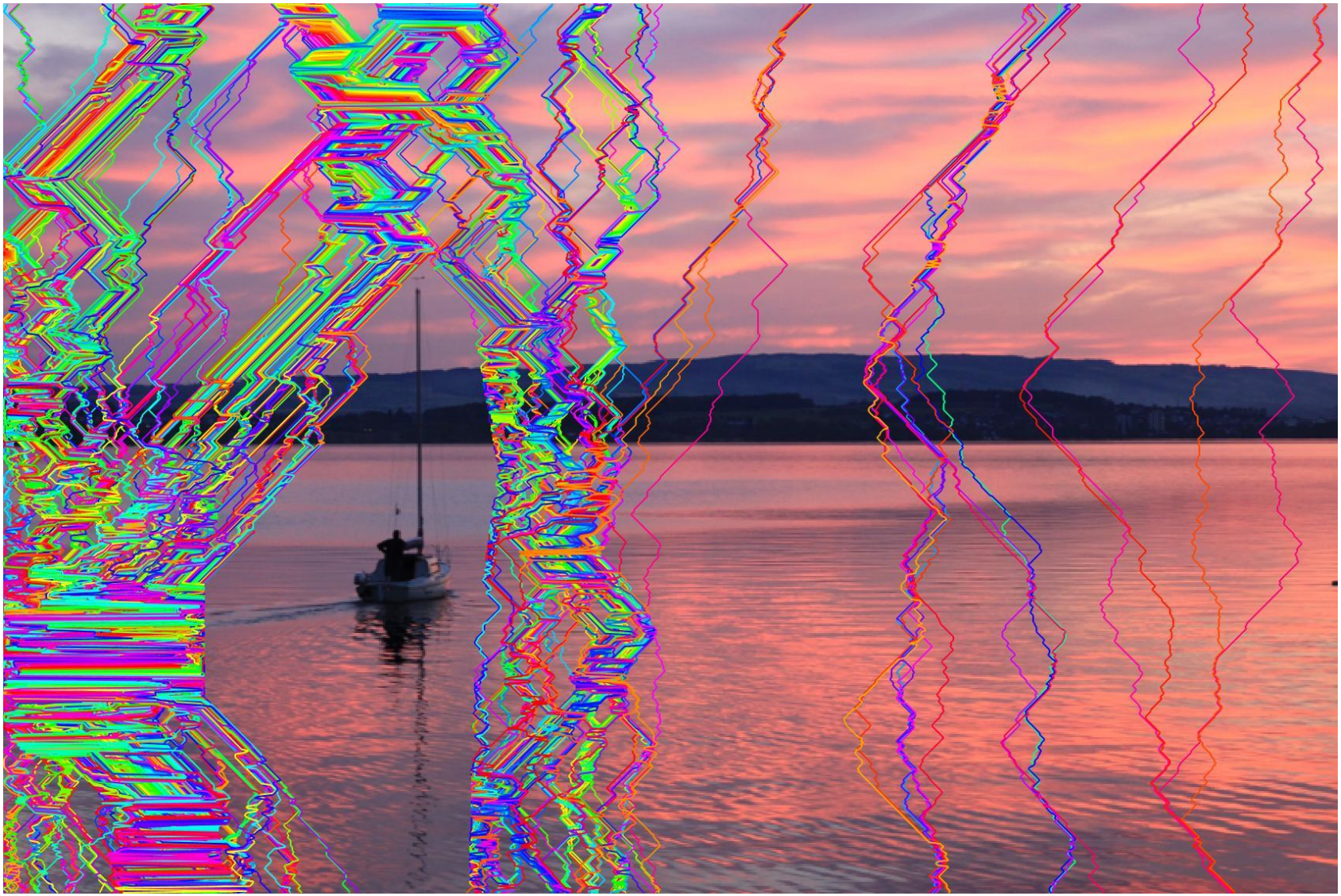




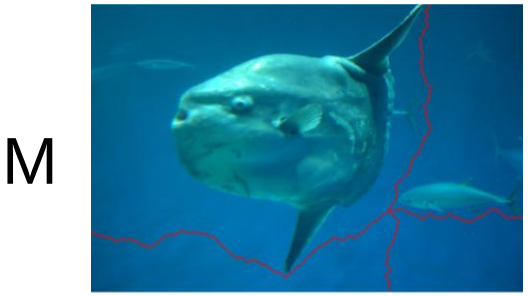
- Guess which one is for carving?

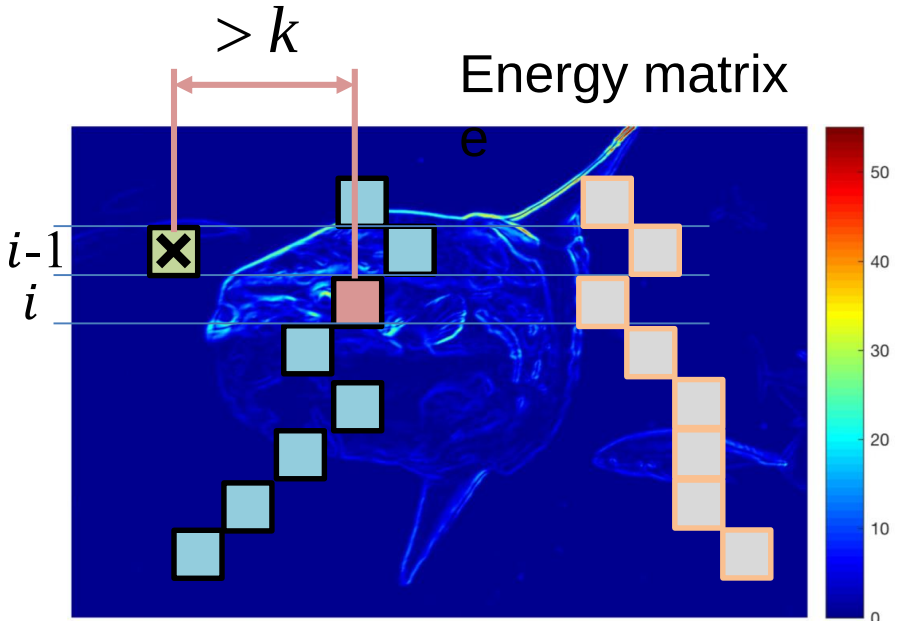
Expanded



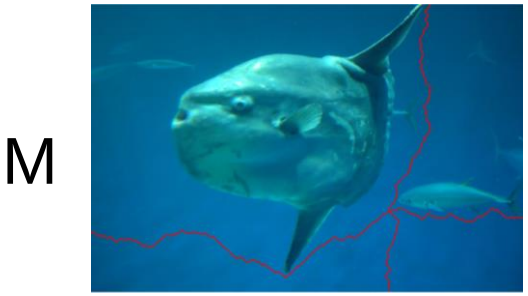


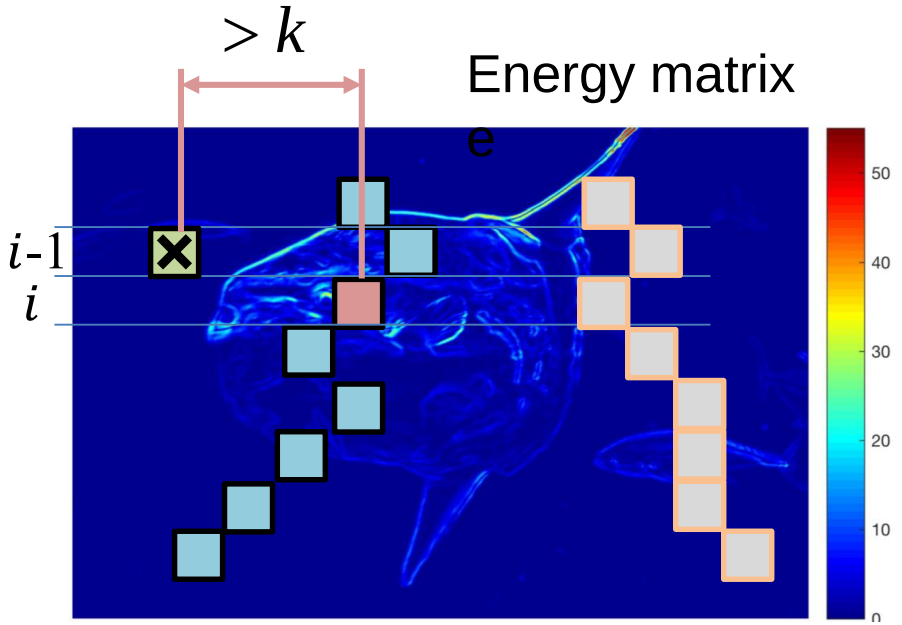
Video 9.2
Jianbo Shi





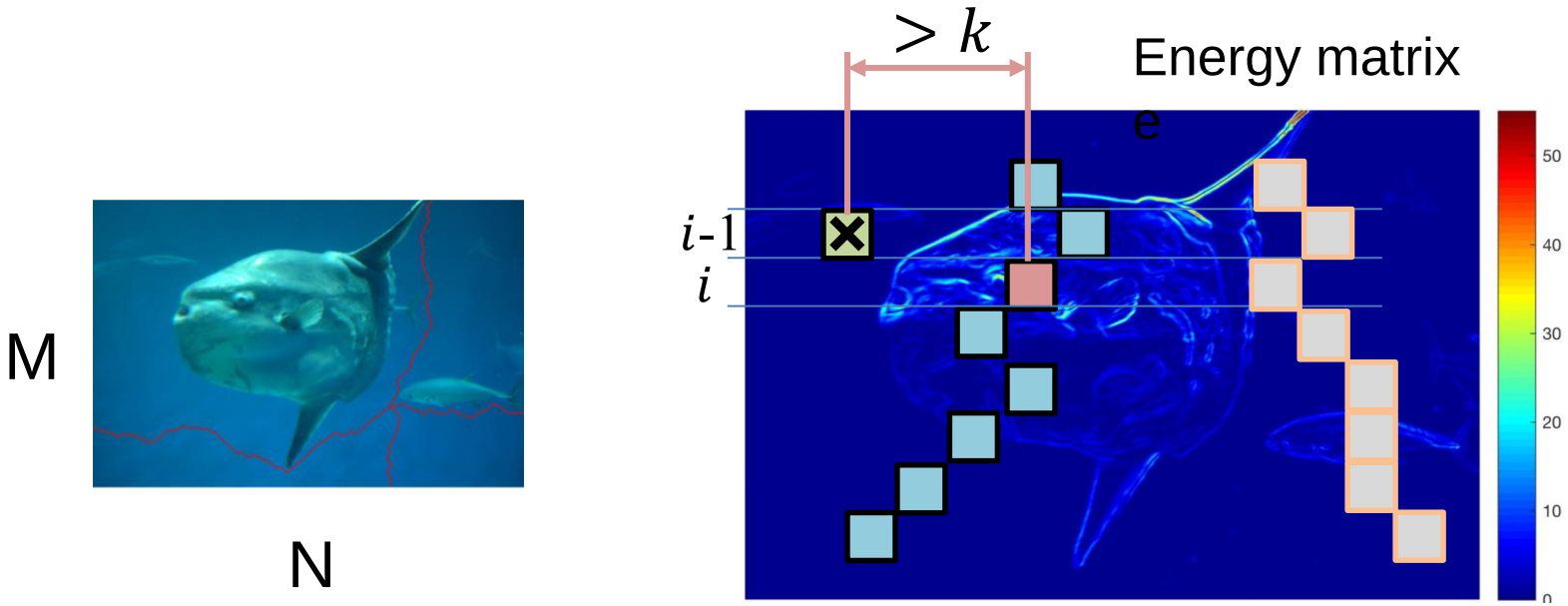
Seam: $S^y : \{(i, y(i)) \mid i = 1,\ldots,M\}$ s.t. $\mid y(i) - y(i-1) \mid \leq k$





$$S^y : \{(i, y(i)) \mid i = 1, \ldots, M\} \text{ s.t. } |y(i) - y(i-1)| \leq k$$

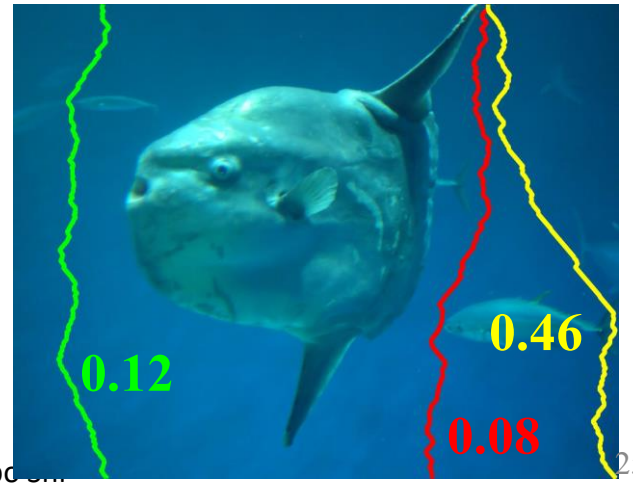
Seam Cost: $E(S^y) = \sum_{i=1}^{M} e(S^y(i))$



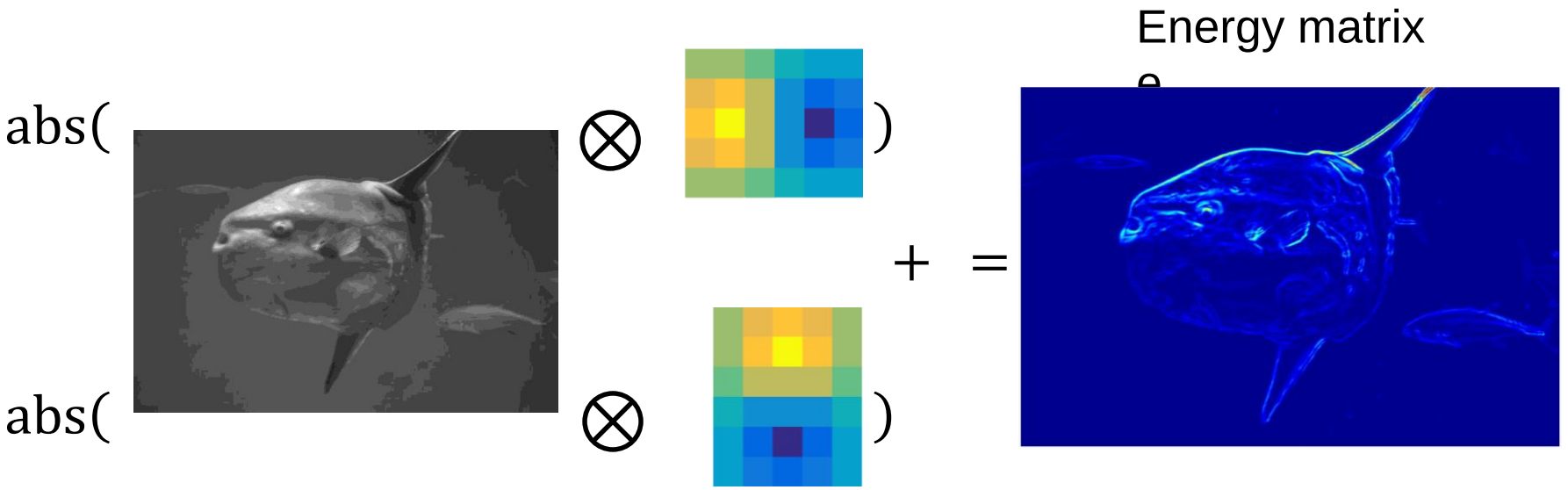
- Seam $S^y: \{(i, y(i))|i = 1, \cdots, M\}$ $s.t.$ $|y(i) - y(i-1)| \leq k$
- **Seam Cost:**

$$E(S^y) = \sum_{i=1}^{M} e(S^y(i))$$

- **Goal:** $S^* = \min E(S^y)$



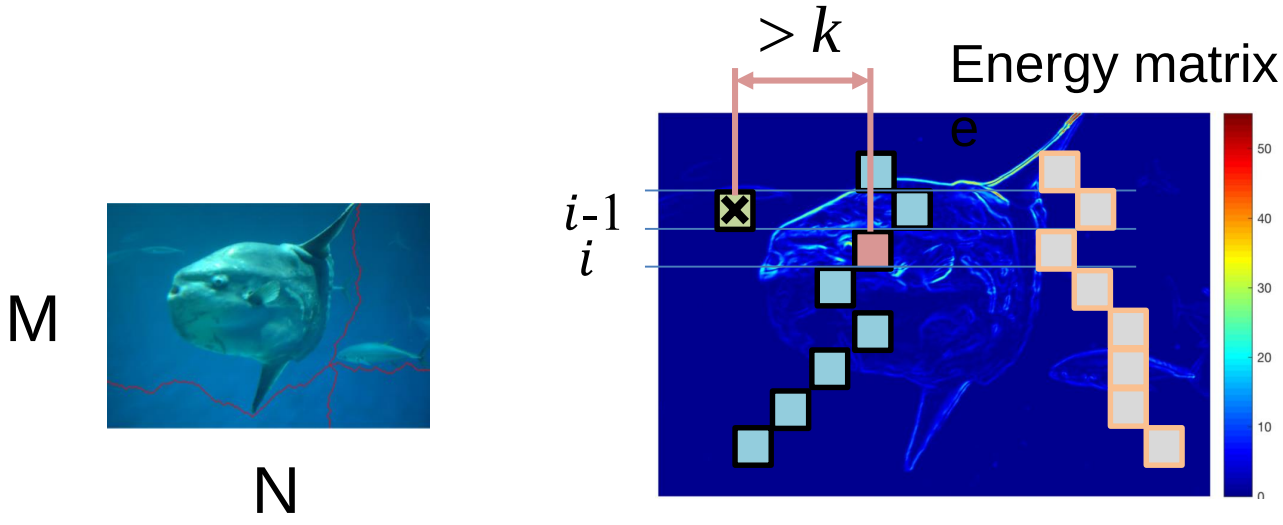
Where does the energy matrix come from?



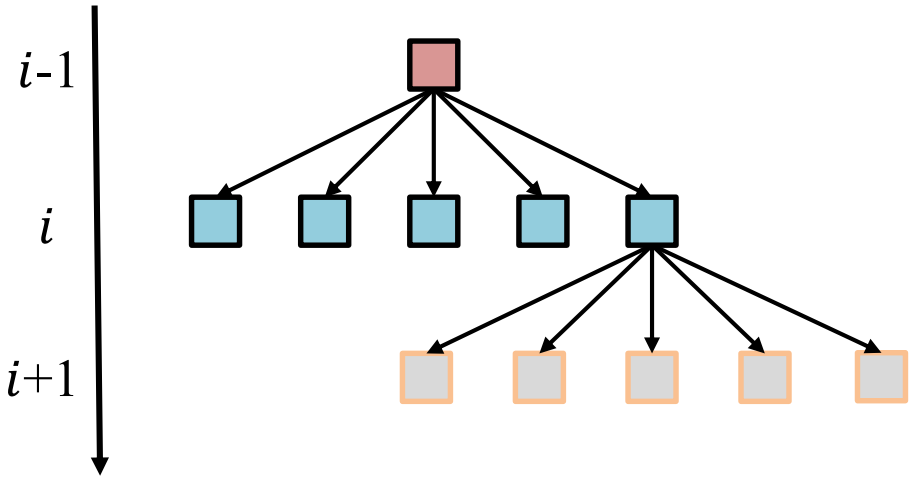
Energy matrix

For example: L1 norm of the edge gradients for the energy function

How to find the best seam?



Idea 1: Brute force search

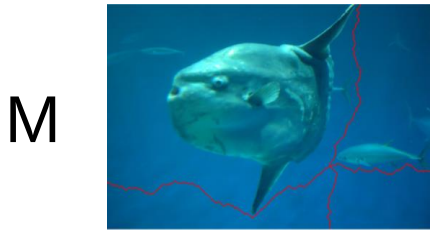


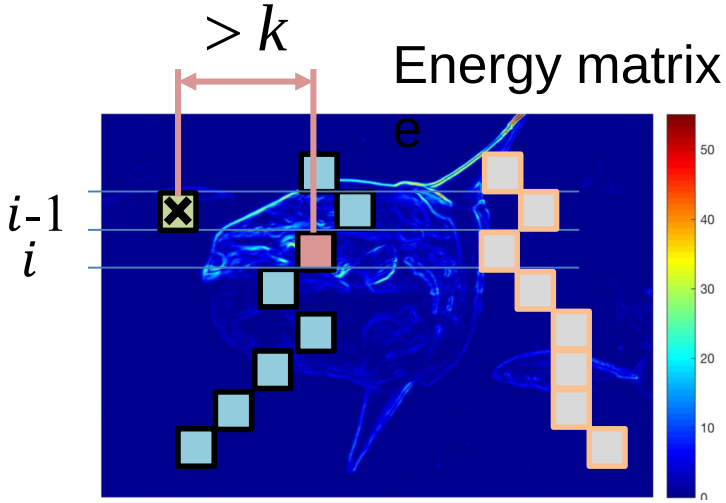
- 1st row: N
- 2nd row: $(2k+1)$N
- 3rd row: $(2k+1)^2$N

……

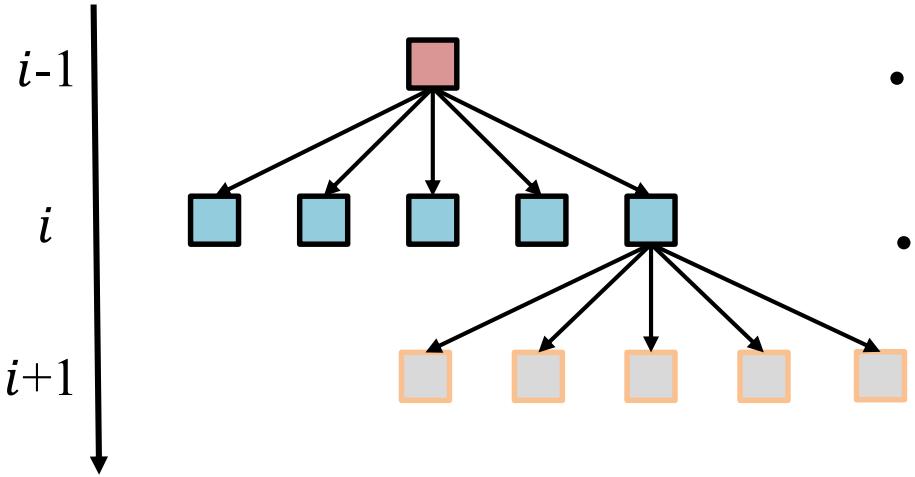
- Mth row: $(2k+1)^{(\mathbf{M}-1)}$N

- How many possibilities for Seam s^y?





Idea 1: Brute force search



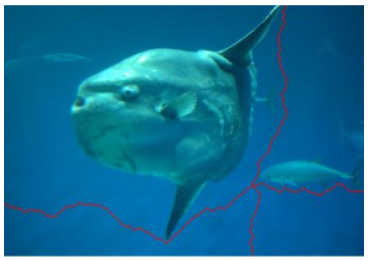
- $\mathbf{M}^{\text{th}}$ row: $(2k+1)^{(\mathbf{M}-1)}\mathrm{N}$
- Given $\mathbf{M} = 768, \mathbf{N} = 1024, k = 1$

- How many possibilities for Seam S^y? 1024×3^{767}

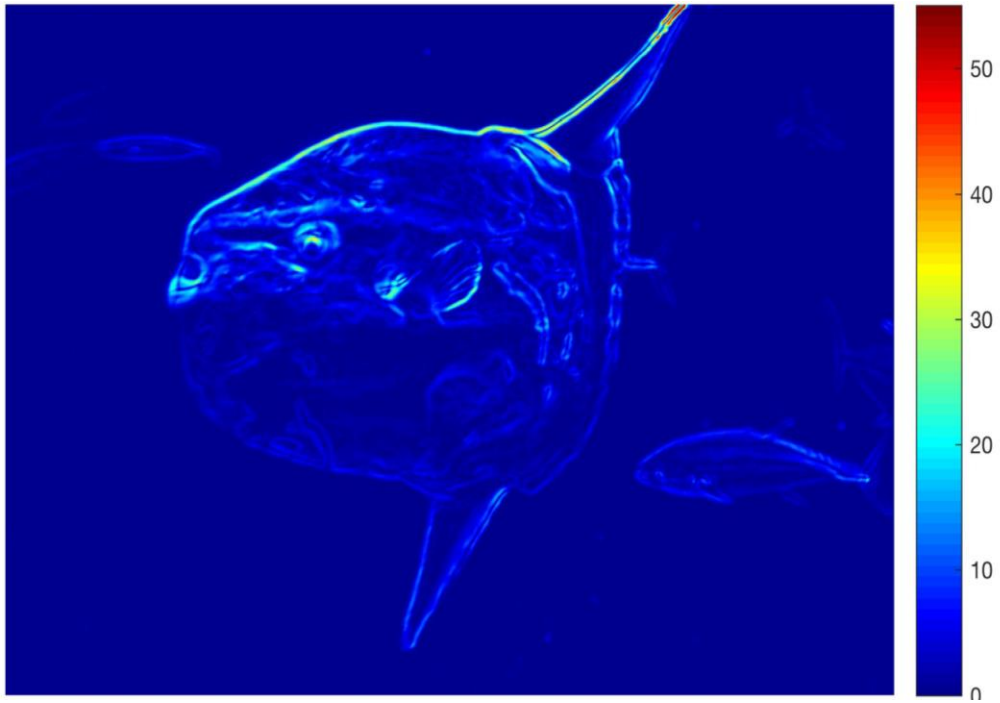
Too many possibilities!

$$1024 \times 3^{767}$$

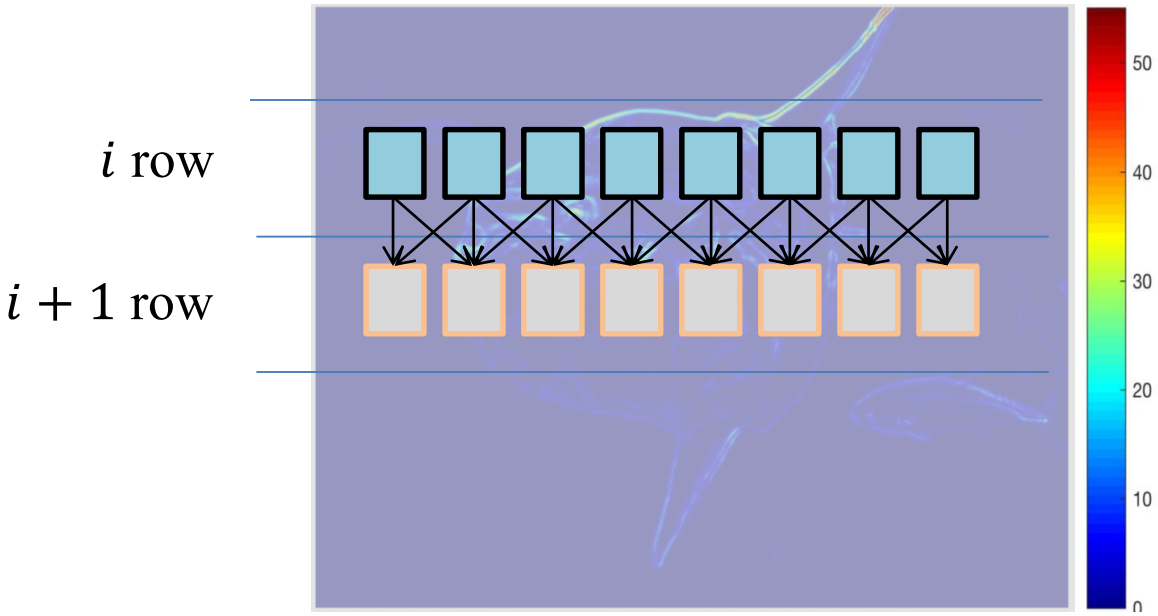
M



N

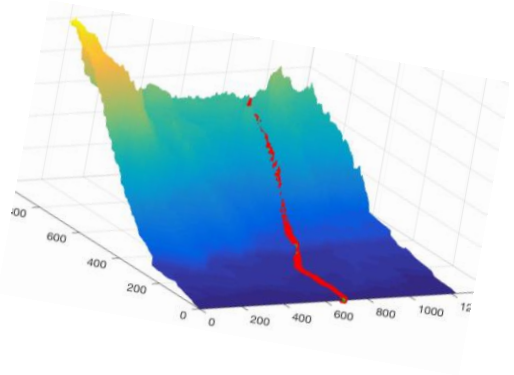


Idea 2: Find the shortest path from first row to the the last row in the energy domain!

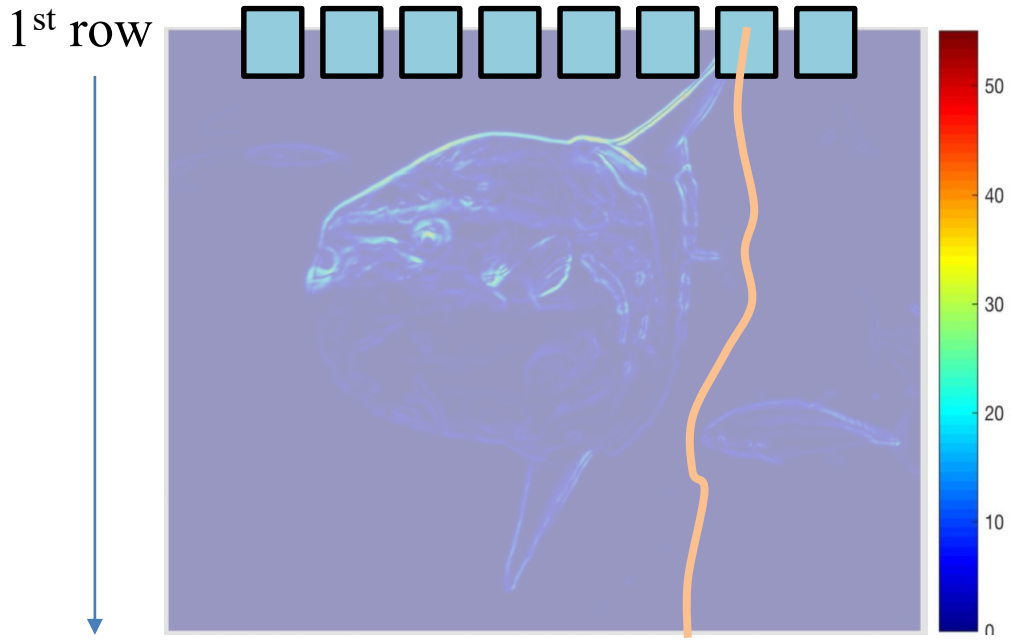


Construct the directed graph where

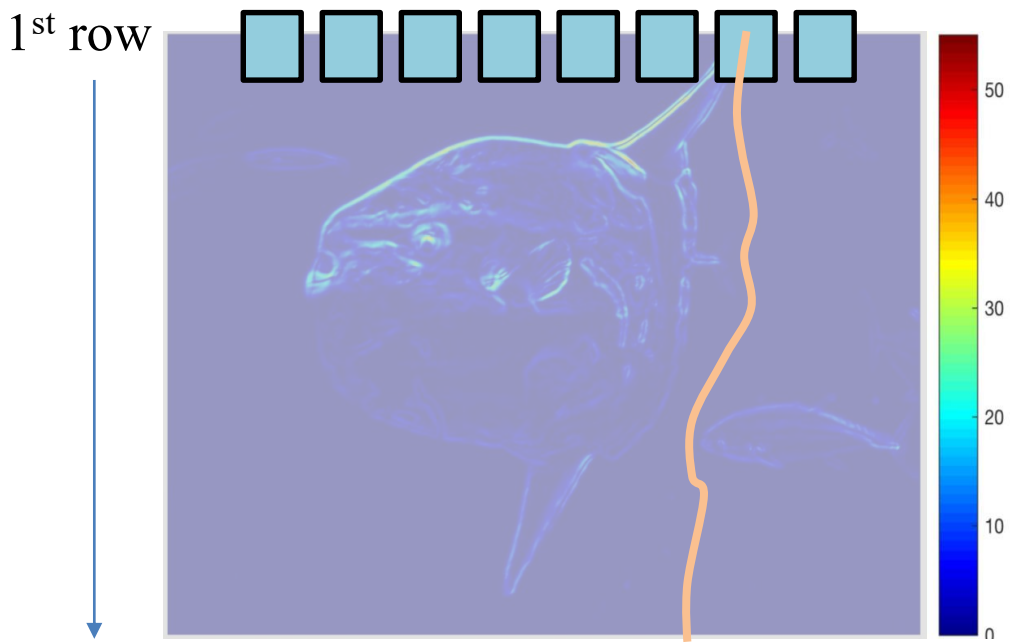
each pixel (node) is connected to the (2k+1) neighbors in the next row



V(u)



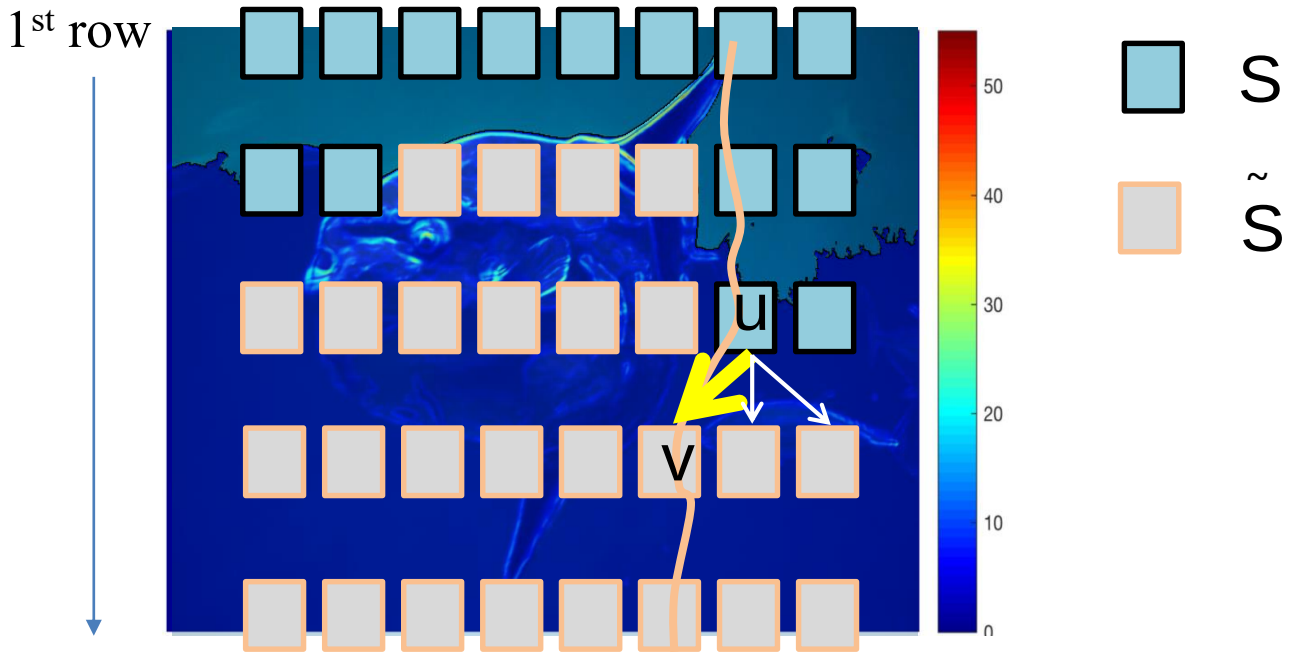
- Create an `interior' set S, initialize with the first row
- Growing S with `least-resistant' step
- Construct a 'Value' matrix with value $V(u)$ encoding the shortest path cost to each node in S



- Initialize S to include the first row, and set the Value function

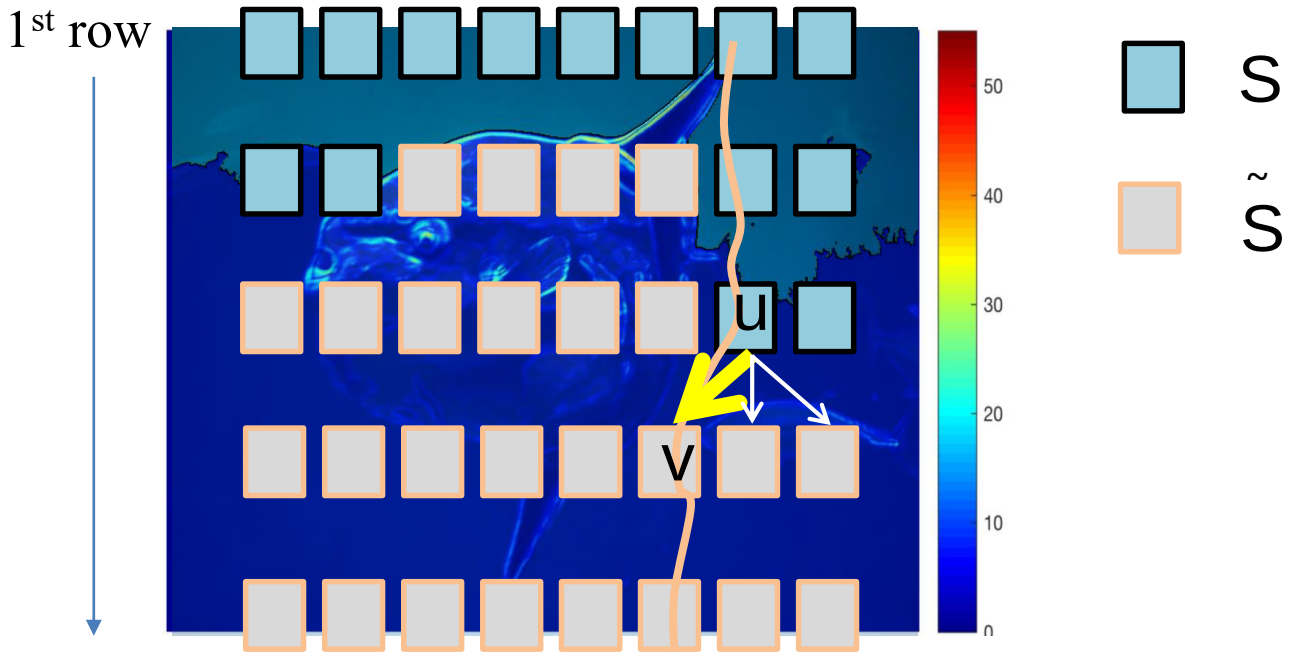
$$V(u) = e(u), u \in S$$

For the first row, the shortest path contains only itself



Iterate: find the step expansion of least resistant from $S \rightarrow \tilde{S}$

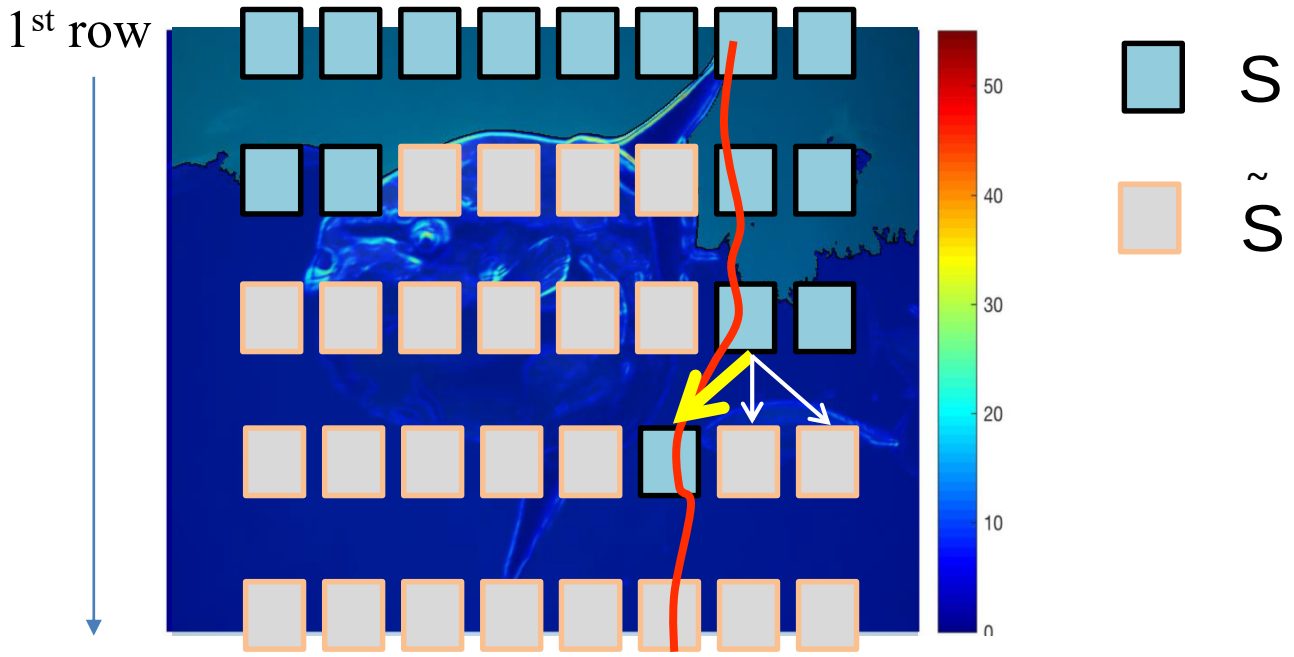
$$v = \underset{u \in S, v \in \tilde{S}}{\operatorname{argmin}} [V(u) + e(v)] \quad \textit{where } (u \rightarrow v) \text{ is a graph edge}$$



Iterate: find the step expansion of least resistant from $S \rightarrow \tilde{S}$

$$v = \underset{u \in S, v \in \tilde{S}}{\operatorname{argmin}} [V(u) + e(v)] \qquad \textit{where } (u \rightarrow v) \text{ is a graph edge}$$

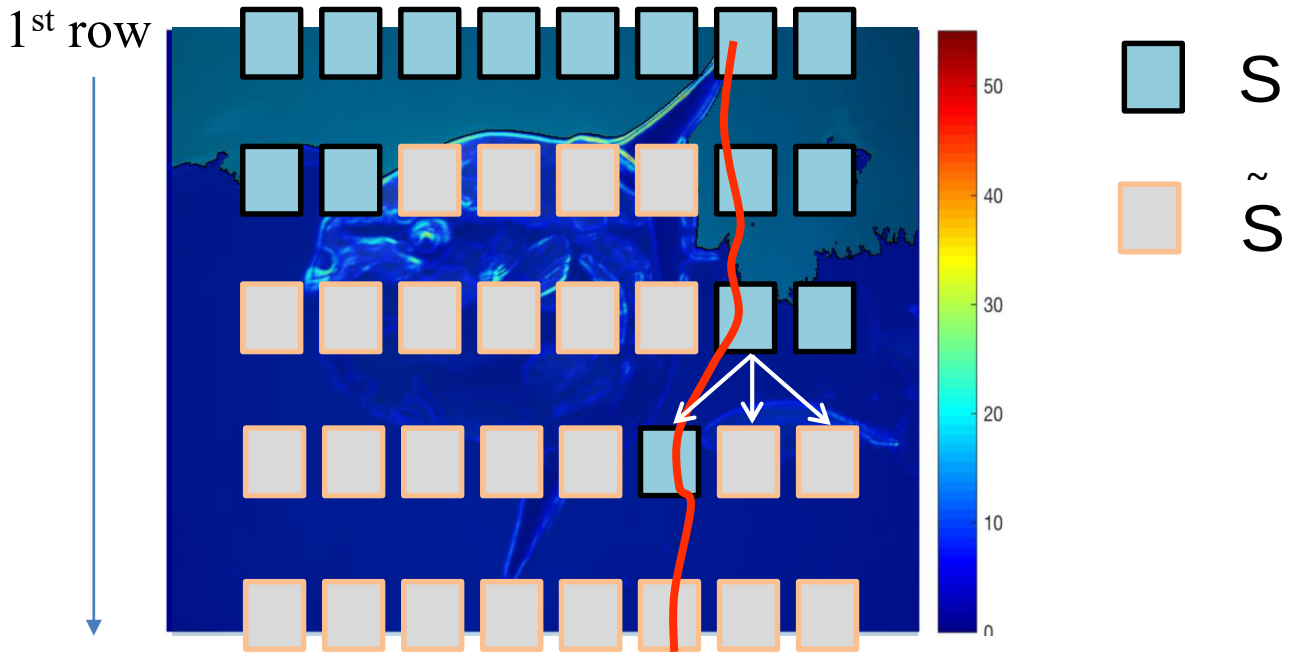
Remember the path back from $(v \rightarrow u)\ P(v) = u$



- Updating the "interior" set: $S = S \cup \{v\}$, $\tilde{S} = \tilde{S} - \{v\}$.

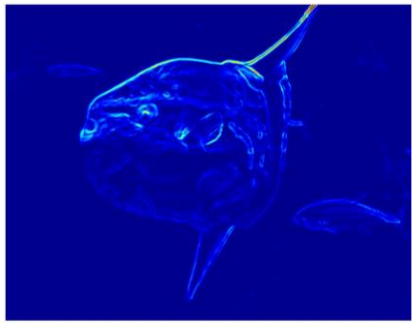
- Updating the Value function $V(v) = \min_{v \in \tilde{S}} [V(u) + e(v)]$

$V(v)$: Is the **contingent cost** of the shortest path connecting the pixel v to the first row

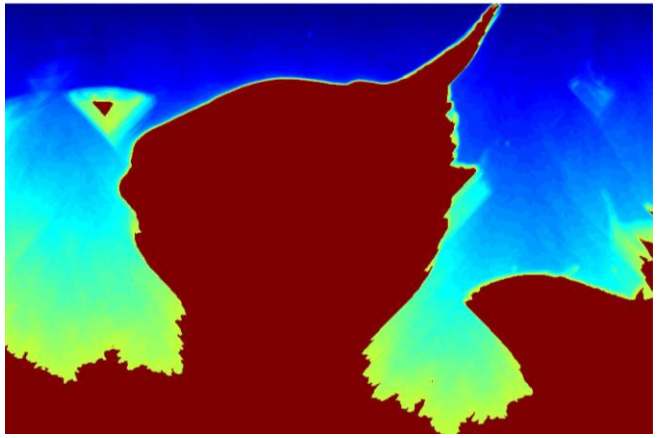


- Updating the "interior" set: $S = S \cup \{v\}$, $\tilde{S} = \tilde{S} - \{v\}$.
- Updating the Value function: $V(v) = \min_{v \in \tilde{S}} [V(u) + e(v)]$
- Until reaching one of the pixels on the last row.

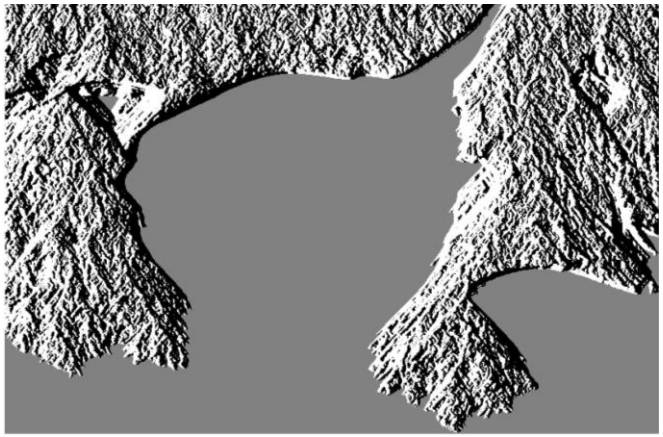
Energy Matrix e



Value function V(u)



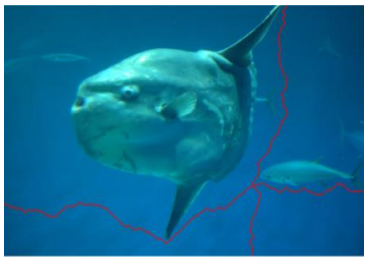
Path Matrix P



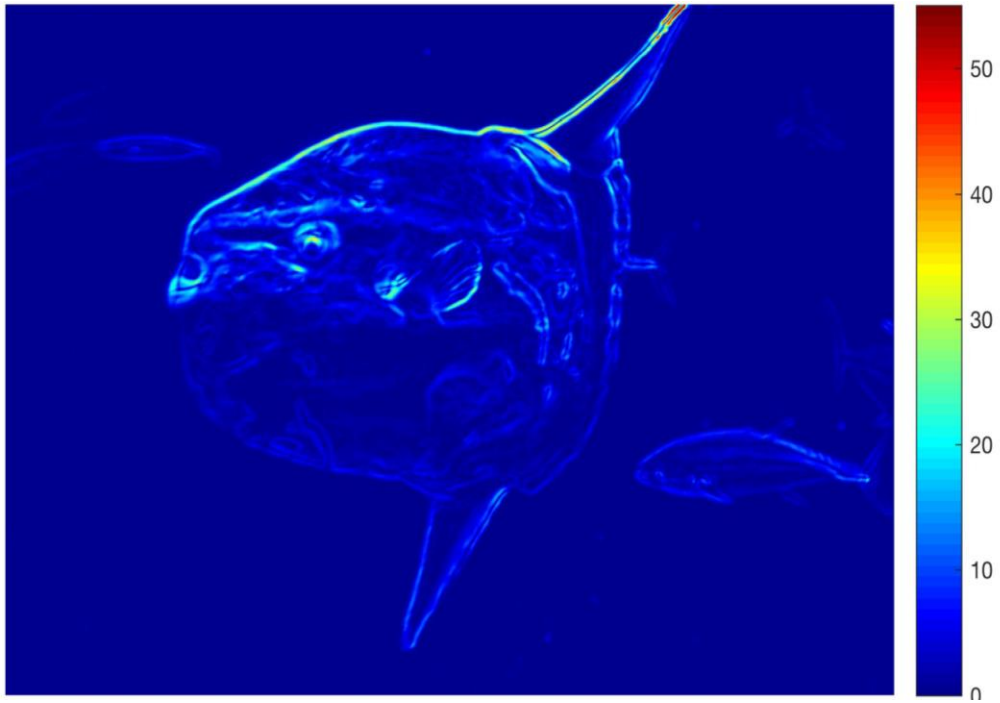
Video 9.3
Jianbo Shi

Dynamic Programming
Frontier growing row by row

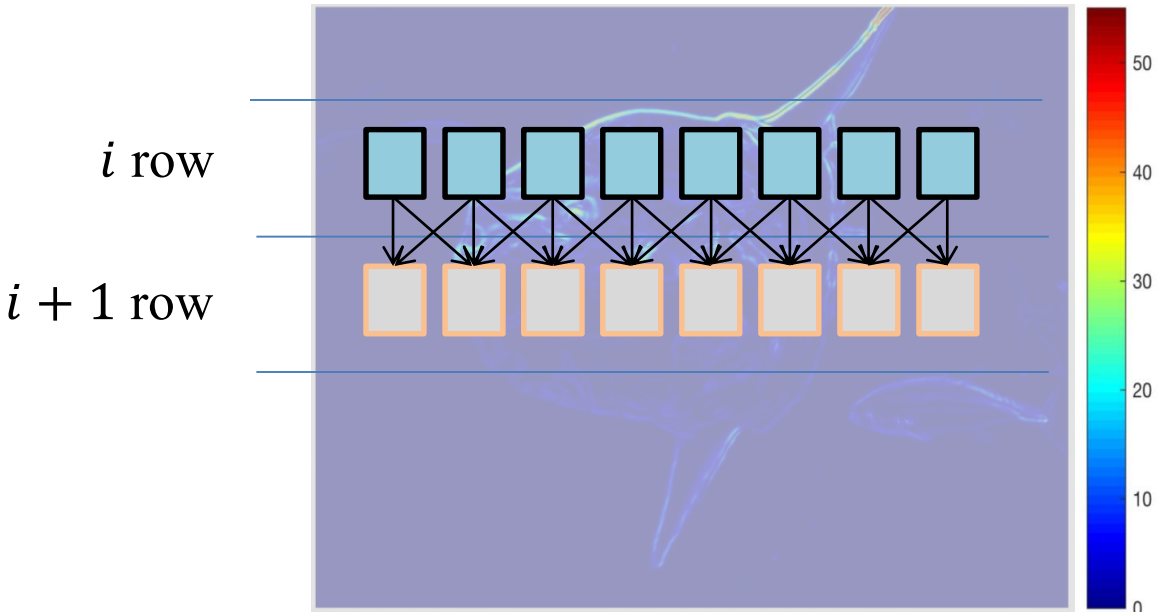
M



N

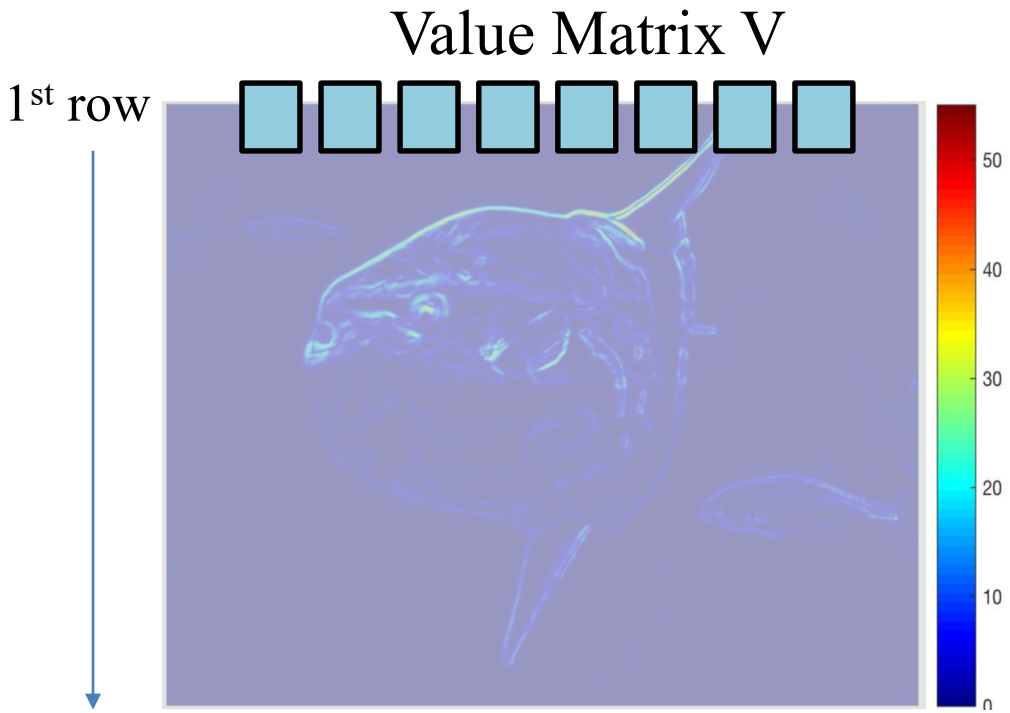


Idea 3: Dynamic programming!



Use the same graph structure

Propagate the frontier row by row to construct the Value Matrix



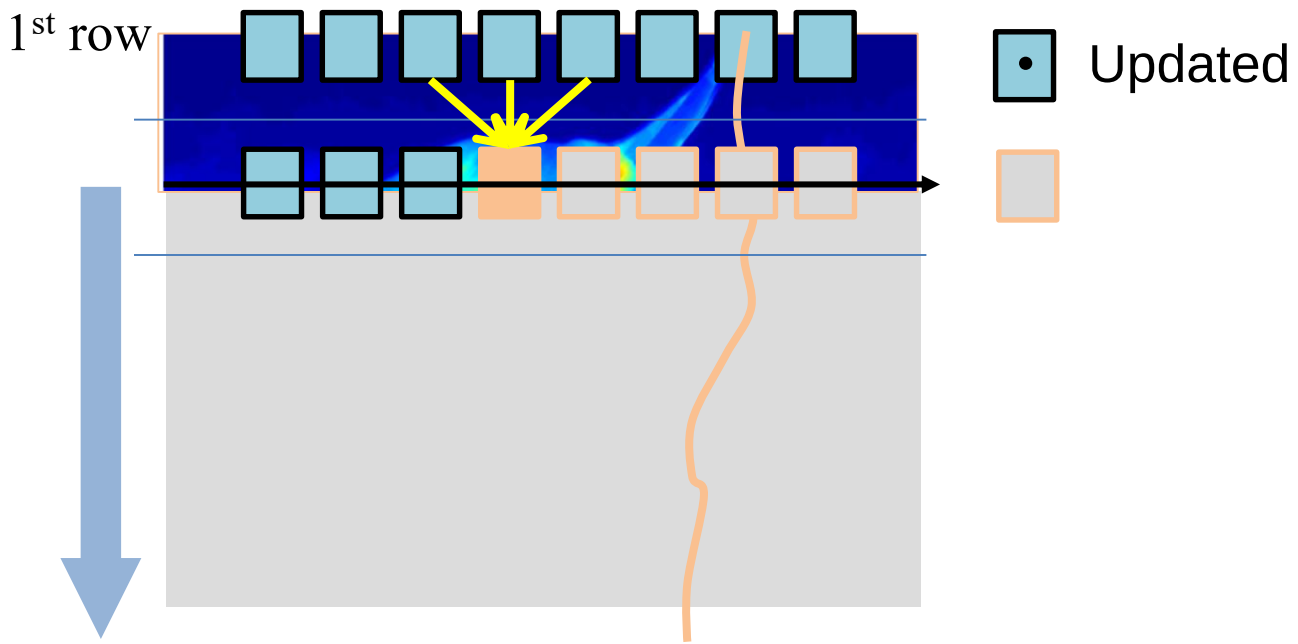
Path Matrix P

- Still start from first row, and initialize the Value function

$$V(1, j) = e(1, j)$$

- Set the Path function

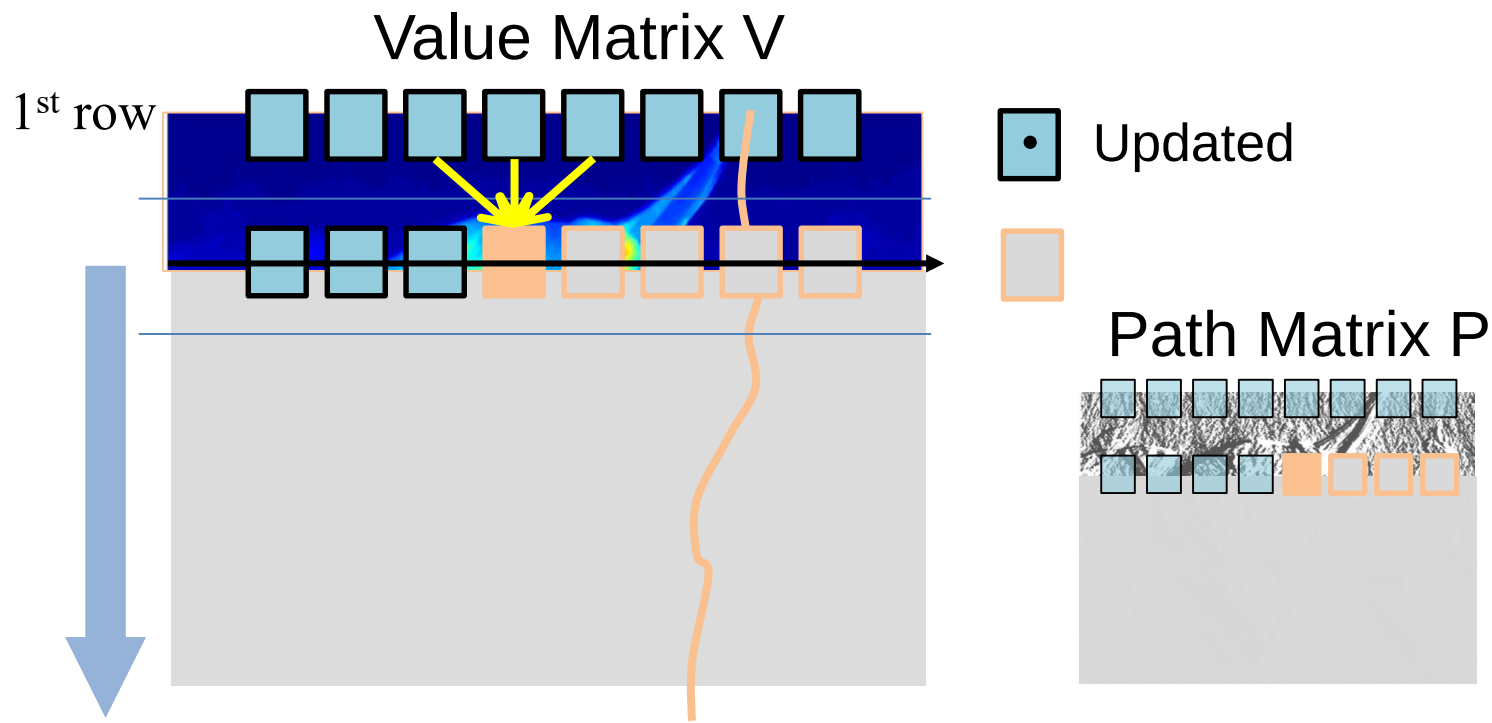
$$P(1, j) = 0$$



- Propagate to the second row, and update the value matrix

$$V(2,j) = e(2,j) + \min(V(1,j-k),\ldots,V(1,j),\ldots,V(1,j+k))$$

$V(2,j)$: Is the **contingent cost** of the shortest path connecting the pixel **(2,j)** to the first row

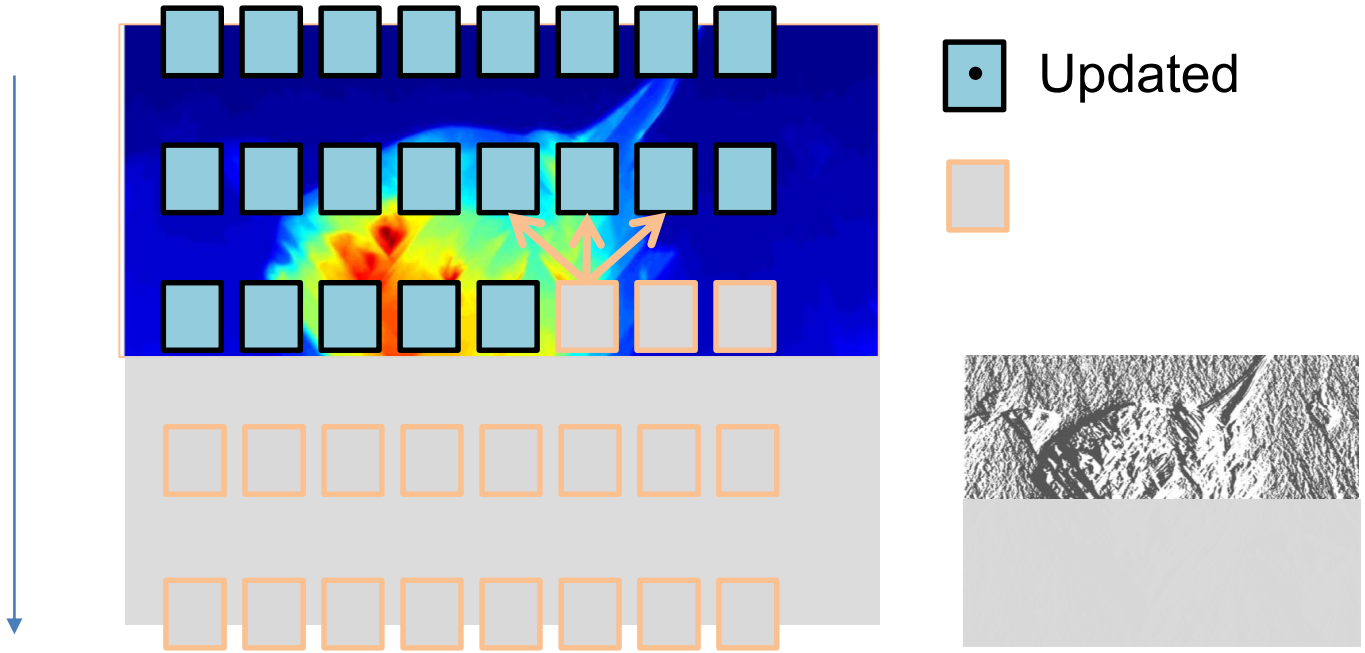


- Propagate to the second row, and update the value matrix

$$V(2,j) = e(2,j) + \min(V(1,j-k),\ldots,V(1,j),\ldots,V(1,j+k))$$

- Set the Path function

$$P(2,j) = \arg\min(V(1,j-k),\ldots,V(1,j),\ldots,V(1,j+k))$$



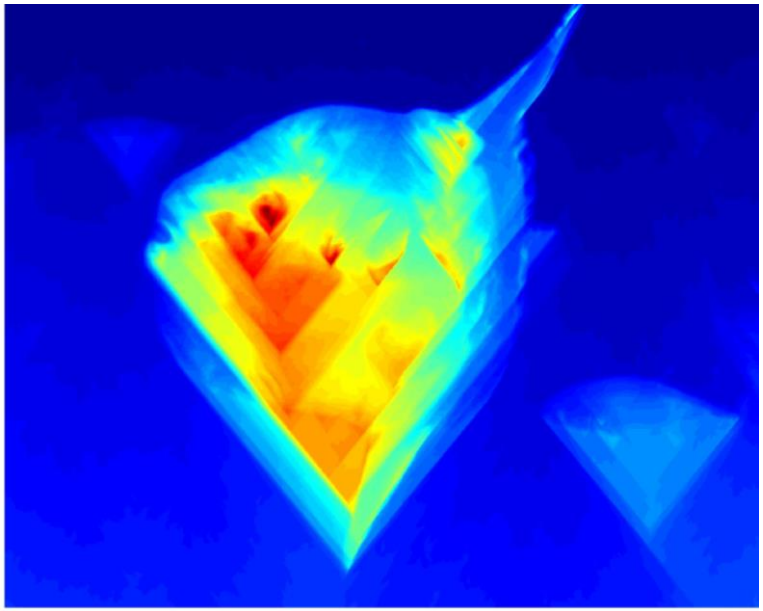
- Iteratively propagate to the **ith** row, and update the value matrix

$$V(i,j) = e(i,j) + \min(V(i-1,j-k),\ldots,V(i-1,j),\ldots,V(i-1,j+k))$$

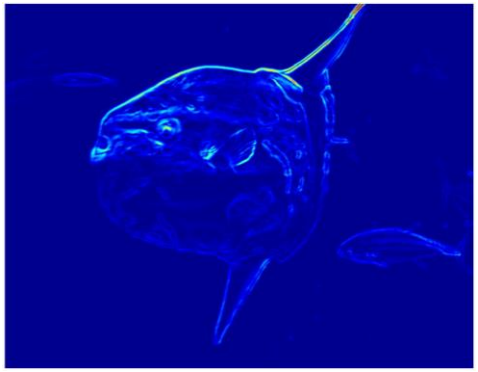
- Set the Path function

$$P(i,j) = \arg\min(V(i-1,j-k),\ldots,V(i-1,j),\ldots,V(i-1,j+k))$$

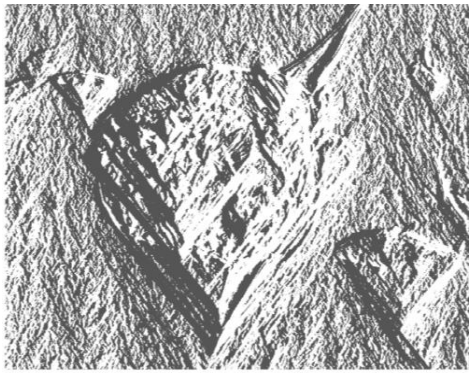
Value Matrix V



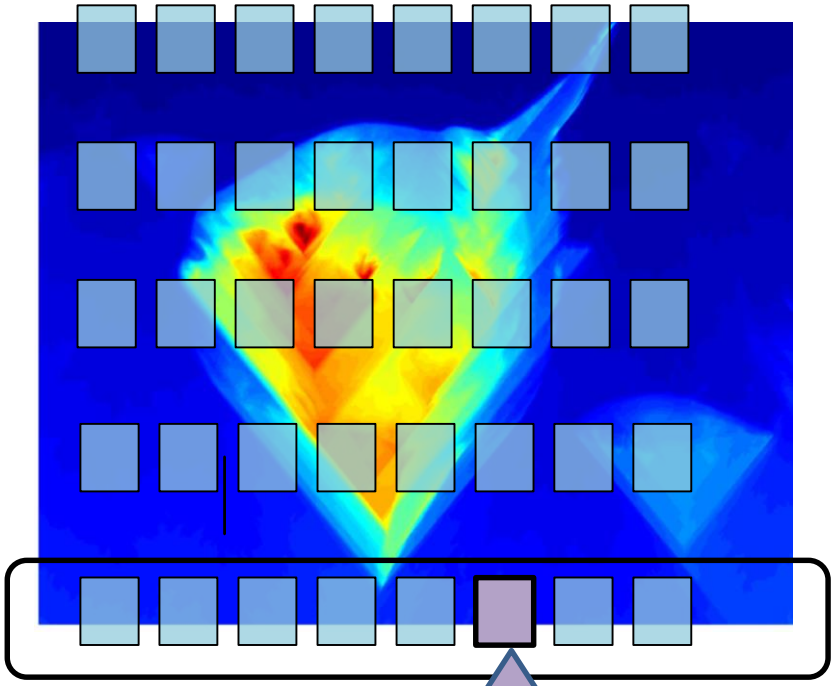
Energy Matrix e



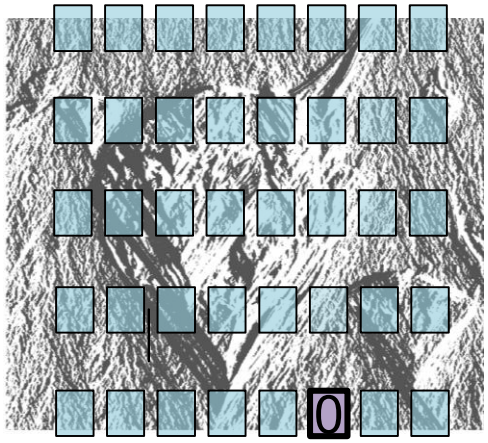
Path Matrix P

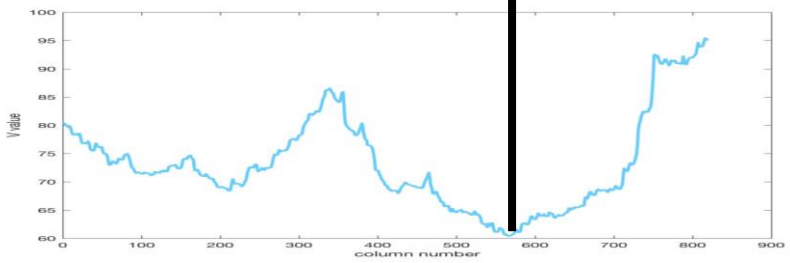


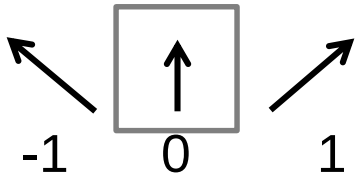
Value Matrix V



Path Matrix P



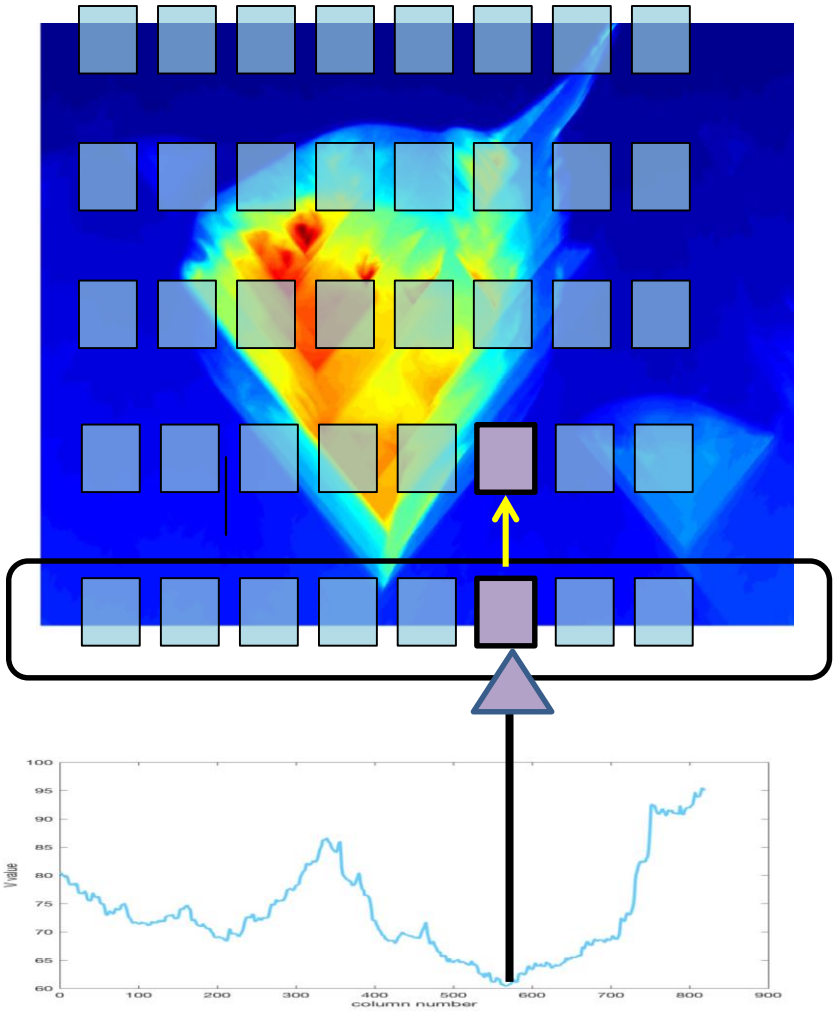




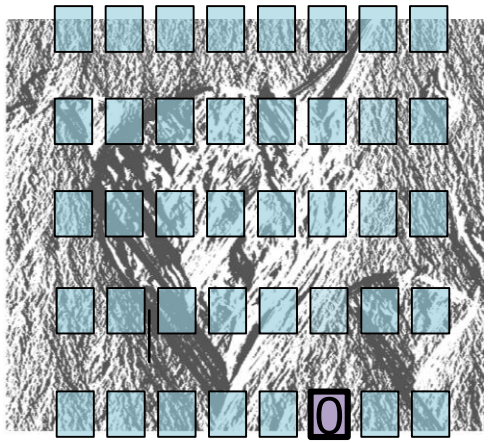
- Localize the minimum of the last row of

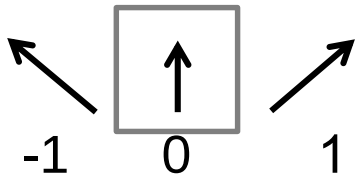
$$\operatorname{argmin} V(M,:)$$

Value Matrix V

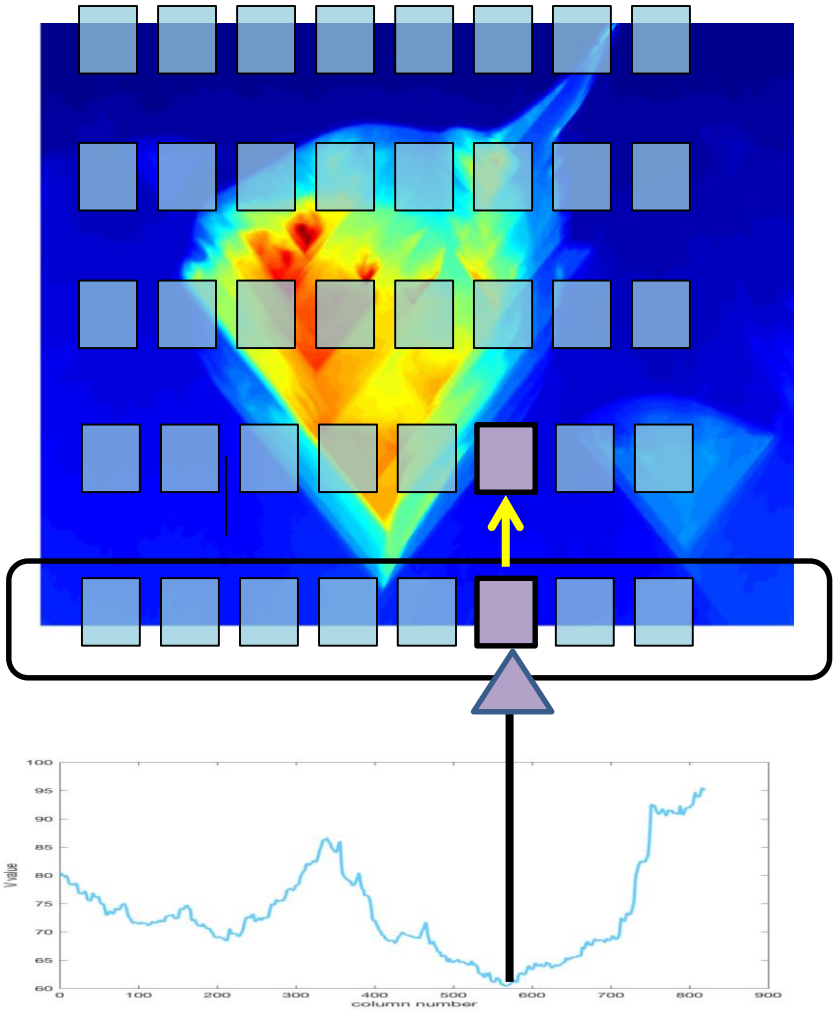


Path Matrix P

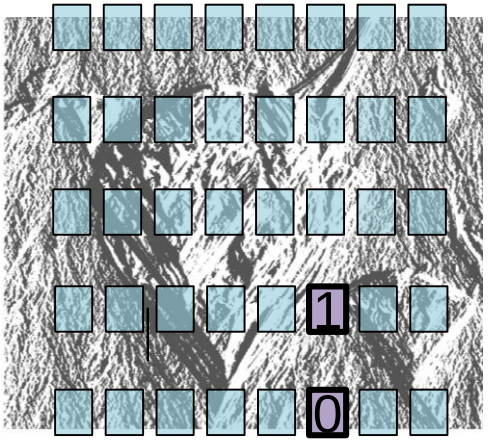


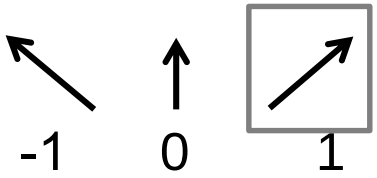


Value Matrix V

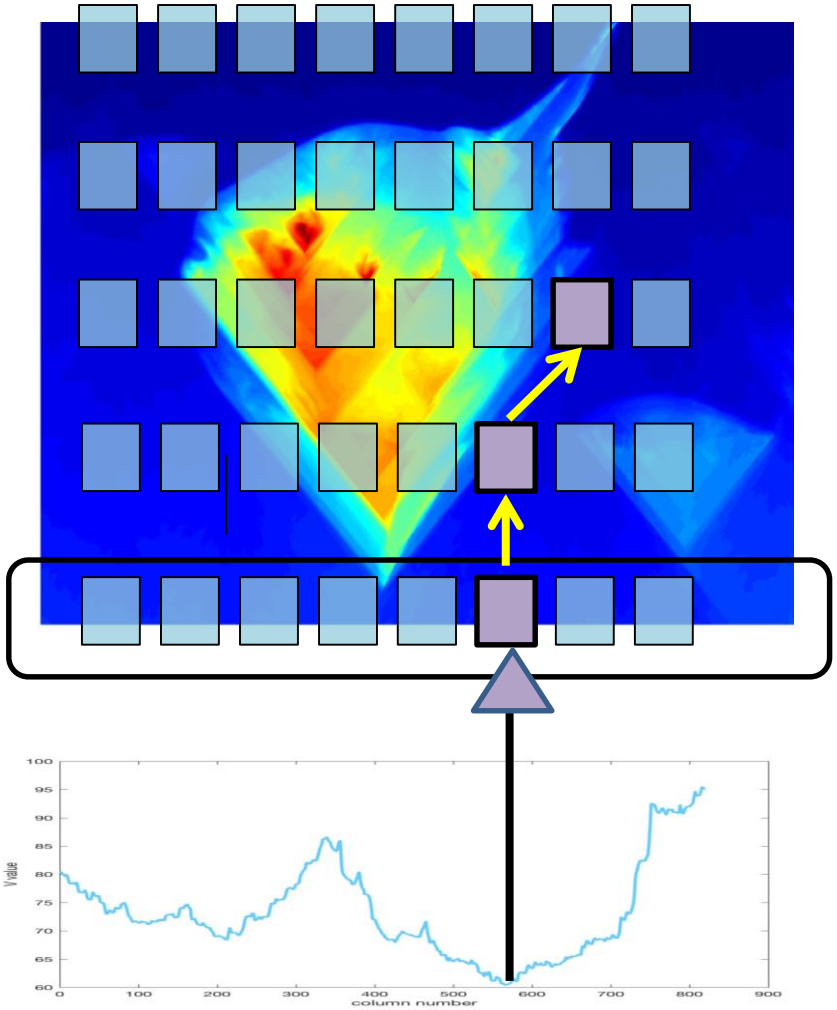


Path Matrix P

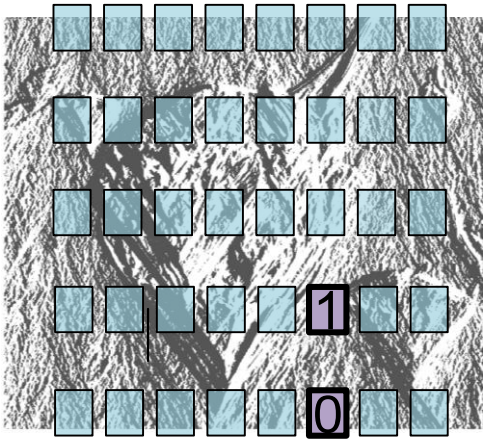


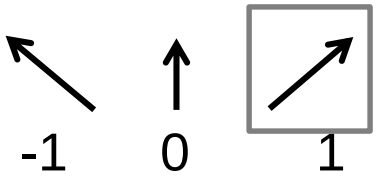


Value Matrix V

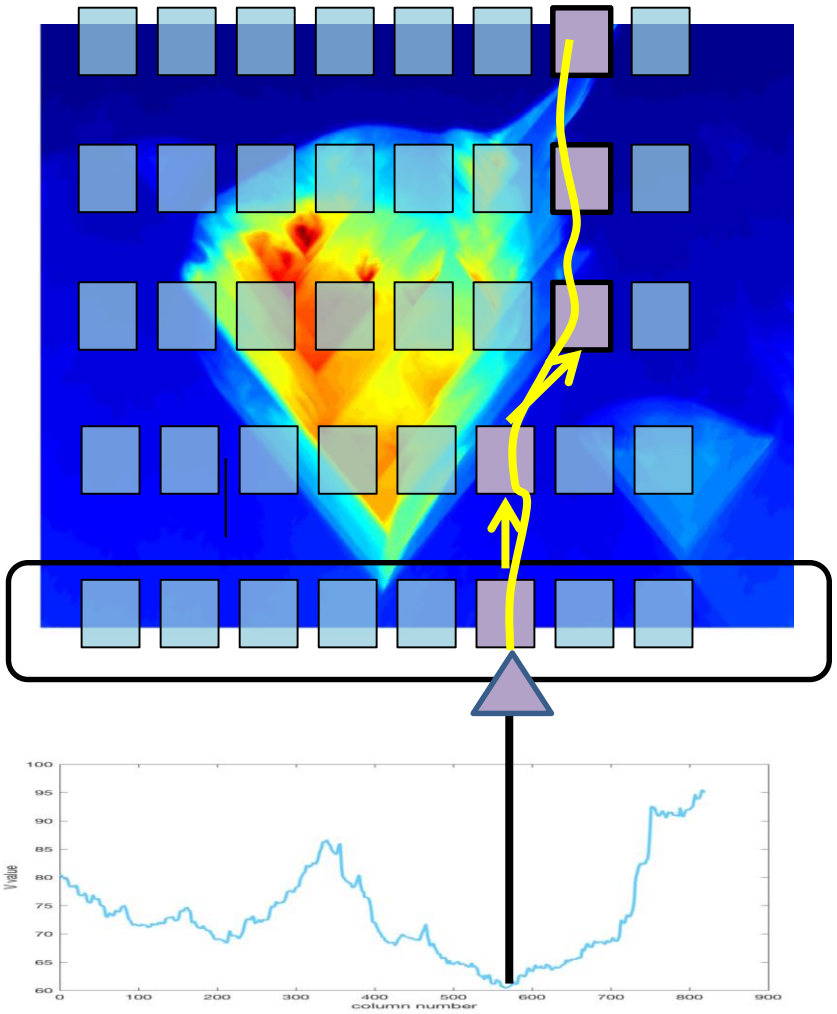


Path Matrix P

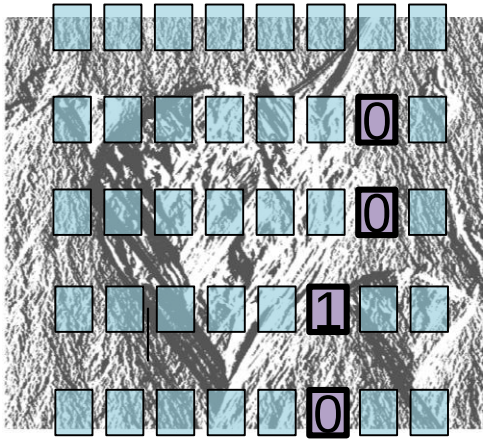


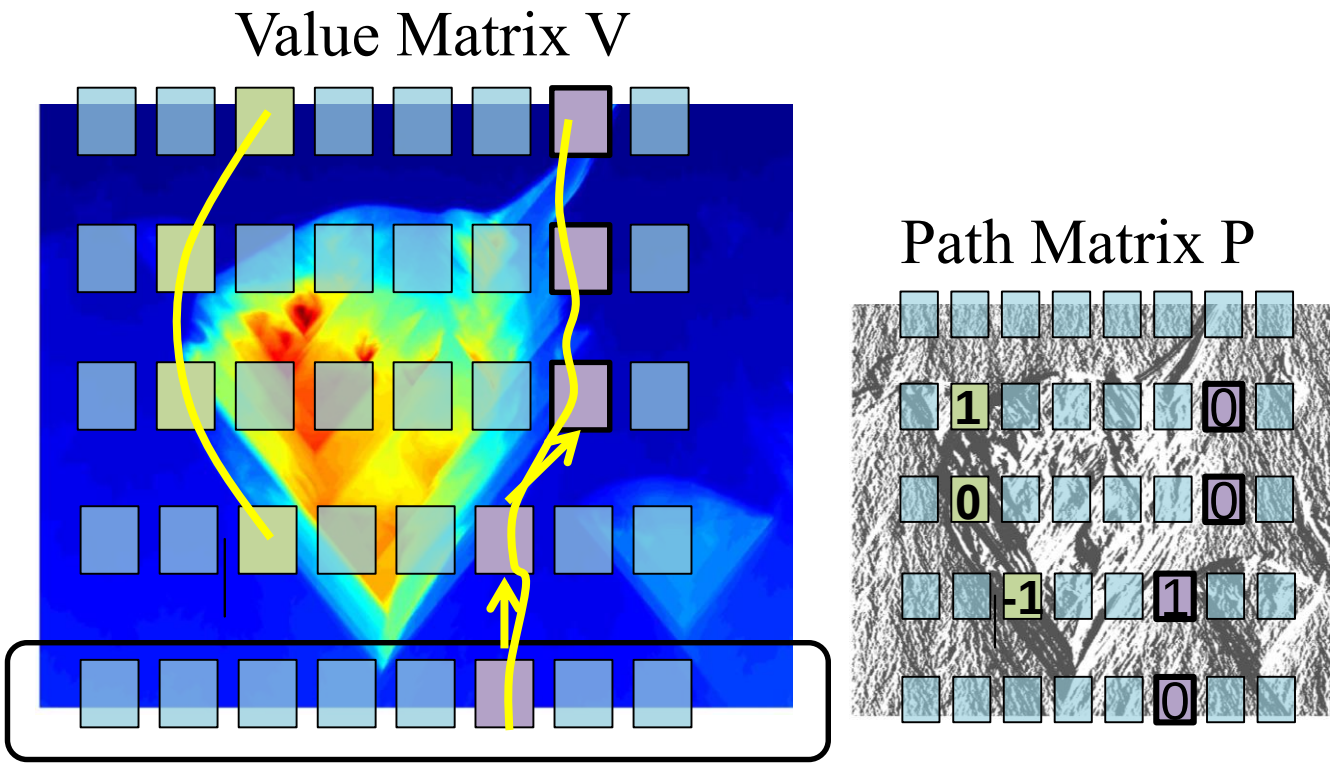


Value Matrix V



Path Matrix P



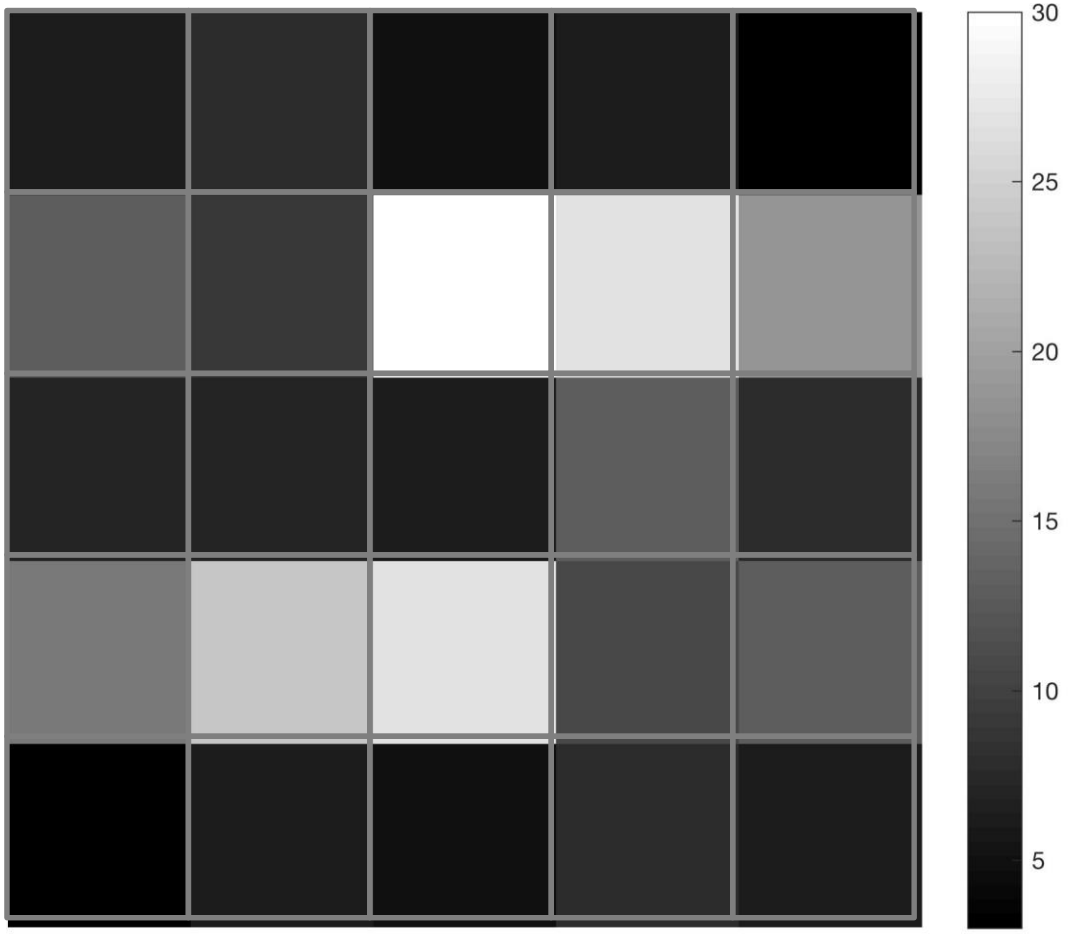


- Since V is the contingency table, we can start from any pixel and find a shortest path to the first row!

Video 9.4
Jianbo Shi

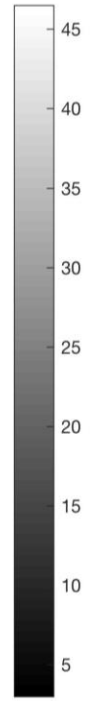
Illustration for synthetic case

Energy matrix



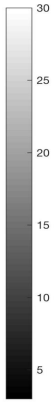
Value matrix

5.5	8	4.5	6	3
18.5	13.5	34.5	30	22
20	20.5	19.5	34.5	30
36	43.5	46.5	30.5	43
39	42	35	38.5	36



Energy matrix

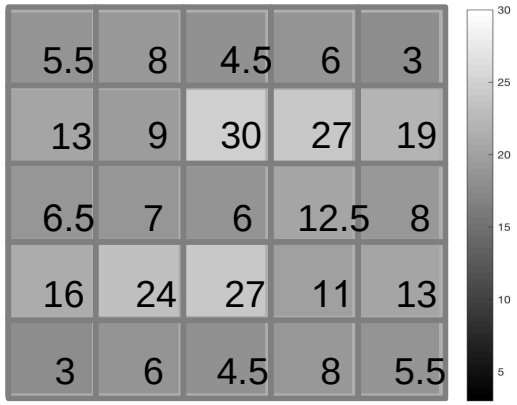
5.5	8	4.5	6	3
13	9	30	27	19
6.5	7	6	12.5	8
16	24	27	11	13
3	6	4.5	8	5.5



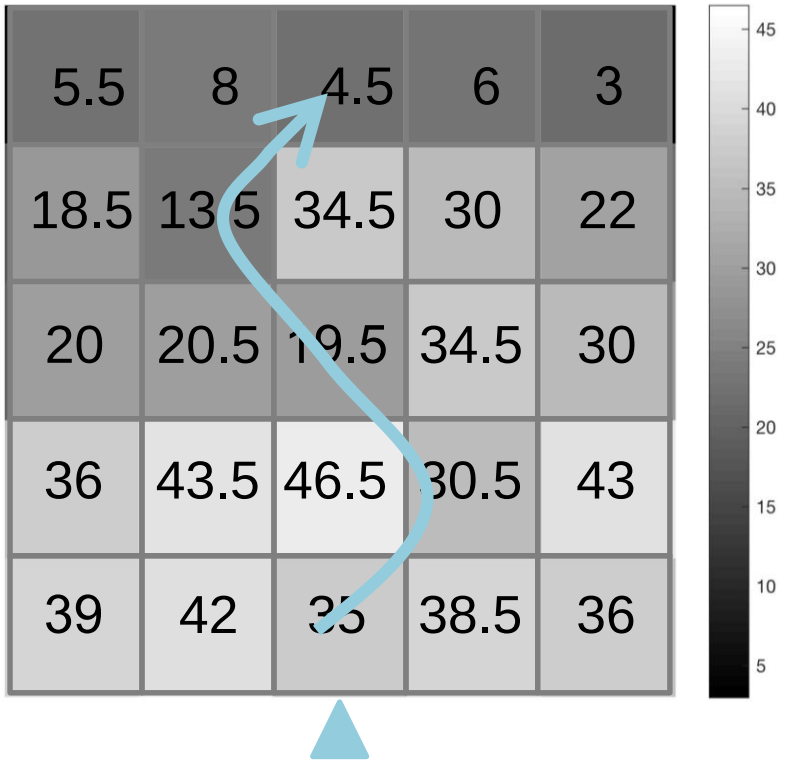
Goal:

- Construct ***value and path matrix*** from the ***energy matrix***
- ***Value matrix*** records the ***energy of the shortest path*** from the starting row to the current pixel

Energy matrix

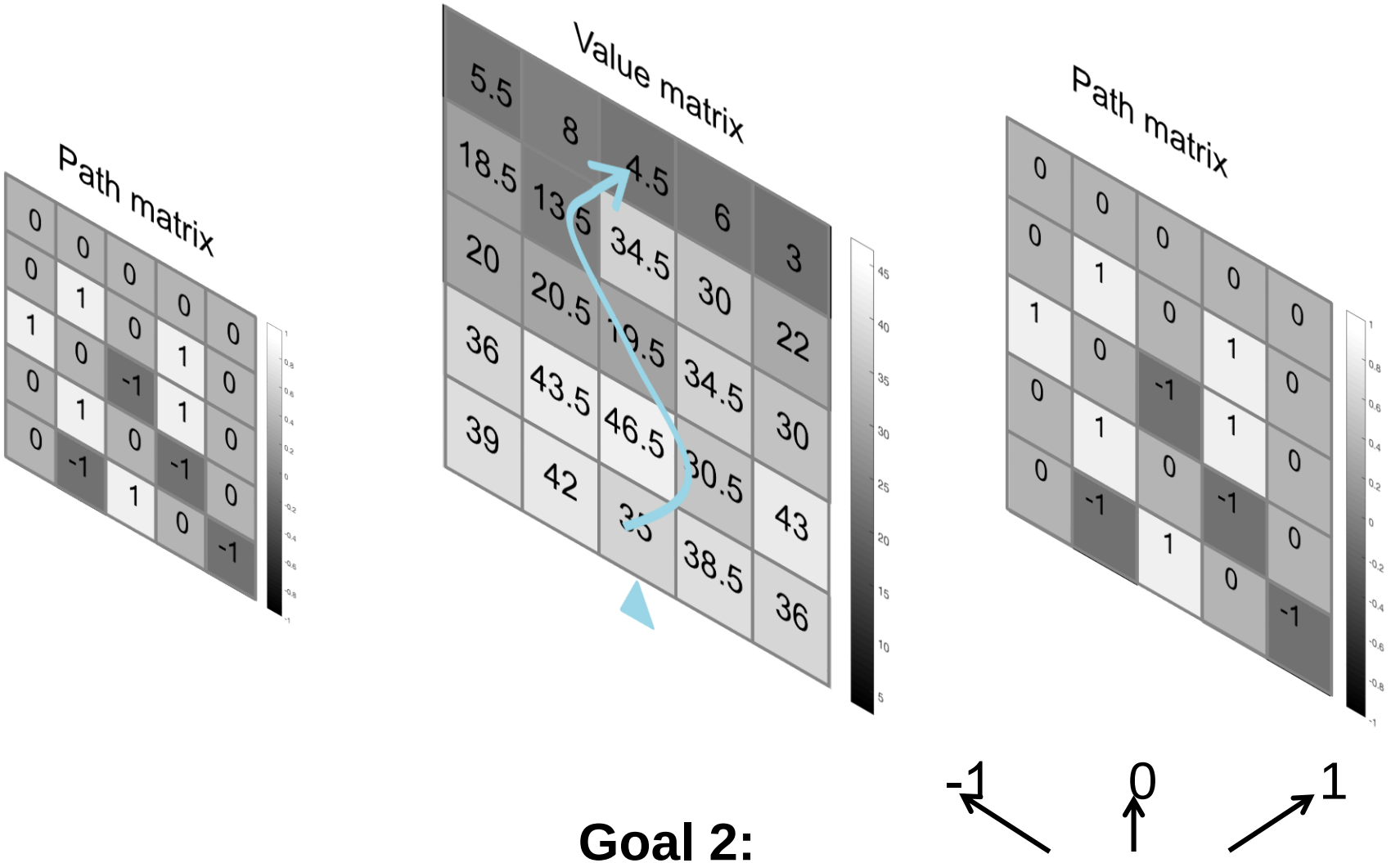


Value matrix



Goal 1:

- Construct ***Value matrix*** from the ***energy matrix***
- ***Value matrix*** records the ***energy of the shortest path*** from the starting row to the current pixel
- Property: every entry encodes the minimal shortest path to that node from starting row



Goal 2:

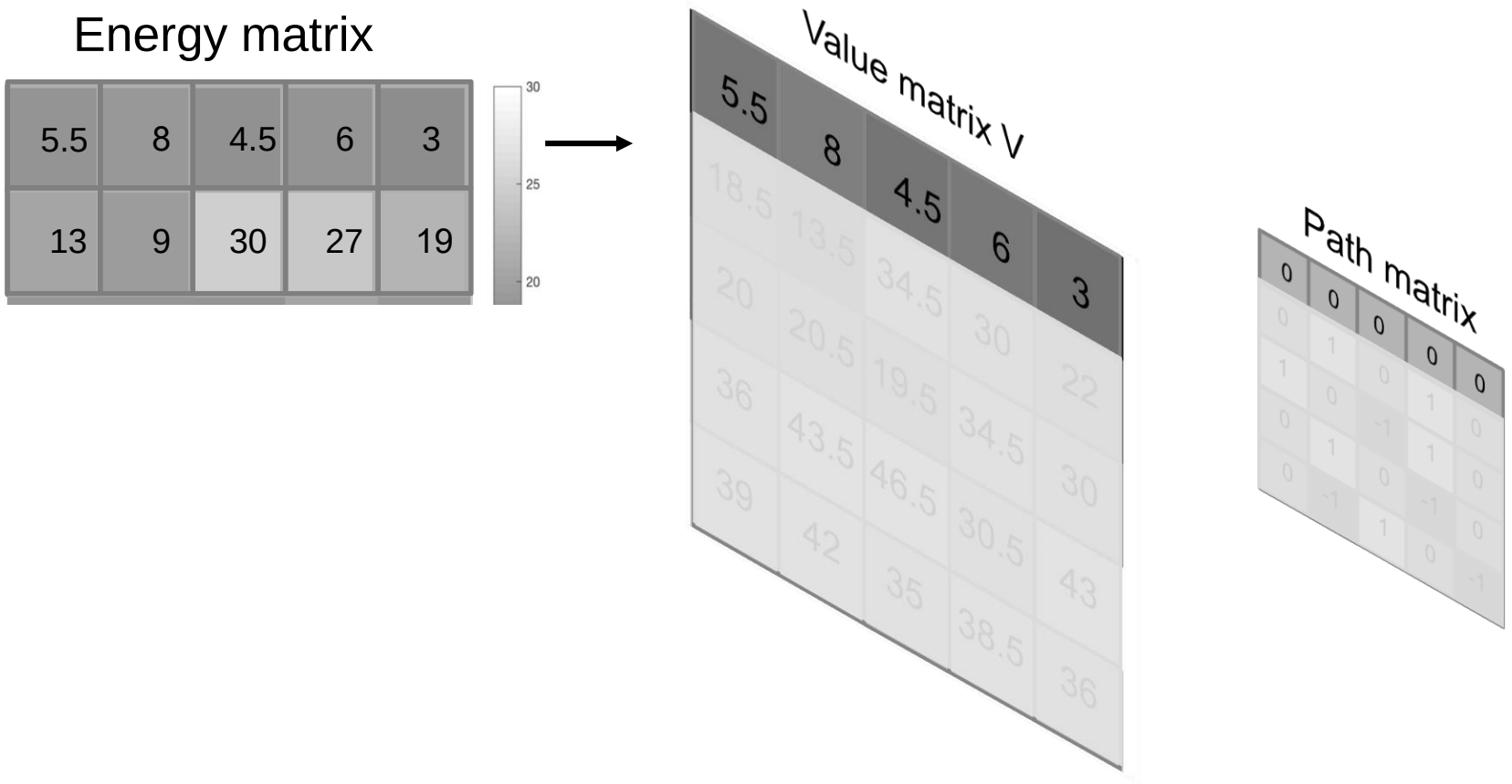
- Construct ***Path matrix***
- The Path matrix records the immediate predecessor in the shortest path, for every node

e: energy function

Energy function

- Energy function records the cost of a pixel.
- Typically it is the image gradient magnitude

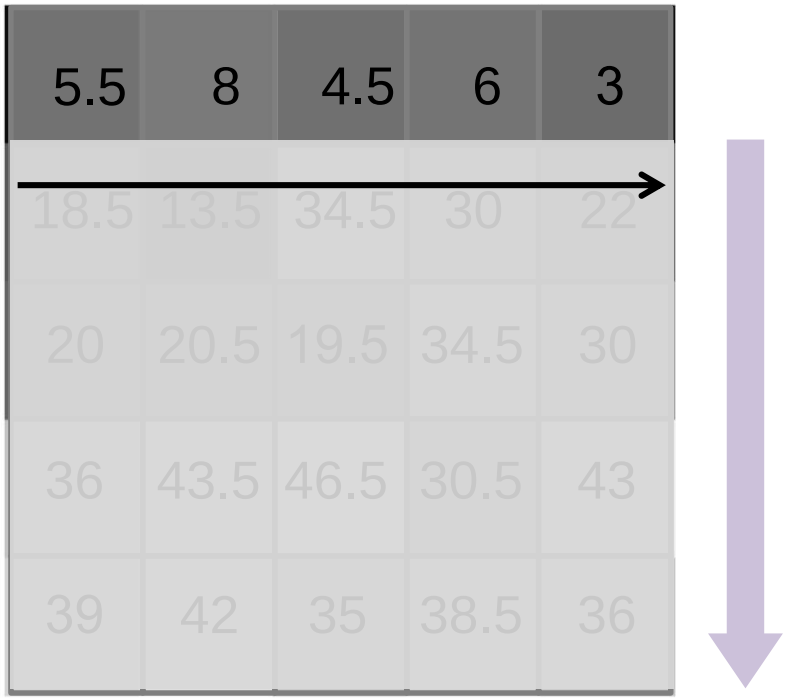
How to generate the two matrices?



Step1: Initializing two matrices

- Set value and path matrix ***the same size*** as the energy matrix
- Initialize its ***first row of Value matrix*** with that of the energy matrix
- Initialize its ***first row of path matrix*** to zero

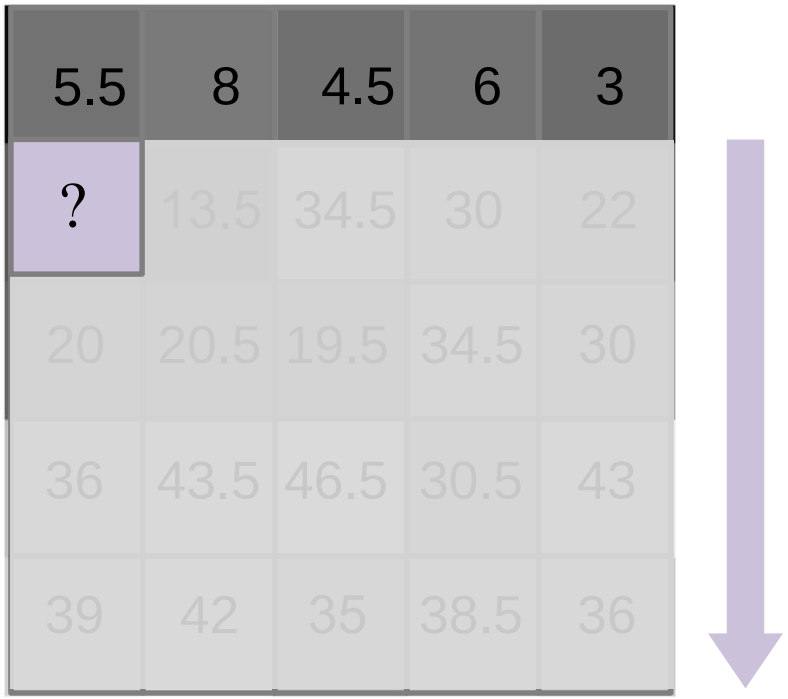
Value matrix V



Step2: Propagation

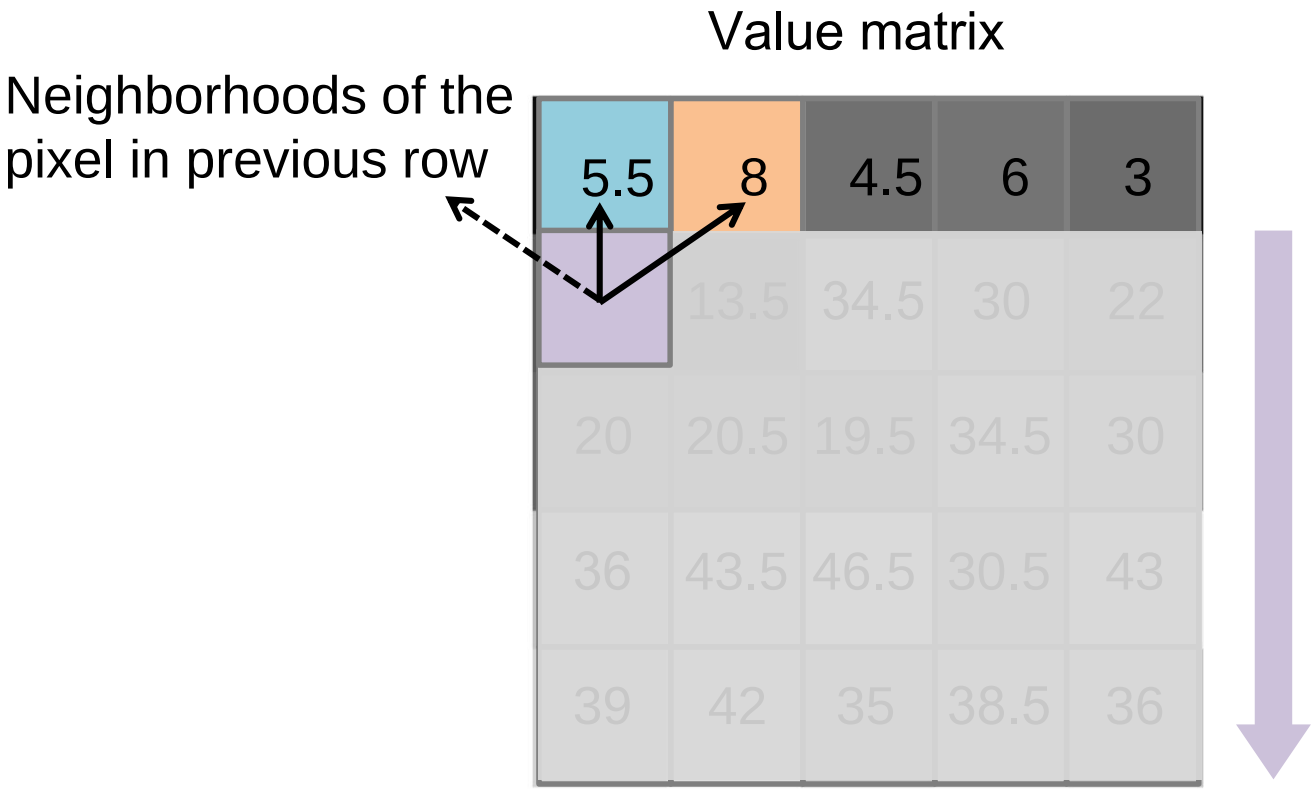
- Start with 2nd row, and propagate row by row

Value matrix V



Step2: Propagation

- Start with 2nd row



Step2: Propagation

- Start with 2nd row
- Find the ***neighbors*** of the pixel in the previous row

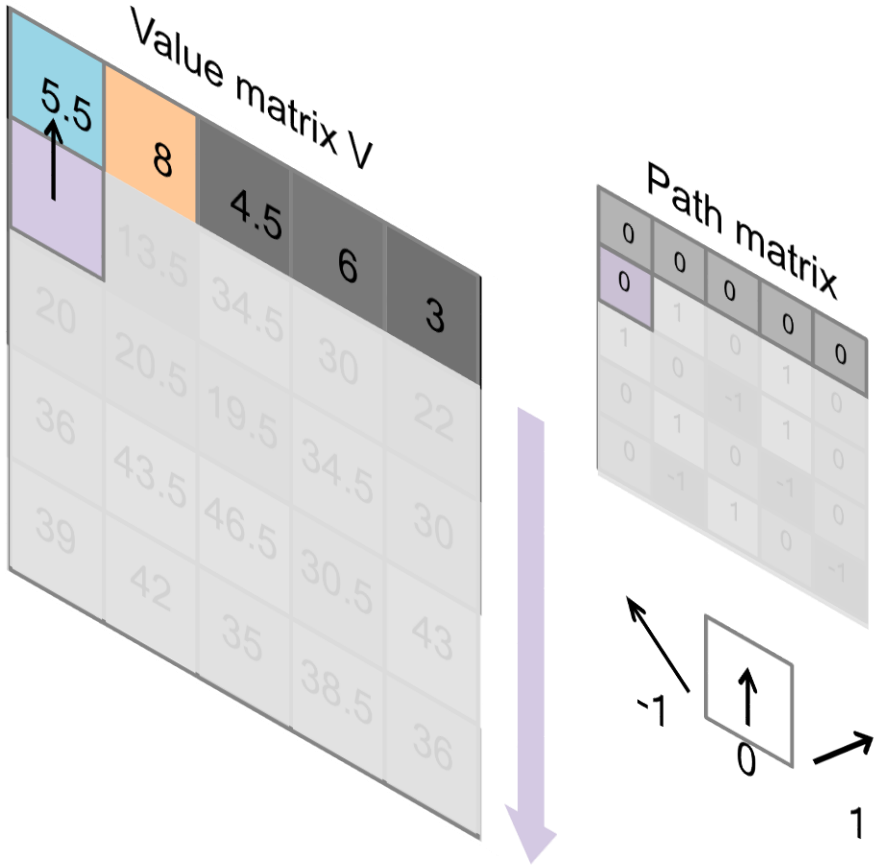
$$V(2,1) = \min \begin{pmatrix} V(1,1) \\ V(1,2) \end{pmatrix} \begin{matrix} 5.5 \\ 8 \end{matrix} \sqrt{} + e(2,1)$$

Value matrix V

5.5	8	4.5	6	3
	13.5	34.5	30	22
20	20.5	19.5	34.5	30
36	43.5	46.5	30.5	43
39	42	35	38.5	36

Step2: Propagation

- Start with 2nd row
- Find the ***neighbors*** of the pixel in the previous row
- Find the ***minimum*** among neighbors

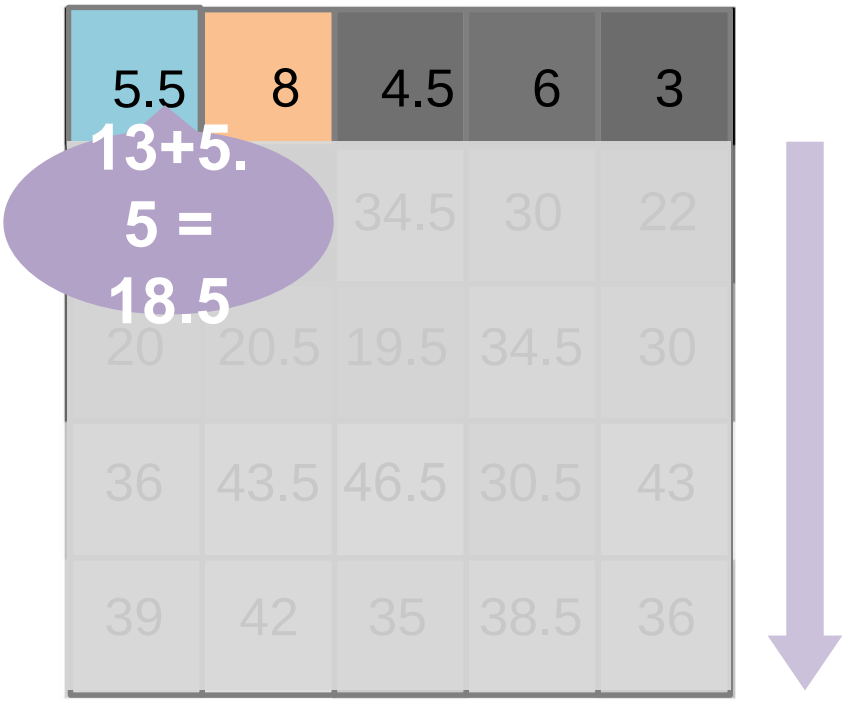


Step2: Propagation

- Start with 2nd row
- Find the ***neighbors*** of the pixel in the previous row
- Find the ***minimum*** among neighbors, record it in Path matrix

Energy matrix

Value matrix V



$$V(2,1) = \min \begin{pmatrix} V(1,1) \\ V(1,2) \end{pmatrix} + e(2,1)$$

$V(1,1)$ = 5.5 √

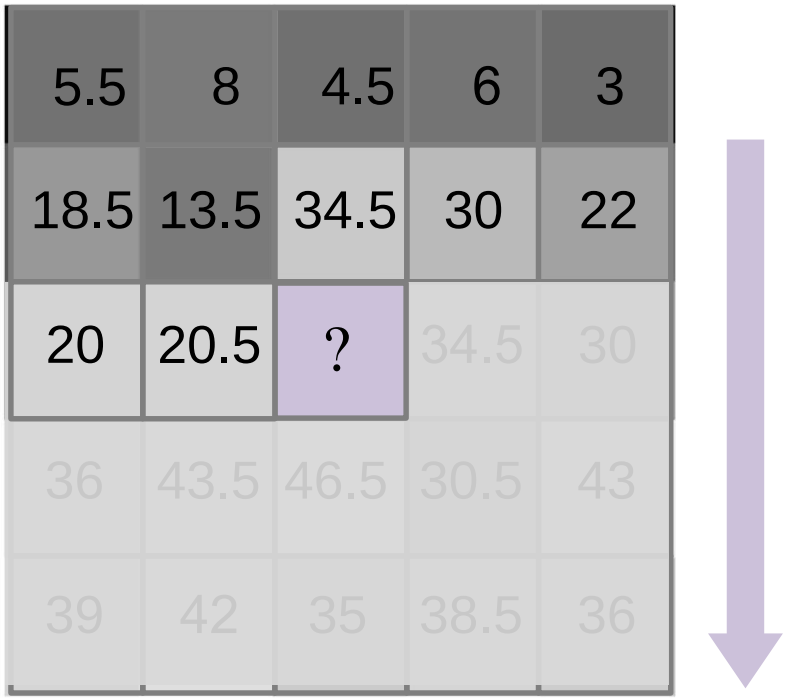
$V(1,2)$ = 8

$e(2,1)$ = 13

Step2: Propagation

- Start with 2nd row,
- Find the ***neighbors*** of the pixel in the previous row
- Find the ***minimum*** among neighbors
- Add the **energy value** of the pixel with the minimum

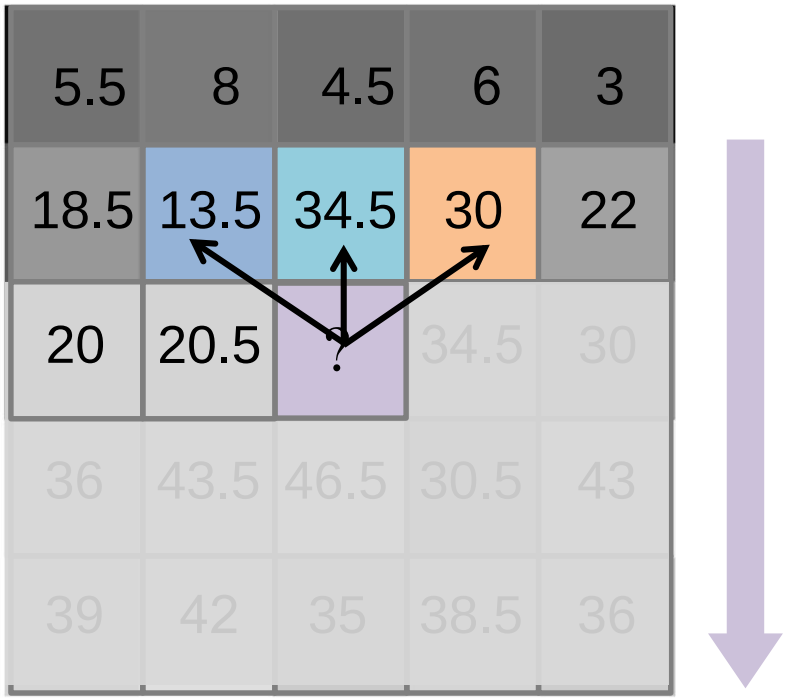
Value matrix V



Step2: Propagation for every row

Value matrix V

Neighborhoods of the pixel in previous row



Step2: Propagation for every row

- Find the ***neighbors*** of the pixel in the previous row

$$\min \begin{pmatrix} V(i-1,j-1) \\ V(i-1,j) \\ V(i-1,j+1) \end{pmatrix} + e(i,j)$$

13.5 ✓

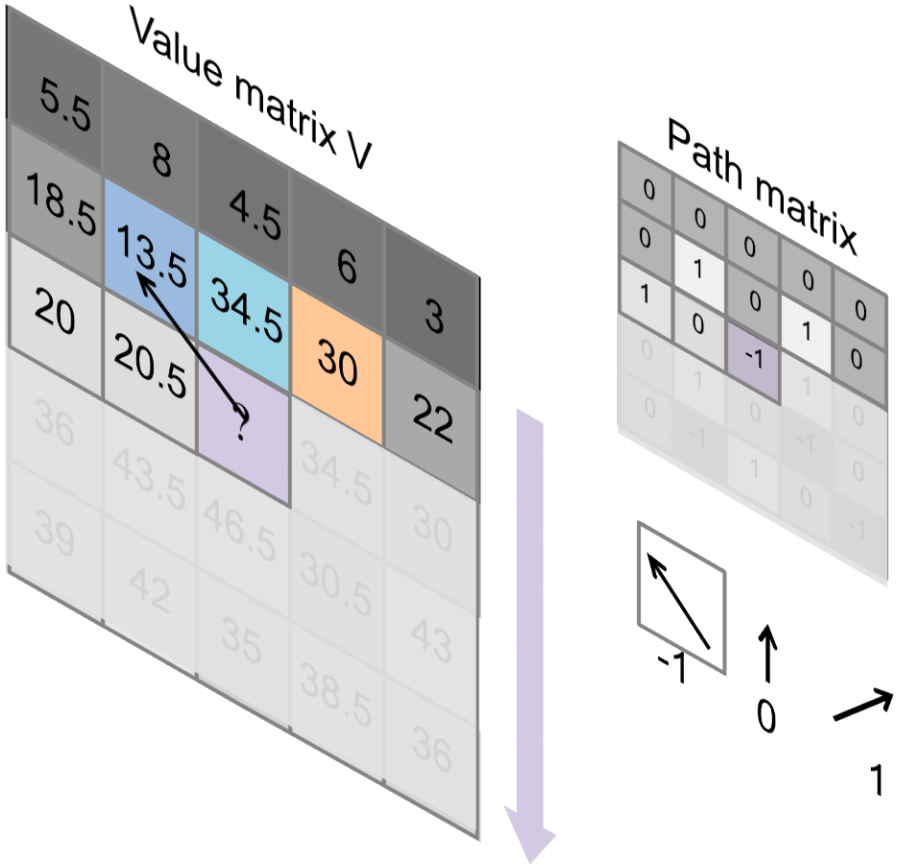
34.5

30

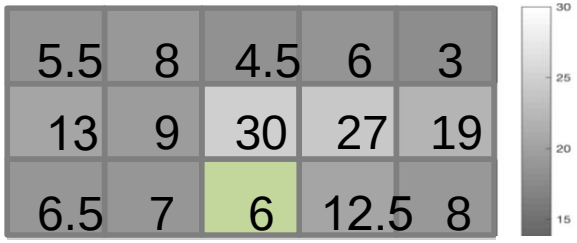
Value matrix V

5.5	8	4.5	6	3
18.5	13.5	34.5	30	22
20	20.5	?	34.5	30
36	43.5	46.5	30.5	43
39	42	35	38.5	36

- Find the ***neighbors*** of the pixel in the previous row
- Find the ***minimum*** among neighbors



- Find the ***neighbors*** of the pixel in the previous row
- Find the ***minimum*** among neighbors, record it in Path matrix



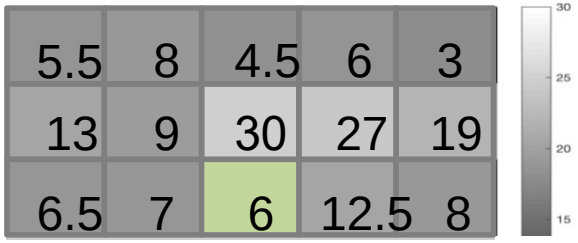
Value matrix V

5.5	8	4.5	6	3
18.5	13.5	34.5	30	22
20	20.5	?	34.5	30
36	43.5	46.5	30.5	43
39	42	35	38.5	36

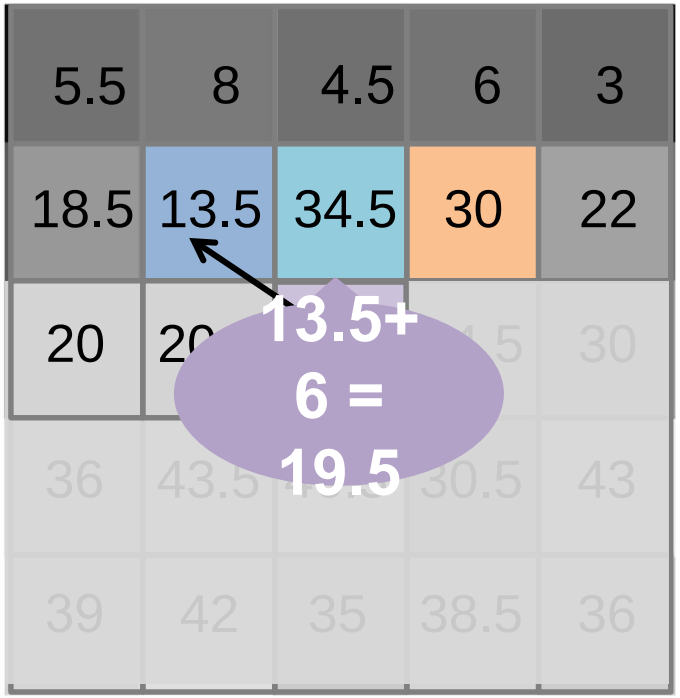
$$\min \begin{pmatrix} V(i-1,j-1) & 13.5 \\ V(i-1,j) & 34.5 \\ V(i-1,j+1) & 30 \end{pmatrix} + e(i,j) \quad 6$$

√

- Find the ***neighbors*** of the pixel in the previous row
- Find the ***minimum*** among neighbors
- Add the **energy value** of the pixel with the minimum



Value matrix V

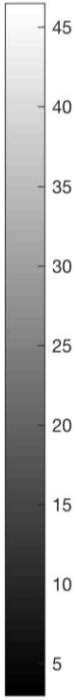


- Find the ***neighbors*** of the pixel in the previous row
- Get the ***minimum*** among neighbors
- Add the **energy value** of the pixel with the minimum

Step3: path resolving

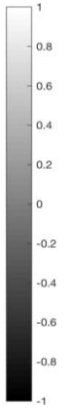
Value matrix

5.5	8	4.5	6	3
18.5	13.5	34.5	30	22
20	20.5	19.5	34.5	30
36	43.5	46.5	30.5	43
39	42	35	38.5	36

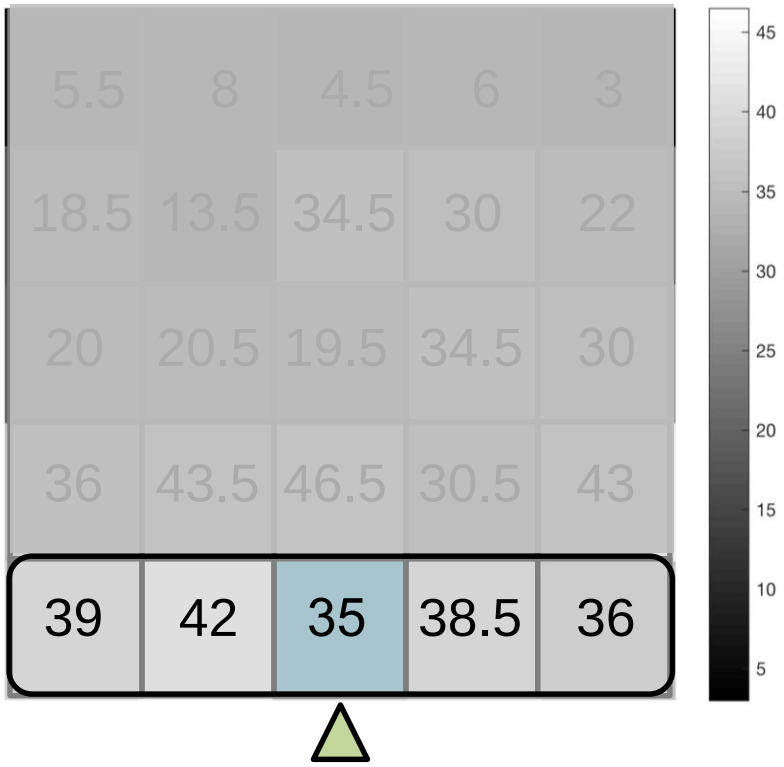


Path matrix

0	0	0	0	0
0	1	0	1	0
1	0	-1	1	0
0	1	0	-1	0
0	-1	1	0	-1

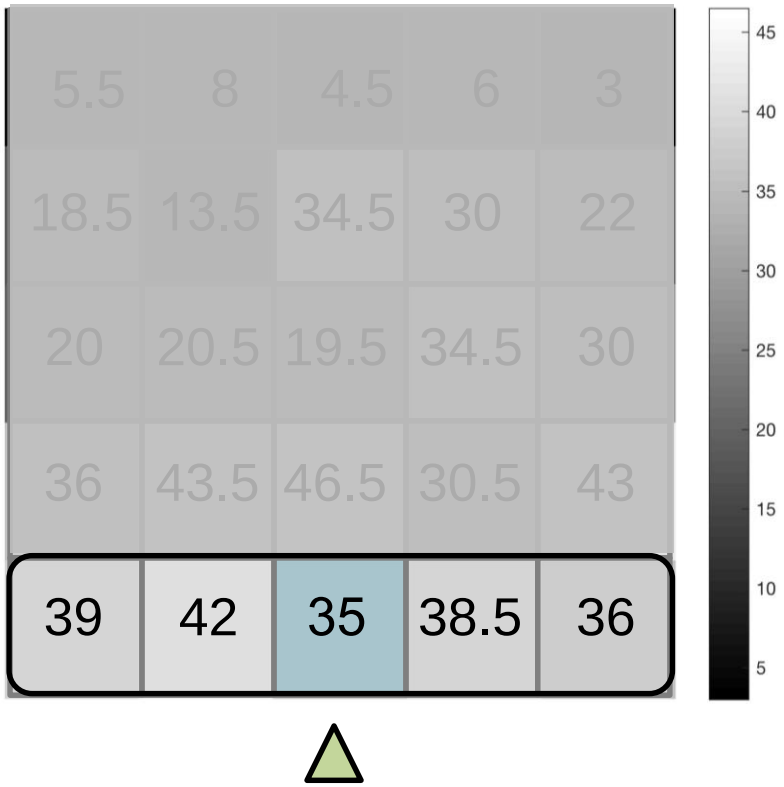


Value matrix

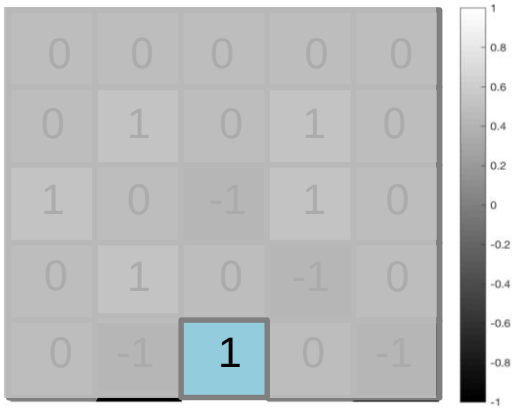


- Find the ***minimum*** of the last row of the Value matrix,
- Find the ***predecessor*** of that pixel

Value matrix

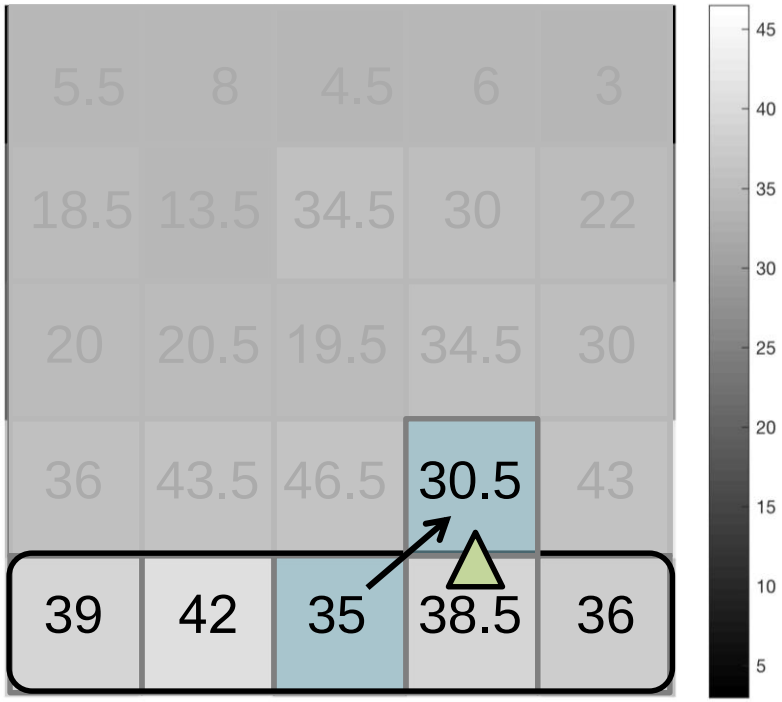


Path matrix

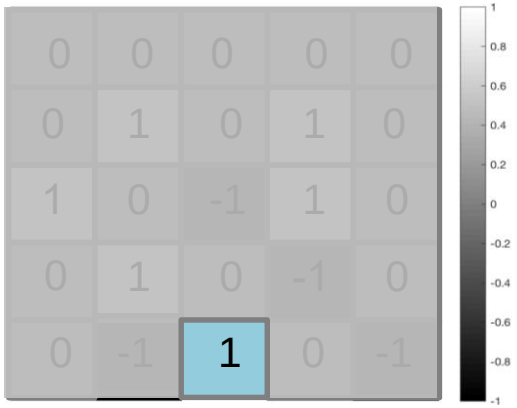


- Find the ***minimum*** of the last row of the Value matrix,
- Find the ***predecessor*** of that pixel, using Path matrix

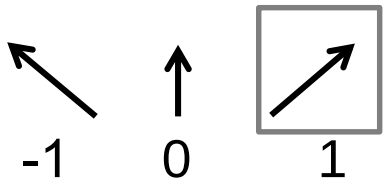
Value matrix



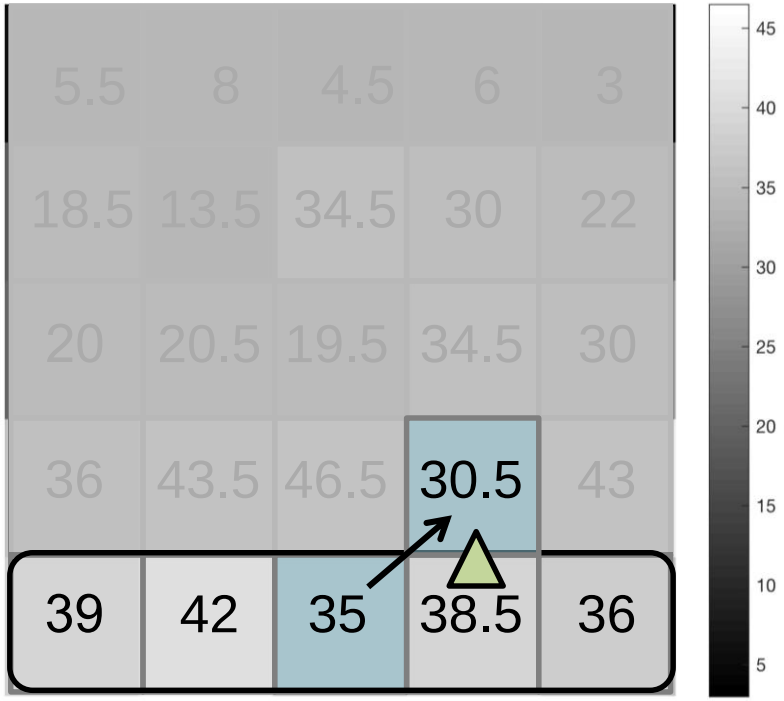
Path matrix



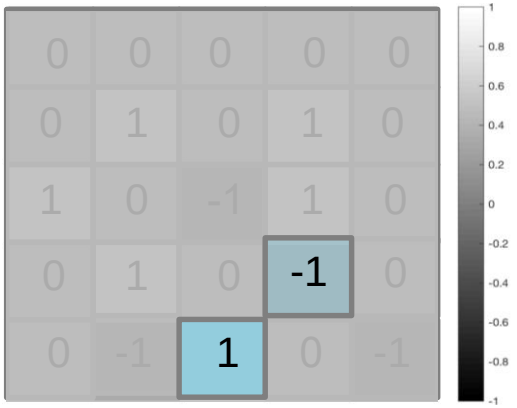
- Move to its ***predecessor***



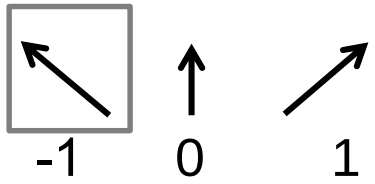
Value matrix



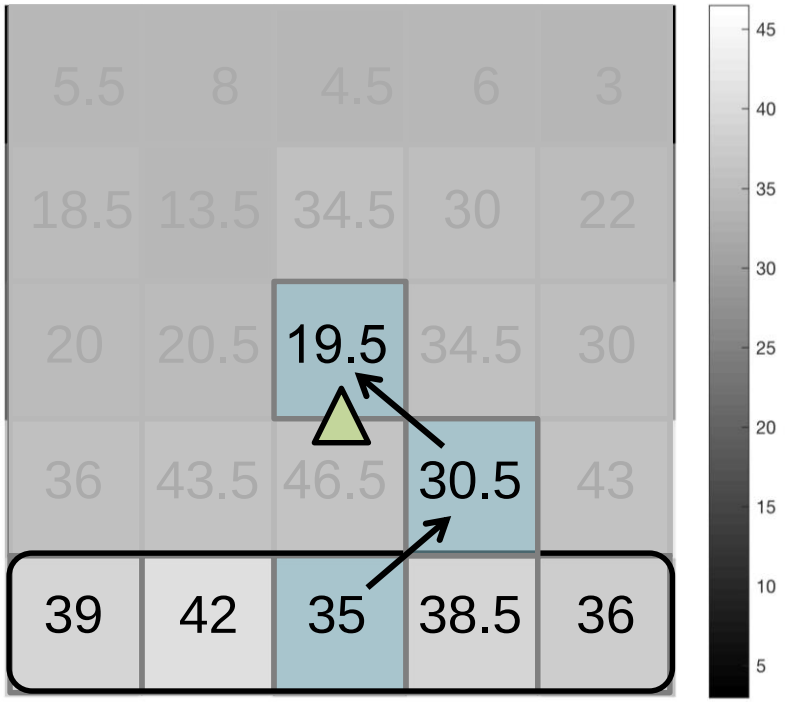
Path matrix



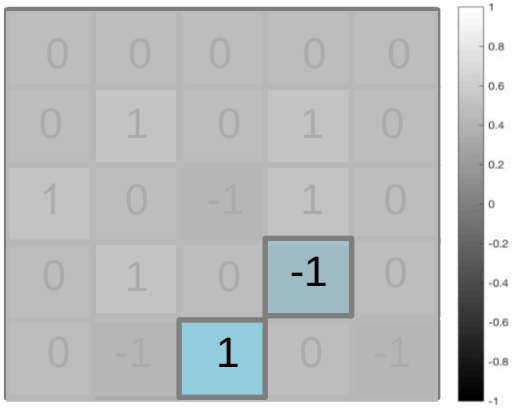
- Follow ***predecessor*** of current pixel,



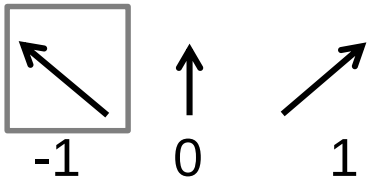
Value matrix



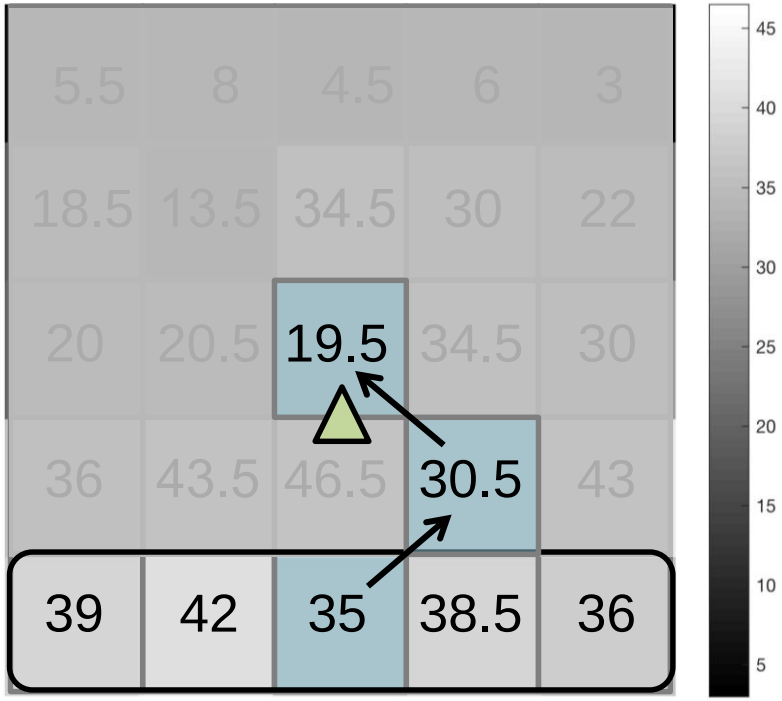
Path matrix



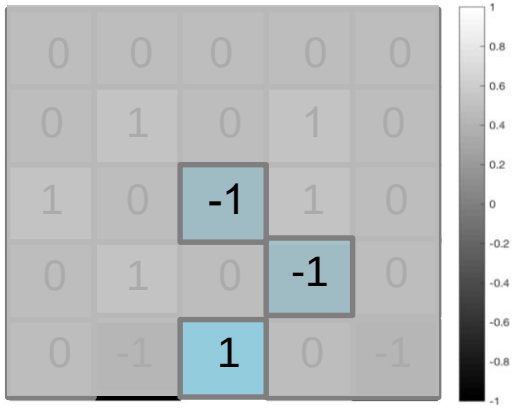
- Move to its ***predecessor***



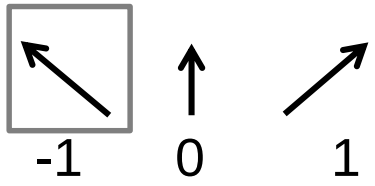
Value matrix



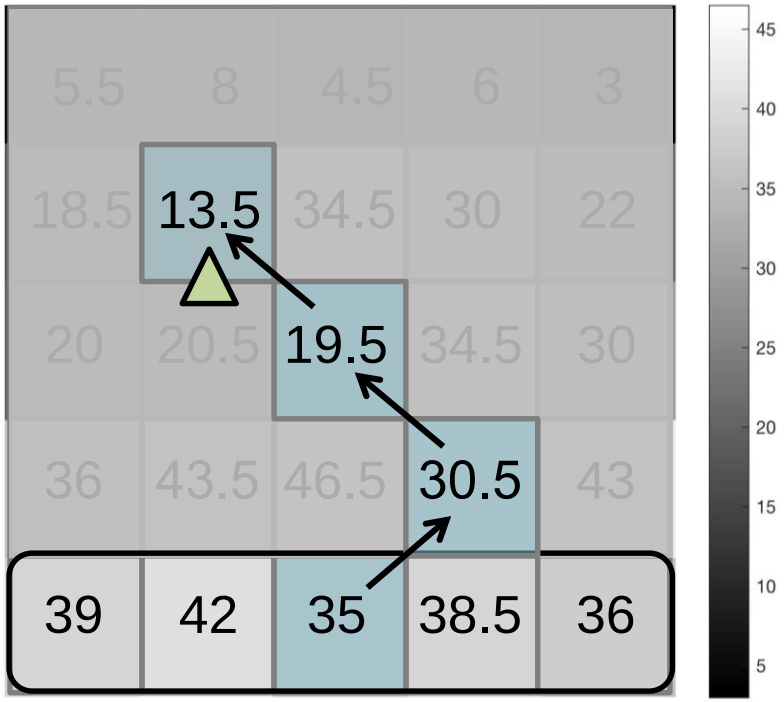
Path matrix



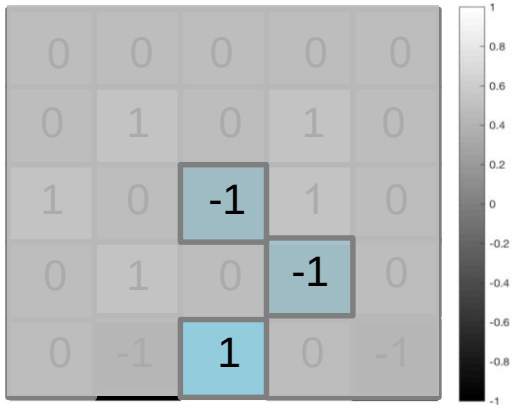
- Find the ***predecessor*** of current pixel



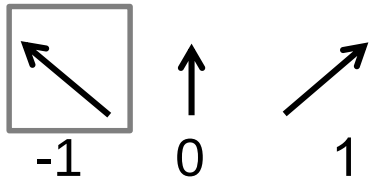
Value matrix



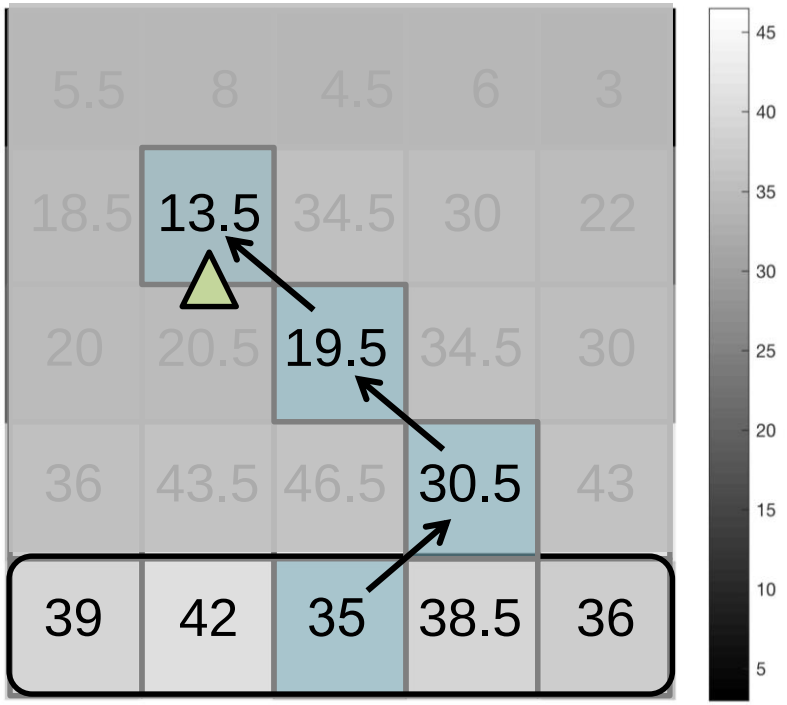
Path matrix



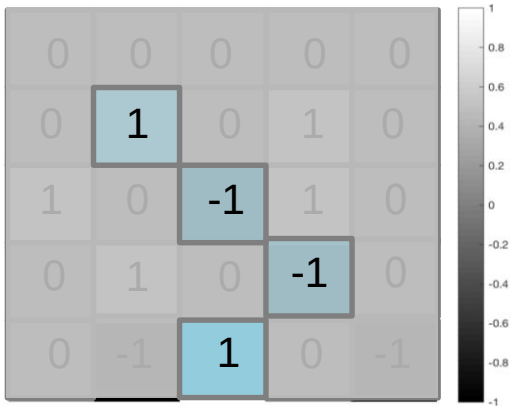
- Move to its ***predecessor***



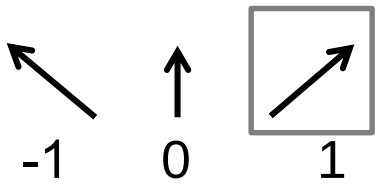
Value matrix



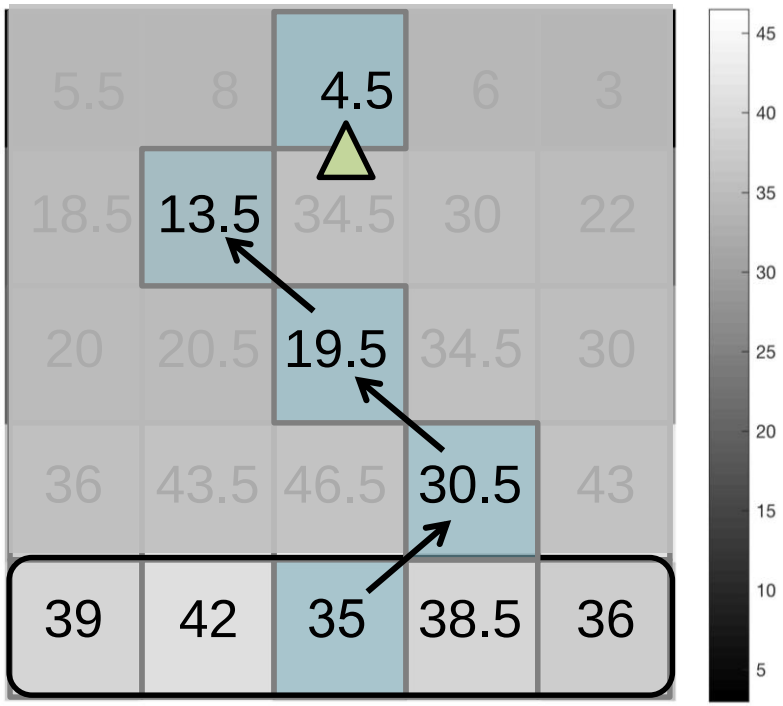
Path matrix



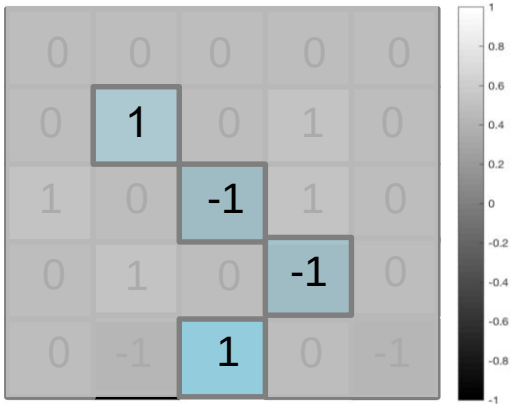
- Find the ***predecessor*** of current pixel



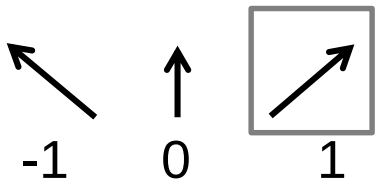
Value matrix



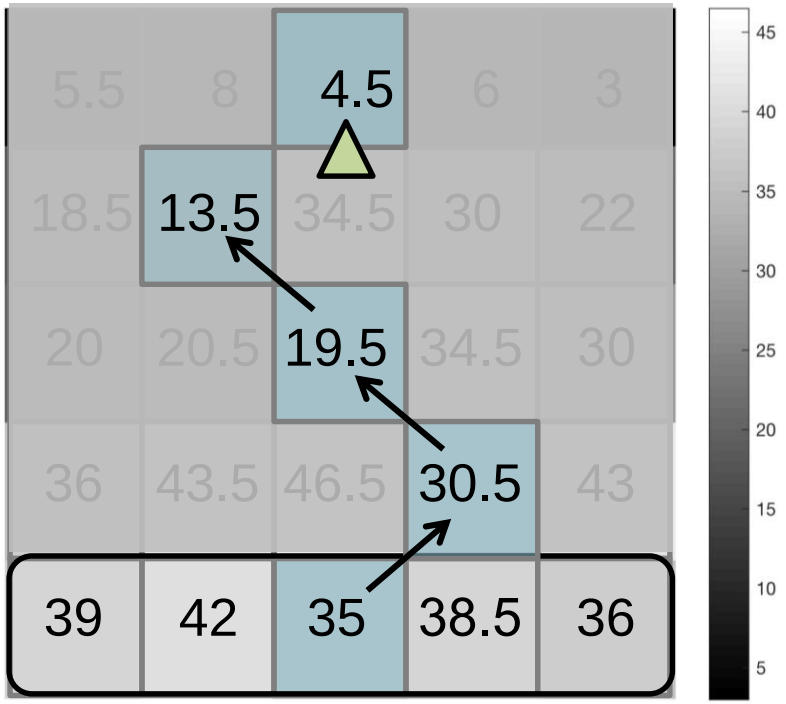
Path matrix



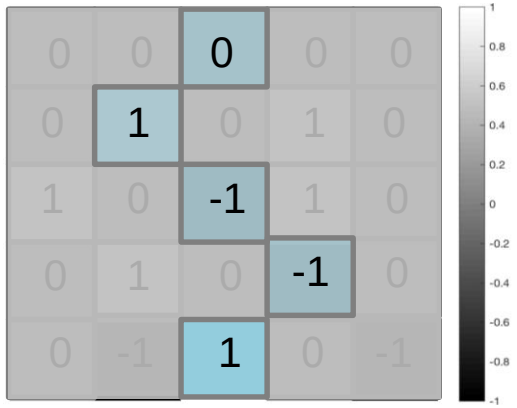
- Stop when reaching the ***first row***



Value matrix

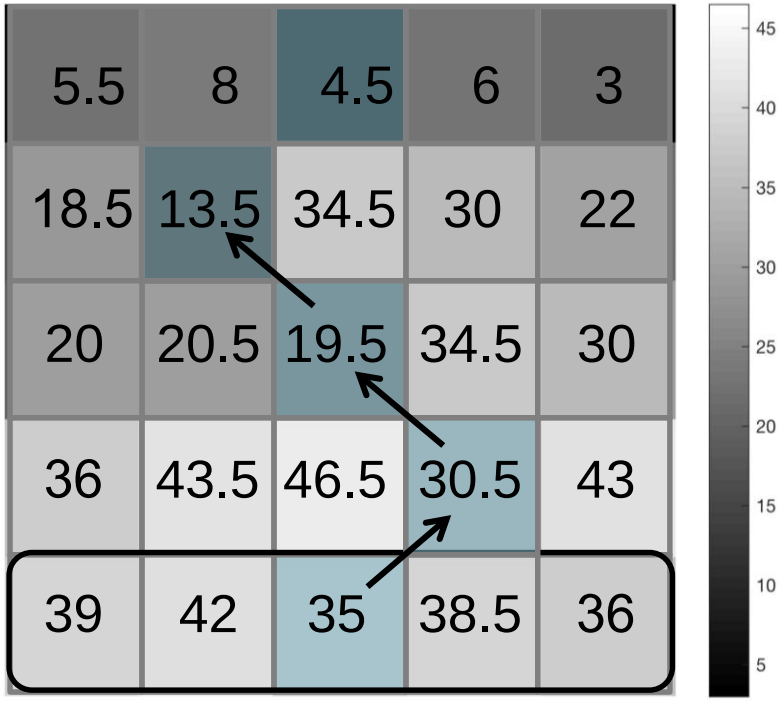


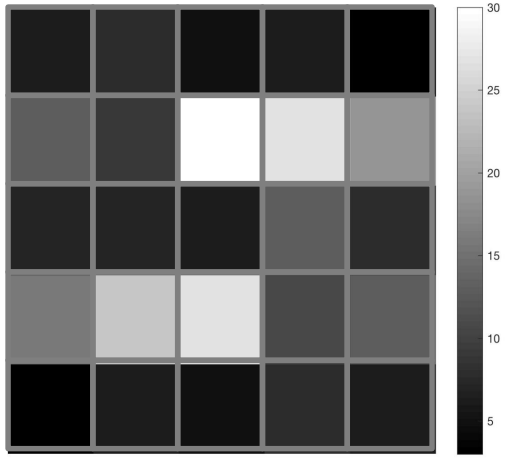
Path matrix



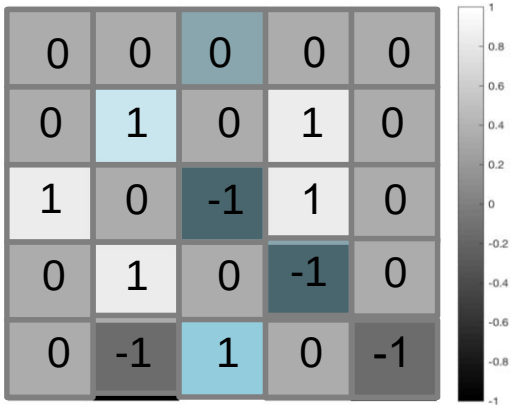
- Seam carving is to delete the path with minimum cost

Value matrix



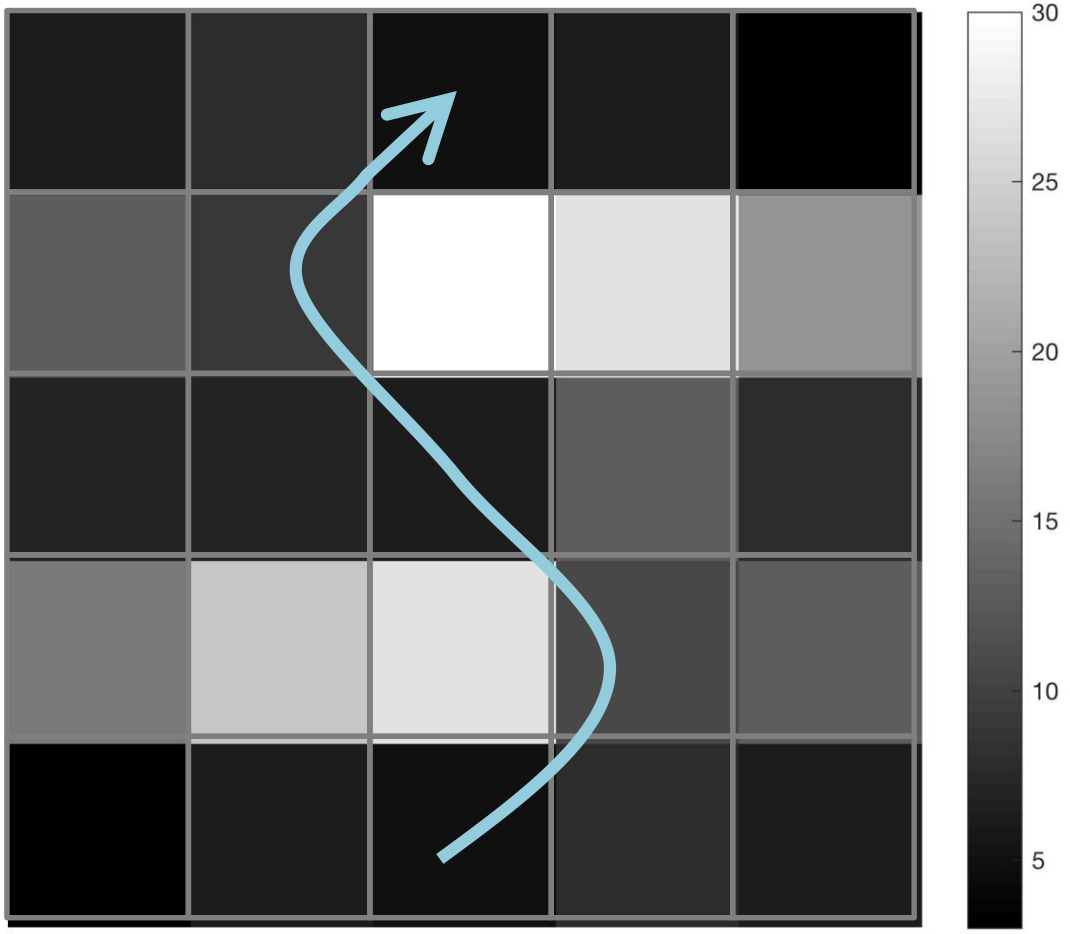


Path matrix



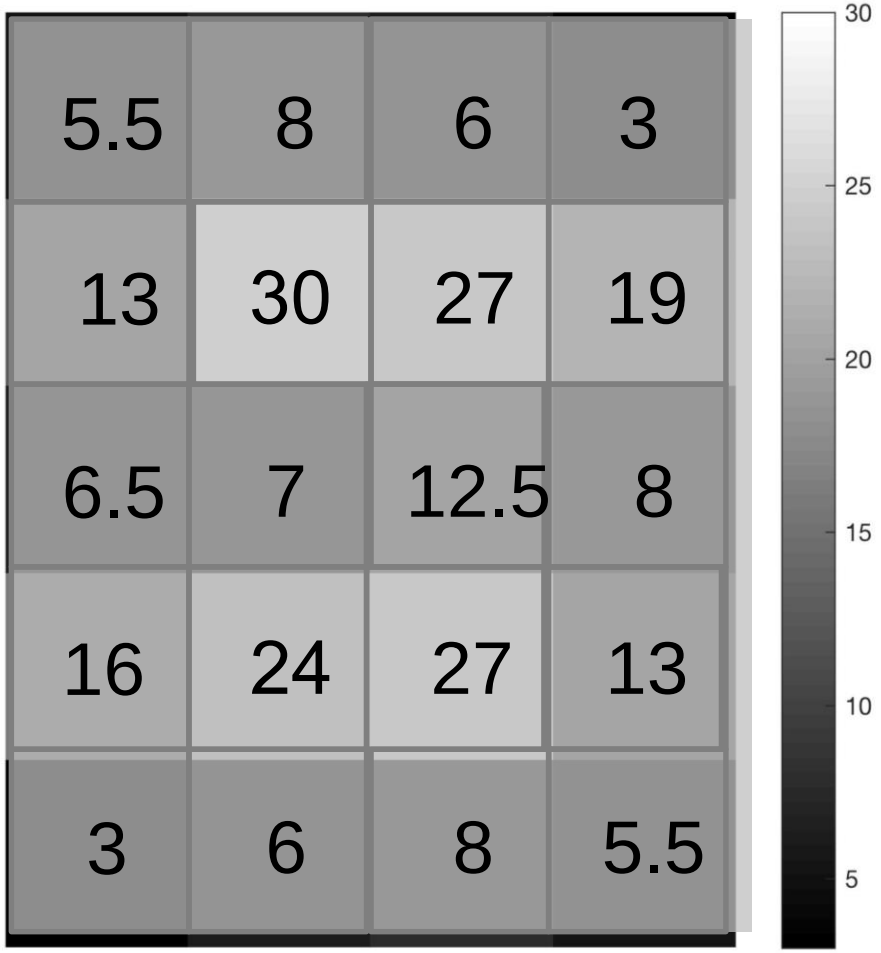
- Seam carving is to delete the path with minimum cost

Energy matrix

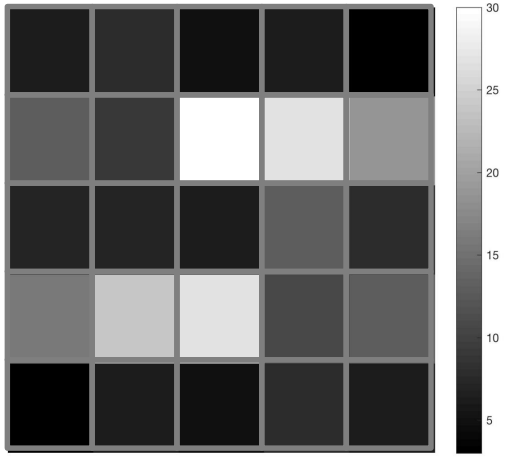


- Seam carving is to delete the path with minimum cost

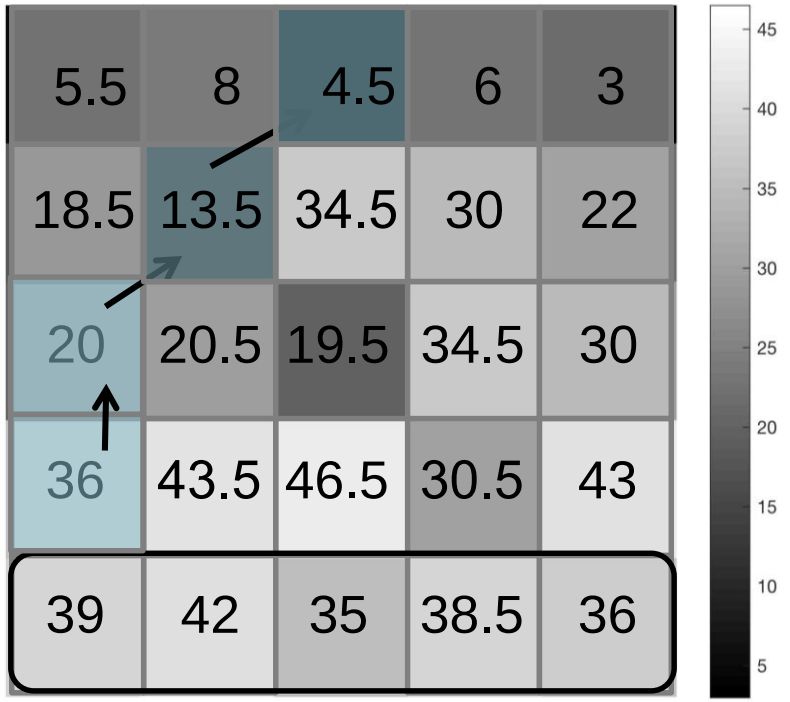
Carved energy matrix



- Seam carving is to delete the path with minimum cost



Value matrix



Path matrix

0	0	0	0	0
0	1	0	1	0
1	0	-1	1	0
0	1	0	-1	0
0	-1	1	0	-1

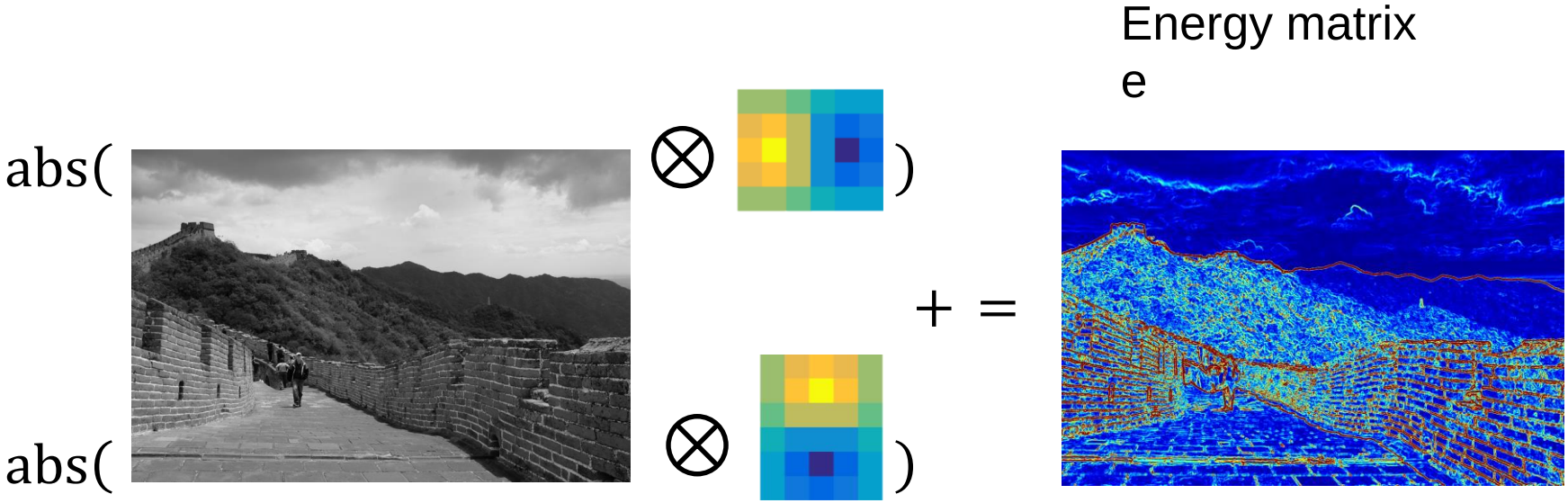
- What if we want to remove a seam that ends on particular pixel?

Video 9.5
Jianbo Shi

Illustration on a real image

Step 0: prepare the energy function

- Transform a color image into gray, Im = rgb2gray(Im)
- Calculate image gradients
- Use the L1 norm of the gradients for the energy function



Step 0: prepare the energy function

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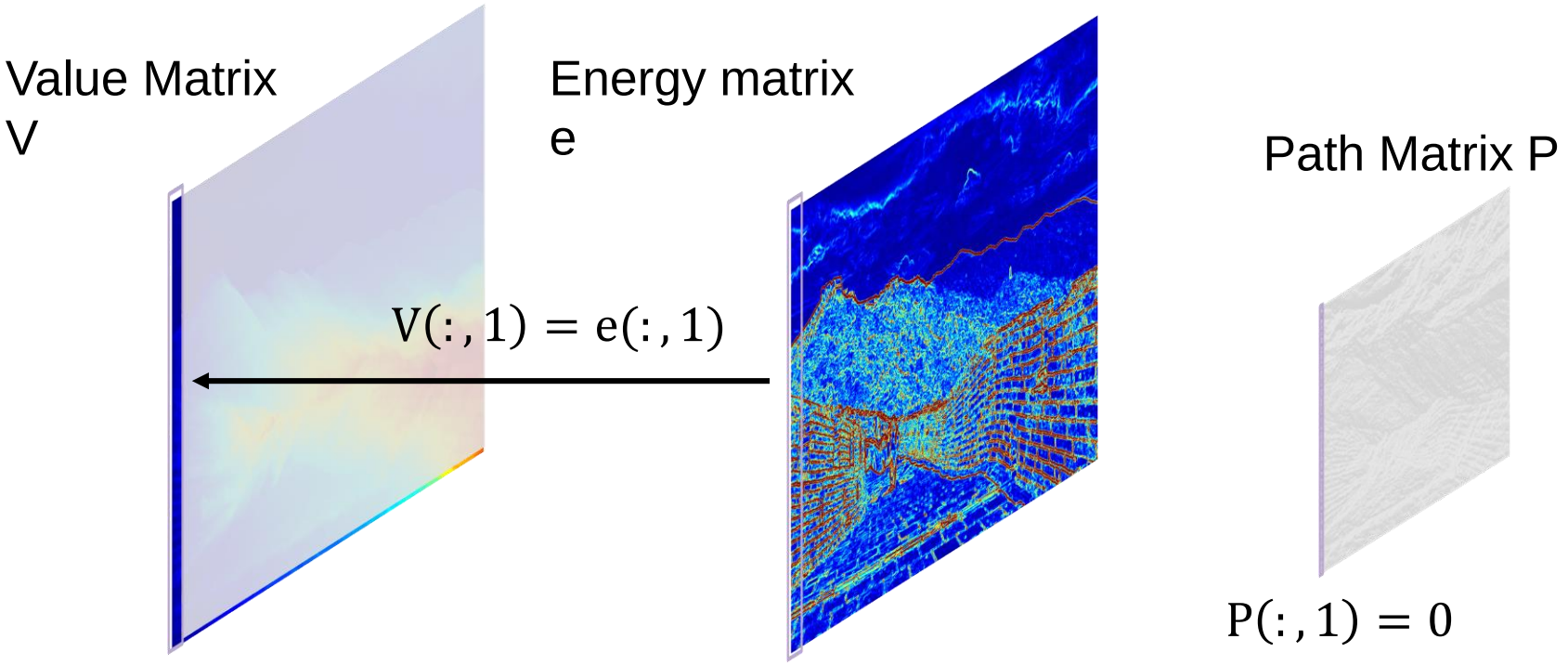
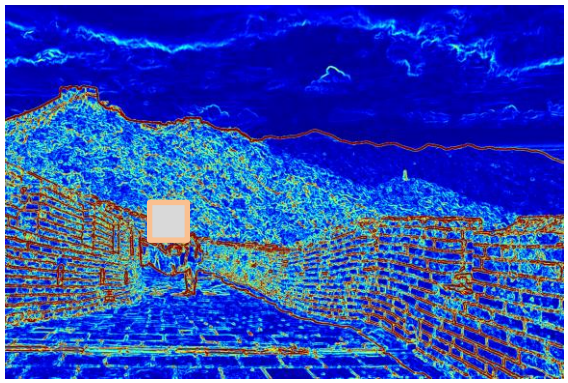


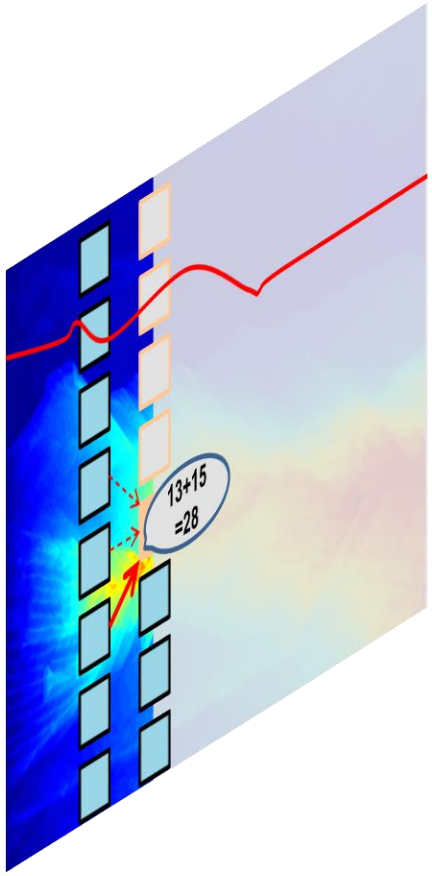
Step 1: Initializing two matrices

- Set value and path matrix the same size as the energy matrix
- Initialize the ***first column*** of value matrix with that of the energy matrix
- Initialize the ***first column*** of path matrix to zero

$$\min \begin{pmatrix} V(i-1,j-1) \\ \mathrm{V}\,(i,j-1) \\ \mathrm{V}(i+1,j-1) \end{pmatrix} + \mathrm{e}(i,j)$$

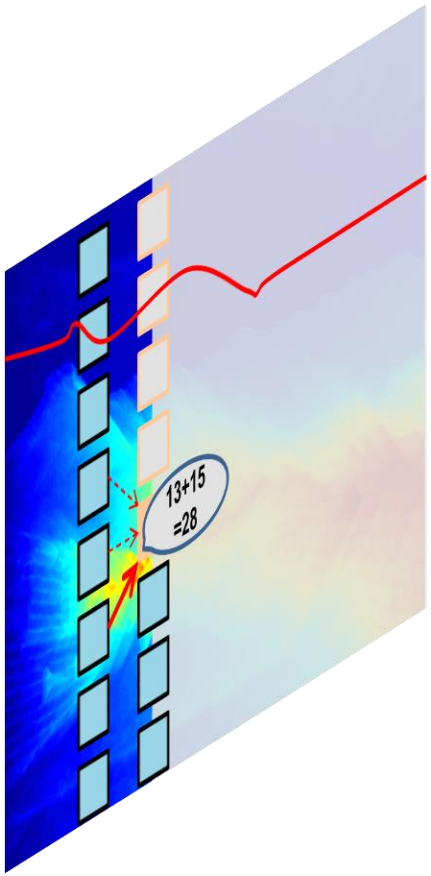
38
24
13 √
15

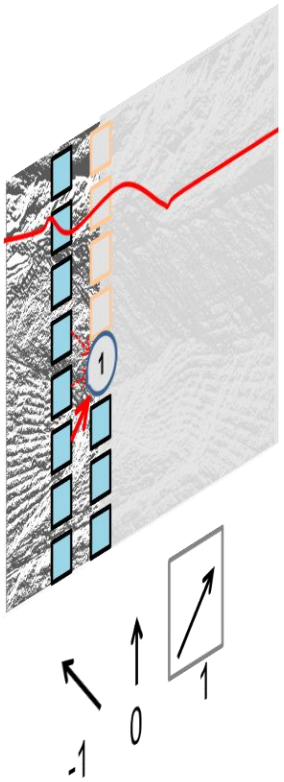




Step 2: Propagation

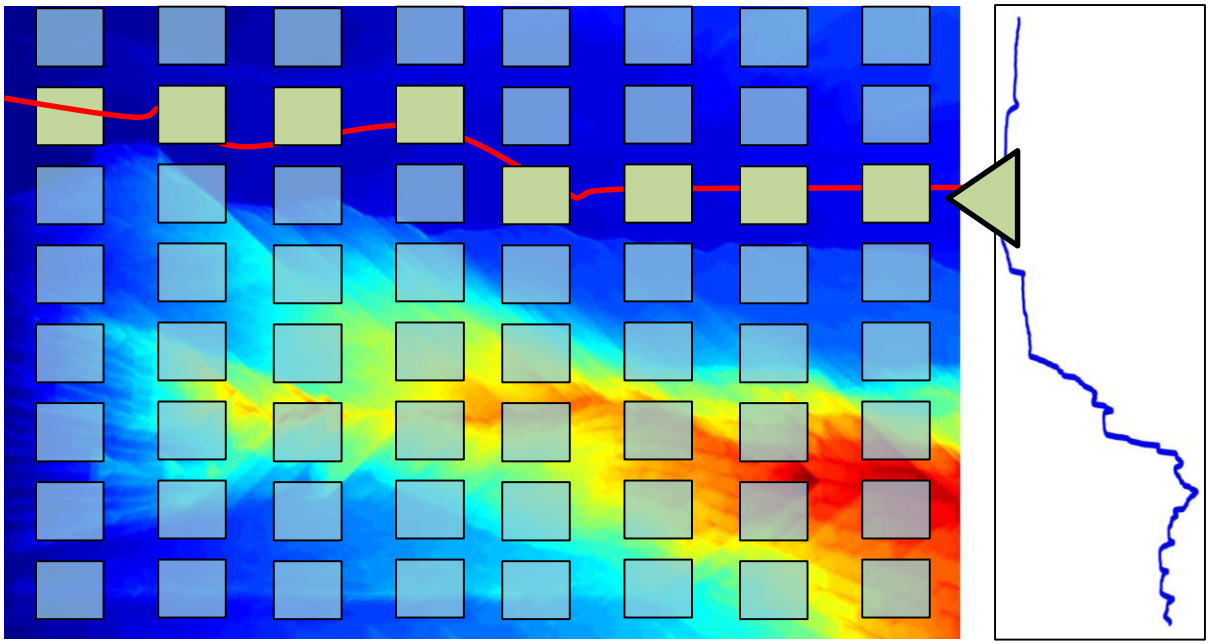
- Start with 2nd column
- Find the ***neighbors*** of the pixel in the previous column
- Find the ***minimum*** among neighbors
- Add the **energy value** of the pixel with the minimum

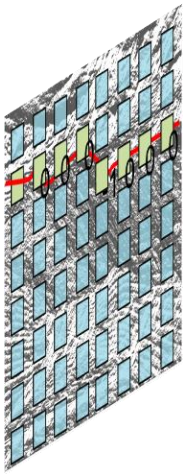




Step 2: Propagation

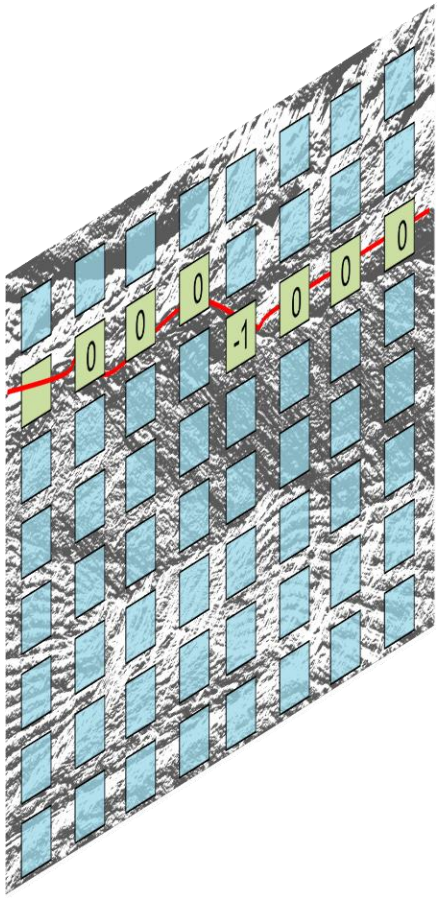
- Start with 2nd column
- Find the ***neighbors*** of the pixel in the previous column
- Find the ***minimum*** among neighbors
- Add the **energy value** of the pixel with the minimum
- Assign the ***direction*** of the minimum to path matrix





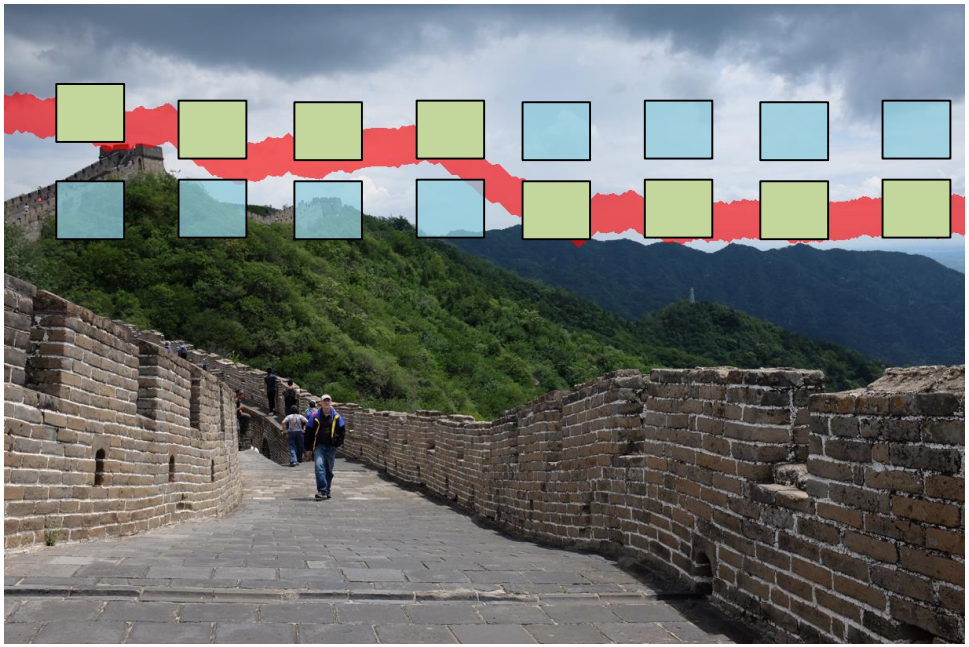
Step 3: Path Resolving

- Find the ***minimum*** of the last column of the Value matrix,
- Find the ***predecessor*** of that pixel, using Path matrix
- Trace back to complete the path using the Path matrix



Step3: Path Resolving

- Find the ***minimum*** of the last column of the Value matrix,
- Find the ***predecessor*** of that pixel, using Path matrix
- Trace back to complete the path using the Path matrix



Step 4: Eliminate the Seam