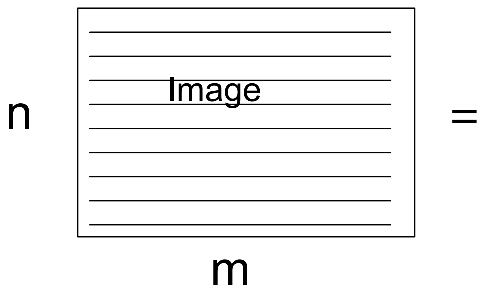
Penn Engineering

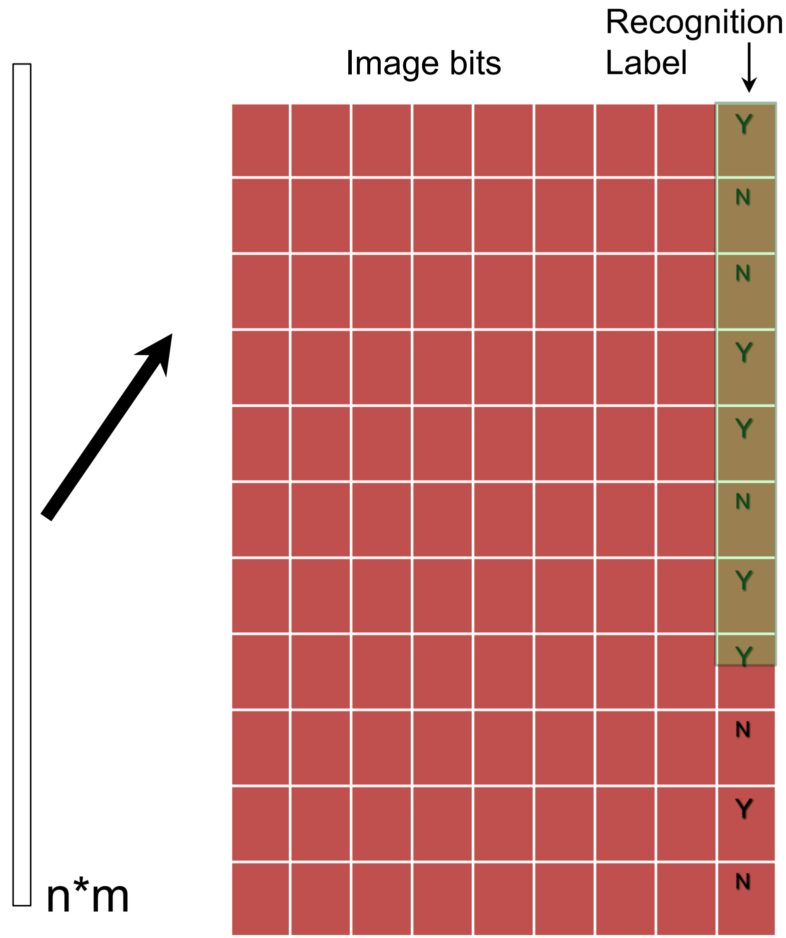
ONLINE LEARNING

Video 12.1

Jianbo Shi

Recognition as a big table lookup





write down all images
in a table and record
the object label

“How many images are there?”

An tiny image example (can you see what it is?)

3

18

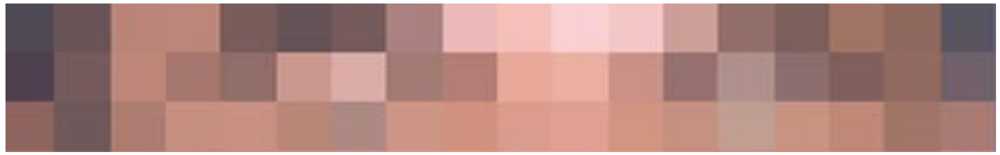
Each pixel has $2^8 = 256$ values

3x18 image above has 54 pixels

Total possible 54 pixel images = $256^{54} = 1.1 \times 10^{130}$

Prof. Kanade’s Theorem: we have not seen anything yet!

3



18

Total possible 54 pixel images $= 1.1 \times 10^{130}$

Compared

number of images seen by all humans ever:

10 billion x 1000 x 100 x 356 x 24 x 60 x 60 x 30 $= 10^{24}$

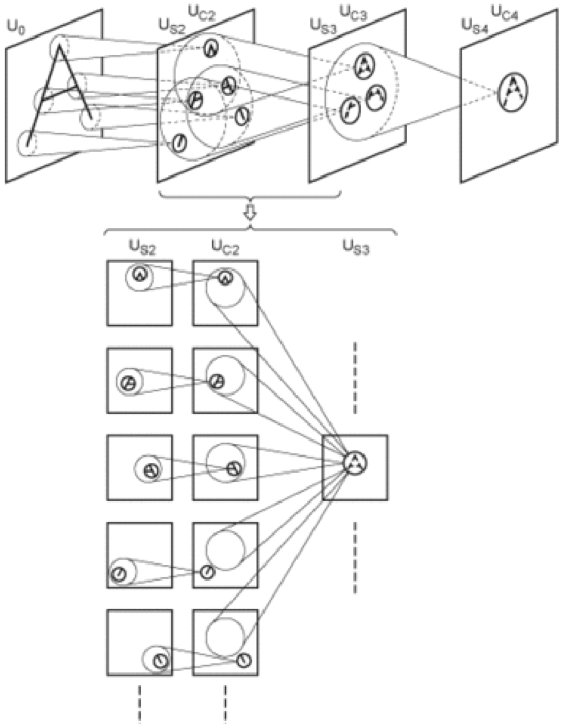
Total population years hours frame rate

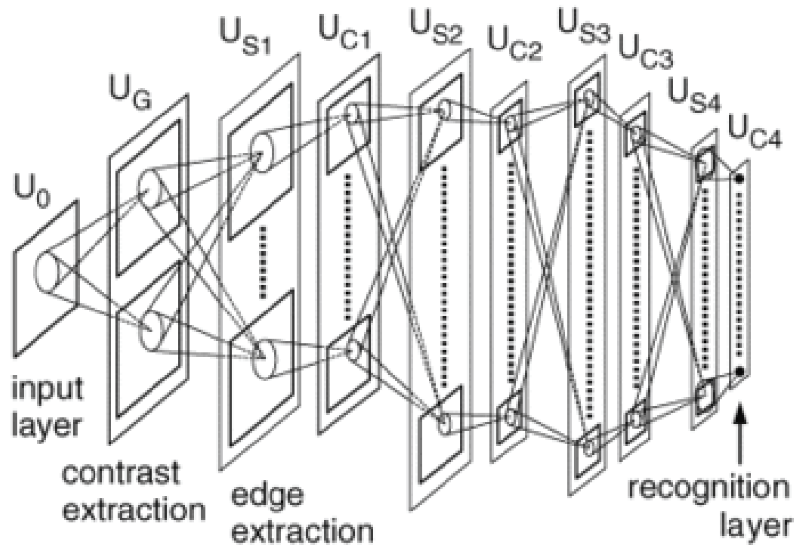
generations days min/sec

We have to be clever in writing down this table!

Earliest "deep" architecture

Neocognitron





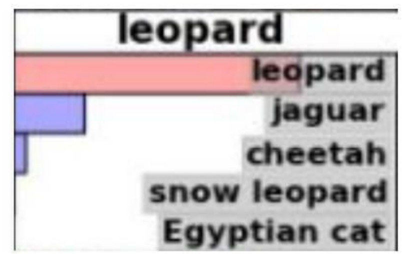
(Fukushima 1974-1982)

Goal: Given an image, we want to identify what class that image belongs to.

Input:

Classification

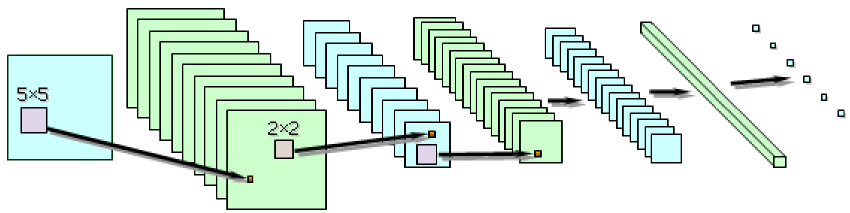
Output:



Pipeline:

Input

Convolutional Neural Network (CNN)

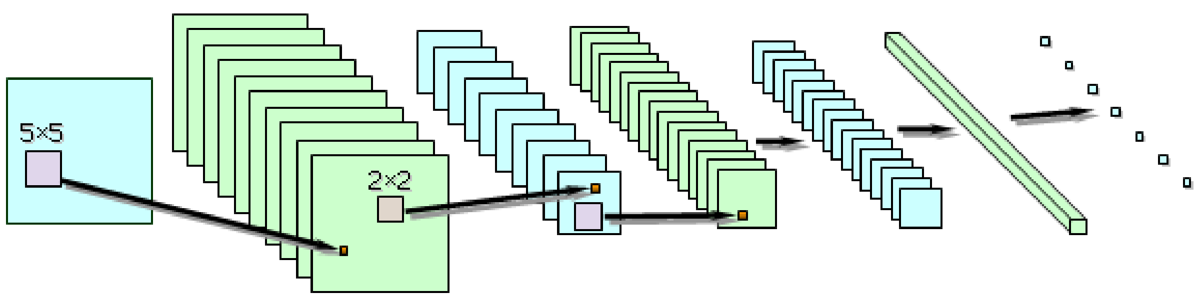


Output

A Monitor

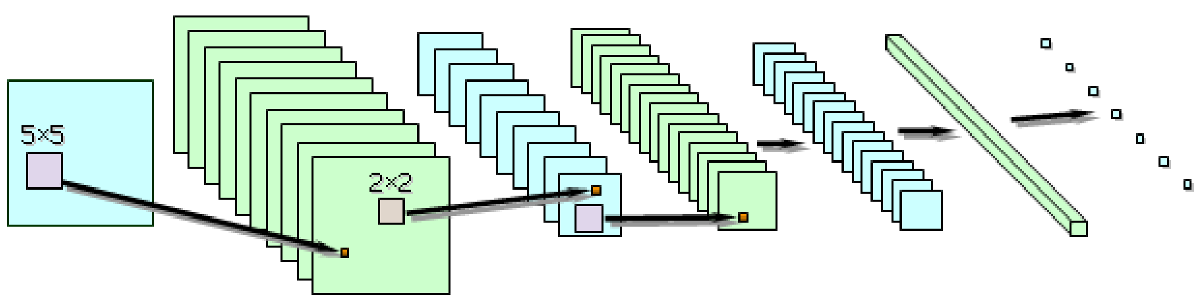
Convolutional Neural Nets (CNNs) in a nutshell:

- A typical CNN takes a raw RGB image as an input.
- It then applies a series of non-linear operations on top of each other.
- These include convolution, sigmoid, matrix multiplication, and pooling (subsampling) operations.
- The output of a CNN is a highly non-linear function of the raw RGB image pixels.



How the key operations are encoded in standard CNNs:

- Convolutional Layers: 2D Convolution
- Fully Connected Layers: Matrix Multiplication
- Sigmoid Layers: Sigmoid function
- Pooling Layers: Subsampling



2D convolution:

$$h = f \otimes g$$

f - the values in a 2D grid that we want to convolve

g - convolutional weights of size MxN

$$h_{ij} = \sum_{m=0}^{M} \sum_{n=0}^{N} f(i-m, j-n) g(m,n)$$

A sliding window operation across the entire grid f .

$f =$ 

$g_1 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

$g_2 = \begin{bmatrix} 0.107 & 0.113 & 0.107 \\ 0.113 & 0.119 & 0.113 \\ 0.107 & 0.113 & 0.107 \end{bmatrix}$

$g_3 = \begin{bmatrix} -1 & 0 & 1 \\ -2 & 0 & 2 \\ -1 & 0 & 1 \end{bmatrix}$

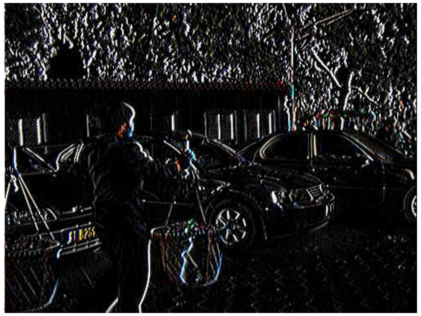
$f \otimes g_1$

Unchanged Image

$f \otimes g_2$

Blurred Image

$f \otimes g_3$



Vertical Edges

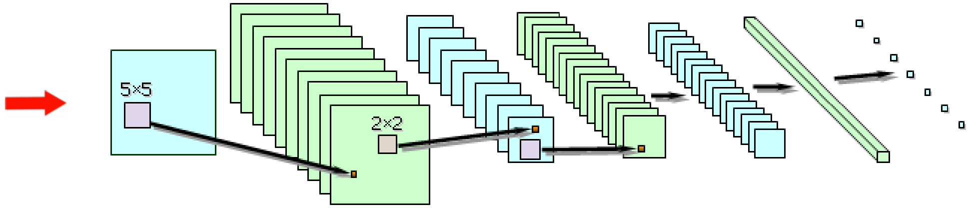
$f =$ 

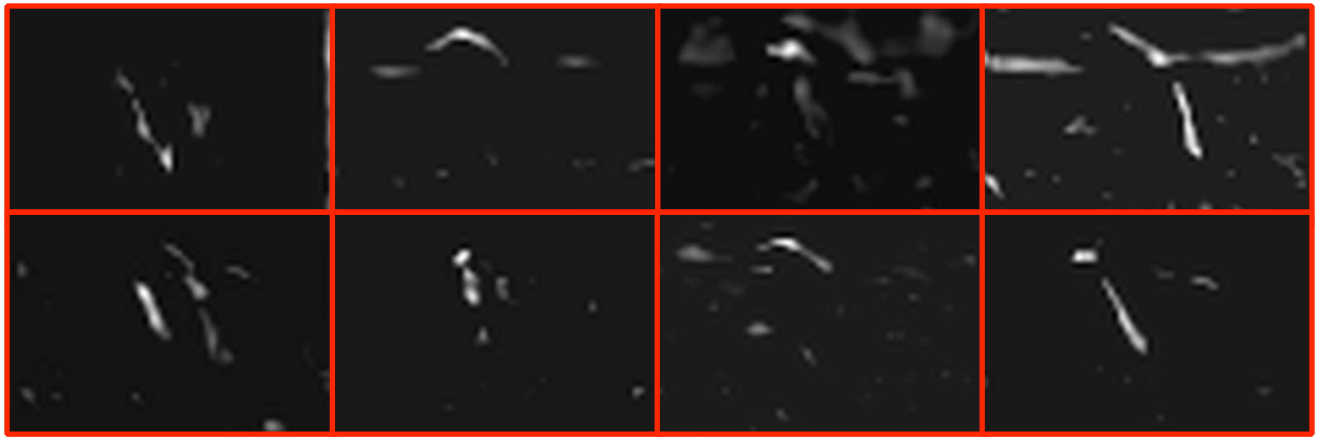
$$g_1 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} \quad g_2 = \begin{bmatrix} 0.107 & 0.113 & 0.107 \\ 0.113 & 0.119 & 0.113 \\ 0.107 & 0.113 & 0.107 \end{bmatrix} \quad g_3 = \begin{bmatrix} -1 & 0 & 1 \\ -2 & 0 & 2 \\ -1 & 0 & 1 \end{bmatrix}$$

CNNs aim to learn convolutional weights directly from the data

Input:

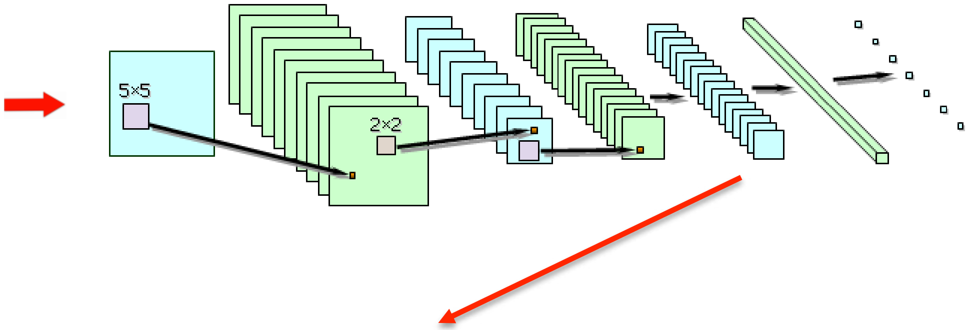
Convolutional Neural Network (CNN)

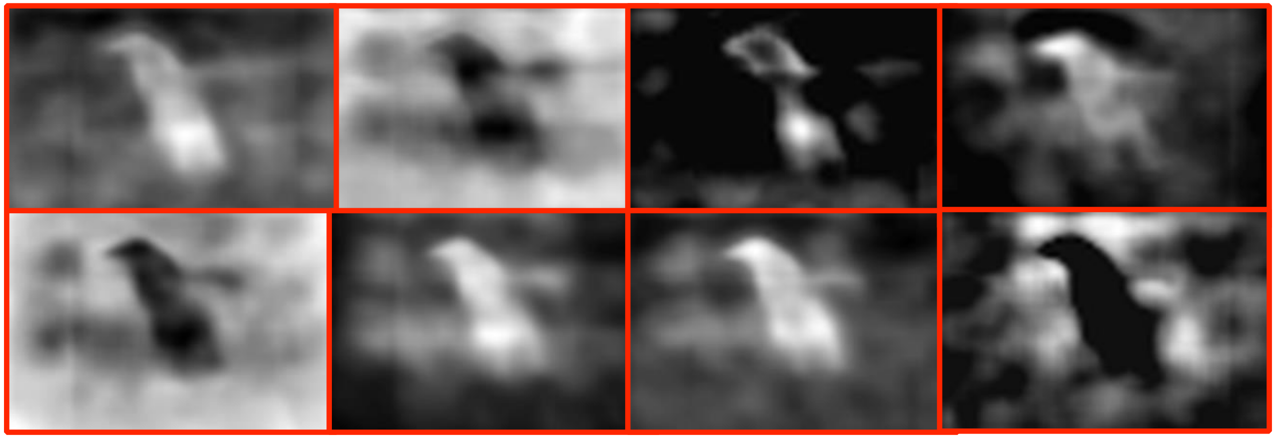




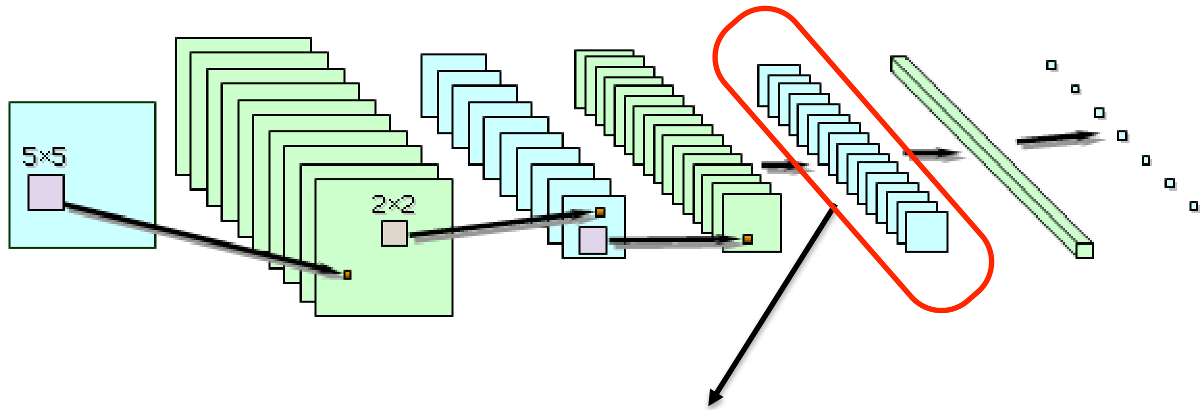
Early layers learn to detect low level structures such as oriented edges, colors and corners

Input: **Convolutional Neural Network (CNN)**

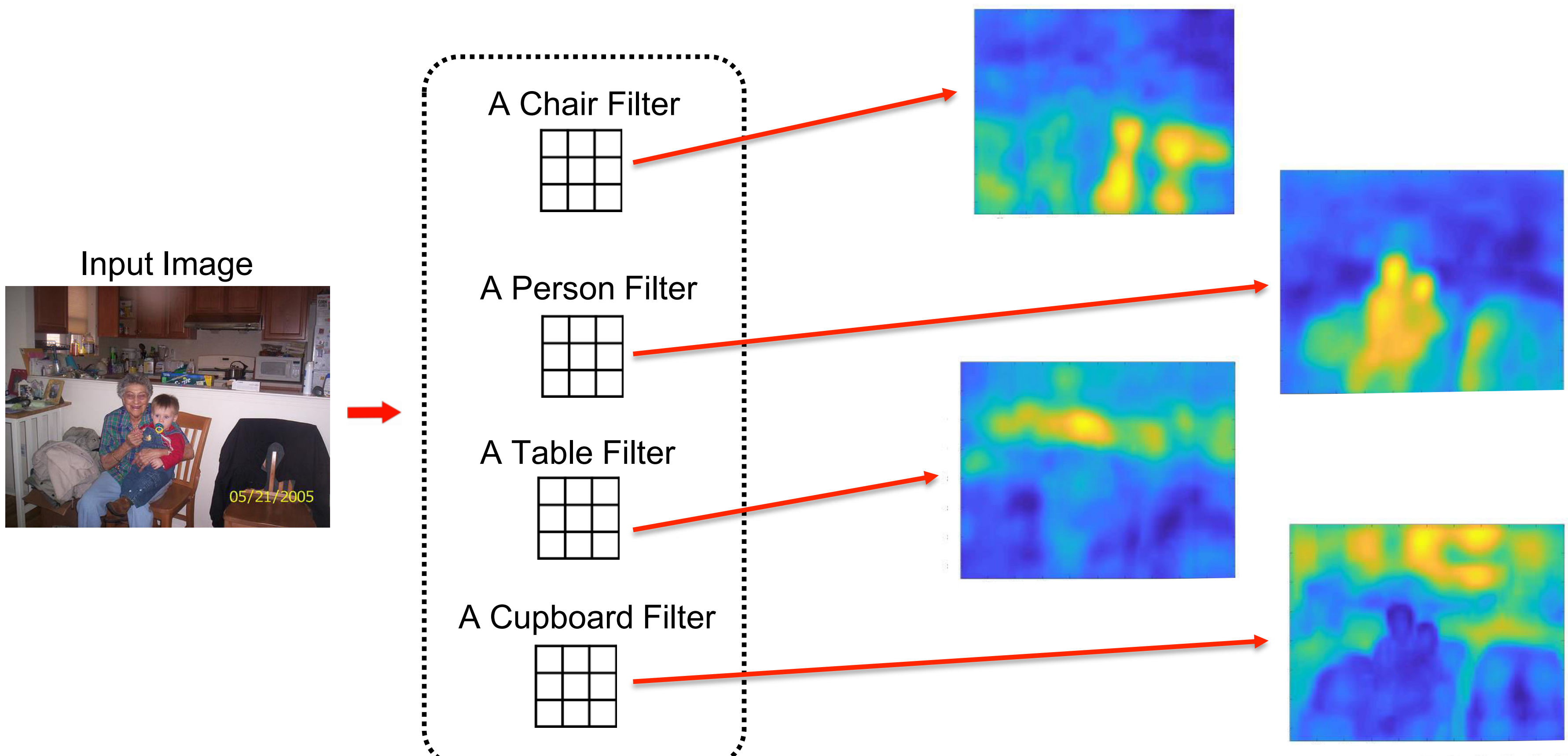




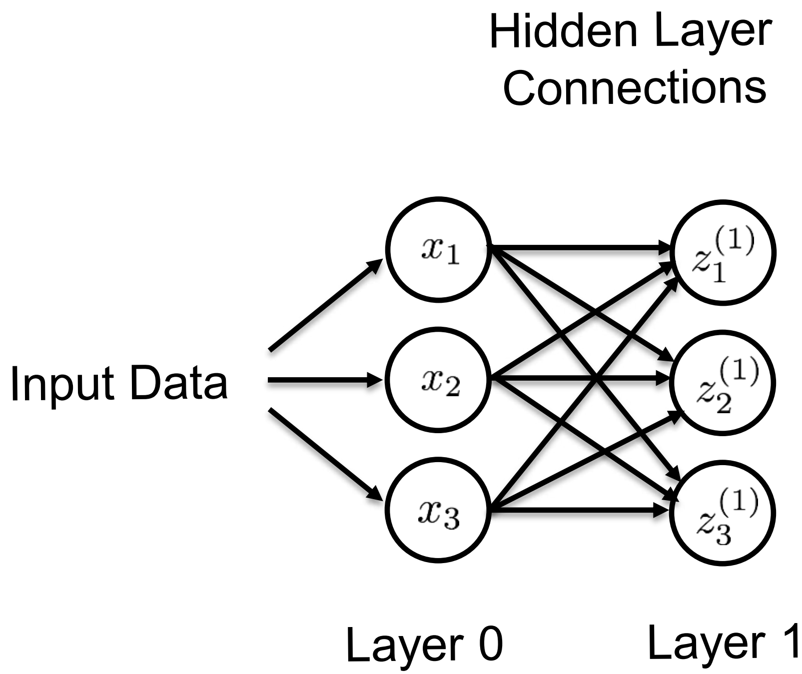
Deep layers learn to detect high-level object structures and their parts.



A Closer Look inside the Convolutional Layer



Fully Connected Layers:

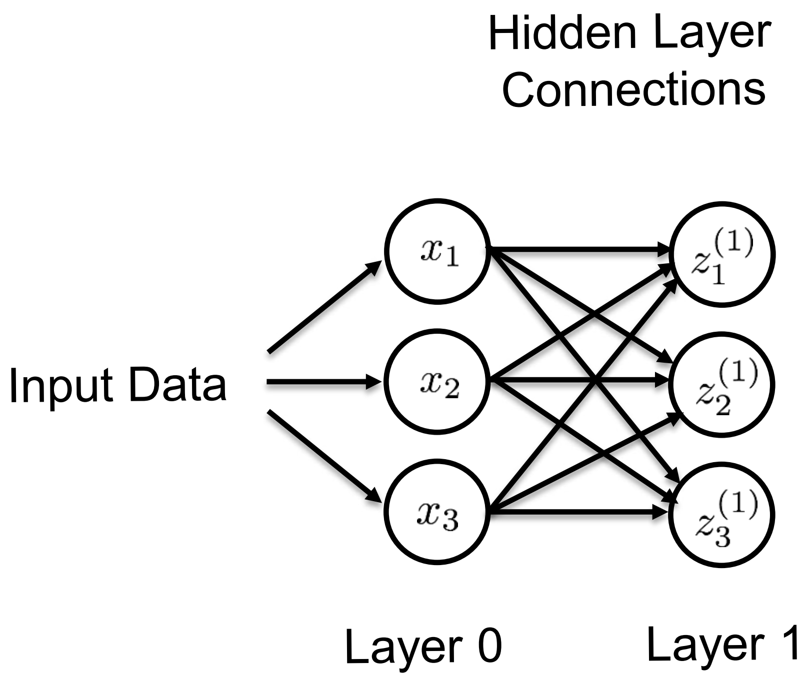


$z_i^{(l)}$ - the output unit i in layer l

$W_{ij}^{(l)}$ - the weight connection between unit j in layer l and unit i in layer $l+1$

$$z_1^{(1)} = W_{11}^{(0)} x_1 + W_{12}^{(0)} x_2 + W_{13}^{(0)} x_3$$
$$z_2^{(1)} = W_{21}^{(0)} x_1 + W_{22}^{(0)} x_2 + W_{23}^{(0)} x_3$$
$$z_3^{(1)} = W_{31}^{(0)} x_1 + W_{32}^{(0)} x_2 + W_{33}^{(0)} x_3$$

Fully Connected Layers:



$z_i^{(l)}$ - the output unit i in layer l

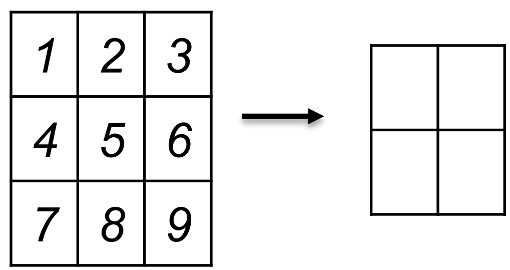
$W_{ij}^{(l)}$ - the weight connection between unit j in layer l and unit i in layer $l+1$

$$z^{(1)} = W^{(0)}x$$

matrix multiplication

Max Pooling Layer:

- Sliding window is applied on a grid of values.
- The maximum is computed using the values in the current window.



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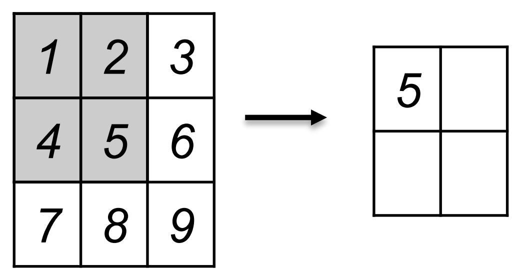

Max Pooling Layer:

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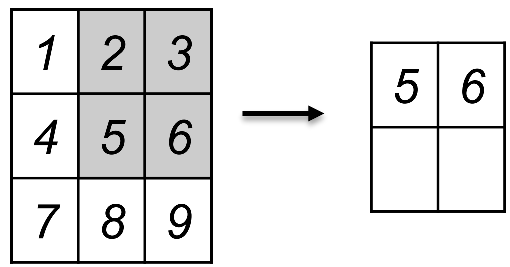
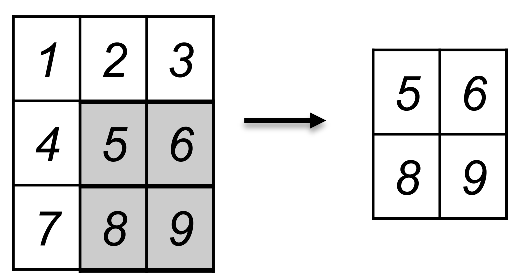


Max Pooling Layer:

- Sliding window is applied on a grid of values.
- The maximum is computed using the values in the current window.



Sigmoid Layer:

- Applies a sigmoid function on an input

$$a^{(l)} = f(z^{(l)}) = \frac{1}{1 + \exp\left(-z^{(l)}\right)}$$

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Video 12.2

Jianbo Shi

Convolutional Networks

Let us now consider a CNN with a specific architecture:

- 2 convolutional layers.
- 2 pooling layers.
- 2 fully connected layers.
- 3 sigmoid layers.

Notation:

- convolutional layer output

- fully connected layer output

- pooling layer

- sigmoid function f

- softmax function

Forward Pass:

x

Notation:

- convolutional layer output

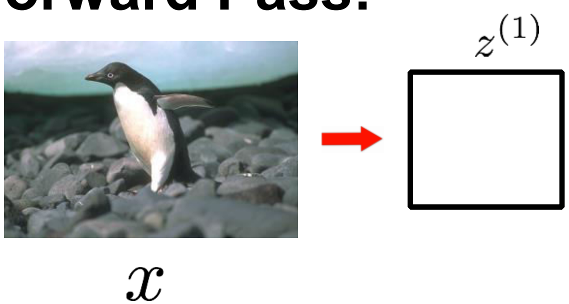
- fully connected layer output

- pooling layer

- sigmoid function f

- softmax function

Forward Pass:



Notation:

- convolutional layer output

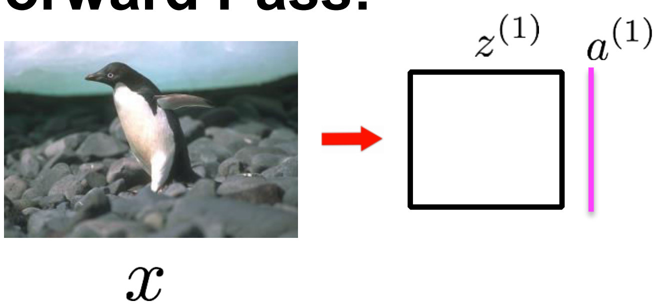
- fully connected layer output

- pooling layer

- sigmoid function f

- softmax function

Forward Pass:



Notation:

- convolutional layer output

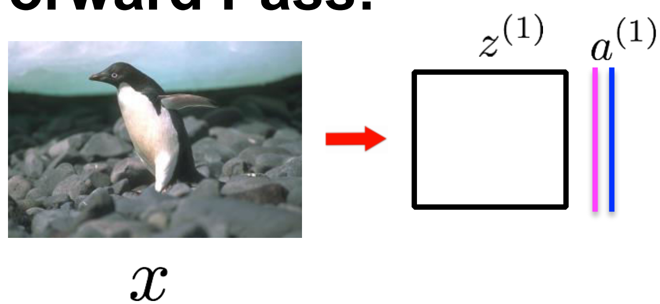
- fully connected layer output

- pooling layer

- sigmoid function f

- softmax function

Forward Pass:



Notation:

- convolutional layer output

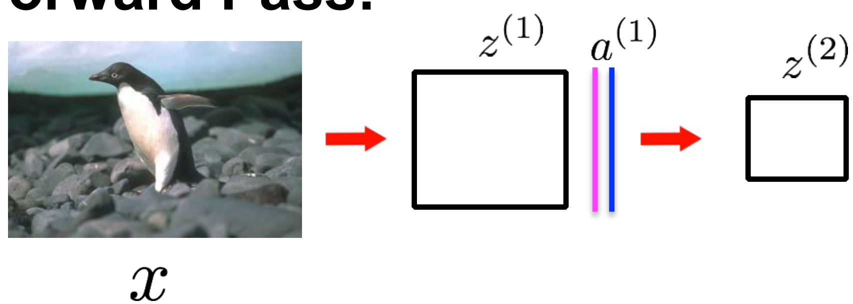
- fully connected layer output

- pooling layer

- sigmoid function f

- softmax function

Forward Pass:



Notation:

- convolutional layer output

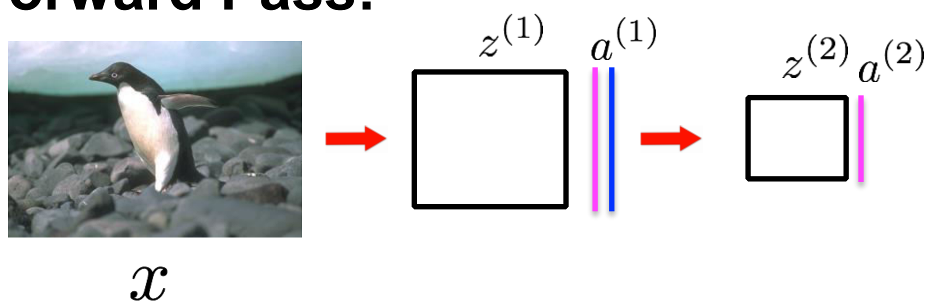
- fully connected layer output

- pooling layer

- sigmoid function f

- softmax function

Forward Pass:



Notation:

- convolutional layer output

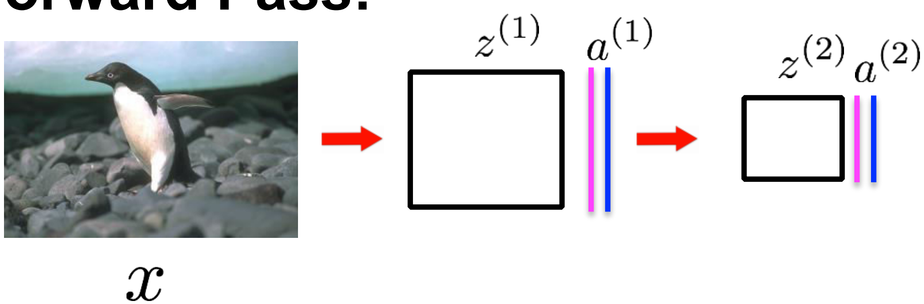
- fully connected layer output

- pooling layer

- sigmoid function f

- softmax function

Forward Pass:



Notation:

- convolutional layer output

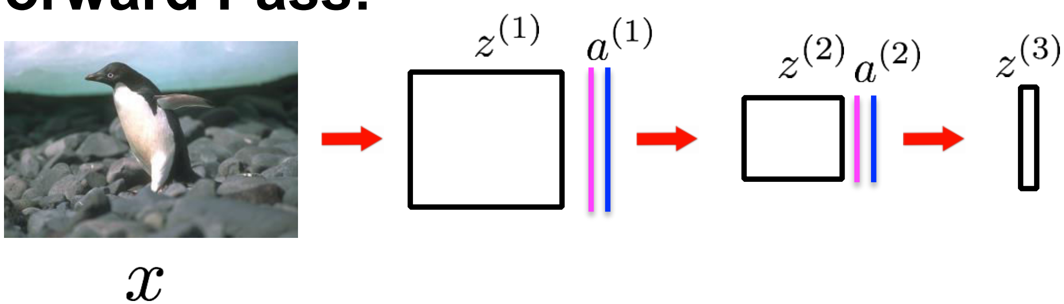
- fully connected layer output

- pooling layer

- sigmoid function f

- softmax function

Forward Pass:



Notation:

- convolutional layer output

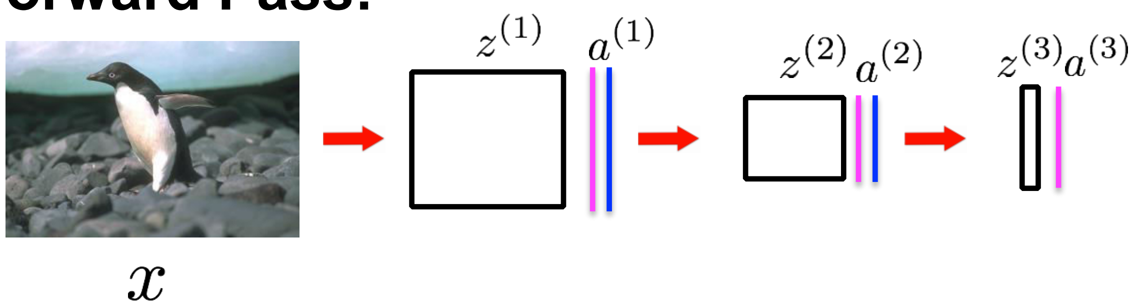
- fully connected layer output

- pooling layer

- sigmoid function f

- softmax function

Forward Pass:

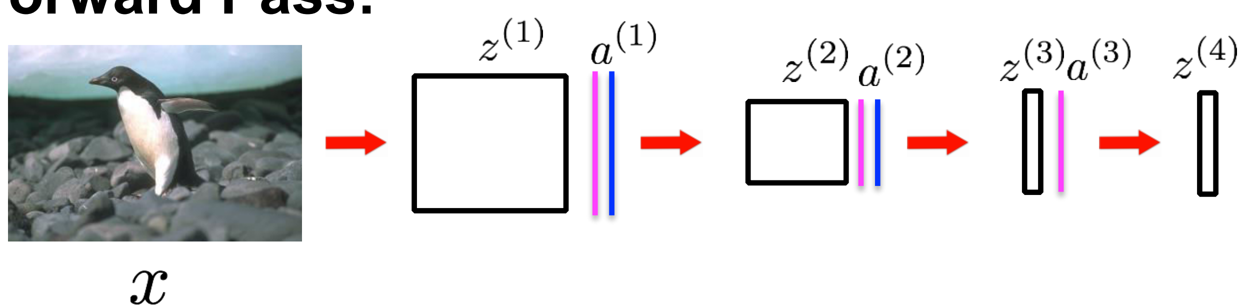


Convolutional Networks

Notation:

- convolutional layer output
- fully connected layer output
- pooling layer
- sigmoid function f
- softmax function

Forward Pass:



Notation:

- convolutional layer output

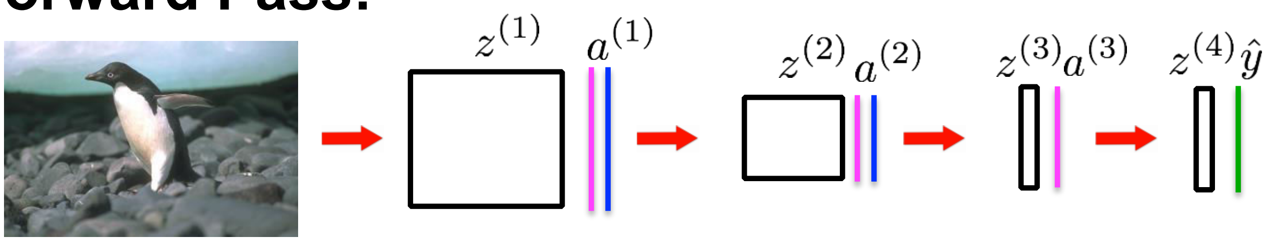
- fully connected layer output

- pooling layer

- sigmoid function f

- softmax function

Forward Pass:



x

Notation:

- convolutional layer output

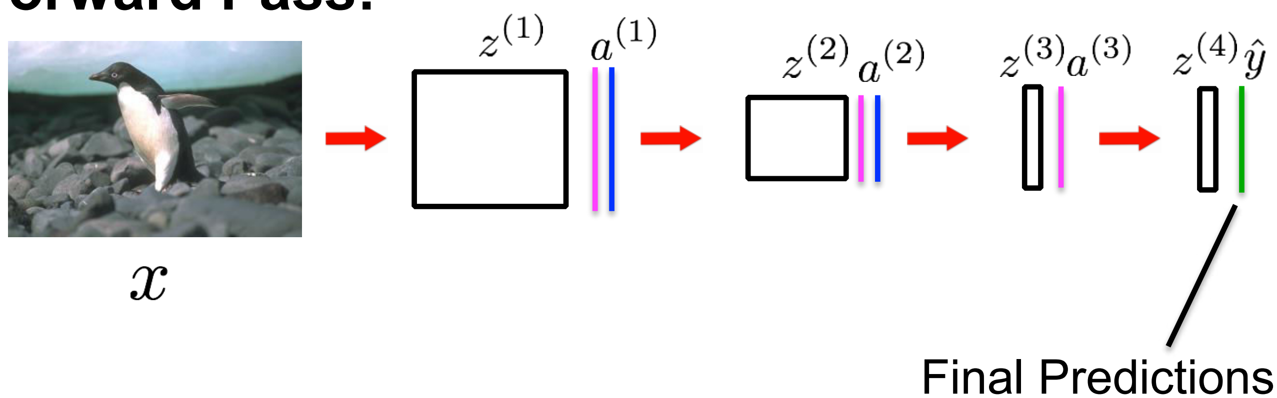
- fully connected layer output

- pooling layer

- sigmoid function f

- softmax function

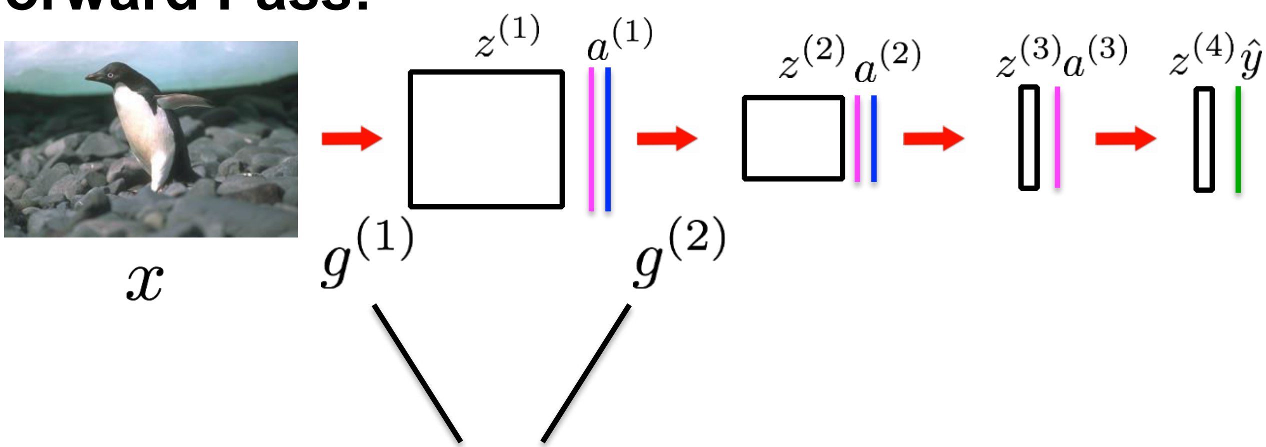
Forward Pass:



Notation:

- convolutional layer output
- fully connected layer output
- pooling layer
- sigmoid function f
- softmax function

Forward Pass:



Convolutional layer parameters in layers 1 and 2

Notation:

- convolutional layer output

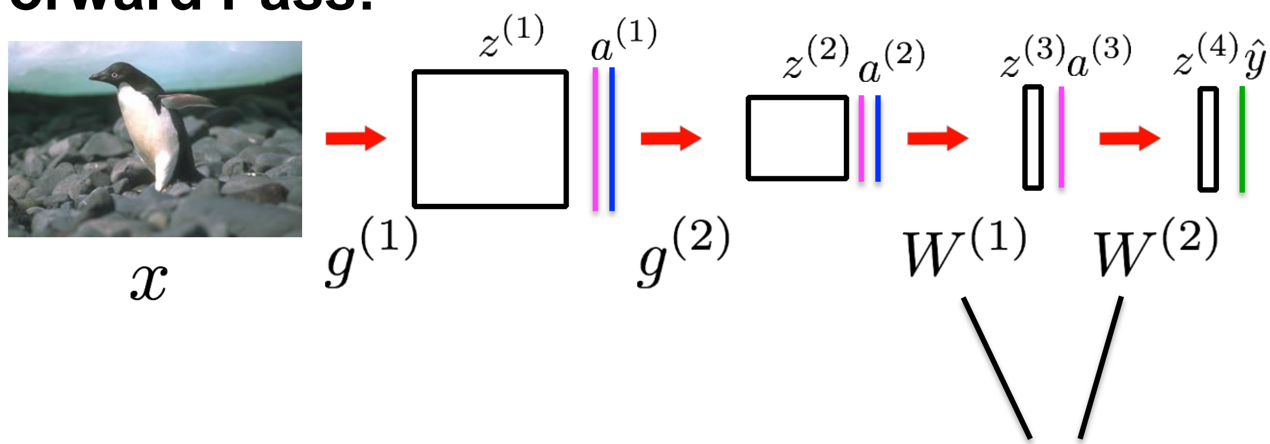
- fully connected layer output

- pooling layer

- sigmoid function f

- softmax function

Forward Pass:



Fully connected layer parameters in the fully connected layers 1 and 2

Notation:

- convolutional layer output

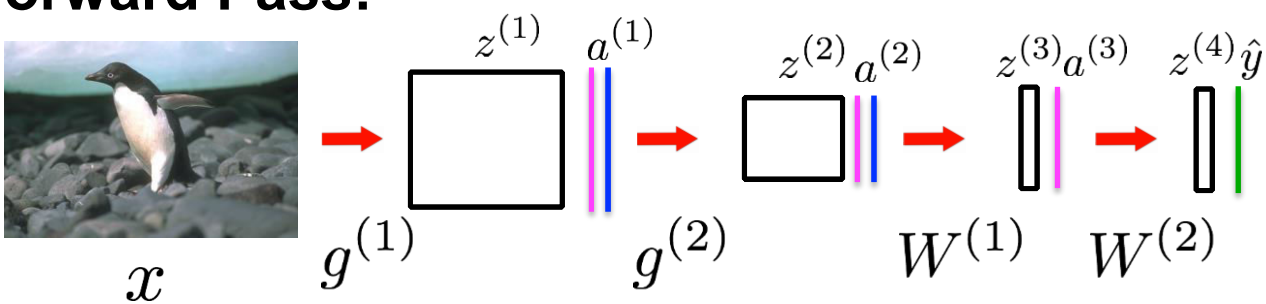
- fully connected layer output

- pooling layer

- sigmoid function f

- softmax function

Forward Pass:



Notation:

- convolutional layer output

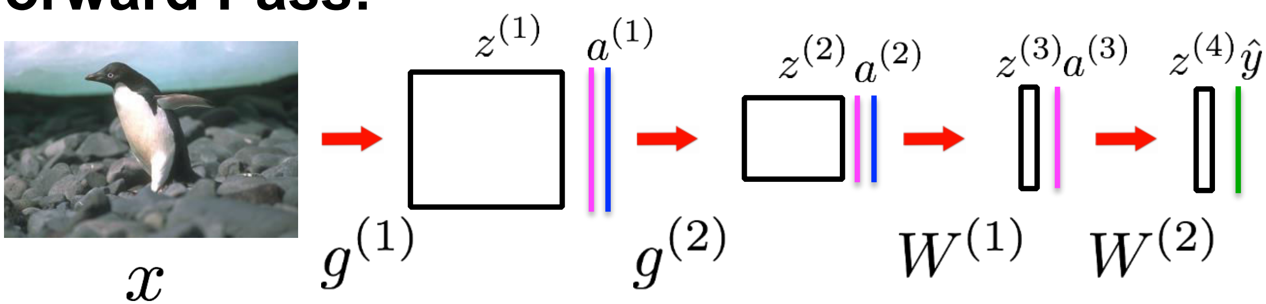
- fully connected layer output

- pooling layer

- sigmoid function f

- softmax function

Forward Pass:



1. $a^{(1)} = pool(f(g^{(1)} * x))$

Notation:

- convolutional layer output

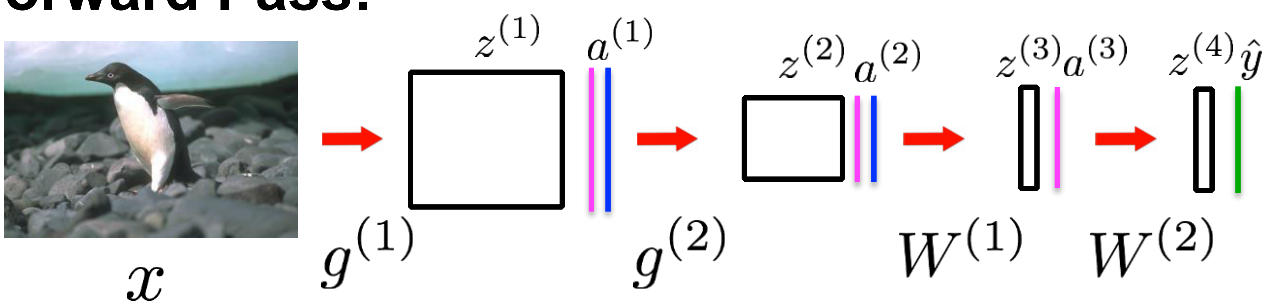
- fully connected layer output

- pooling layer

- sigmoid function f

- softmax function

Forward Pass:

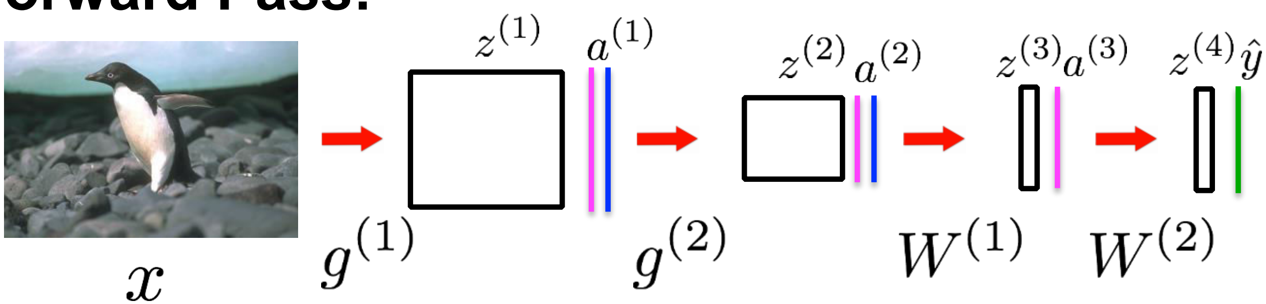


1. $a^{(1)} = pool(f(g^{(1)} * x))$
2. $a^{(2)} = pool(f(g^{(2)} * a^{(1)}))$

Notation:

- convolutional layer output
- fully connected layer output
- pooling layer
- sigmoid function f
- softmax function

Forward Pass:

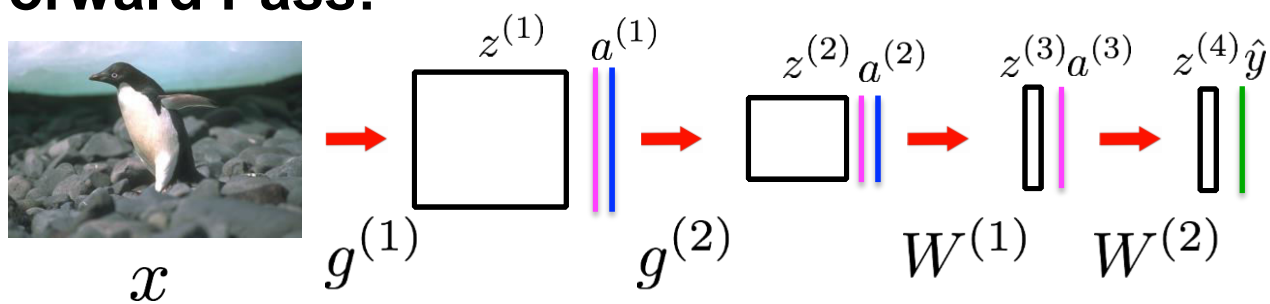


1. $a^{(1)} = pool(f(g^{(1)} * x))$
2. $a^{(2)} = pool(f(g^{(2)} * a^{(1)}))$
3. $a^{(3)} = f(W^{(1)} a^{(2)})$

Notation:

- convolutional layer output
- fully connected layer output
- pooling layer
- sigmoid function f
- softmax function

Forward Pass:



1. $a^{(1)} = pool(f(g^{(1)} * x))$
2. $a^{(2)} = pool(f(g^{(2)} * a^{(1)}))$
3. $a^{(3)} = f(W^{(1)} a^{(2)})$
4. $\hat{y} = softmax(W^{(2)} a^{(3)})$

Notation:

- convolutional layer output

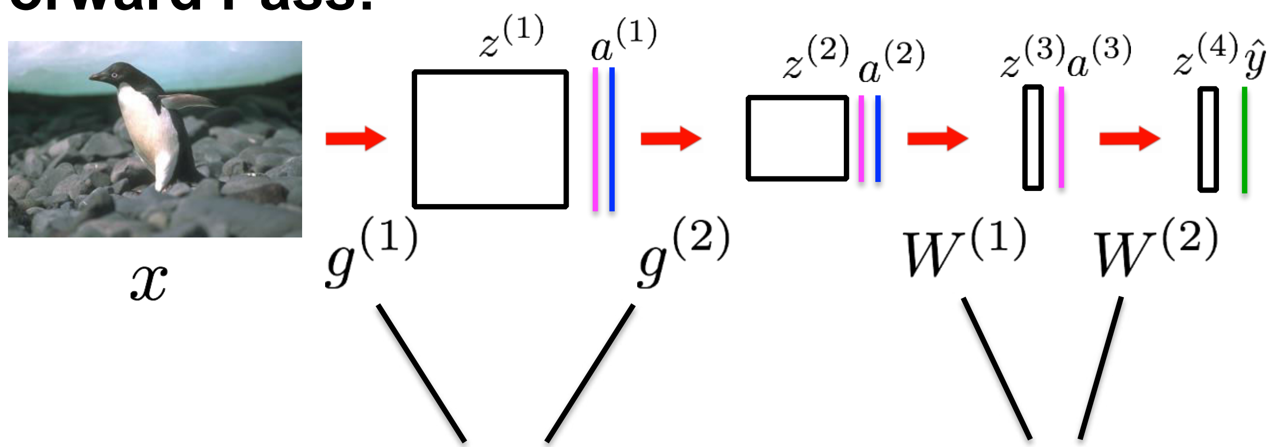
- fully connected layer output

- pooling layer

- sigmoid function f

- softmax function

Forward Pass:

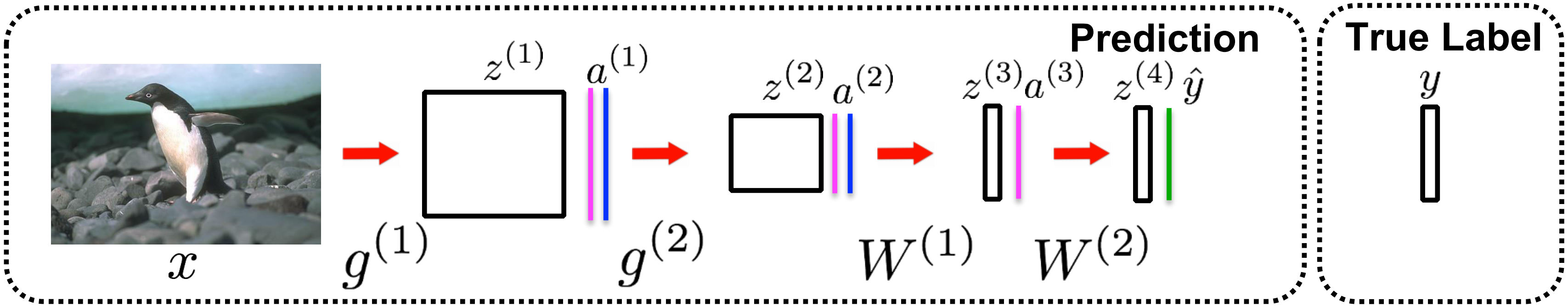


Key Question: How to learn the parameters from the data?

Backpropagation

for

Convolutional Neural Networks



How to learn the parameters of a CNN?

- Assume that we are a given a **labeled** training dataset

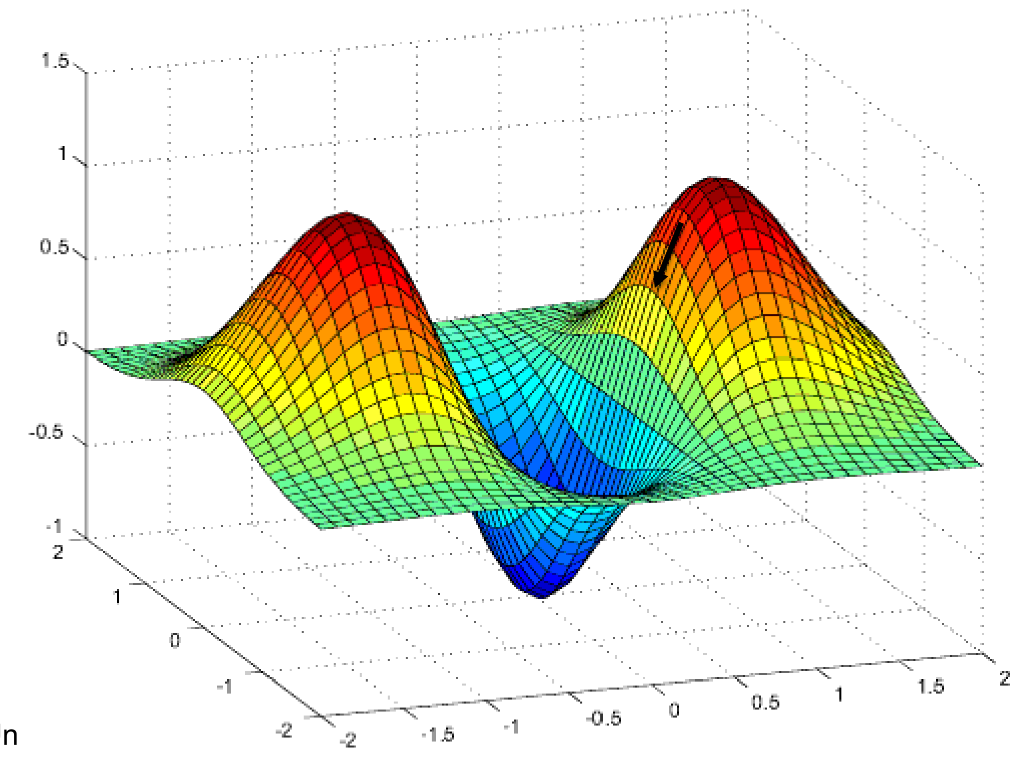
$$\{(x^{(1)}, y^{(1)}), \ldots, (x^{(n)}, y^{(n)})\}$$

- We want to adjust the parameters of a CNN such that CNN's predictions would be as close to true labels as possible.
- This is difficult to do because the learning objective is highly non-linear.

Gradient descent:

- Iteratively minimizes the objective function.
- The function needs to be differentiable.

$$\theta = \theta - \alpha \frac{\partial L(\theta)}{\partial \theta}$$



Gradient descent:

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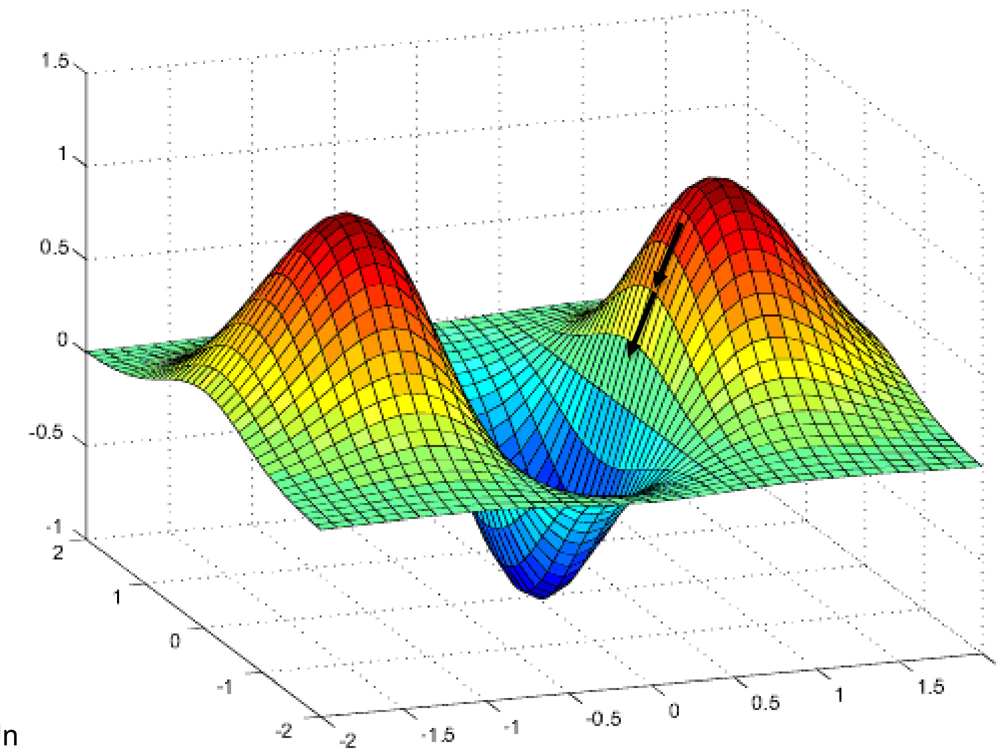

Gradient descent:

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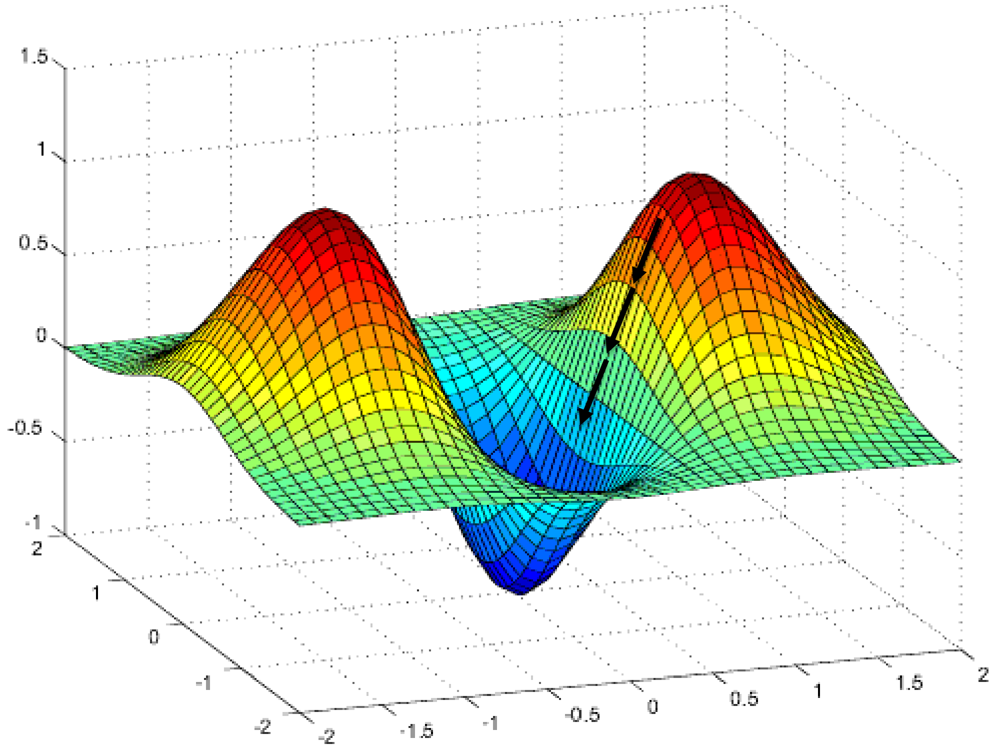

Gradient descent:

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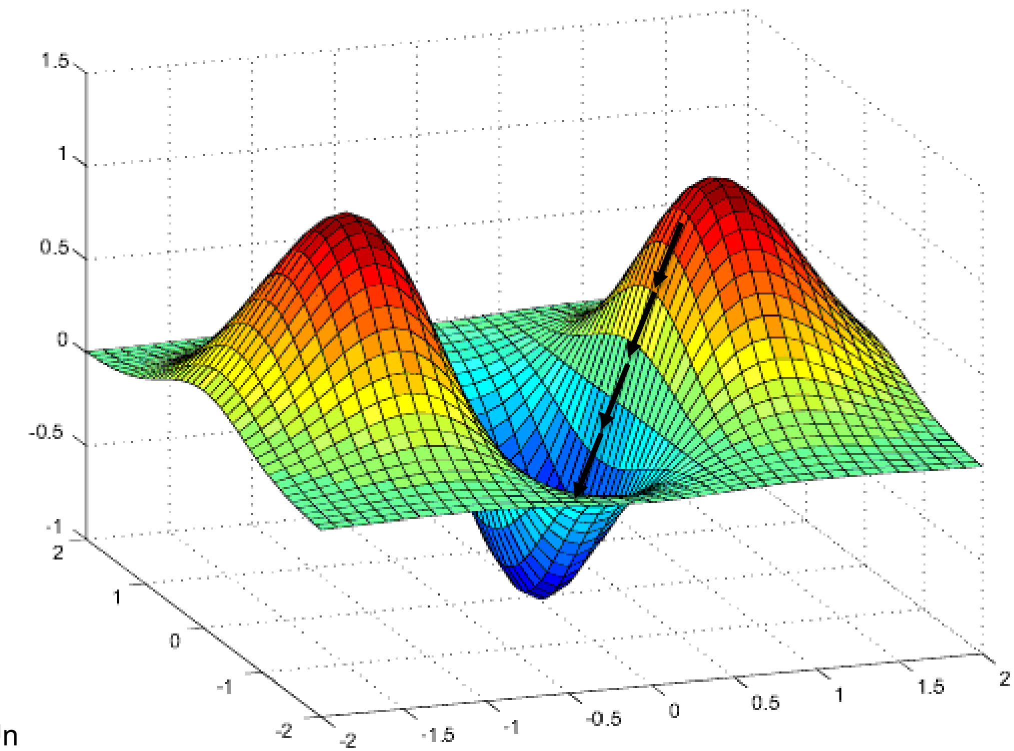
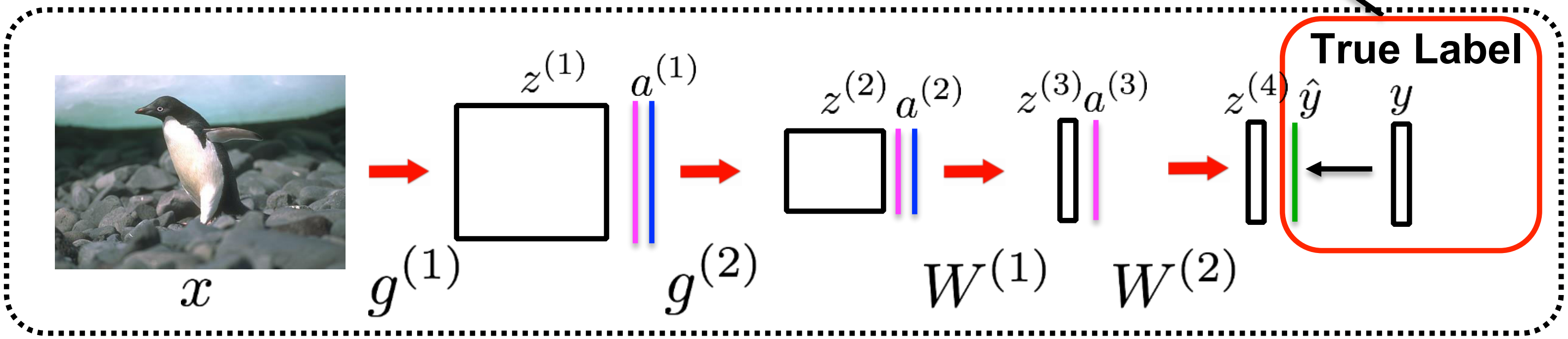
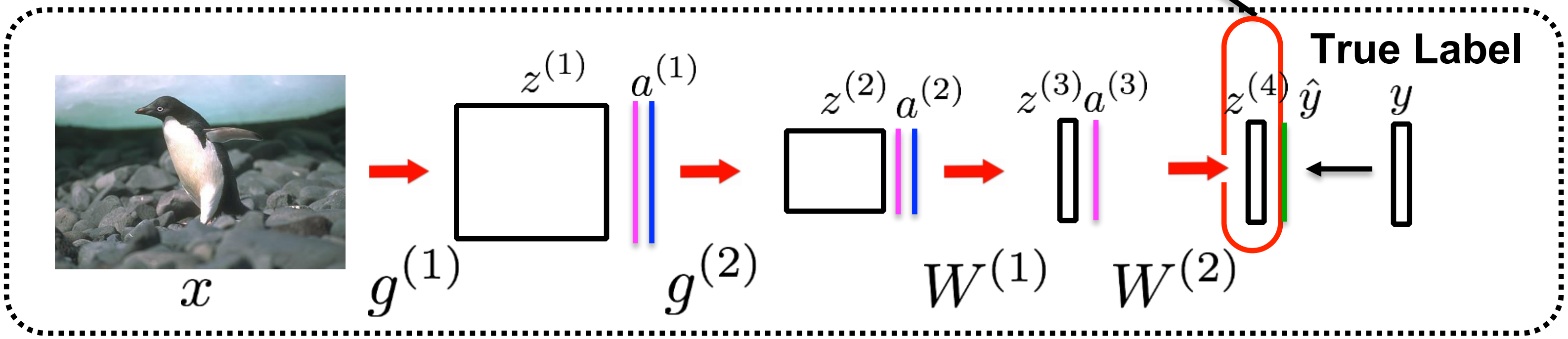
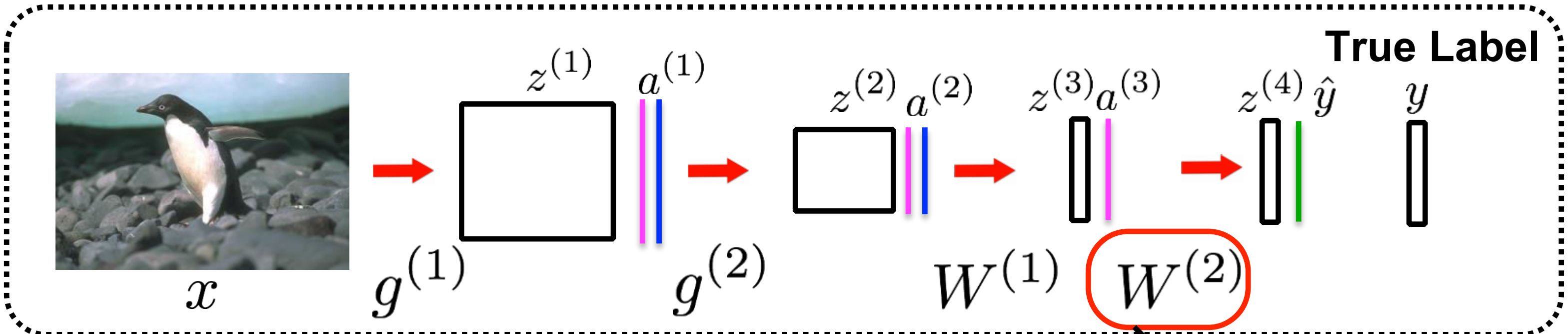


1. Compute the gradients of the overall loss w.r.t. to our predictions and propagate it back: $\frac{\partial L}{\partial \hat{y}}$



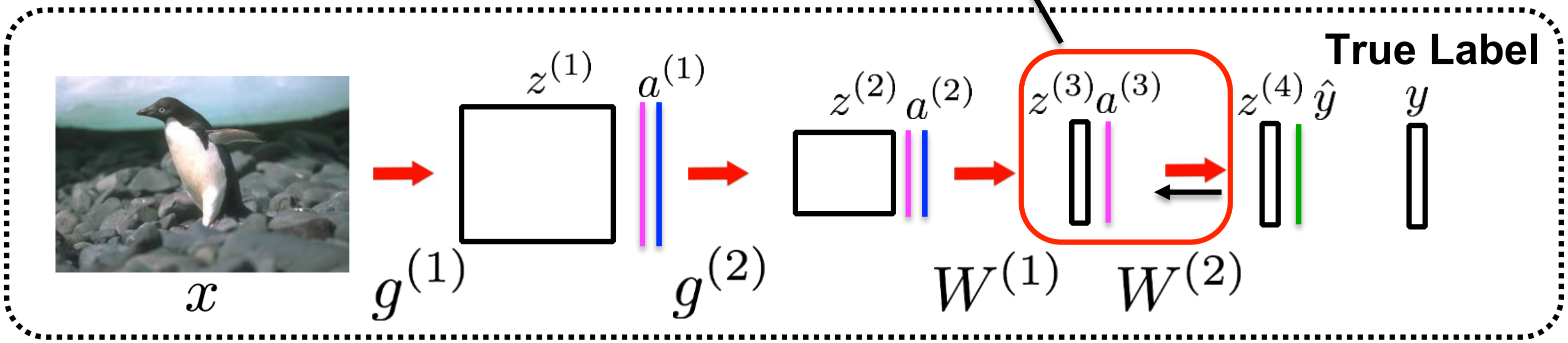
2. Compute the gradients of the overall loss and propagate it back: $\frac{\partial L}{\partial z^{(4)}}$

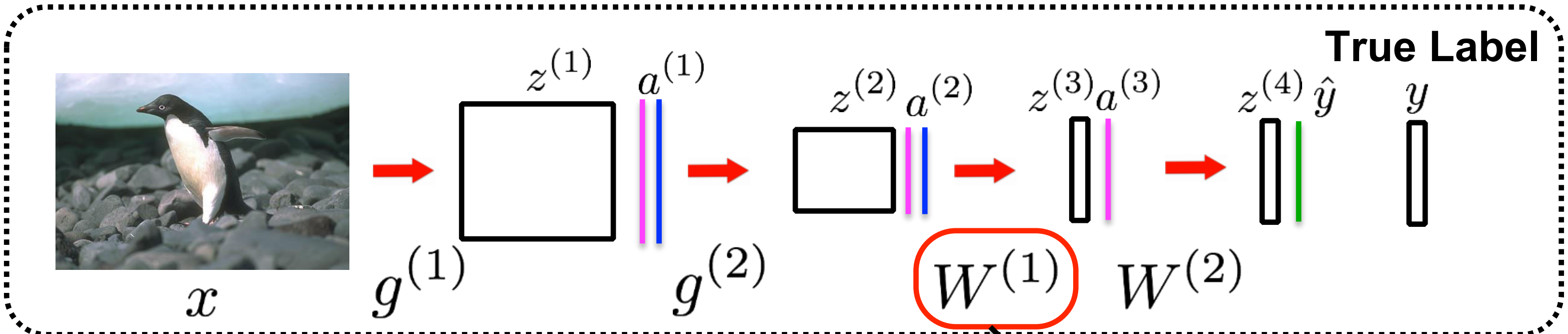




3. Compute the gradients to adjust the weights: $\frac{\partial L}{\partial W^{(2)}}$

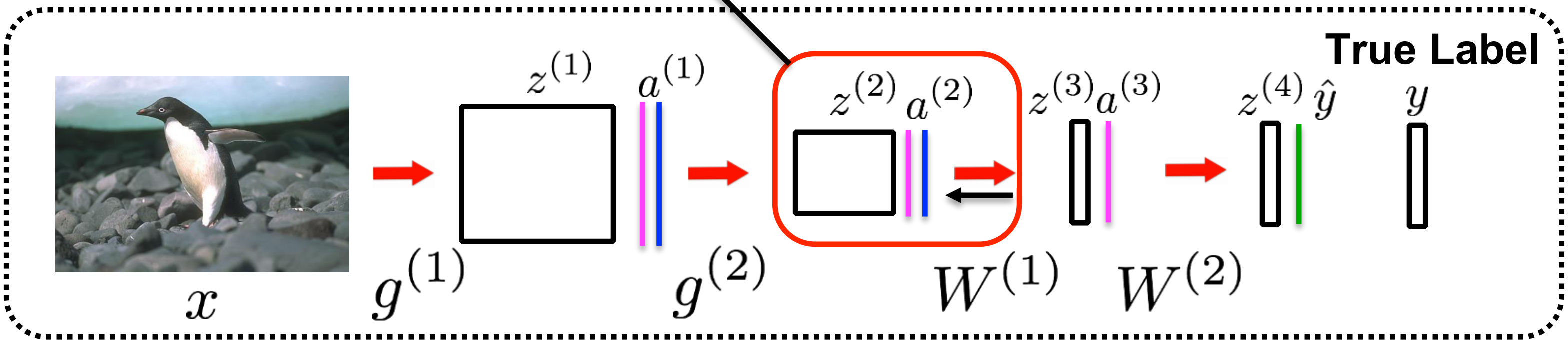
4. Backpropagate the gradients to previous layers: $\dfrac{\partial L}{\partial z^{(3)}}$

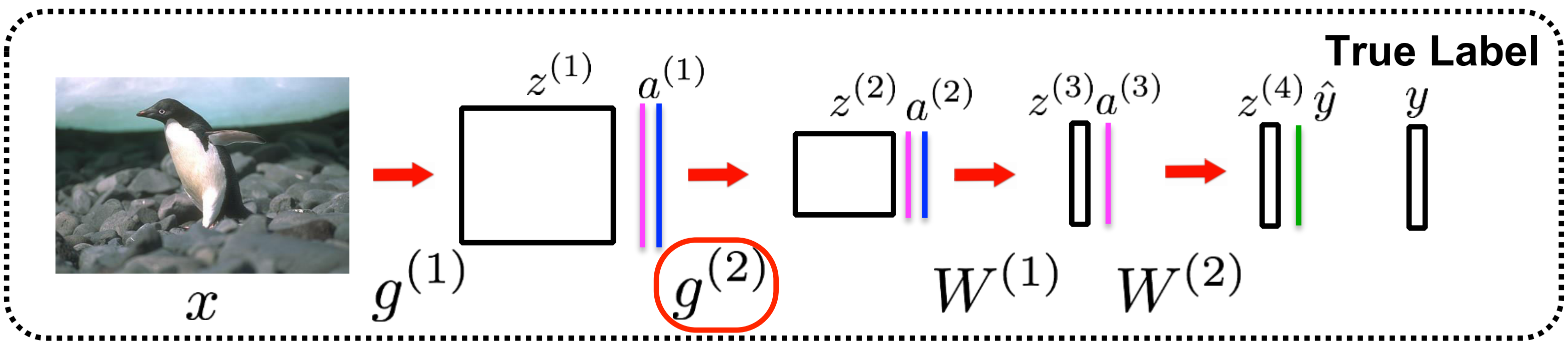




5. Compute the gradients to adjust the weights: $\frac{\partial L}{\partial W^{(1)}}$

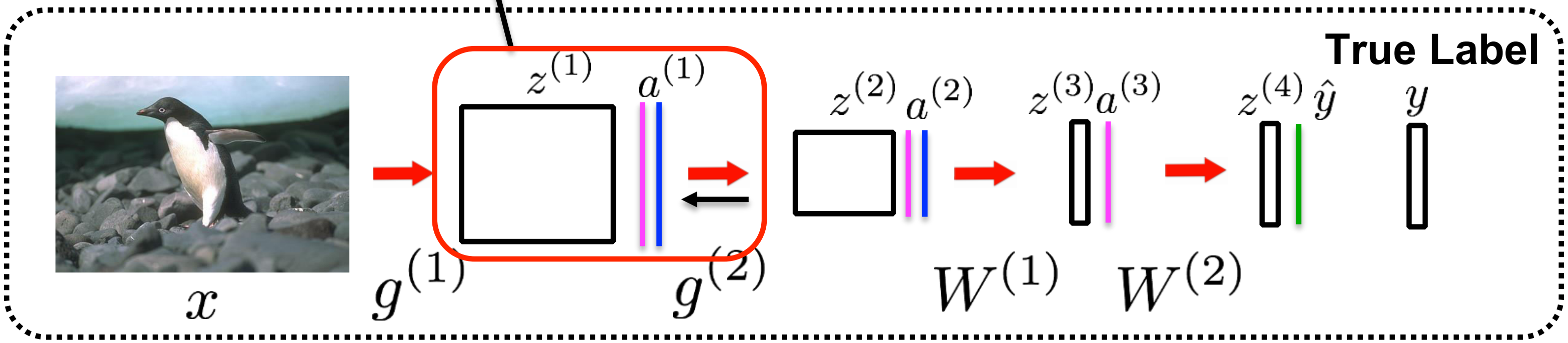
6. Backpropagate the gradients to previous layers: $\dfrac{\partial L}{\partial z^{(2)}}$

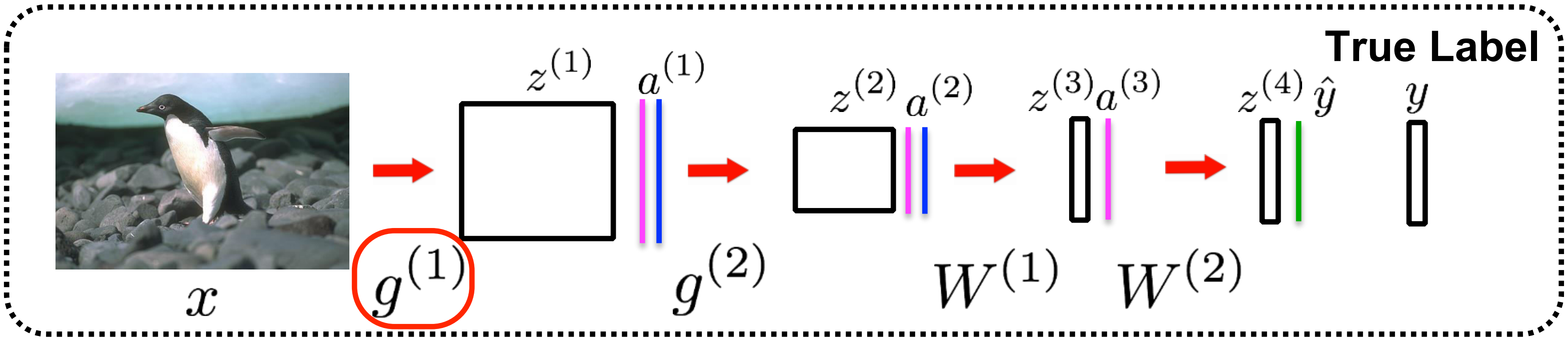




7. Compute the gradients to adjust the weights: $\dfrac{\partial L}{\partial g^{(2)}}$

8. Backpropagate the gradients to previous layers: $\frac{\partial L}{\partial z^{(1)}}$

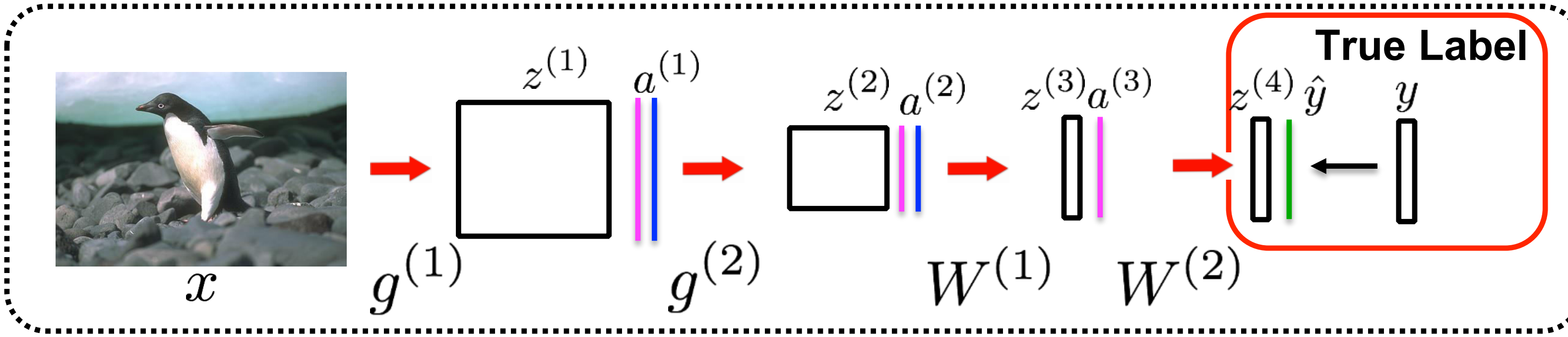




9. Compute the gradients to adjust the weights: $\dfrac{\partial L}{\partial g^{(1)}}$

Penn Engineering
ONLINE LEARNING

Video 12.3
Jianbo Shi

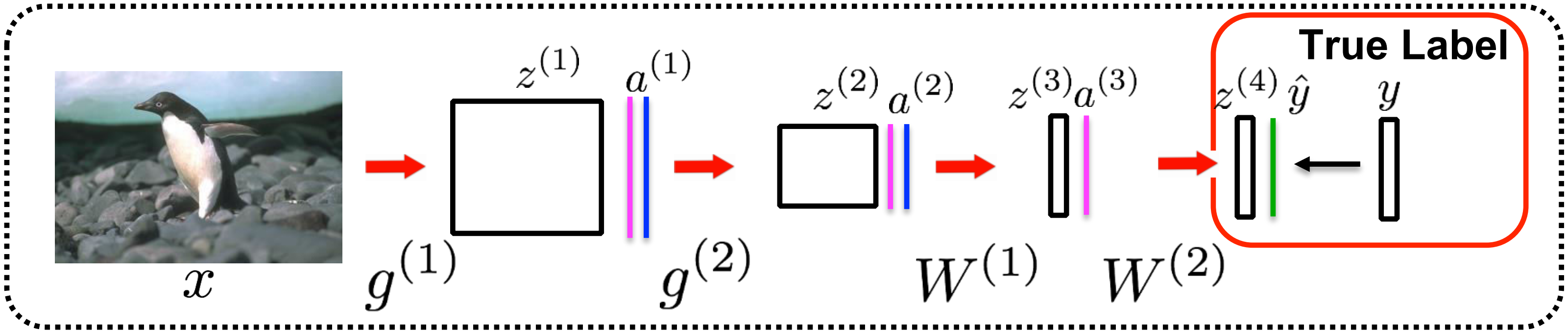


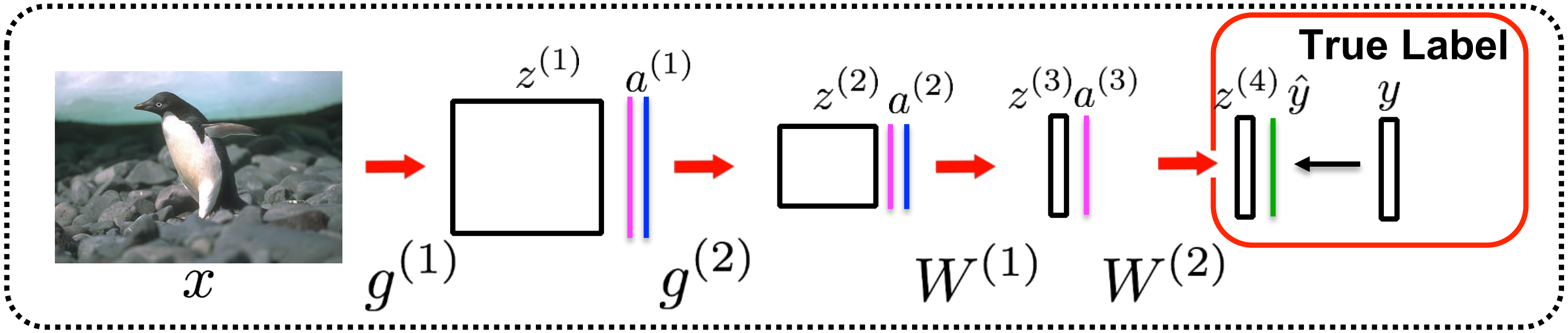
Assume that we have K=5 object classes:

Class 1: **Penguin**
Class 2: **Building**
Class 3: **Chair**
Class 4: **Person**
Class 5: **Bird**

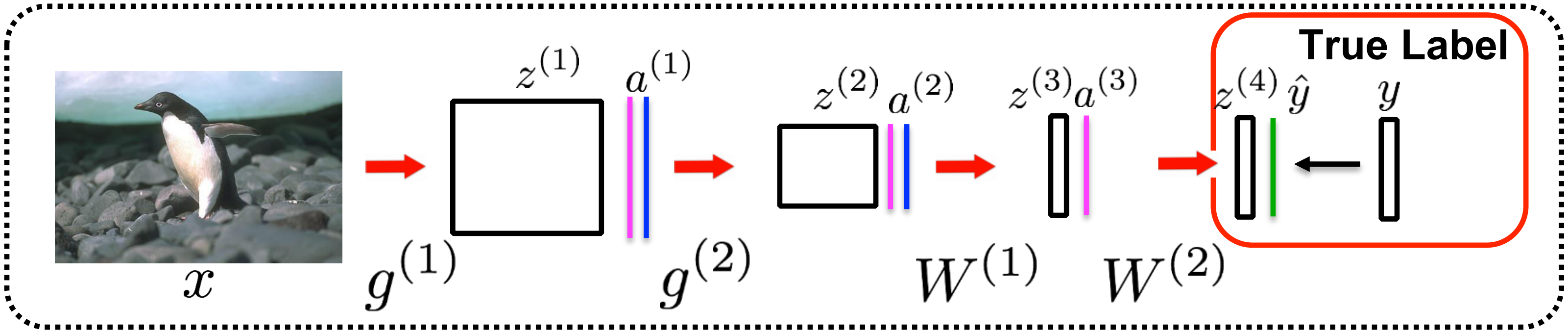
$$\hat{y} = \begin{array}{|c|c|c|c|c|} \hline 0.5 & 0 & 0.1 & 0.2 & 0.1 \\ \hline \end{array}$$

$$y = \begin{array}{|c|c|c|c|c|} \hline 1 & 0 & 0 & 0 & 0 \\ \hline \end{array}$$


True Label
$z^{(1)}$
$a^{(1)}$
$z^{(2)}$
$a^{(2)}$
$z^{(3)}$
$a^{(3)}$
$z^{(4)}$
$\hat{y}$
y
x
$g^{(1)}$
$g^{(2)}$
$W^{(1)}$
$W^{(2)}$

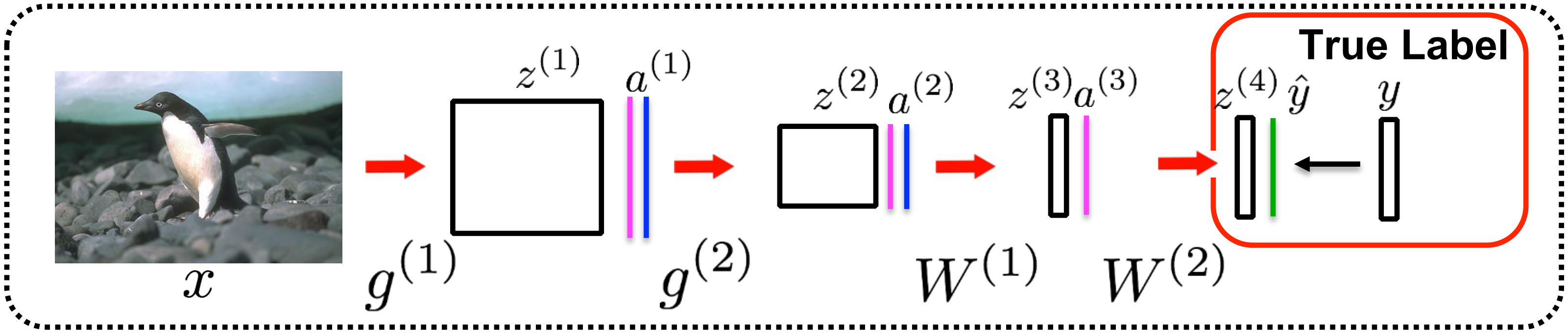


$$L = -\sum_{i=1}^{K} y_i \log\left(\hat{y}_i\right) \quad \text{where} \quad \hat{y}_i = \frac{\exp\left(z_i^{(4)}\right)}{\sum_{j=1}^{K} \exp\left(z_j^{(4)}\right)}$$



$$L = -\sum_{i=1}^{K} y_i \log(\hat{y}_i)$$ where $$\hat{y}_i = \frac{\exp(z_i^{(4)})}{\sum_{j=1}^{K} \exp(z_j^{(4)})}$$

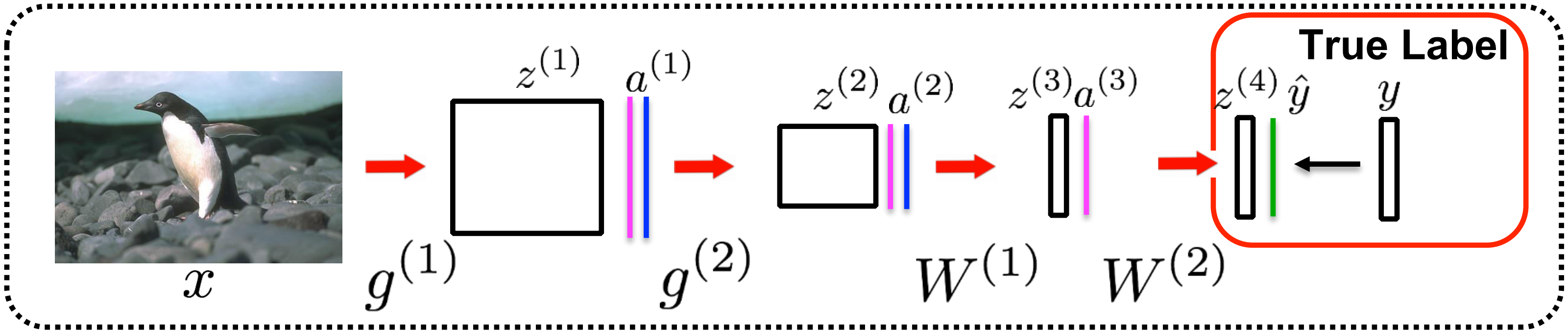
$$\frac{\partial L}{\partial z_i^{(4)}} = \frac{\partial L}{\partial \hat{y}} \frac{\partial \hat{y}}{\partial z_i^{(4)}}$$



$$L = -\sum_{i=1}^{K} y_i \log\left(\hat{y}_i\right) \quad \text{where} \quad \hat{y}_i = \frac{\exp\left(z_i^{(4)}\right)}{\sum_{j=1}^{K} \exp\left(z_j^{(4)}\right)}$$

$$\frac{\partial L}{\partial z_i^{(4)}} = \boxed{\frac{\partial L}{\partial \hat{y}}} \frac{\partial \hat{y}}{\partial z_i^{(4)}}$$

$$\boxed{\frac{\partial L}{\partial \hat{y}_i} = -\frac{y_i}{\hat{y}_i}}$$

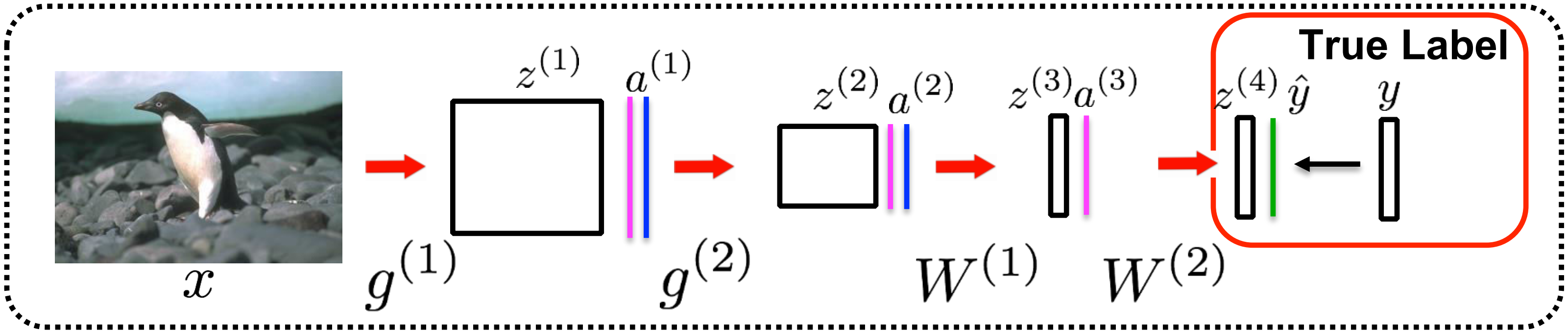


$$L = -\sum_{i=1}^{K} y_i \log\left(\hat{y}_i\right) \quad \text{where} \quad \hat{y}_i = \frac{\exp\left(z_i^{(4)}\right)}{\sum_{j=1}^{K} \exp\left(z_j^{(4)}\right)}$$

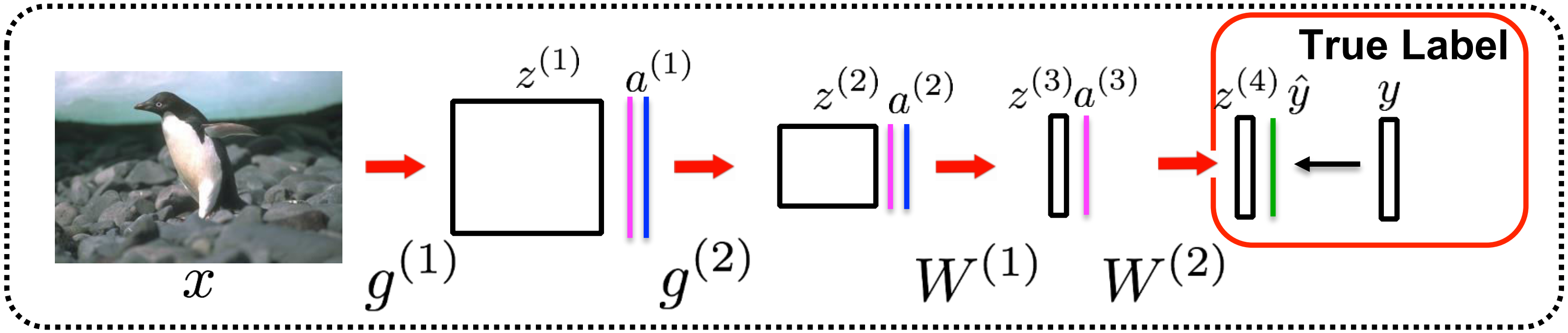
$$\frac{\partial L}{\partial z_i^{(4)}} = \boxed{\frac{\partial L}{\partial \hat{y}}} \boxed{\frac{\partial \hat{y}}{\partial z_i^{(4)}}}$$

$$\boxed{\frac{\partial L}{\partial \hat{y}_i} = -\frac{y_i}{\hat{y}_i}}$$

$$\boxed{\frac{\partial \hat{y}_i}{\partial z_j^{(4)}} = \begin{cases} \hat{y}_i(1-\hat{y}_i), & \text{if} \quad i = j \\ -\hat{y}_i \hat{y}_j, & \text{if} \quad i \neq j \end{cases}}$$

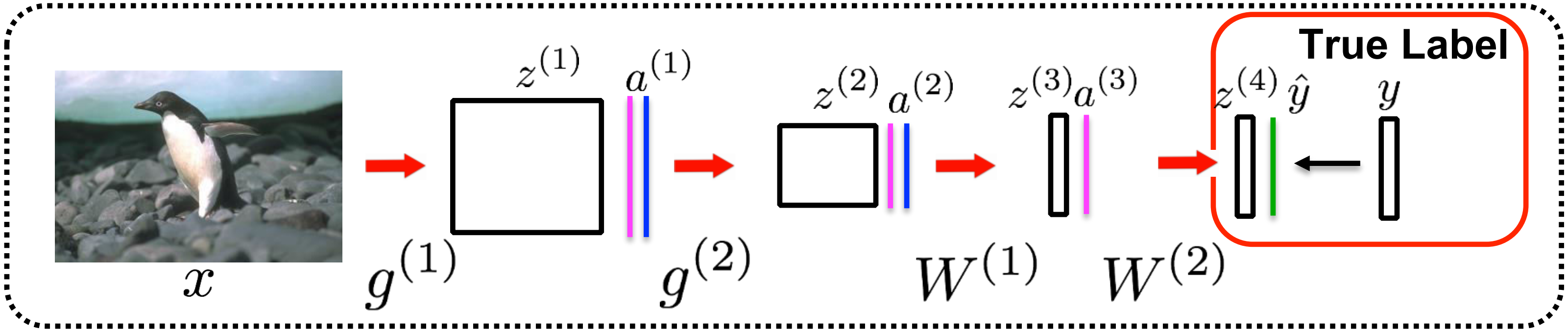


$$\frac{\partial L}{\partial z_i^{(4)}} = \frac{\partial L}{\partial \hat{y}} \frac{\partial \hat{y}}{\partial z_i^{(4)}}$$



$$\frac{\partial L}{\partial z_i^{(4)}} = \frac{\partial L}{\partial \hat{y}} \frac{\partial \hat{y}}{\partial z_i^{(4)}}$$

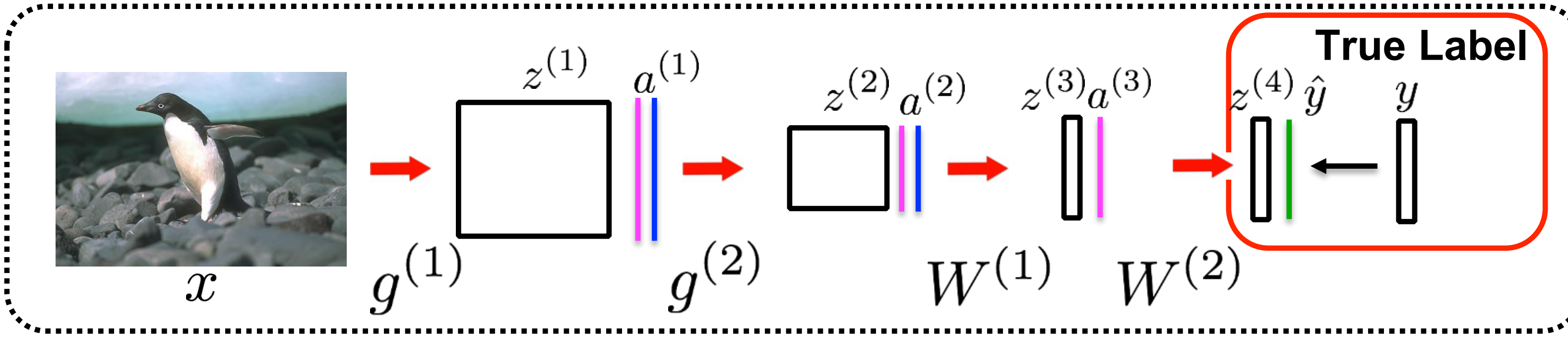
$$= \frac{\partial L}{\partial \hat{y}_i} \frac{\partial \hat{y}_i}{\partial z_i^{(4)}} + \sum_{i \neq j} \frac{\partial L}{\partial \hat{y}_j} \frac{\partial \hat{y}_j}{\partial z_i^{(4)}}$$



$$\frac{\partial L}{\partial z_i^{(4)}} = \frac{\partial L}{\partial \hat{y}} \frac{\partial \hat{y}}{\partial z_i^{(4)}}$$

$$= \frac{\partial L}{\partial \hat{y}_i} \frac{\partial \hat{y}_i}{\partial z_i^{(4)}} + \sum_{i \neq j} \frac{\partial L}{\partial \hat{y}_j} \frac{\partial \hat{y}_j}{\partial z_i^{(4)}}$$

$$\boxed{= \hat{y}_i - y_i}$$

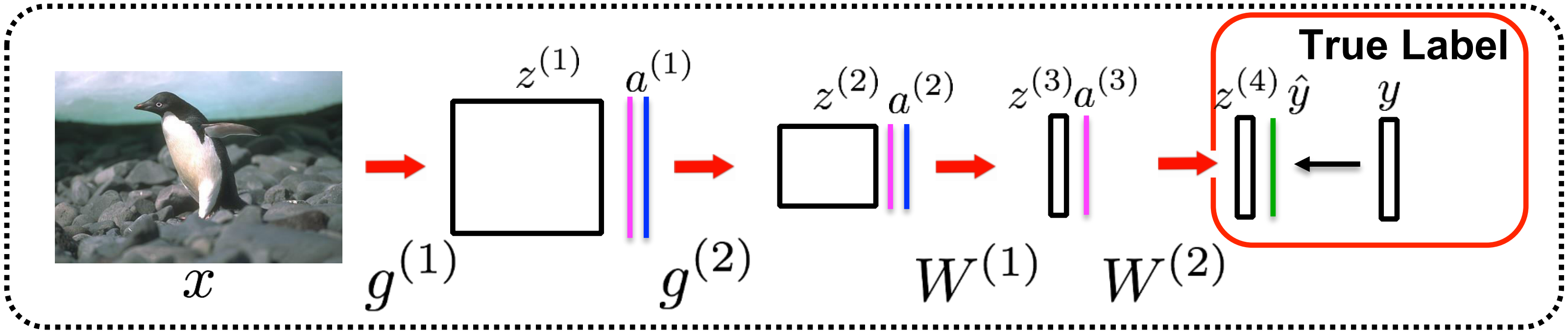


Assume that we have K=5 object classes:

Class 1: **Penguin**
Class 2: **Building**
Class 3: **Chair**
Class 4: **Person**
Class 5: **Bird**

$$\hat{y} = \boxed{0.5 \mid 0 \mid 0.1 \mid 0.2 \mid 0.1}$$

$$y = \boxed{1 \mid 0 \mid 0 \mid 0 \mid 0}$$



Assume that we have K=5 object classes:

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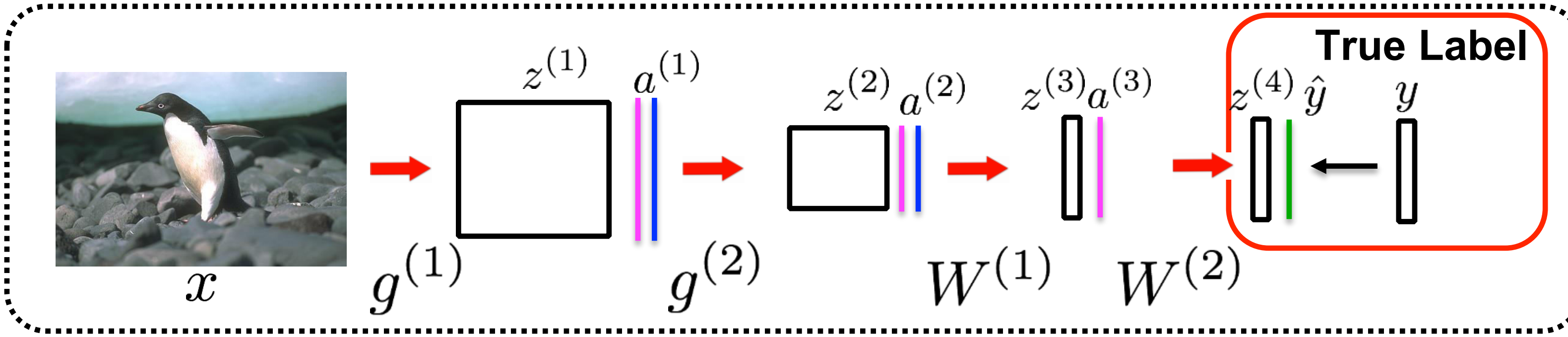
$$\hat{y} = \begin{array}{|c|c|c|c|c|}\hline 0.5 & 0 & 0.1 & 0.2 & 0.1 \\ \hline\end{array}$$

$$y = \begin{array}{|c|c|c|c|c|}\hline 1 & 0 & 0 & 0 & 0 \\ \hline\end{array}$$

$$\frac{\partial L}{\partial z^{(4)}} = \begin{array}{|c|c|c|c|c|}\hline -0.5 & 0 & 0.1 & 0.2 & 0.1 \\ \hline\end{array}$$



Assume that we have K=5 object classes:

Class 1: **Penguin**
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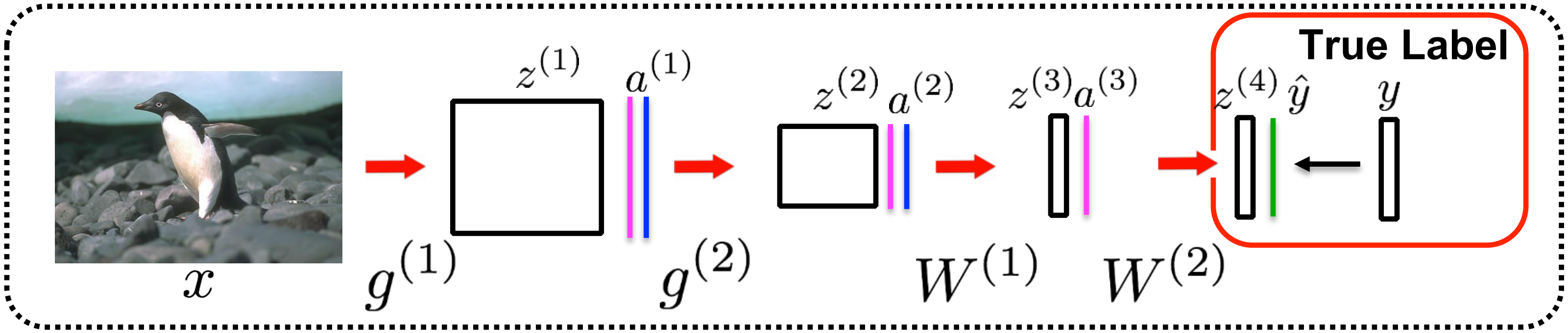
$$\hat{y} = \begin{bmatrix} 0.5 & 0 & 0.1 & 0.2 & 0.1 \end{bmatrix}$$

$$y = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\frac{\partial L}{\partial z^{(4)}} = \begin{bmatrix} \boxed{-0.5} & 0 & 0.1 & 0.2 & 0.1 \end{bmatrix}$$

Increasing the score corresponding to the true class decreases the loss.



Assume that we have K=5 object classes:

Class 1: **Penguin**
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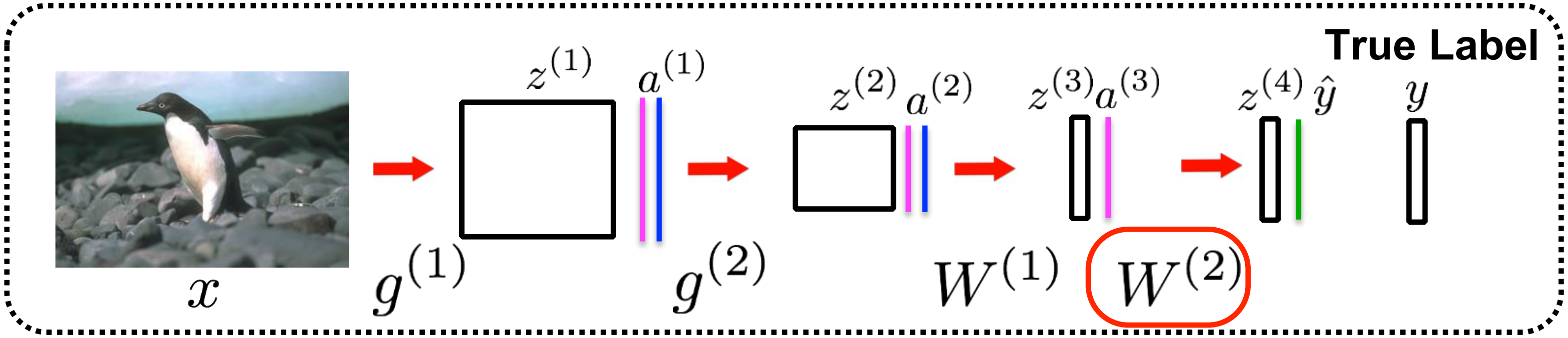
$$\hat{y} = \begin{array}{|c|c|c|c|c|}\hline 0.5 & 0 & 0.1 & 0.2 & 0.1 \\ \hline\end{array}$$

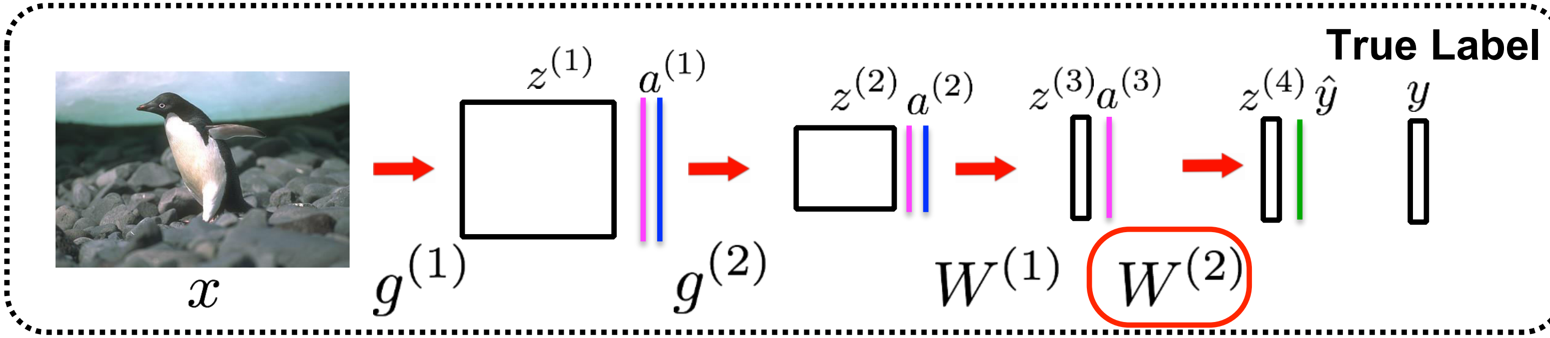
$$y = \begin{array}{|c|c|c|c|c|}\hline 1 & 0 & 0 & 0 & 0 \\ \hline\end{array}$$

$$\frac{\partial L}{\partial z^{(4)}} = \begin{array}{|c|c|c|c|c|}\hline -0.5 & 0 & 0.1 & 0.2 & 0.1 \\ \hline\end{array}$$

Decreasing the score of other classes also decreases the loss.

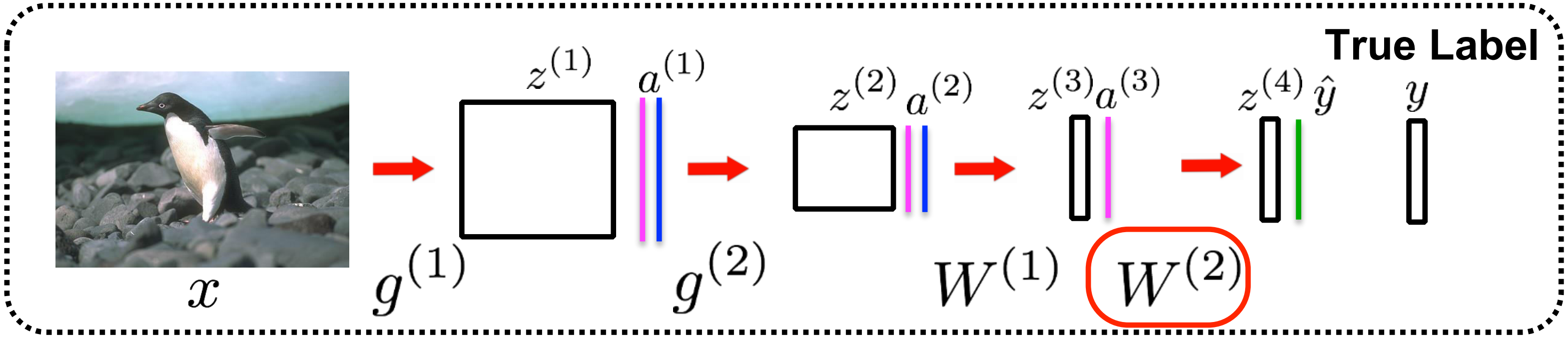


Adjusting the weights:



Adjusting the weights:

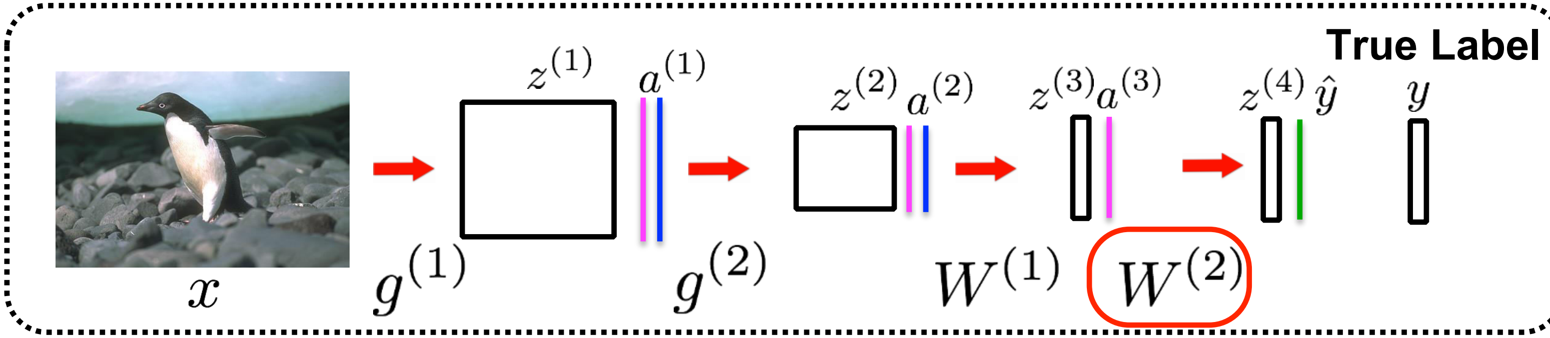
Need to compute the following gradient $$\frac{\partial L}{\partial W^{(2)}} = \frac{\partial L}{\partial z^{(4)}} \frac{\partial z^{(4)}}{\partial W^{(2)}}$$



Adjusting the weights:

Need to compute the following gradient

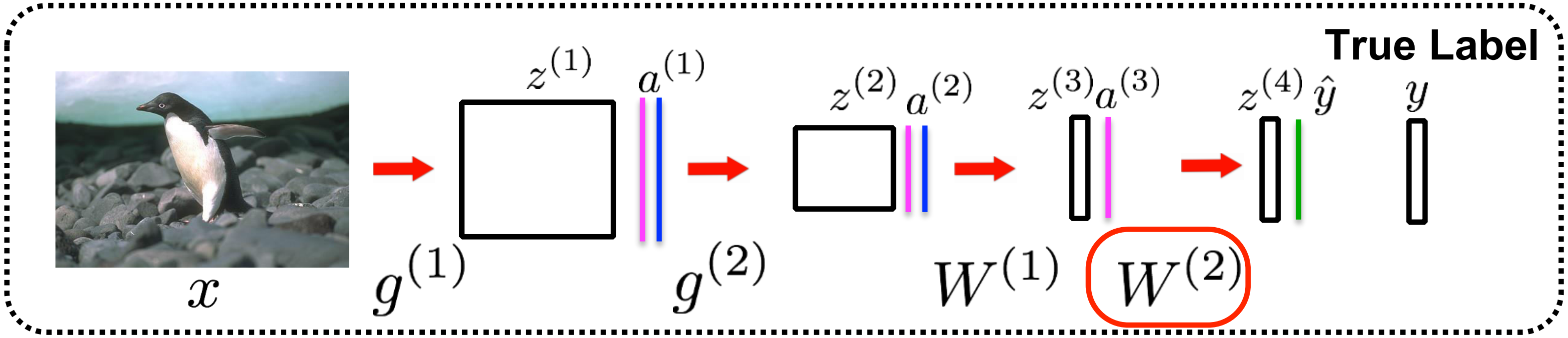
$$\frac{\partial L}{\partial W^{(2)}} = \boxed{\frac{\partial L}{\partial z^{(4)}}} \frac{\partial z^{(4)}}{\partial W^{(2)}}$$



Adjusting the weights:

Need to compute the following gradient $$\frac{\partial L}{\partial W^{(2)}} = \boxed{\frac{\partial L}{\partial z^{(4)}}} \frac{\partial z^{(4)}}{\partial W^{(2)}}$$

$\boxed{\frac{\partial L}{\partial z^{(4)}}}$ was already computed in the previous step



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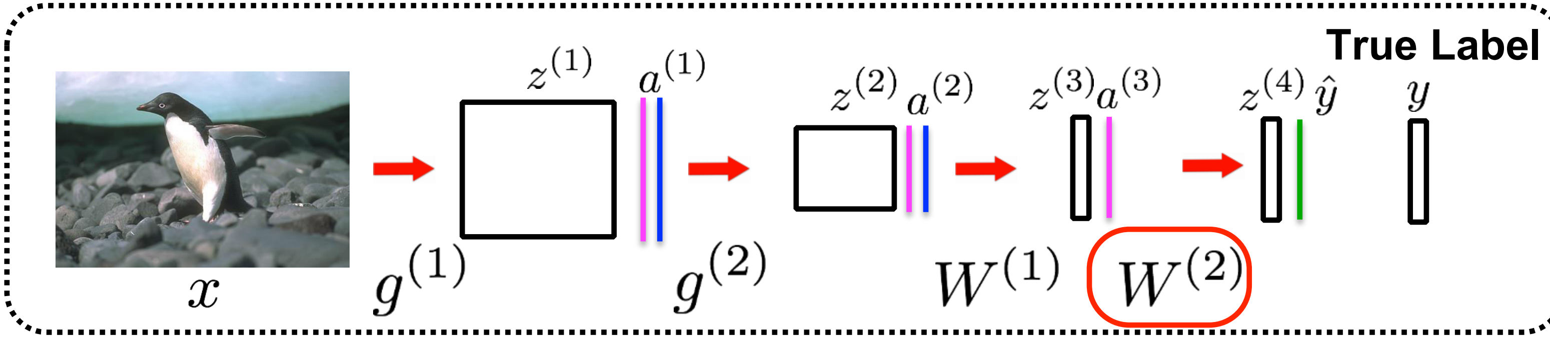
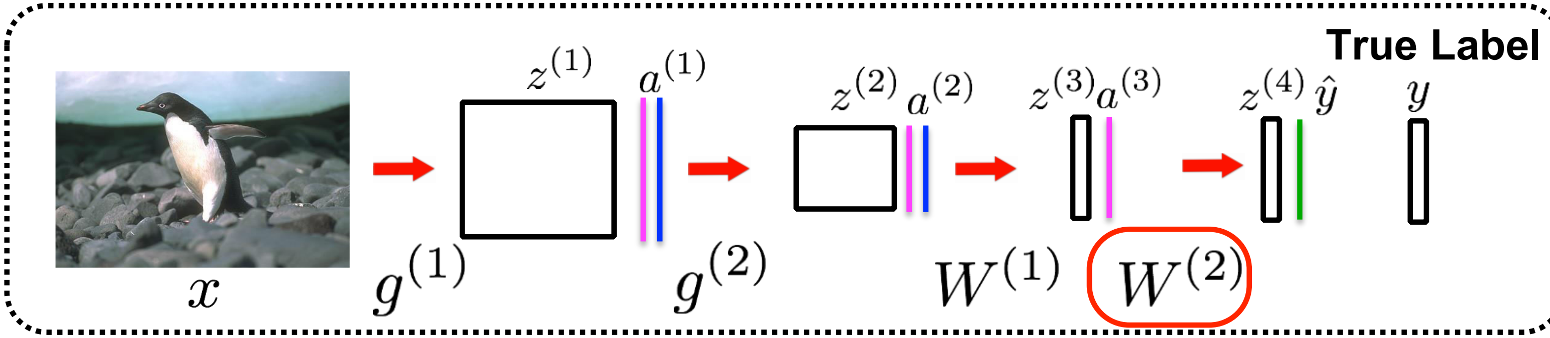


Adjusting the weights:

Need to compute the following gradient $$\frac{\partial L}{\partial W^{(2)}} = \frac{\partial L}{\partial z^{(4)}} \boxed{\frac{\partial z^{(4)}}{\partial W^{(2)}}}$$

$$z_i^{(4)} = \sum_{k=1}^{N} W_{ik}^{(2)} f(z_k^{(3)})$$ where $$f(z^{(3)}) = a^{(3)}$$

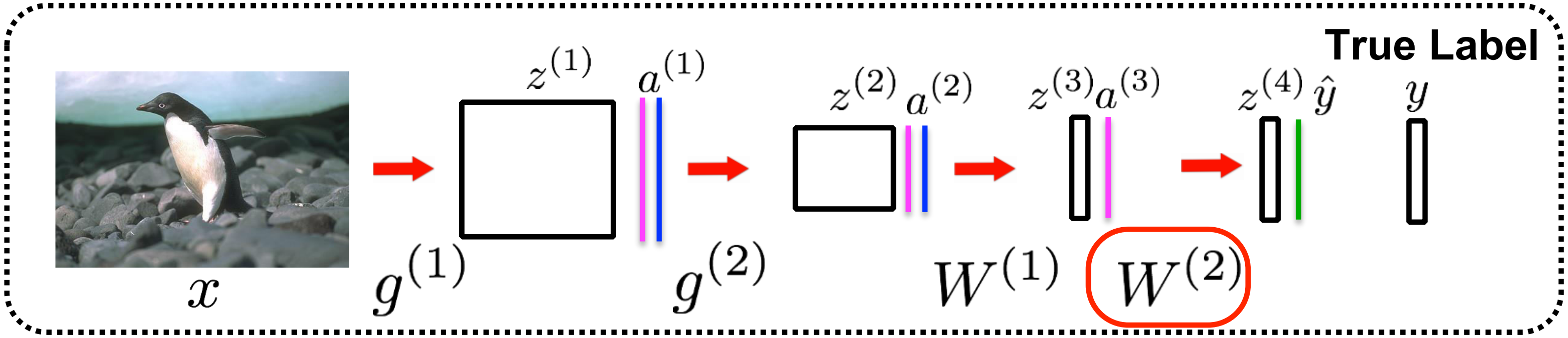


Adjusting the weights:

Need to compute the following gradient $\frac{\partial L}{\partial W^{(2)}} = \frac{\partial L}{\partial z^{(4)}} \boxed{\frac{\partial z^{(4)}}{\partial W^{(2)}}}$

$$z_i^{(4)} = \sum_{k=1}^{N} W_{ik}^{(2)} f(z_k^{(3)}) \quad \text{where} \quad f(z^{(3)}) = a^{(3)}$$

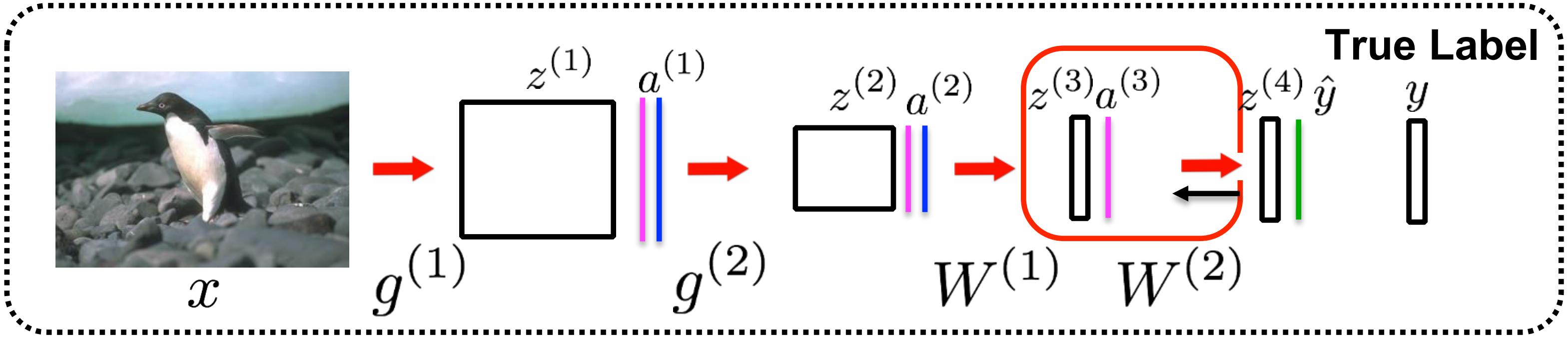
$$\boxed{\frac{\partial z_i^{(4)}}{\partial W_{ij}^{(2)}} = f(z_j^{(3)})}$$



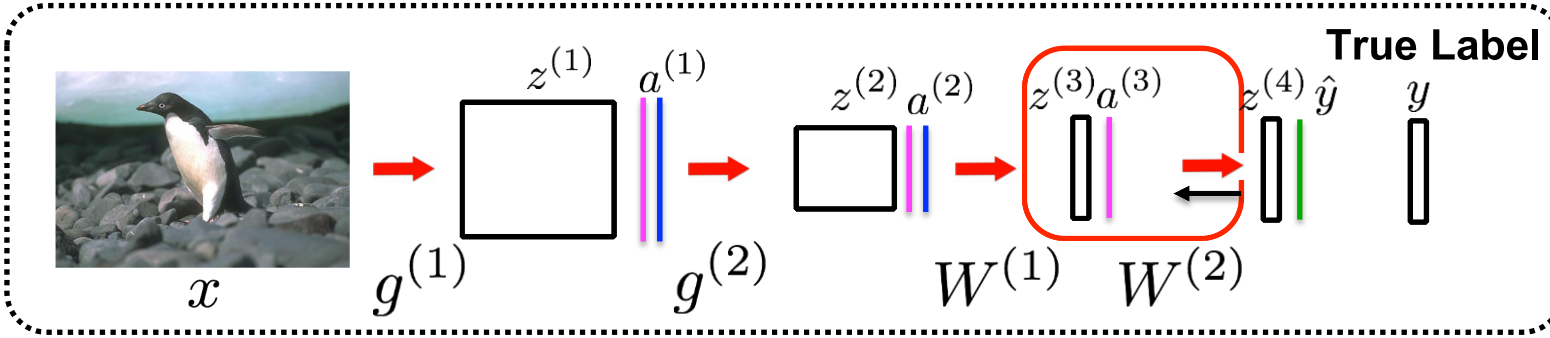
Adjusting the weights:

Need to compute the following gradient $$\frac{\partial L}{\partial W^{(2)}} = \frac{\partial L}{\partial z^{(4)}} \frac{\partial z^{(4)}}{\partial W^{(2)}}$$

Update rule: $$W_{ij}^{(2)} = W_{ij}^{(2)} - \alpha \frac{\partial L}{\partial W_{ij}^{(2)}}$$

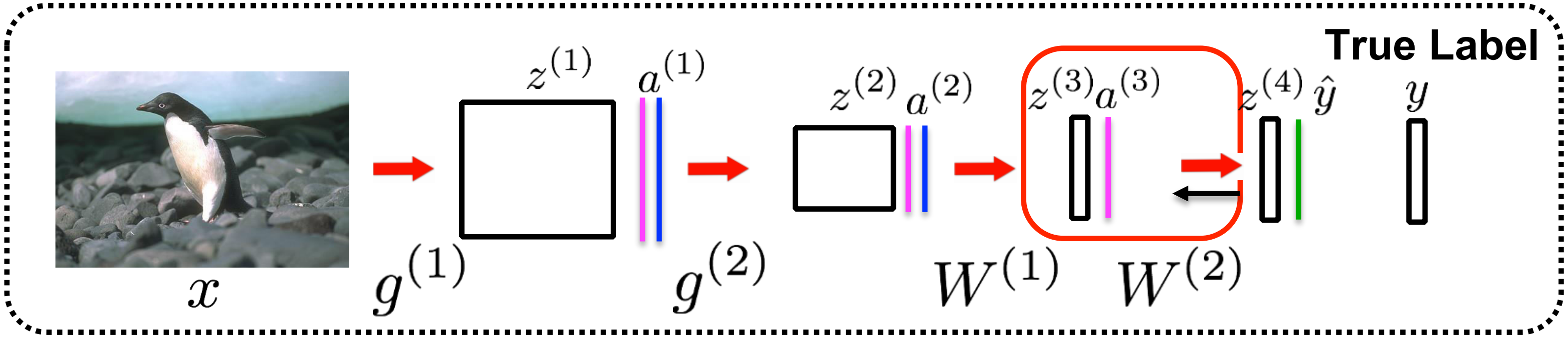


Backpropagating the gradients:



Backpropagating the gradients:

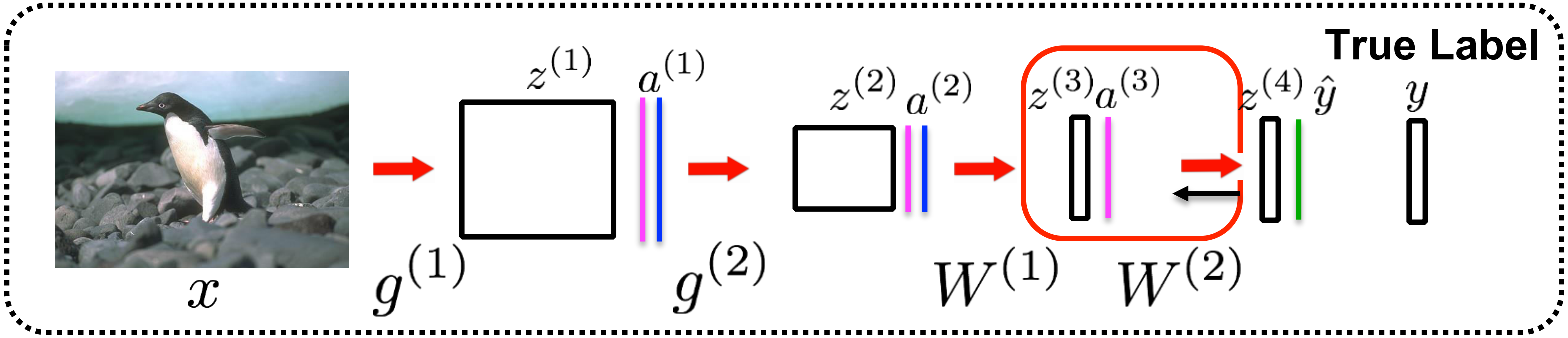
Need to compute the following gradient: $$\frac{\partial L}{\partial z^{(3)}} = \frac{\partial L}{\partial z^{(4)}} \frac{\partial z^{(4)}}{\partial f(z^{(3)})} \frac{\partial f(z^{(3)})}{\partial z^{(3)}}$$



Backpropagating the gradients:

Need to compute the following gradient:

$$\frac{\partial L}{\partial z^{(3)}} = \boxed{\frac{\partial L}{\partial z^{(4)}}} \frac{\partial z^{(4)}}{\partial f(z^{(3)})} \frac{\partial f(z^{(3)})}{\partial z^{(3)}}$$



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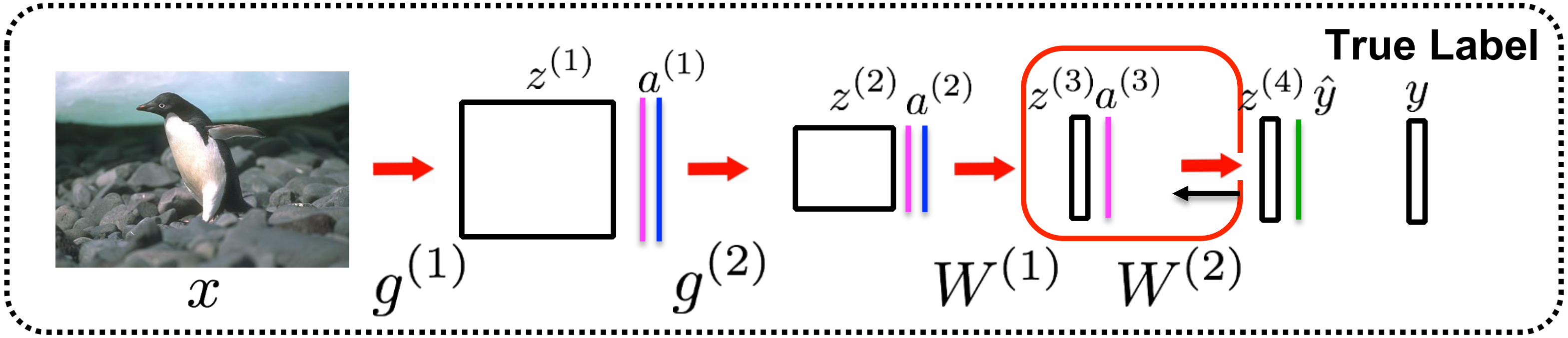
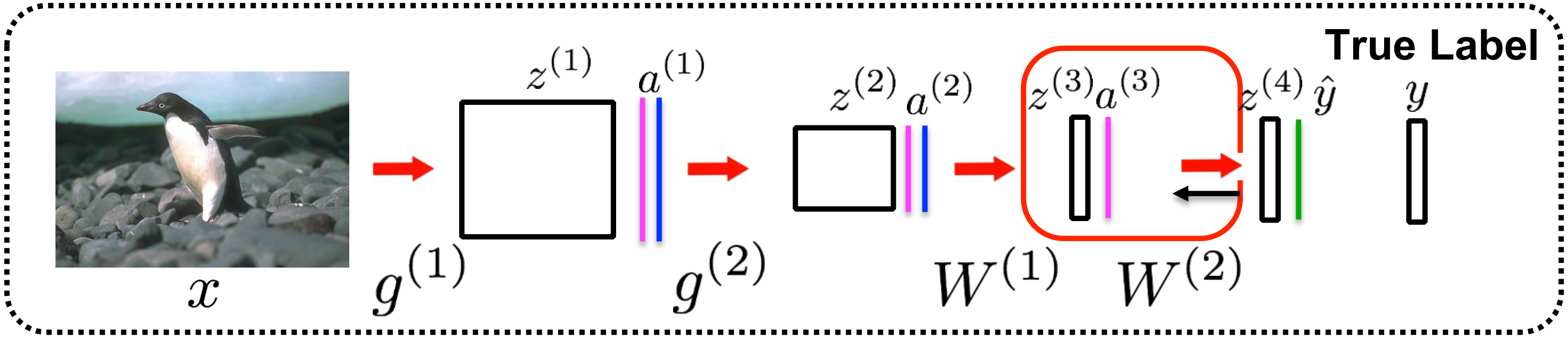
$\boxed{\frac{\partial L}{\partial z^{(4)}}}$ was already computed in the previous step



Backpropagating the gradients:

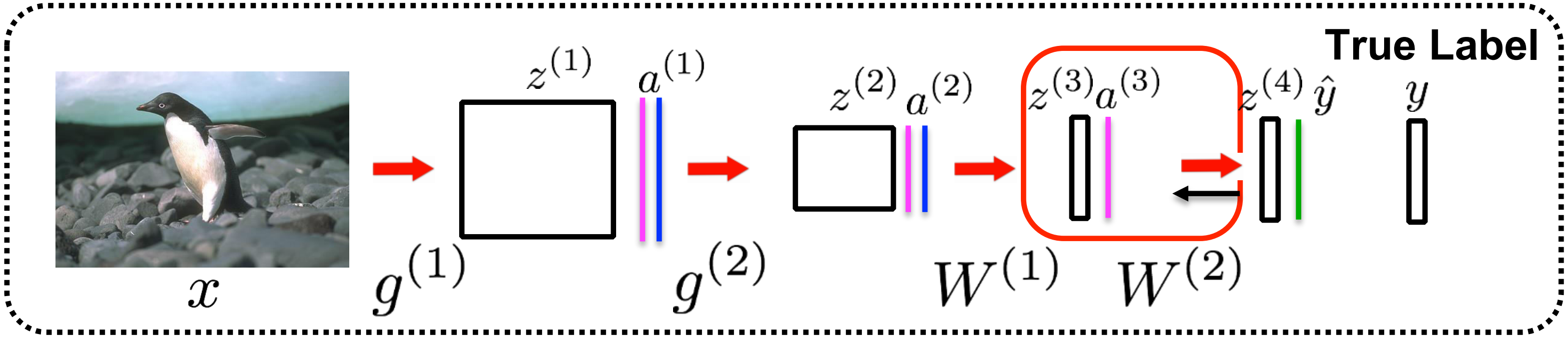
Need to compute the following gradient: $$\frac{\partial L}{\partial z^{(3)}} = \frac{\partial L}{\partial z^{(4)}} \boxed{\frac{\partial z^{(4)}}{\partial f(z^{(3)})}} \frac{\partial f(z^{(3)})}{\partial z^{(3)}}$$



Backpropagating the gradients:

Need to compute the following gradient: $$\frac{\partial L}{\partial z^{(3)}} = \frac{\partial L}{\partial z^{(4)}} \boxed{\frac{\partial z^{(4)}}{\partial f(z^{(3)})}} \frac{\partial f(z^{(3)})}{\partial z^{(3)}}$$

$$z_i^{(4)} = \sum_{k=1}^{N} W_{ik}^{(2)} f(z_k^{(3)}) \quad \text{where} \quad f(z^{(3)}) = a^{(3)}$$

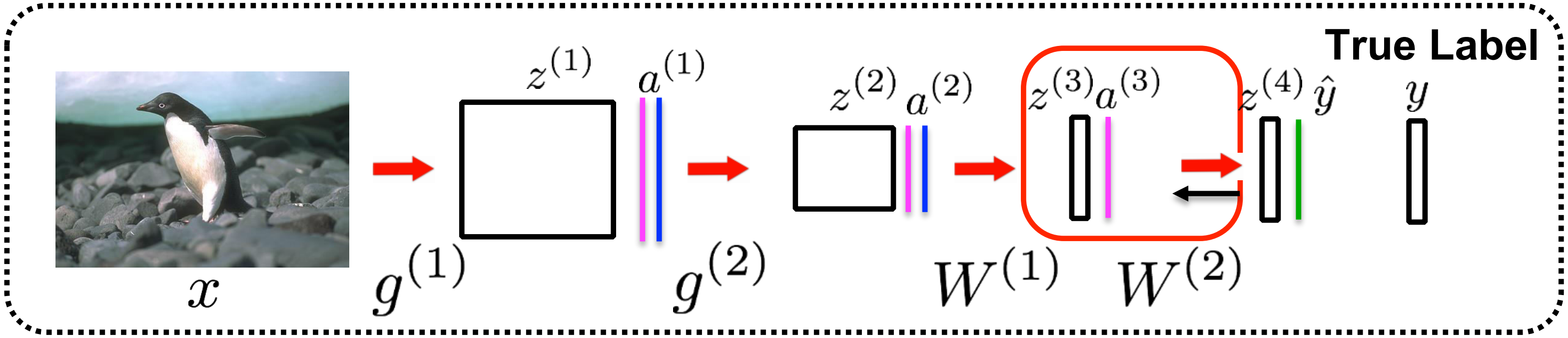


Backpropagating the gradients:

Need to compute the following gradient: $$\frac{\partial L}{\partial z^{(3)}} = \frac{\partial L}{\partial z^{(4)}} \boxed{\frac{\partial z^{(4)}}{\partial f(z^{(3)})}} \frac{\partial f(z^{(3)})}{\partial z^{(3)}}$$

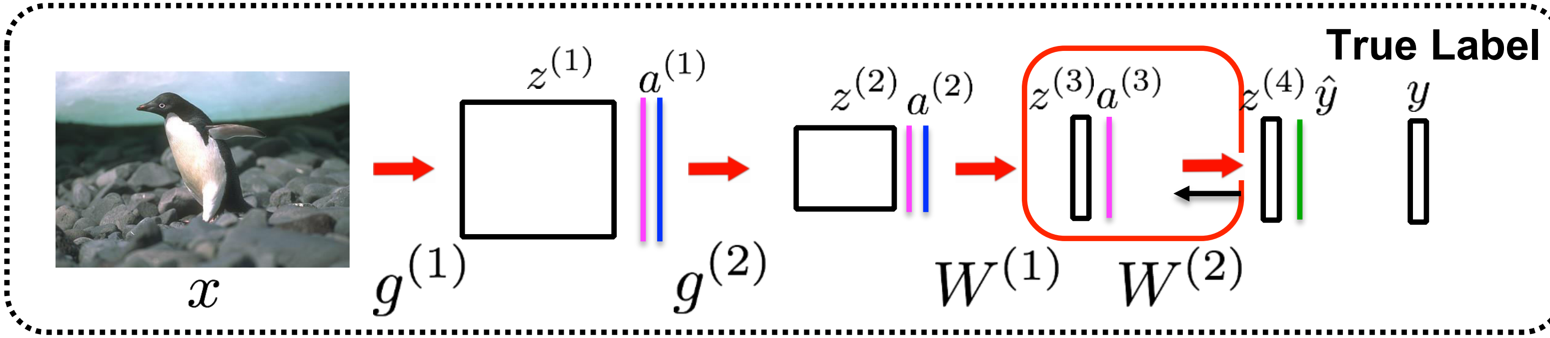
$$z_i^{(4)} = \sum_{k=1}^{N} W_{ik}^{(2)} f(z_k^{(3)})$$ where $$f(z^{(3)}) = a^{(3)}$$

$$\boxed{\frac{\partial z_i^{(4)}}{\partial f(z_j^{(3)})} = W_{ij}^{(2)}}$$



Backpropagating the gradients:

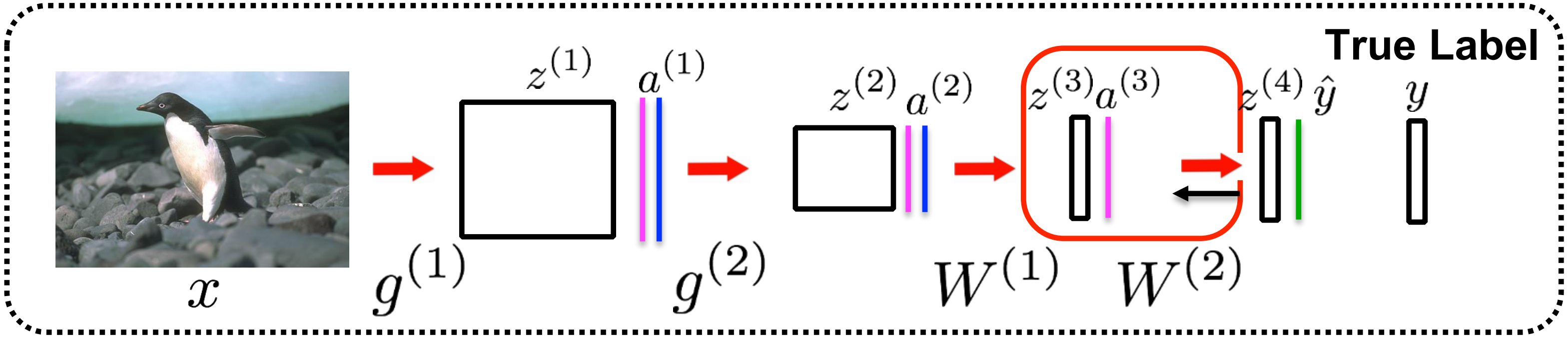
Need to compute the following gradient: $$\frac{\partial L}{\partial z^{(3)}} = \frac{\partial L}{\partial z^{(4)}} \frac{\partial z^{(4)}}{\partial f(z^{(3)})} \boxed{\frac{\partial f(z^{(3)})}{\partial z^{(3)}}}$$



Backpropagating the gradients:

Need to compute the following gradient: $$\frac{\partial L}{\partial z^{(3)}} = \frac{\partial L}{\partial z^{(4)}} \frac{\partial z^{(4)}}{\partial f(z^{(3)})} \boxed{\frac{\partial f(z^{(3)})}{\partial z^{(3)}}}$$

$$f(z^{(3)}) = \frac{1}{1 + \exp(-z^{(3)})}$$

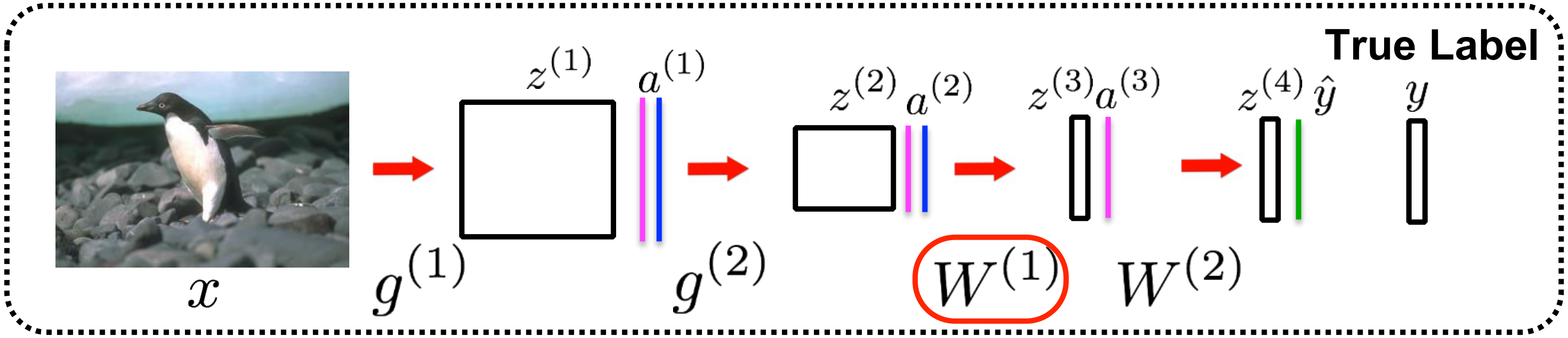


Backpropagating the gradients:

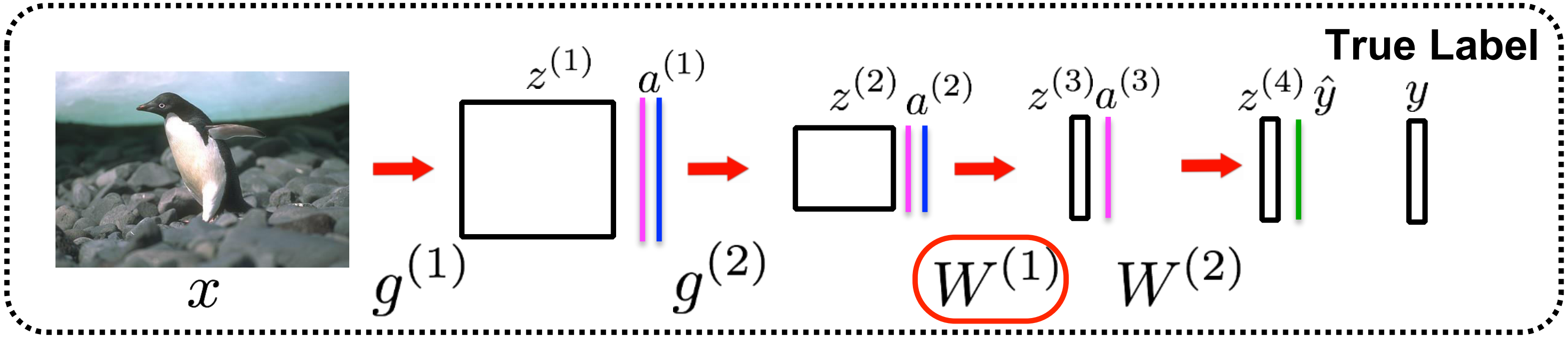
Need to compute the following gradient: $$\frac{\partial L}{\partial z^{(3)}} = \frac{\partial L}{\partial z^{(4)}} \frac{\partial z^{(4)}}{\partial f(z^{(3)})} \boxed{\frac{\partial f(z^{(3)})}{\partial z^{(3)}}}$$

$$f(z^{(3)}) = \frac{1}{1 + \exp(-z^{(3)})}$$

$$\boxed{\frac{\partial f(z^{(3)})}{\partial z^{(3)}} = f(z^{(3)})(1 - f(z^{(3)}))}$$



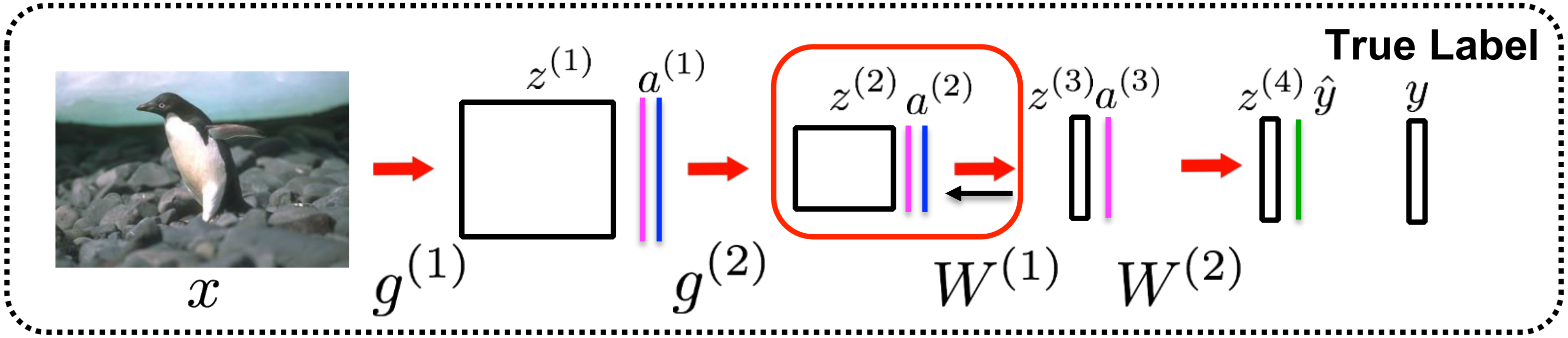
Adjusting the weights:



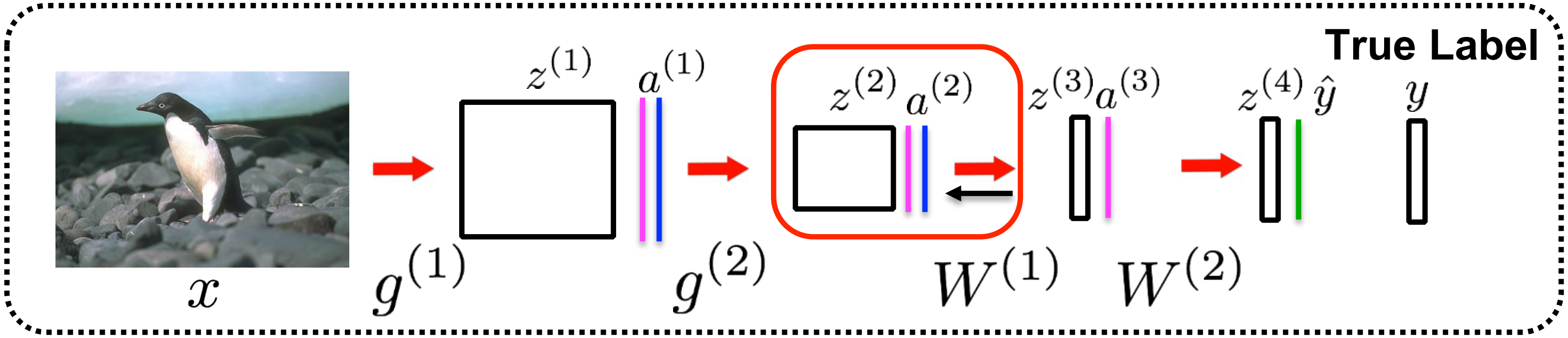
Adjusting the weights:

Need to compute the following gradient $\displaystyle \frac{\partial L}{\partial W^{(1)}} = \frac{\partial L}{\partial z^{(3)}} \frac{\partial z^{(3)}}{\partial W^{(1)}}$

Update rule: $\displaystyle W_{ij}^{(1)} = W_{ij}^{(1)} - \alpha \frac{\partial L}{\partial W_{ij}^{(1)}}$

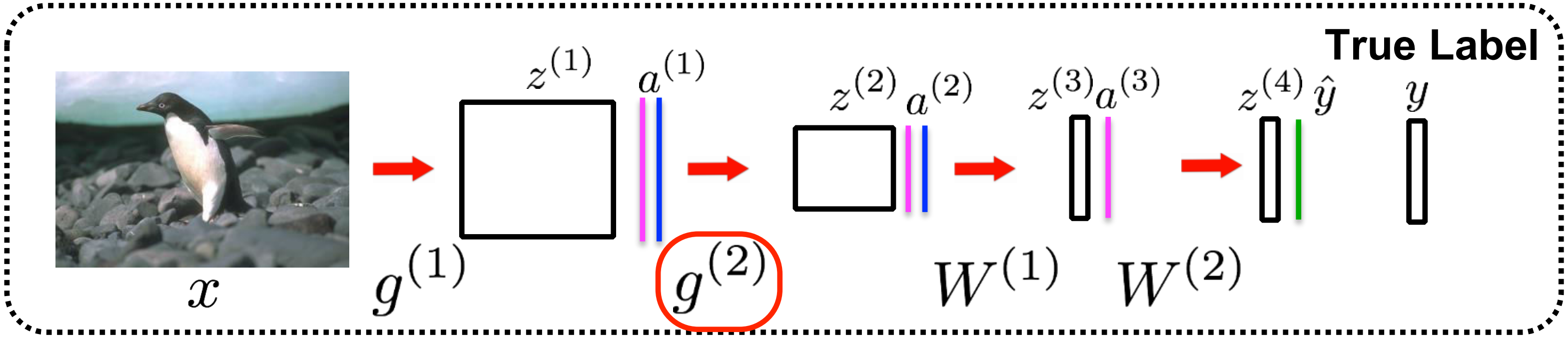


Backpropagating the gradients:

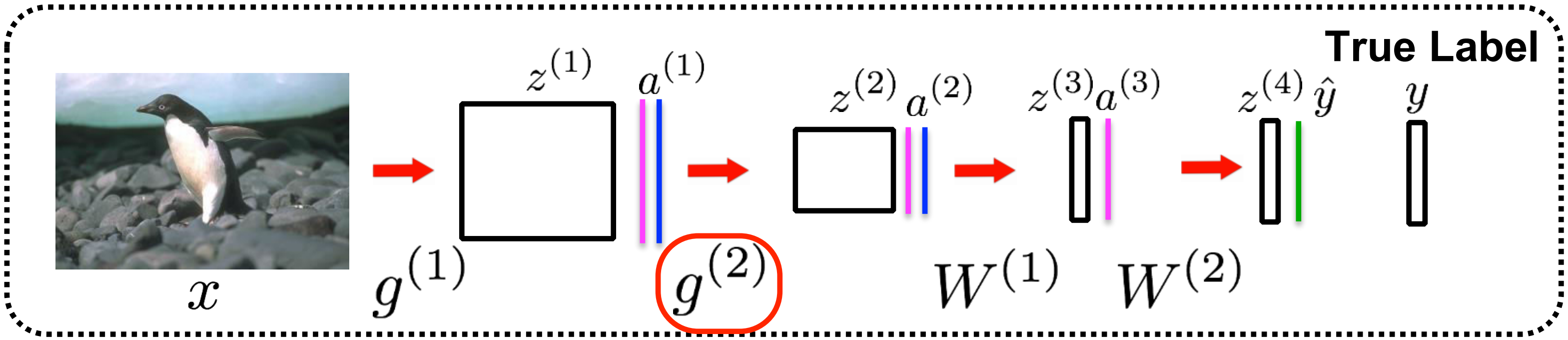


Backpropagating the gradients:

Need to compute the following gradient: $$\frac{\partial L}{\partial z^{(2)}} = \frac{\partial L}{\partial z^{(3)}} \frac{\partial z^{(3)}}{\partial f(z^{(2)})} \frac{\partial f(z^{(2)})}{\partial z^{(2)}}$$

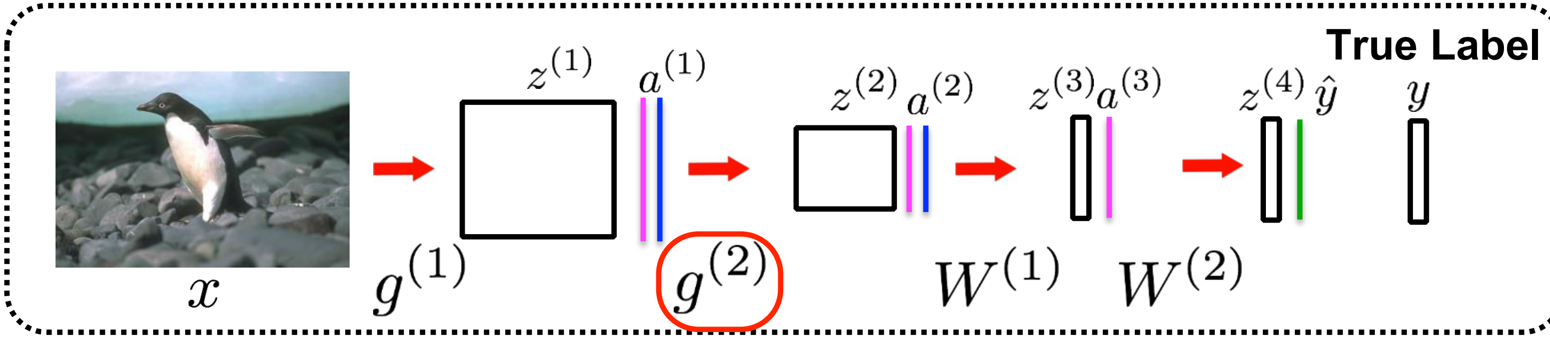


Adjusting the weights:



Adjusting the weights:

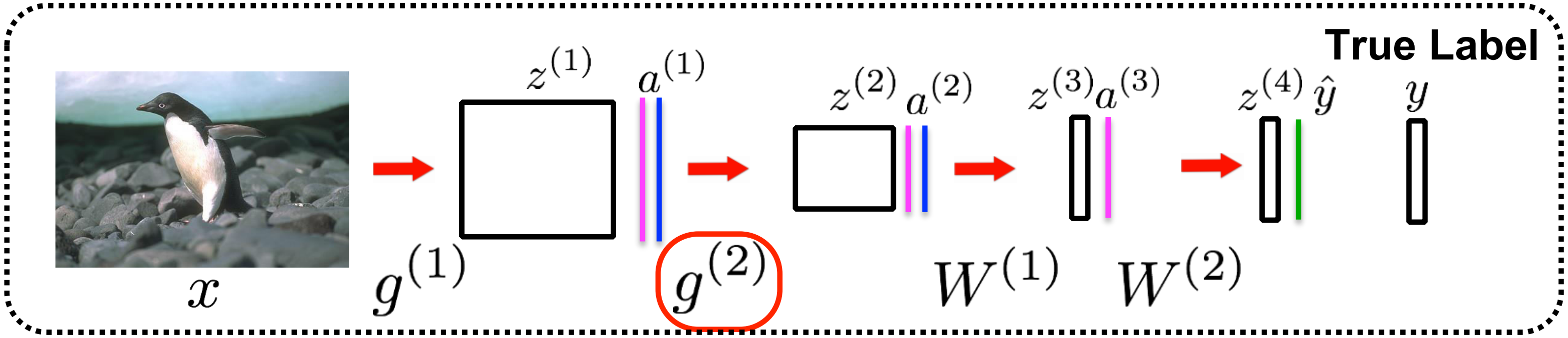
Need to compute the following gradient
$$\frac{\partial L}{\partial g^{(2)}} = \frac{\partial L}{\partial z^{(2)}} \frac{\partial z^{(2)}}{\partial g^{(2)}}$$



Adjusting the weights:

Need to compute the following gradient

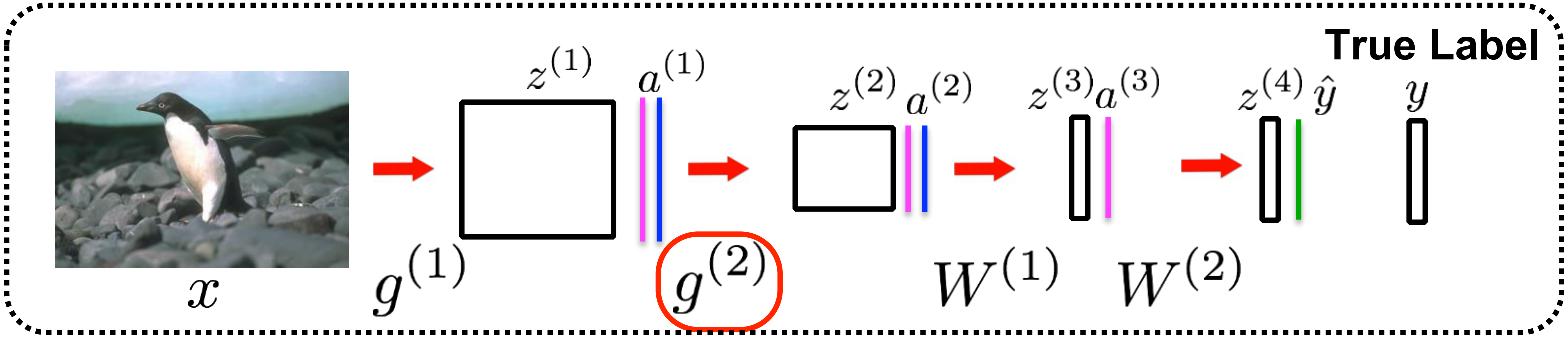
$$\frac{\partial L}{\partial g^{(2)}} = \boxed{\frac{\partial L}{\partial z^{(2)}}} \frac{\partial z^{(2)}}{\partial g^{(2)}}$$



Adjusting the weights:

Need to compute the following gradient $\frac{\partial L}{\partial g^{(2)}} = \boxed{\frac{\partial L}{\partial z^{(2)}}} \frac{\partial z^{(2)}}{\partial g^{(2)}}$

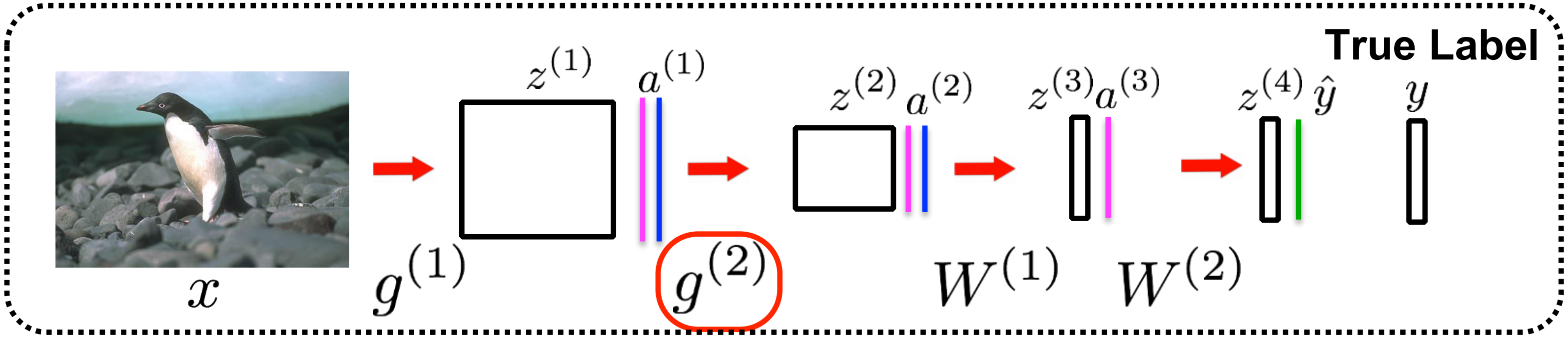
$\boxed{\frac{\partial L}{\partial z^{(2)}}}$ was already computed in the previous step



Adjusting the weights:

Need to compute the following gradient

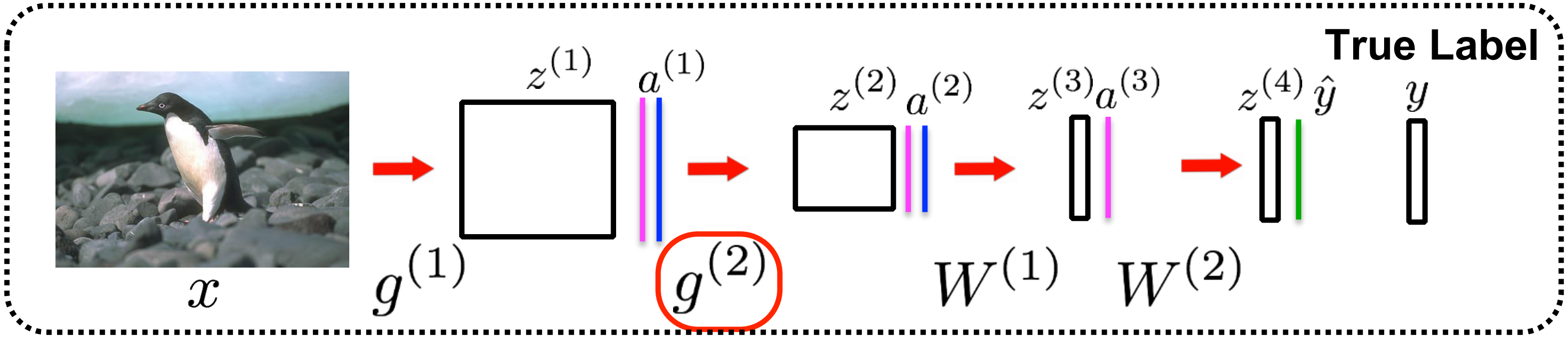
$$\frac{\partial L}{\partial g^{(2)}} = \frac{\partial L}{\partial z^{(2)}} \boxed{\frac{\partial z^{(2)}}{\partial g^{(2)}}}$$



<u>Adjusting the weights:</u>

Need to compute the following gradient $\frac{\partial L}{\partial g^{(2)}} = \frac{\partial L}{\partial z^{(2)}} \boxed{\frac{\partial z^{(2)}}{\partial g^{(2)}}}$

$z_{ij}^{(2)} = \sum_{u=0}^{M} \sum_{v=0}^{N} g_{uv}^{(2)} a_{(i-u)(j-v)}^{(1)}$ where $f(z^{(1)}) = a^{(1)}$



Adjusting the weights:

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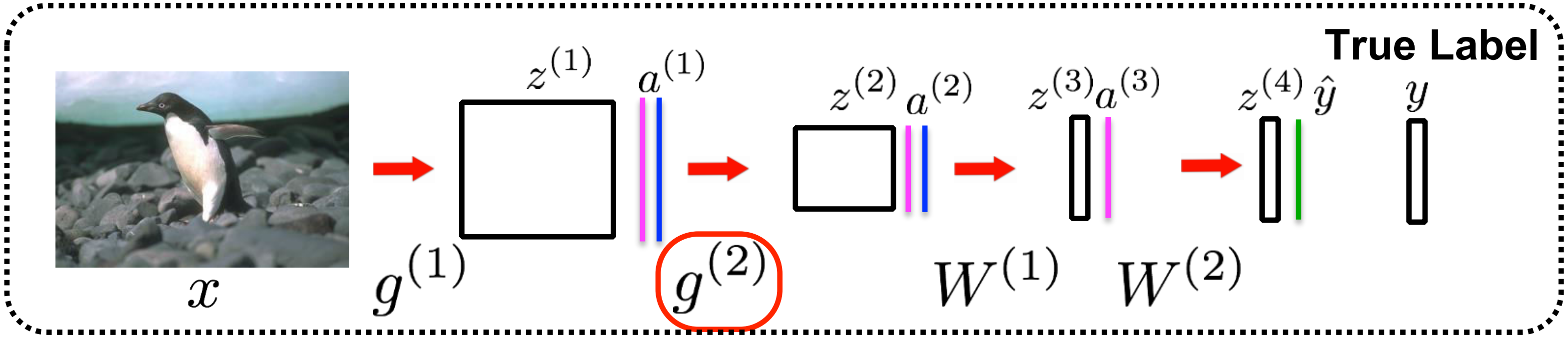
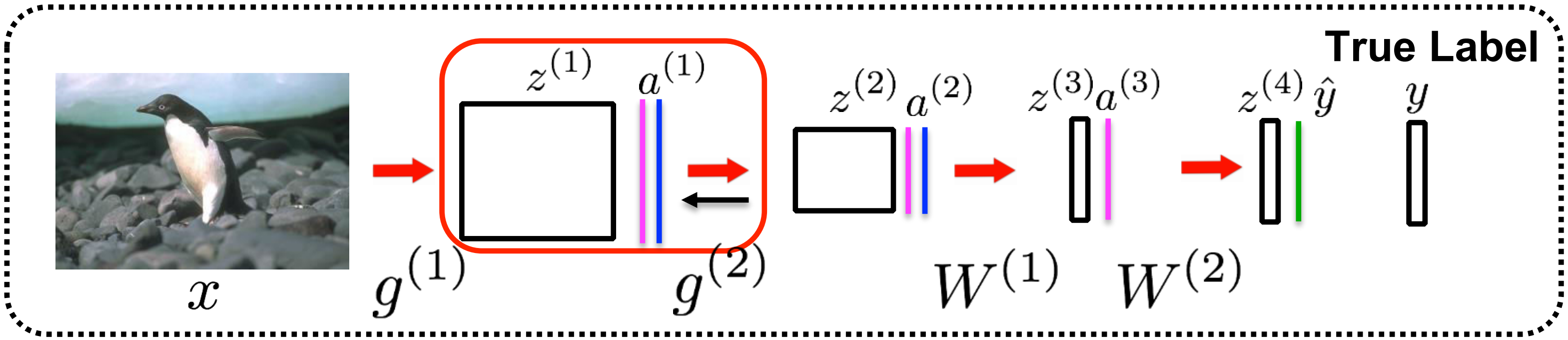
$$\boxed{\frac{\partial z_{ij}^{(2)}}{\partial g_{mn}^{(2)}} = a_{(i-m)(j-n)}^{(1)}}$$



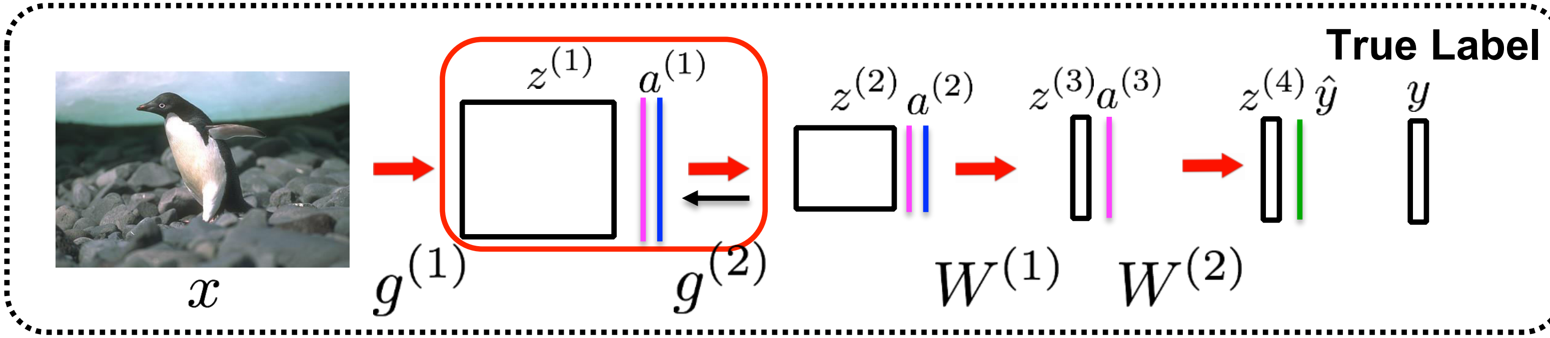
Adjusting the weights:

Need to compute the following gradient $$\frac{\partial L}{\partial g^{(2)}} = \frac{\partial L}{\partial z^{(2)}} \frac{\partial z^{(2)}}{\partial g^{(2)}}$$

Update rule: $$g^{(2)}_{mn} = g^{(2)}_{mn} - \alpha \frac{\partial L}{\partial g^{(2)}_{mn}}$$



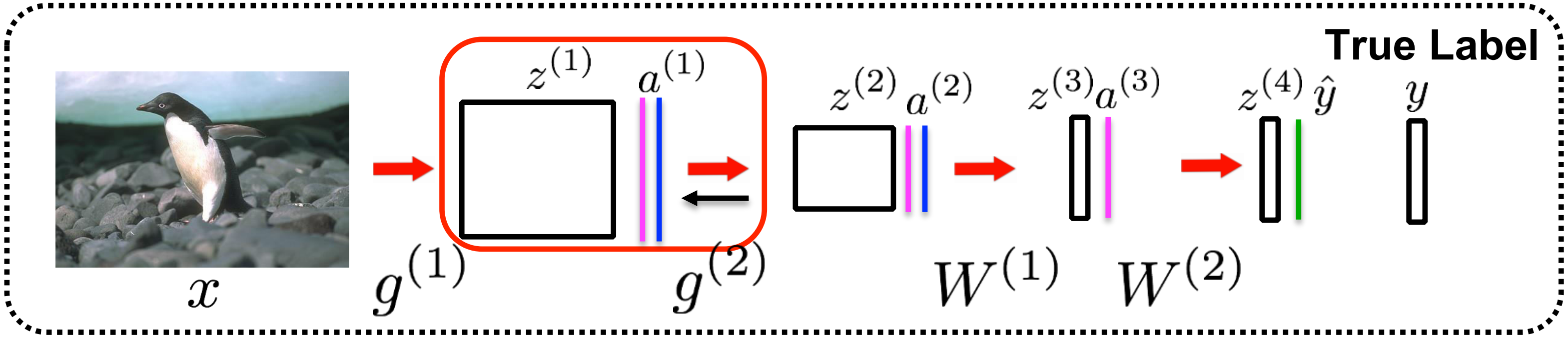
<u>Backpropagating the gradients:</u>



Backpropagating the gradients:

Need to compute the following gradient:

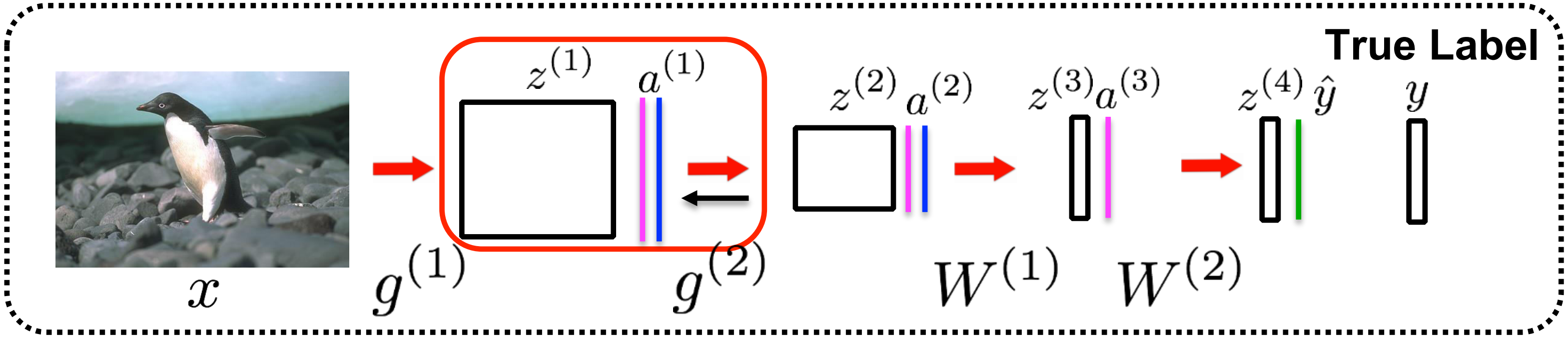
$$\frac{\partial L}{\partial z^{(1)}} = \frac{\partial L}{\partial z^{(2)}} \frac{\partial z^{(2)}}{\partial f(z^{(1)})} \frac{\partial f(z^{(1)})}{\partial z^{(1)}}$$



Backpropagating the gradients:

Need to compute the following gradient:

$$\frac{\partial L}{\partial z^{(1)}} = \boxed{\frac{\partial L}{\partial z^{(2)}}} \frac{\partial z^{(2)}}{\partial f(z^{(1)})} \frac{\partial f(z^{(1)})}{\partial z^{(1)}}$$



Backpropagating the gradients:

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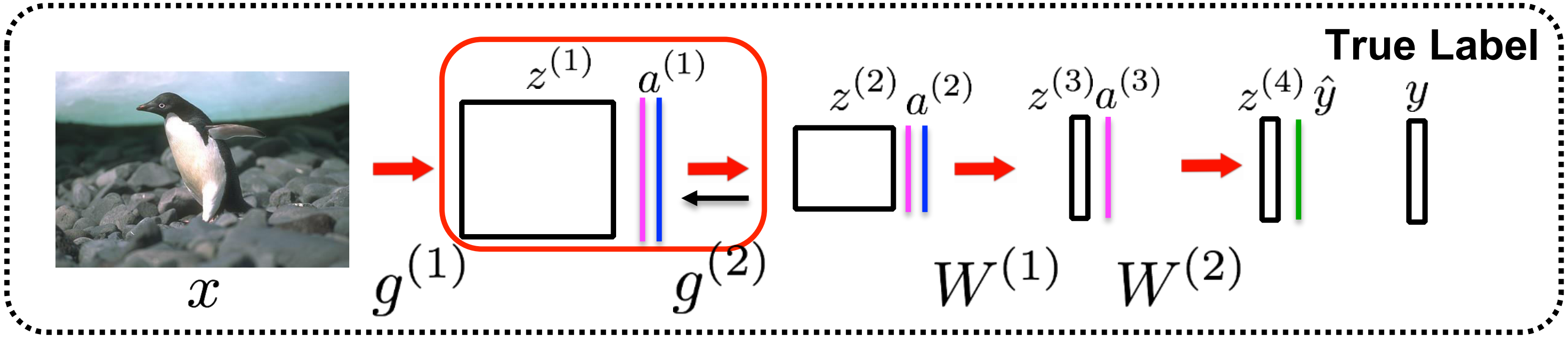
$\boxed{\frac{\partial L}{\partial z^{(2)}}}$ was already computed in the previous step



Backpropagating the gradients:

Need to compute the following gradient:

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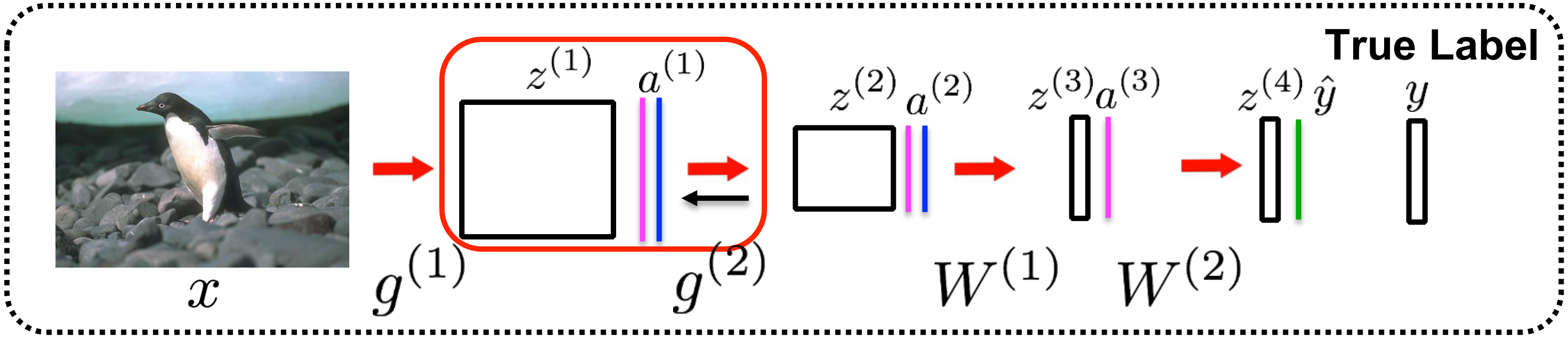
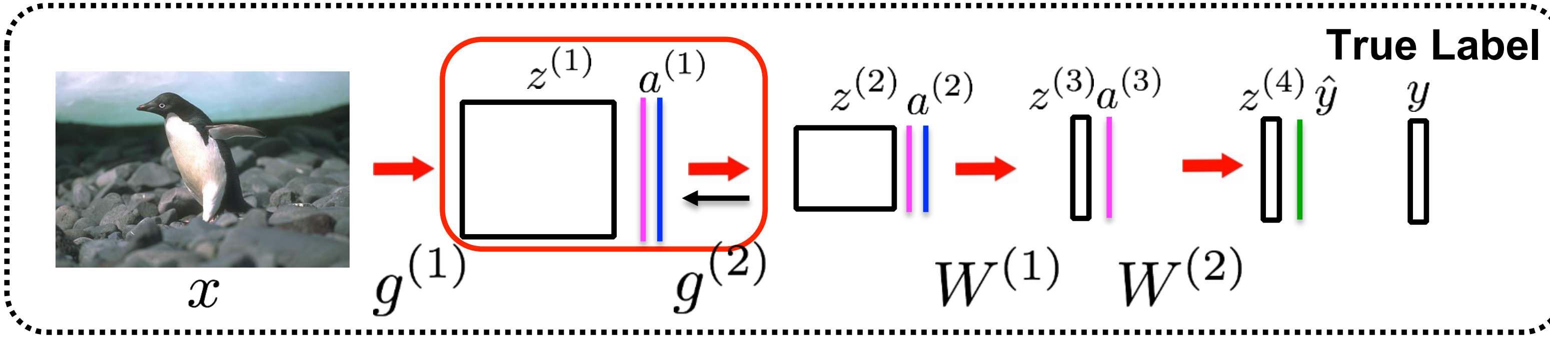


Backpropagating the gradients:

Need to compute the following gradient: $$\frac{\partial L}{\partial z^{(1)}} = \frac{\partial L}{\partial z^{(2)}} \boxed{\frac{\partial z^{(2)}}{\partial f(z^{(1)})}} \frac{\partial f(z^{(1)})}{\partial z^{(1)}}$$

$$z_{ij}^{(2)} = \sum_{u=0}^{M} \sum_{v=0}^{N} g_{uv}^{(2)} a_{(i-u)(j-v)}^{(1)} \quad \text{where} \quad f(z^{(1)}) = a^{(1)}$$

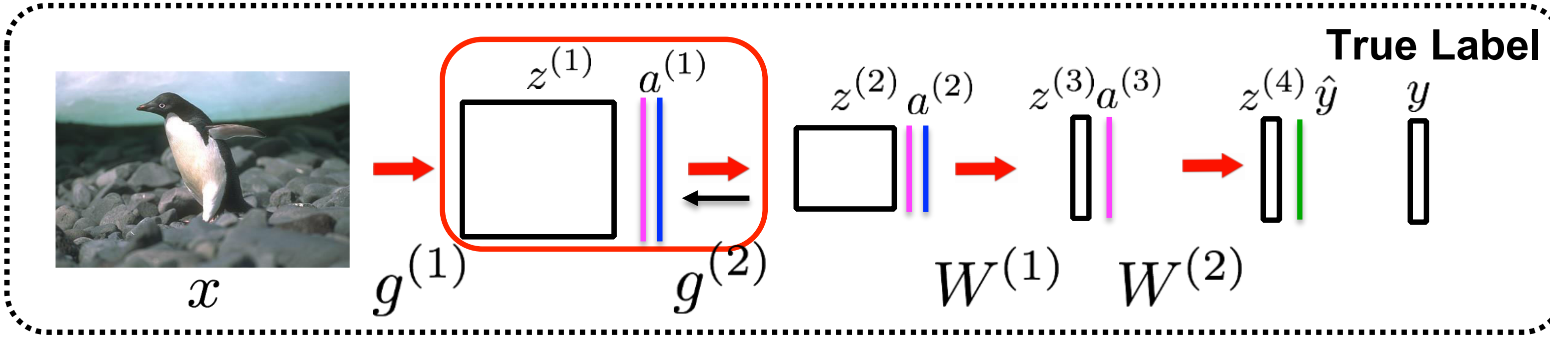


Backpropagating the gradients:

Need to compute the following gradient: $$\frac{\partial L}{\partial z^{(1)}} = \frac{\partial L}{\partial z^{(2)}} \boxed{\frac{\partial z^{(2)}}{\partial f(z^{(1)})}} \frac{\partial f(z^{(1)})}{\partial z^{(1)}}$$

$$z_{ij}^{(2)} = \sum_{u=0}^{M} \sum_{v=0}^{N} g_{uv}^{(2)} a_{(i-u)(j-v)}^{(1)} \quad \text{where} \quad f(z^{(1)}) = a^{(1)}$$

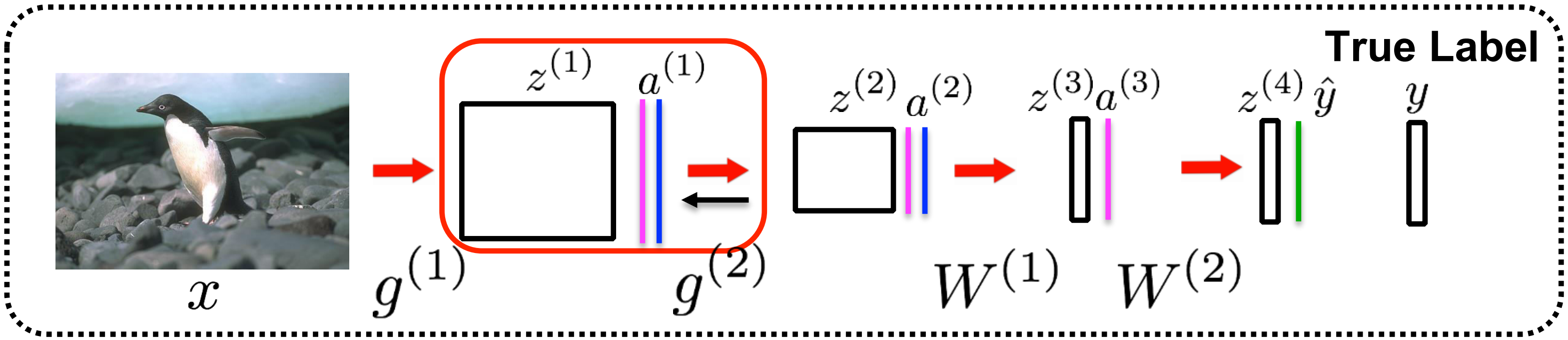
$$\boxed{\frac{\partial z_{ij}^{(2)}}{\partial a_{(i-m)(j-n)}^{(1)}} = g_{mn}^{(2)}}$$



Backpropagating the gradients:

Need to compute the following gradient:

$$\frac{\partial L}{\partial z^{(1)}} = \frac{\partial L}{\partial z^{(2)}} \frac{\partial z^{(2)}}{\partial f(z^{(1)})} \boxed{\frac{\partial f(z^{(1)})}{\partial z^{(1)}}}$$

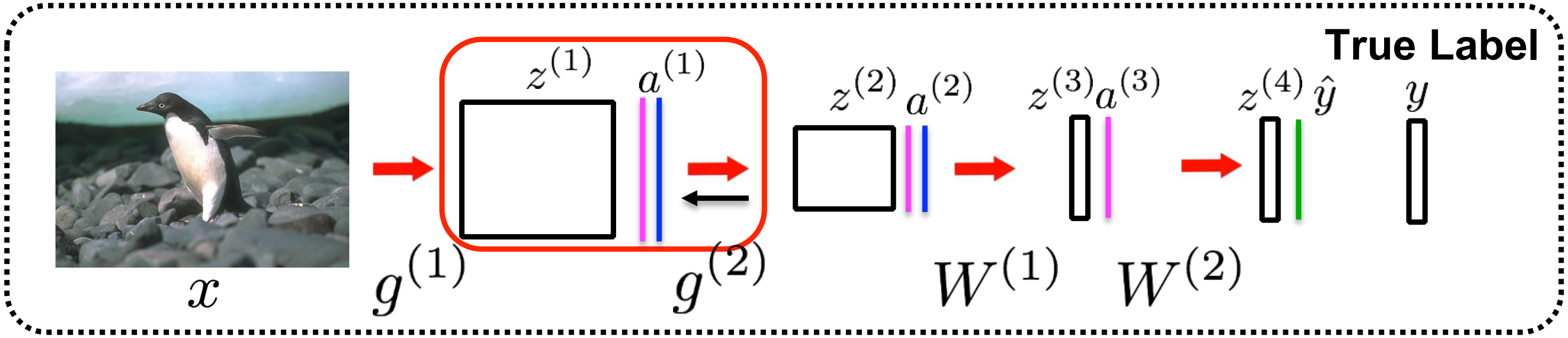


Backpropagating the gradients:

Need to compute the following gradient:

$$\frac{\partial L}{\partial z^{(1)}} = \frac{\partial L}{\partial z^{(2)}} \frac{\partial z^{(2)}}{\partial f(z^{(1)})} \boxed{\frac{\partial f(z^{(1)})}{\partial z^{(1)}}}$$

$$f(z^{(1)}) = \frac{1}{1 + \exp(-z^{(1)})}$$



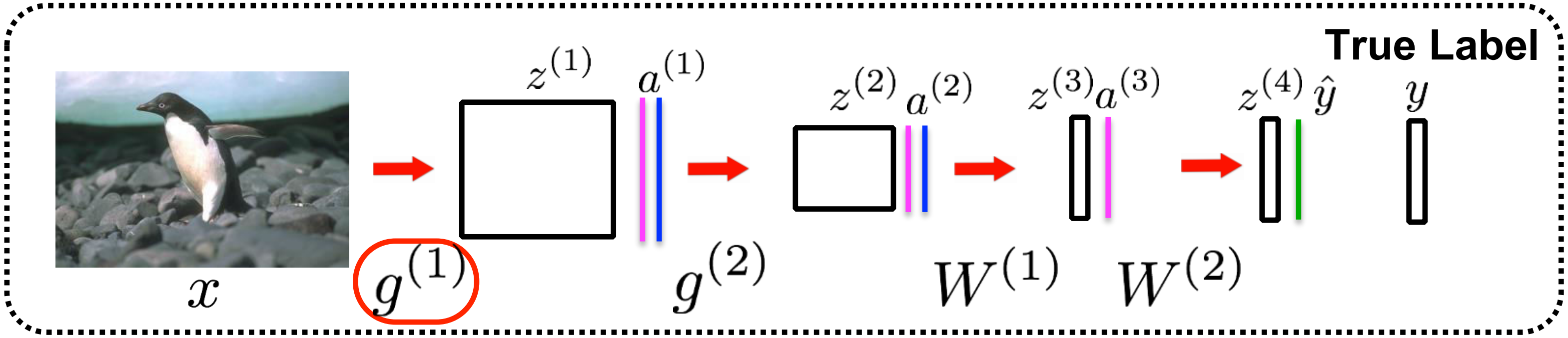
Backpropagating the gradients:

Need to compute the following gradient:

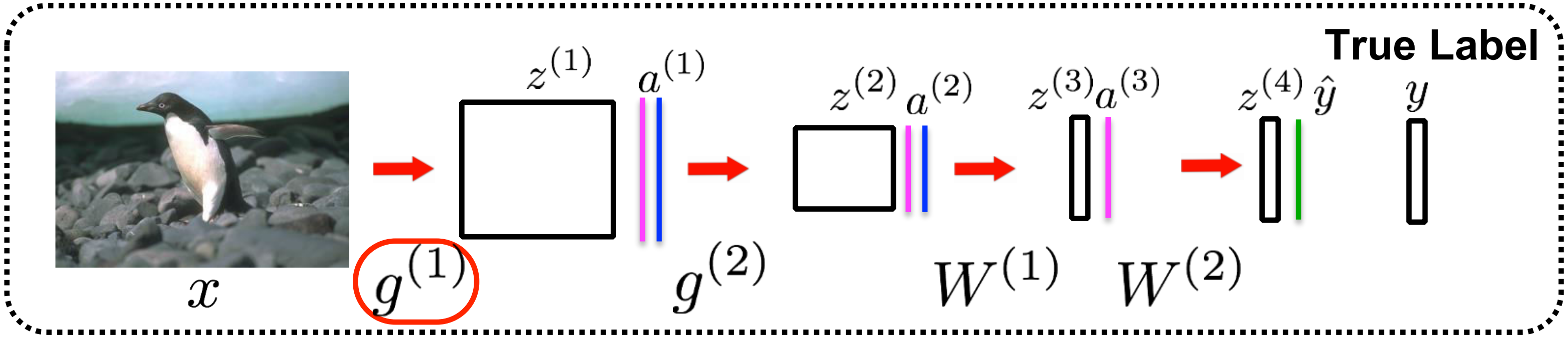
$$\frac{\partial L}{\partial z^{(1)}} = \frac{\partial L}{\partial z^{(2)}} \frac{\partial z^{(2)}}{\partial f(z^{(1)})} \boxed{\frac{\partial f(z^{(1)})}{\partial z^{(1)}}}$$

$$f(z^{(1)}) = \frac{1}{1 + \exp\left(-z^{(1)}\right)}$$

$$\boxed{\frac{\partial f(z^{(1)})}{\partial z^{(1)}} = f(z^{(1)})(1 - f(z^{(1)}))}$$



Adjusting the weights:



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Update rule: $$g^{(1)}_{mn} = g^{(1)}_{mn} - \alpha \frac{\partial L}{\partial g^{(1)}_{mn}}$$

Video 12.4

Jianbo Shi

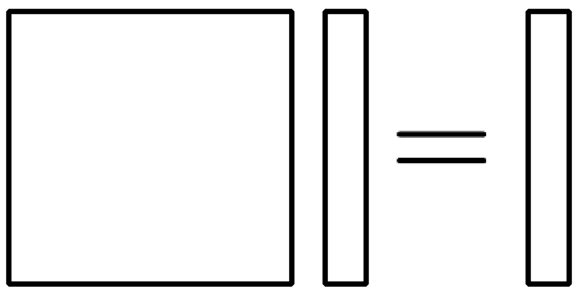
Visual illustration

Backpropagation

Convolutional Neural Networks

Fully Connected Layers:

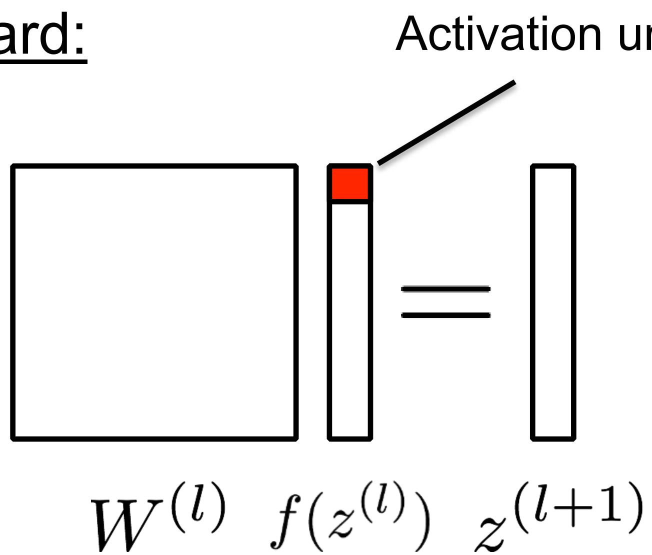
Forward:



$$W^{(l)} \quad f(z^{(l)}) \quad z^{(l+1)}$$

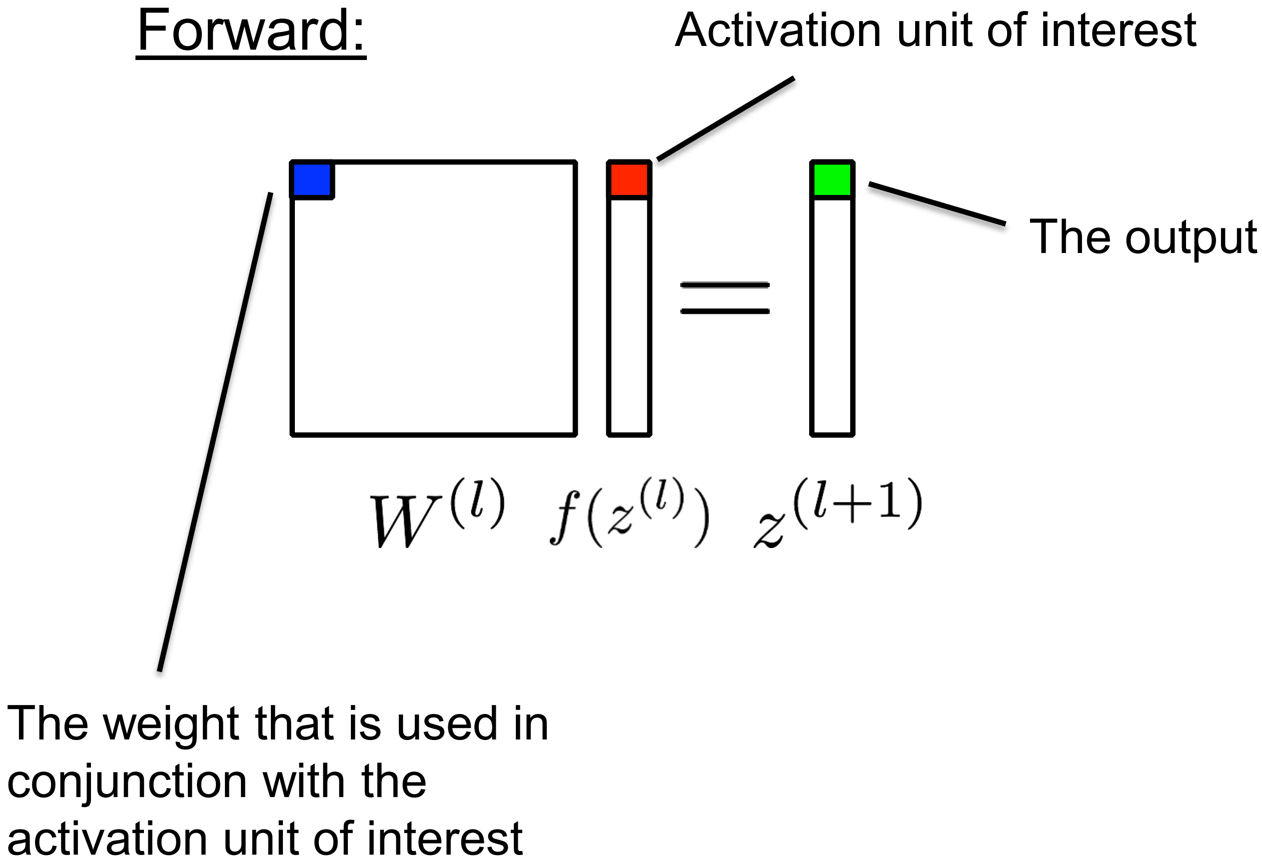
Fully Connected Layers:

Forward:



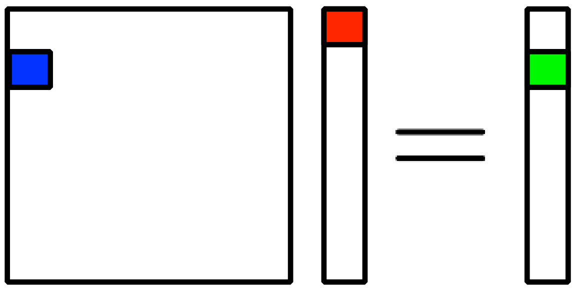
Fully Connected Layers:

Forward:



Fully Connected Layers:

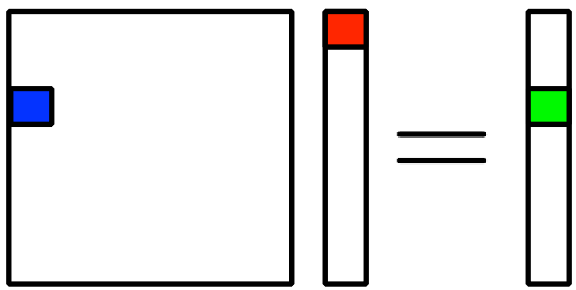
Forward:



$W^{(l)}$ $f(z^{(l)})$ $z^{(l+1)}$

Fully Connected Layers:

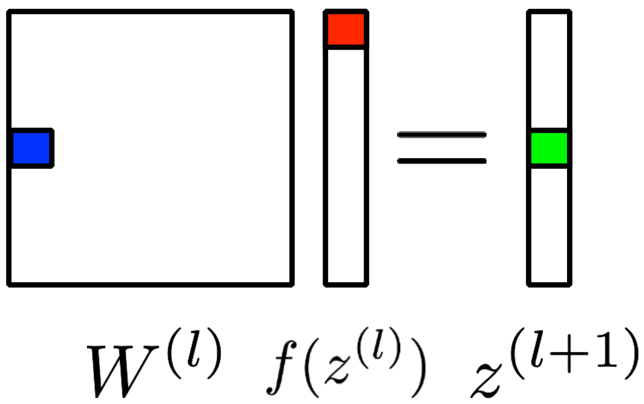
Forward:



$W^{(l)} \quad f(z^{(l)}) \quad z^{(l+1)}$

Fully Connected Layers:

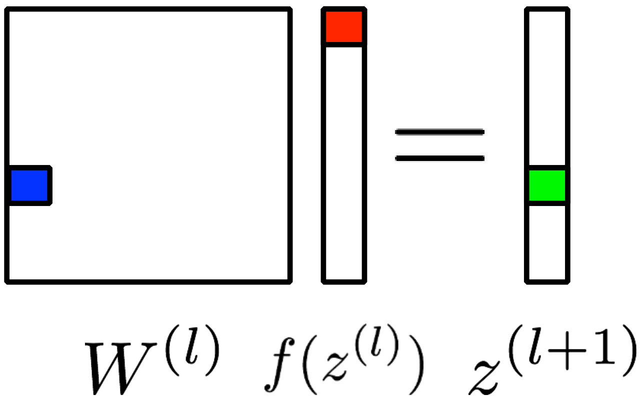
Forward:



Backpropagation

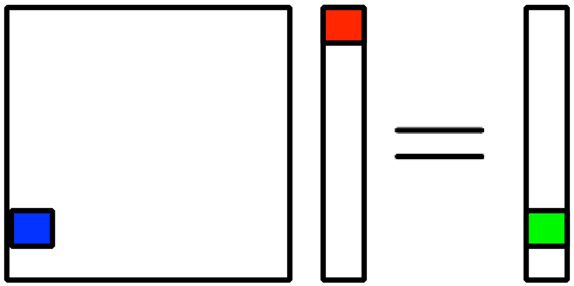
Fully Connected Layers:

Forward:



Fully Connected Layers:

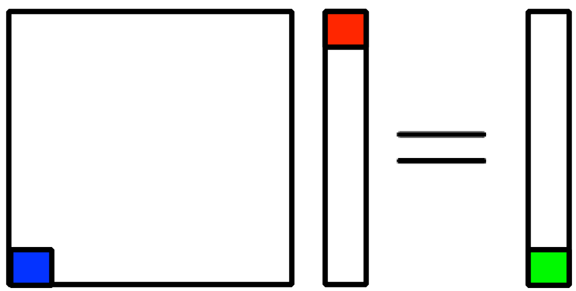
Forward:



$W^{(l)} \quad f(z^{(l)}) \quad z^{(l+1)}$

Fully Connected Layers:

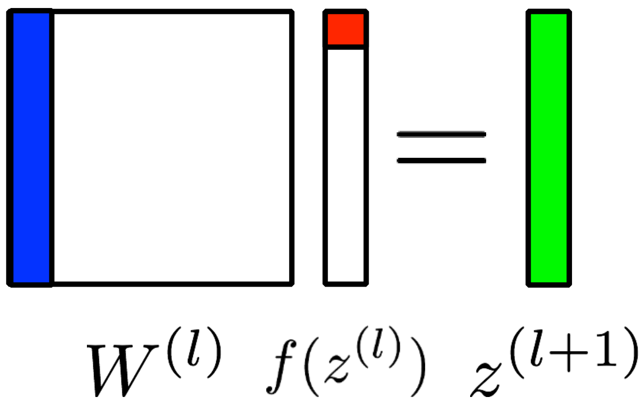
Forward:



$$W^{(l)} \quad f(z^{(l)}) \quad z^{(l+1)}$$

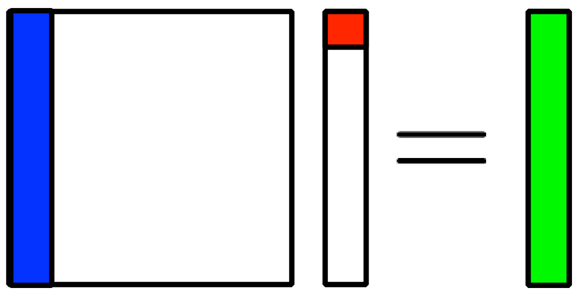
Fully Connected Layers:

Forward:



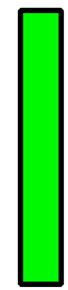
Fully Connected Layers:

Forward:



$W^{(l)}$ $f(z^{(l)})$ $z^{(l+1)}$

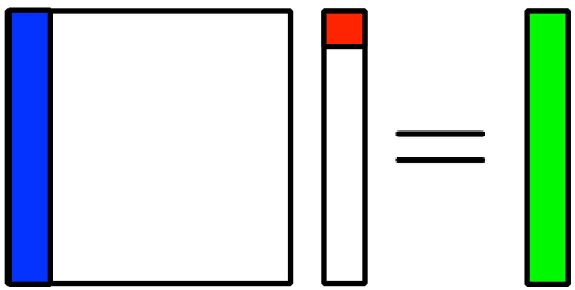
Backward:



$$\frac{\partial L}{\partial z^{(l+1)}}$$

Fully Connected Layers:

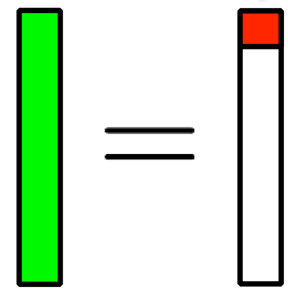
Forward:



$W^{(l)} \quad f(z^{(l)}) \quad z^{(l+1)}$

A measure how much an activation unit contributed to the loss

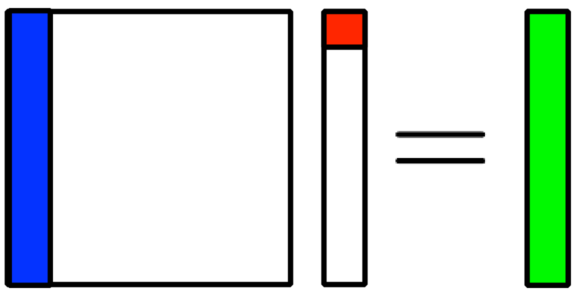
Backward:



$\frac{\partial L}{\partial z^{(l+1)}} \quad \frac{\partial L}{\partial f(z^{(l)})}$

Fully Connected Layers:

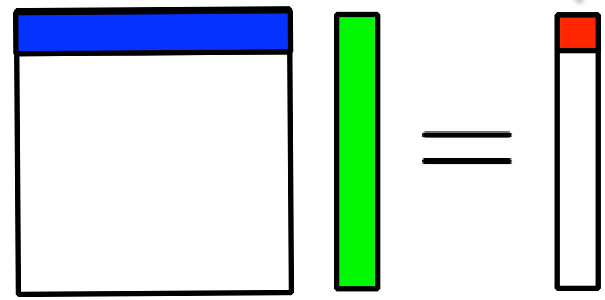
Forward:



$$W^{(l)} \quad f(z^{(l)}) \quad z^{(l+1)}$$

Backward:

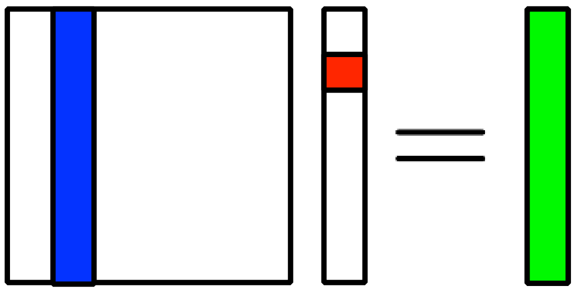
A measure how much an activation unit contributed to the loss



$$(W^{(l)})^T \quad \frac{\partial L}{\partial z^{(l+1)}} \quad \frac{\partial L}{\partial f(z^{(l)})}$$

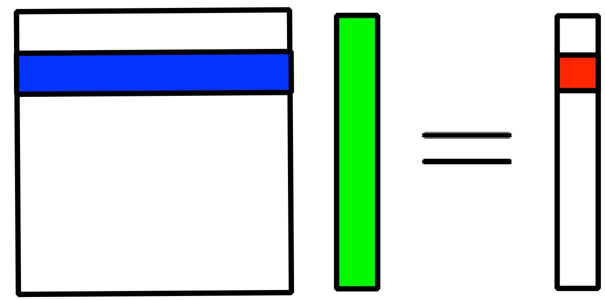
Fully Connected Layers:

Forward:



$$W^{(l)} \; f(z^{(l)}) = z^{(l+1)}$$

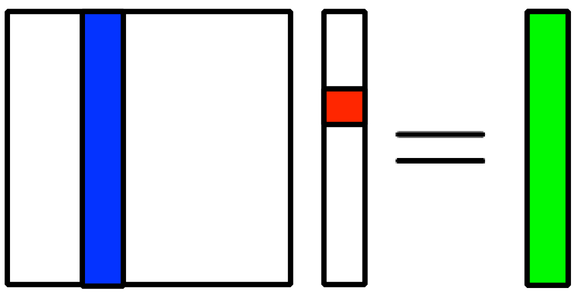
Backward:



$$(W^{(l)})^T \frac{\partial L}{\partial z^{(l+1)}} = \frac{\partial L}{\partial f(z^{(l)})}$$

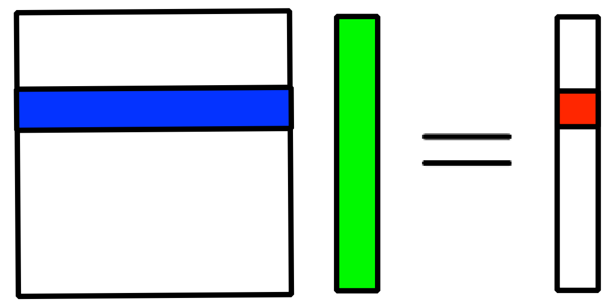
Fully Connected Layers:

Forward:



$W^{(l)} \quad f(z^{(l)}) \quad z^{(l+1)}$

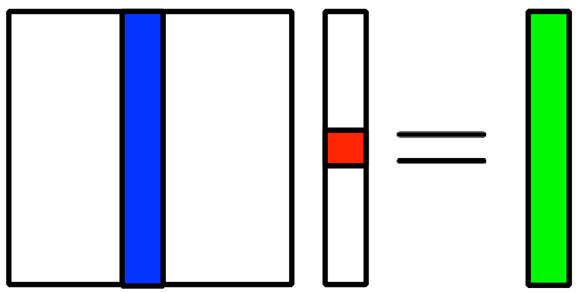
Backward:



$(W^{(l)})^T \quad \dfrac{\partial L}{\partial z^{(l+1)}} \quad \dfrac{\partial L}{\partial f(z^{(l)})}$

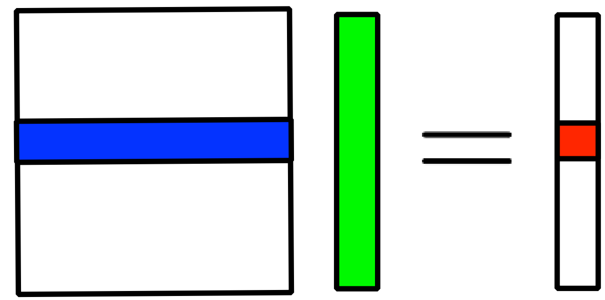
Fully Connected Layers:

Forward:



$$W^{(l)} \quad f(z^{(l)}) \quad z^{(l+1)}$$

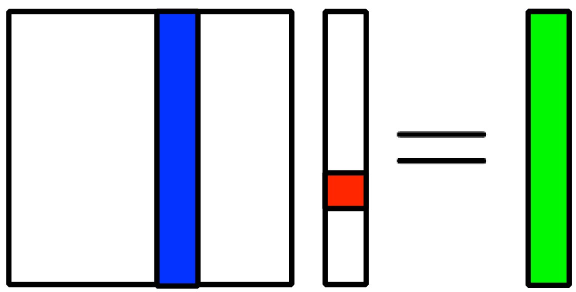
Backward:



$$(W^{(l)})^T \quad \frac{\partial L}{\partial z^{(l+1)}} \quad \frac{\partial L}{\partial f(z^{(l)})}$$

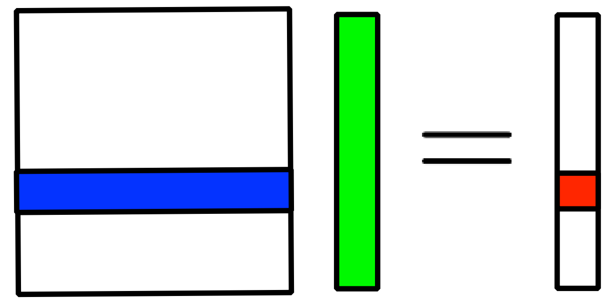
Fully Connected Layers:

Forward:



$$W^{(l)} \quad f(z^{(l)}) \quad z^{(l+1)}$$

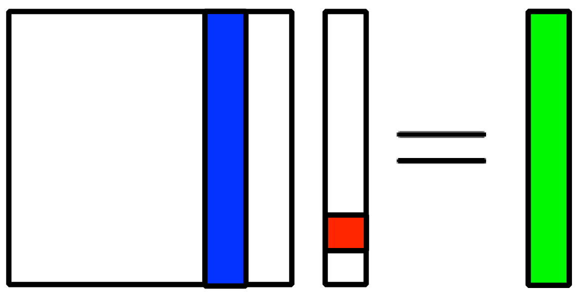
Backward:



$$(W^{(l)})^T \quad \frac{\partial L}{\partial z^{(l+1)}} \quad \frac{\partial L}{\partial f(z^{(l)})}$$

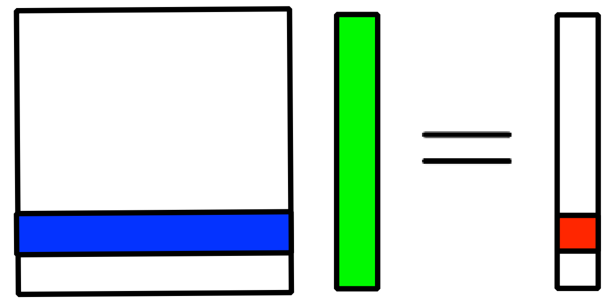
Fully Connected Layers:

Forward:



$$W^{(l)} \quad f(z^{(l)}) \quad z^{(l+1)}$$

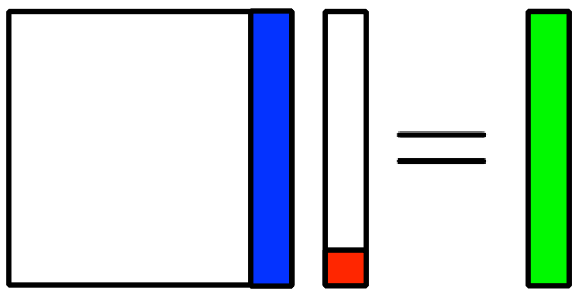
Backward:



$$(W^{(l)})^T \quad \frac{\partial L}{\partial z^{(l+1)}} \quad \frac{\partial L}{\partial f(z^{(l)})}$$

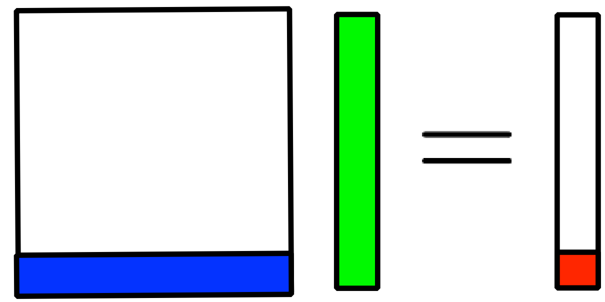
<u>**Fully Connected Layers:**</u>

<u>Forward:</u>



$$W^{(l)} \quad f(z^{(l)}) \quad z^{(l+1)}$$

<u>Backward:</u>



$$(W^{(l)})^T \quad \frac{\partial L}{\partial z^{(l+1)}} \quad \frac{\partial L}{\partial f(z^{(l)})}$$

Summary for fully connected layers

Backpropagation

Convolutional Neural Networks

Summary:

1. Let $\frac{\partial L}{\partial z_i^{(n)}} = \hat{y}_i - y_i$, where n denotes the number of layers in the network.

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2. For each fully connected layer l :

- For each node i in layer l set:

$$\frac{\partial L}{\partial z_i^{(l)}} = (\sum_{j=1}^{s^{l+1}} W_{ji}^{(l)} \frac{\partial L}{\partial z_j^{(l+1)}}) \frac{\partial f(z_i^{(l)})}{\partial z_i^{(l)}}$$

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1. Let $\frac{\partial L}{\partial z_i^{(n)}} = \hat{y}_i - y_i$, where n denotes the number of layers in the network.

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- Compute partial derivatives: $\frac{\partial L}{\partial W_{ij}^{(l)}} = f(z_j^{(l)}) \frac{\partial L}{\partial z_i^{(l+1)}}$

Summary:

1. Let $\frac{\partial L}{\partial z_i^{(n)}} = \hat{y}_i - y_i$, where n denotes the number of layers in the network.

2. For each fully connected layer l:

- For each node i in layer l set:

$$\frac{\partial L}{\partial z_i^{(l)}} = \left(\sum_{j=1}^{s^{l+1}} W_{ji}^{(l)} \frac{\partial L}{\partial z_j^{(l+1)}}\right) \frac{\partial f(z_i^{(l)})}{\partial z_i^{(l)}}$$

- Compute partial derivatives: $\frac{\partial L}{\partial W_{ij}^{(l)}} = f(z_j^{(l)}) \frac{\partial L}{\partial z_i^{(l+1)}}$

- Update the parameters: $W_{ij}^{(l)} = W_{ij}^{(l)} - \alpha \frac{\partial L}{\partial W_{ij}^{(l)}}$

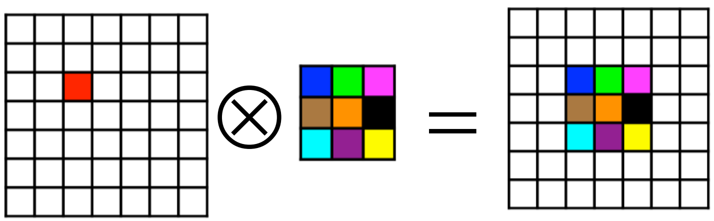
Visual illustration

Backpropagation

Convolutional Neural Networks

Convolutional Layers:

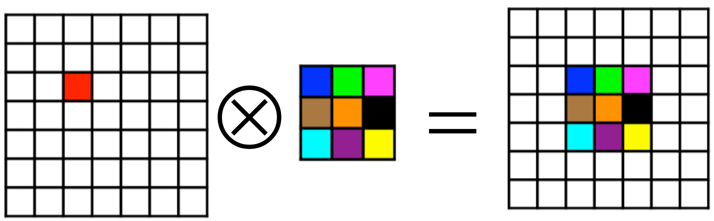
Forward:



$$a^{(l)} \otimes g^{(l)} = z^{(l+1)}$$

Convolutional Layers:

Forward:



$$a^{(l)} \otimes g^{(l)} = z^{(l+1)}$$

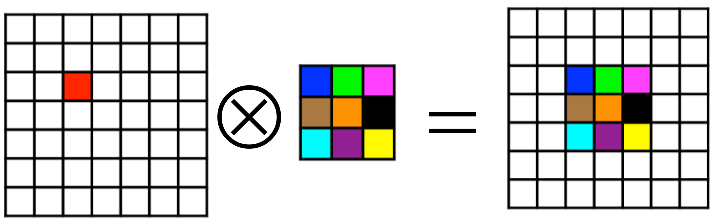
A measure how much an activation unit contributed to the loss

Backward:

$$\frac{\partial L}{\partial z^{(l+1)}} \qquad \frac{\partial L}{\partial f\left(z^{(l)}\right)}$$

Convolutional Layers:

Forward:



$$a^{(l)} \otimes g^{(l)} = z^{(l+1)}$$

Backward:

$$sum\ (\ g^{(l)} \odot \frac{\partial L}{\partial z^{(l+1)}}\ ) = \frac{\partial L}{\partial f(z^{(l)})}$$

Summary:

1. Let $\frac{\partial L}{\partial z^{(c)}}$, where c denotes the index of a first fully connected layer.

2. For each convolutional layer l:

   - For each node ij in layer l set

$$\frac{\partial L}{\partial z_{ij}^{(l)}} = \left(\sum_{m=0}^{M} \sum_{n=0}^{N} g_{mn}^{(l)} \frac{\partial L}{\partial z_{(i+m)(j+n)}^{(l+1)}}\right) \frac{\partial f(z_{ij}^{(l)})}{\partial z_{ij}^{(l)}}$$

Summary:

1. Let $\frac{\partial L}{\partial z^{(c)}}$, where c denotes the index of a first fully connected layer.

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- Compute partial derivatives:

$$\frac{\partial L}{\partial g_{ij}^{(l)}} = \sum_{y=0}^{H}\sum_{x=0}^{W} \frac{\partial L}{\partial z_{yx}^{(l+1)}} f(z_{(y-i)(x-j)}^{(l)})$$

Summary:

1. Let $\frac{\partial L}{\partial z^{(c)}}$, where c denotes the index of a first fully connected layer.

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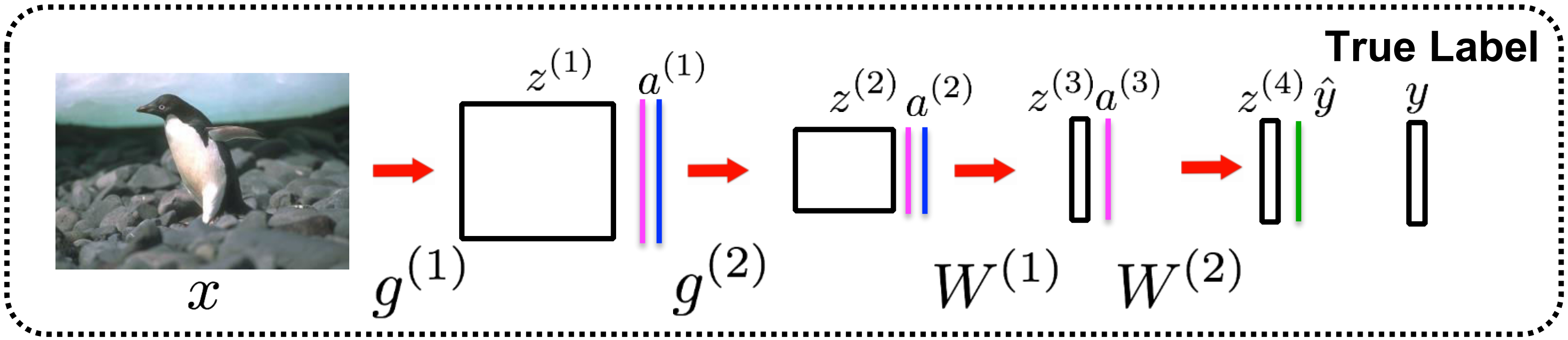
- Compute partial derivatives:

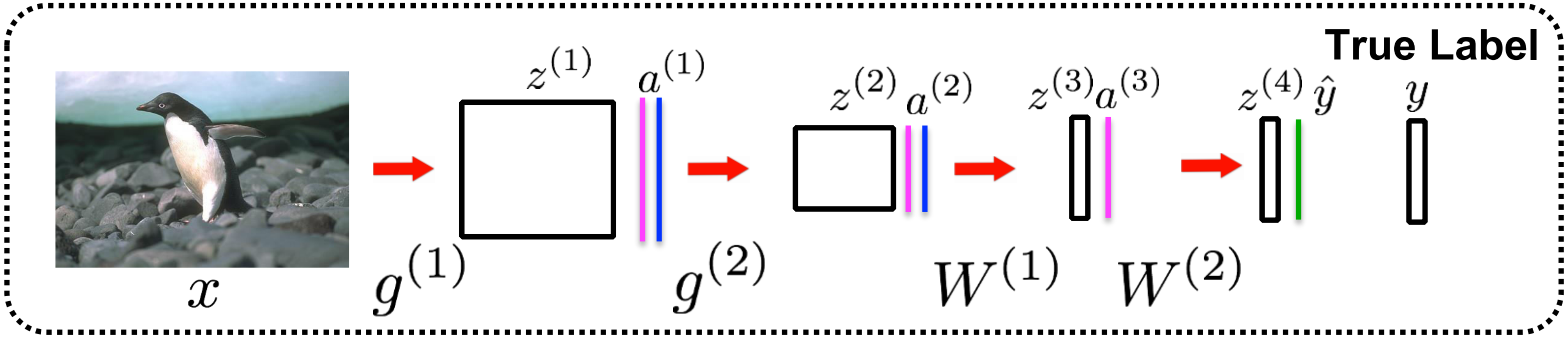
$$\frac{\partial L}{\partial g_{ij}^{(l)}} = \sum_{y=0}^{H}\sum_{x=0}^{W} \frac{\partial L}{\partial z_{yx}^{(l+1)}} f(z_{(y-i)(x-j)}^{(l)})$$

- Update the parameters: $g_{ij}^{(l)} = g_{ij}^{(l)} - \alpha \frac{\partial L}{\partial g_{ij}^{(l)}}$



Gradient in pooling layers:

- There is no learning done in the pooling layers
- The error that is backpropagated to the pooling layer, is sent back from to the node where it came from.



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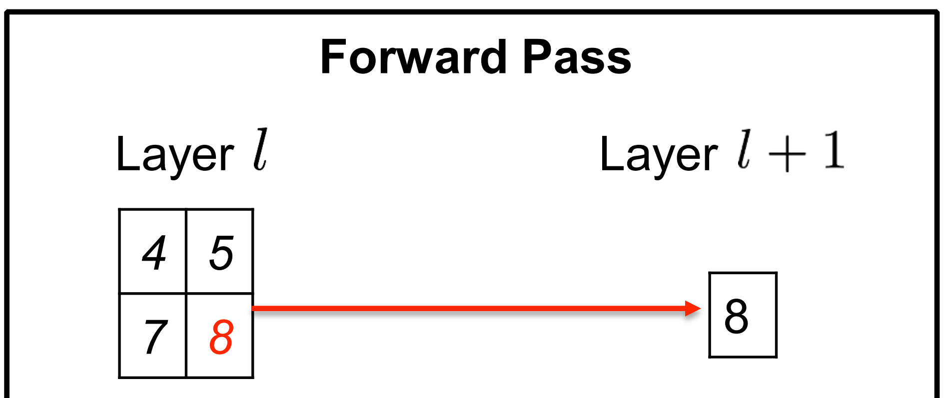

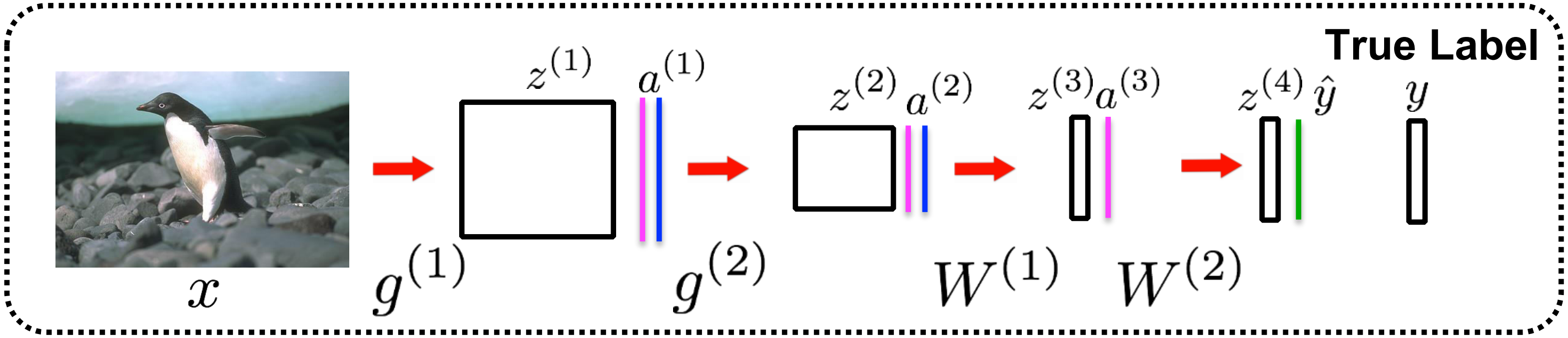

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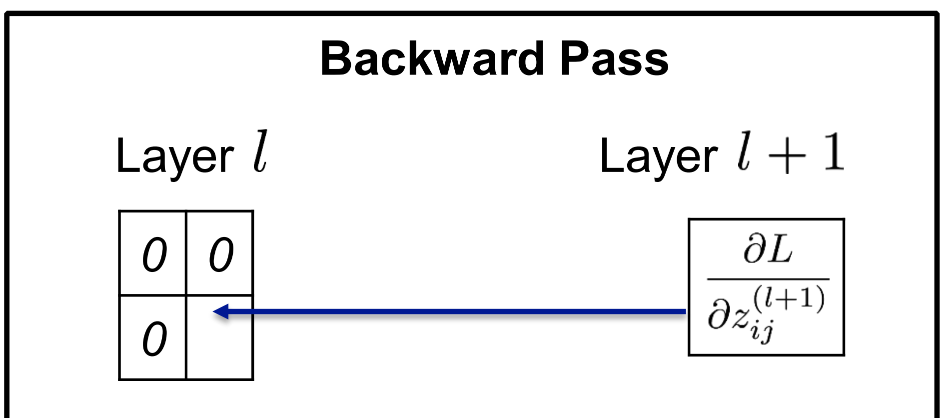
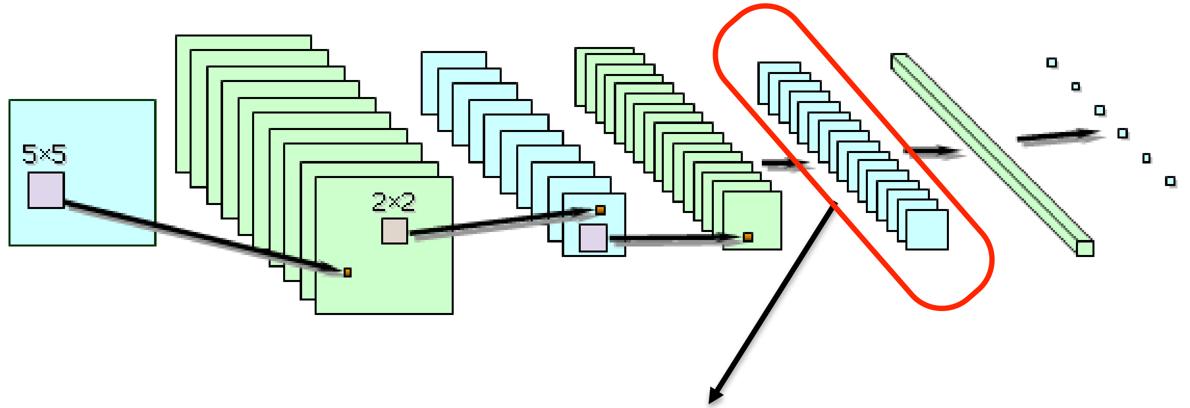
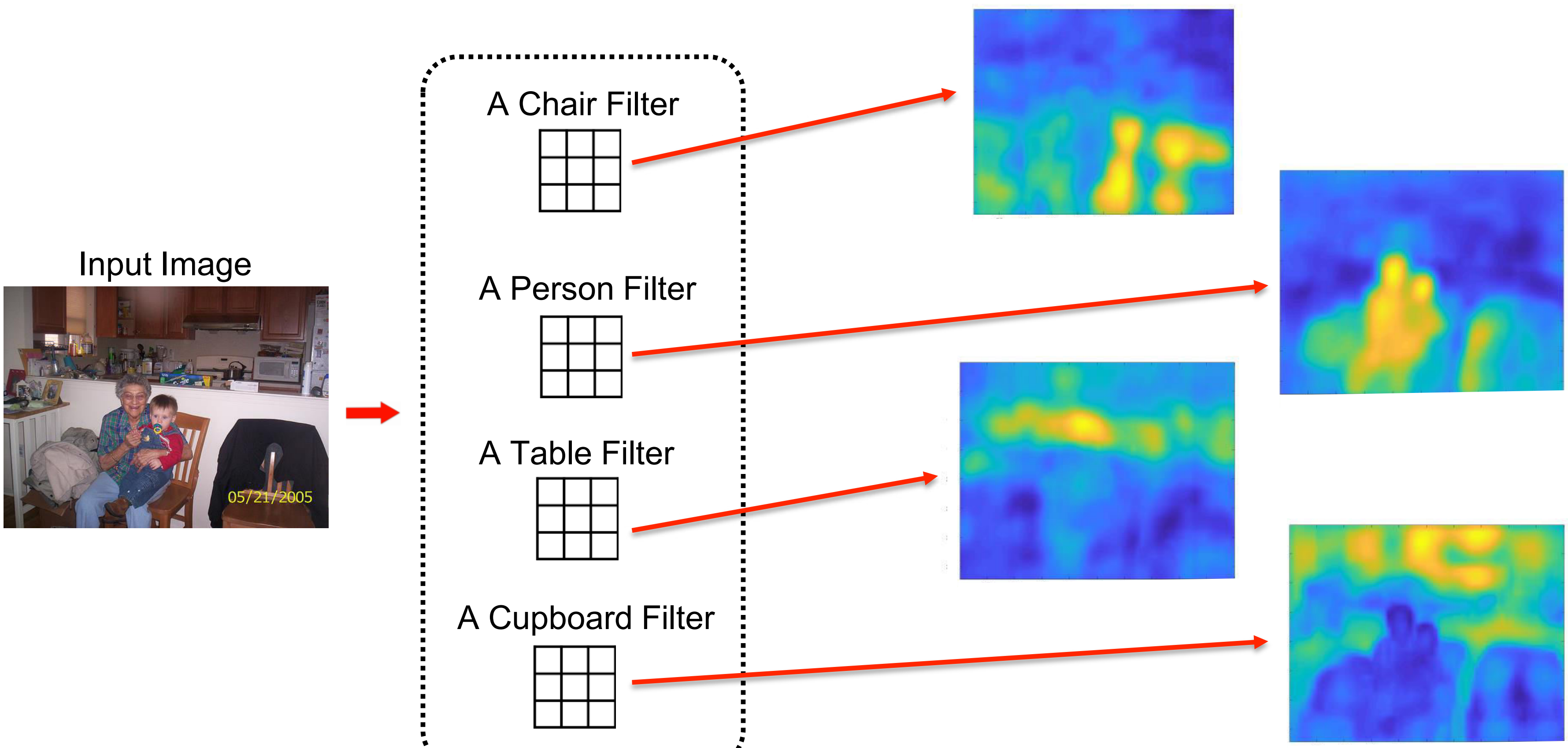


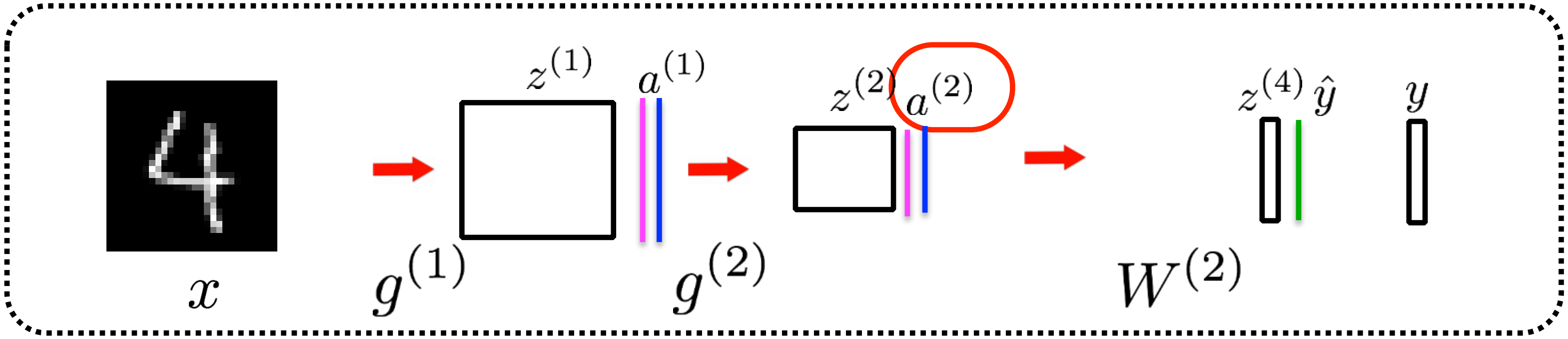
Video 12.5

Jianbo Shi

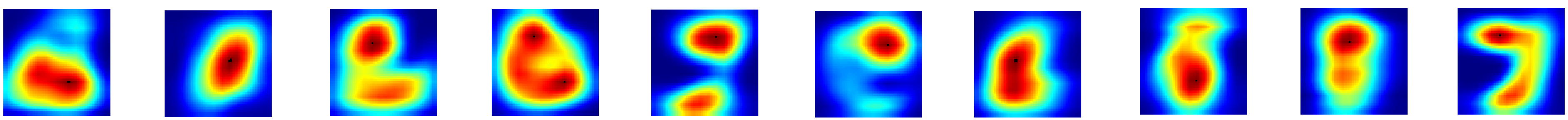


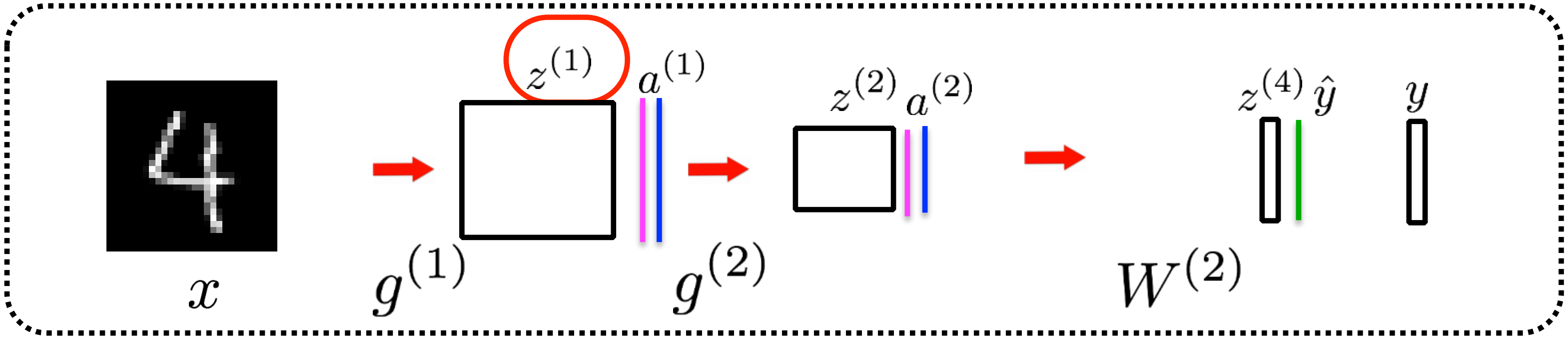
A Closer Look inside the Convolutional Layer





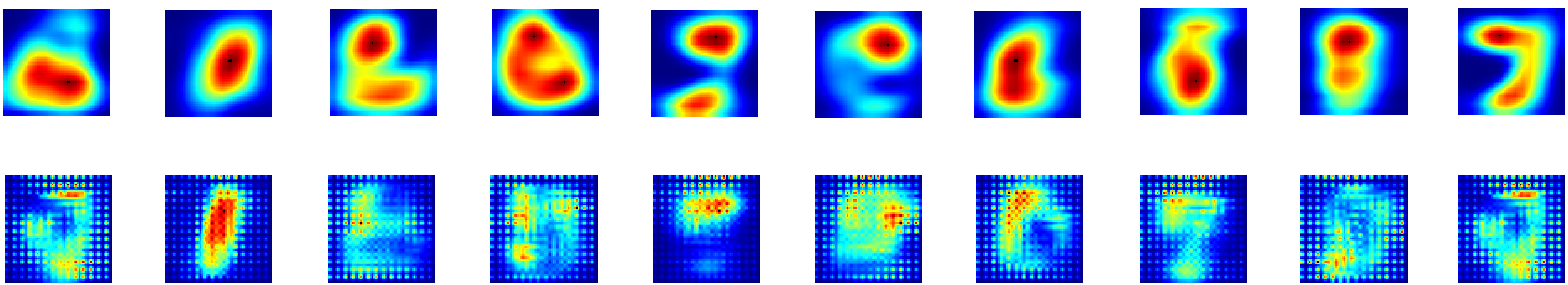
A Closer Look inside the Convolutional Layer :

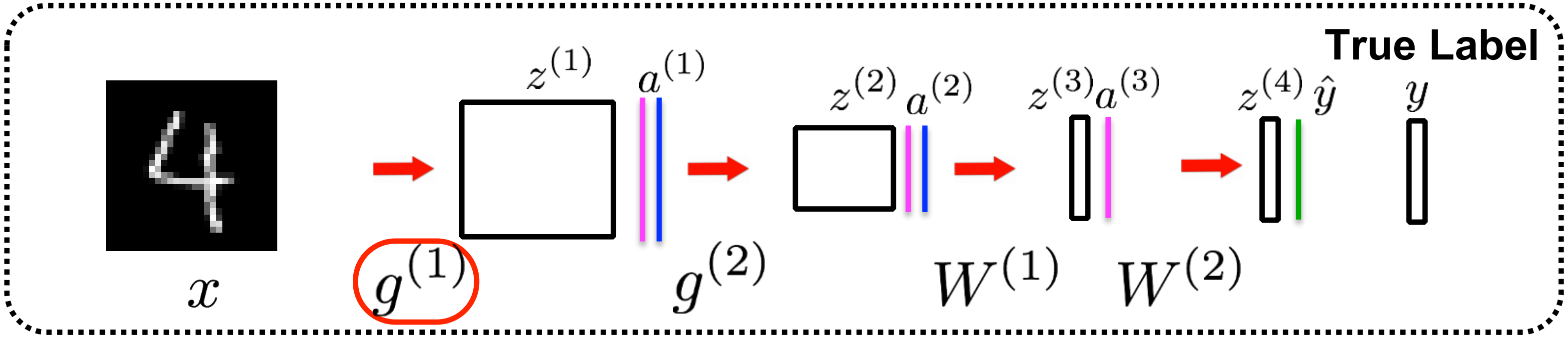




A Closer Look inside the Back Propagation Convolutional Layer :

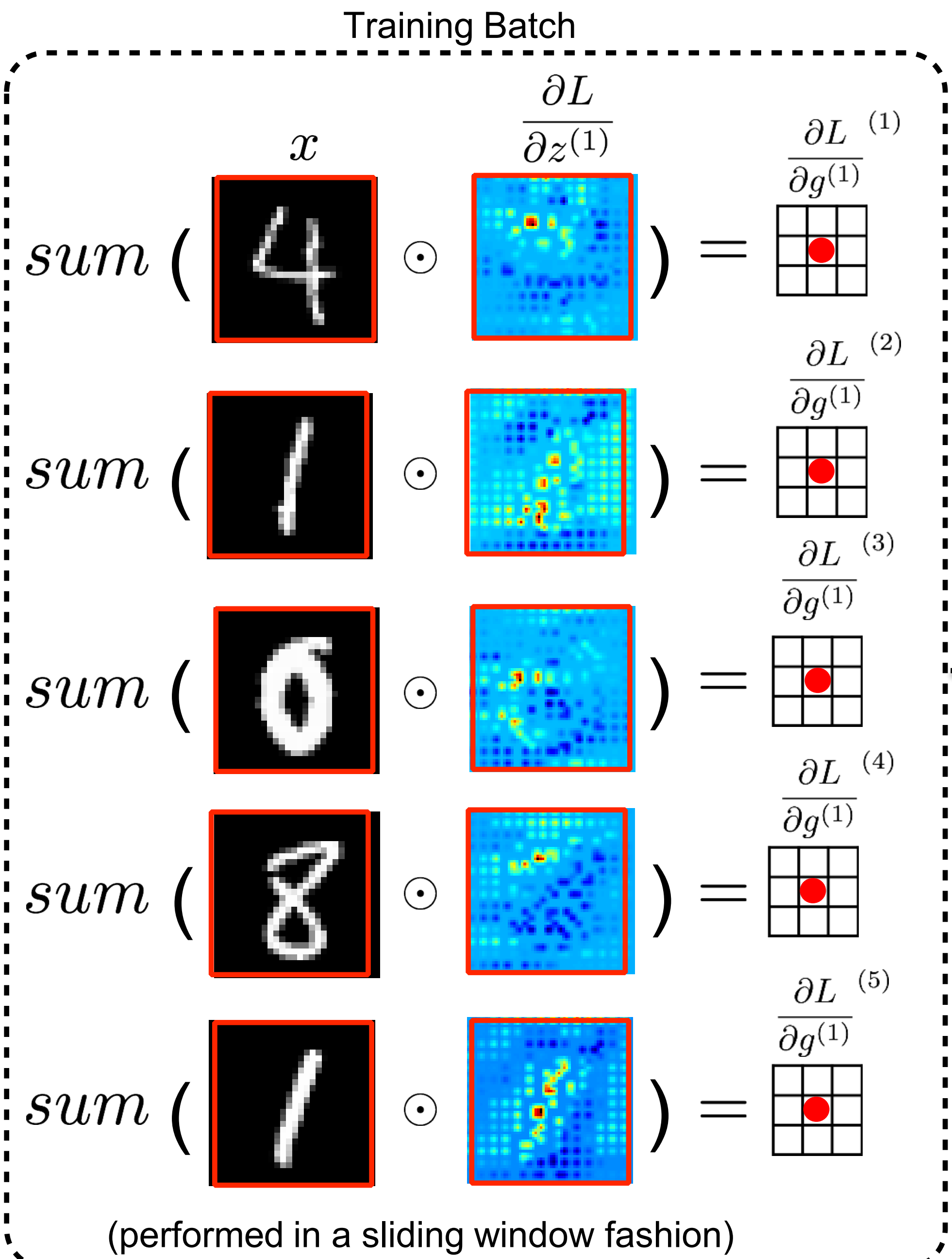
$$\frac{\partial L}{\partial z_{ij}^{(l)}} = \left(\sum_{m=0}^{M}\sum_{n=0}^{N} g_{mn}^{(l)} \frac{\partial L}{\partial z_{(i+m)(j+n)}^{(l+1)}}\right) \frac{\partial f(z_{ij}^{(l)})}{\partial z_{ij}^{(l)}}$$

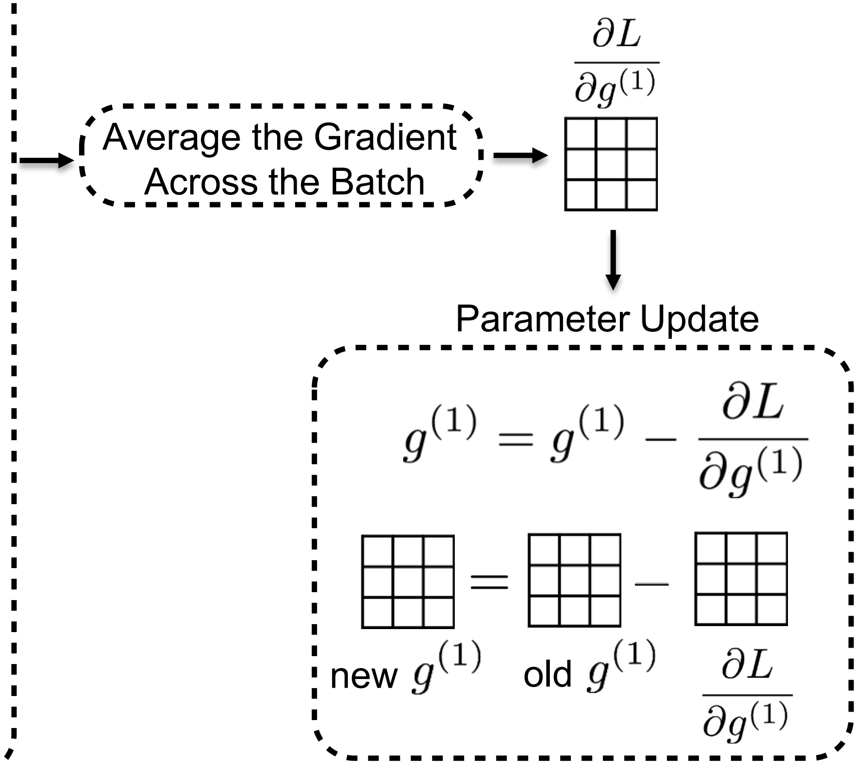




Adjusting the weights:

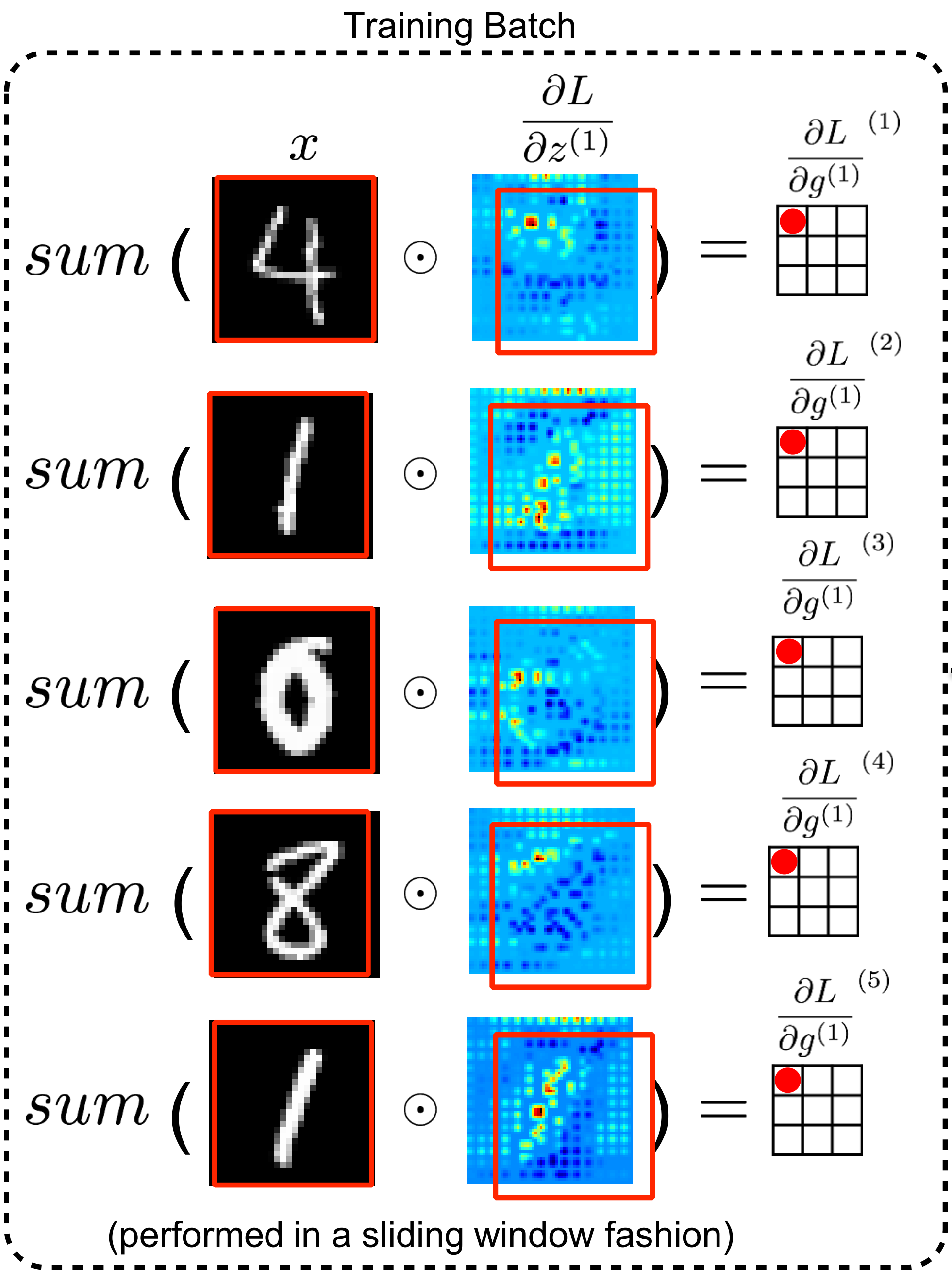
$$\frac{\partial L}{\partial g_{ij}^{(l)}} = \sum_{y=0}^{H}\sum_{x=0}^{W} \frac{\partial L}{\partial z_{yx}^{(l+1)}} f(z_{(y-i)(x-j)}^{(l)})$$

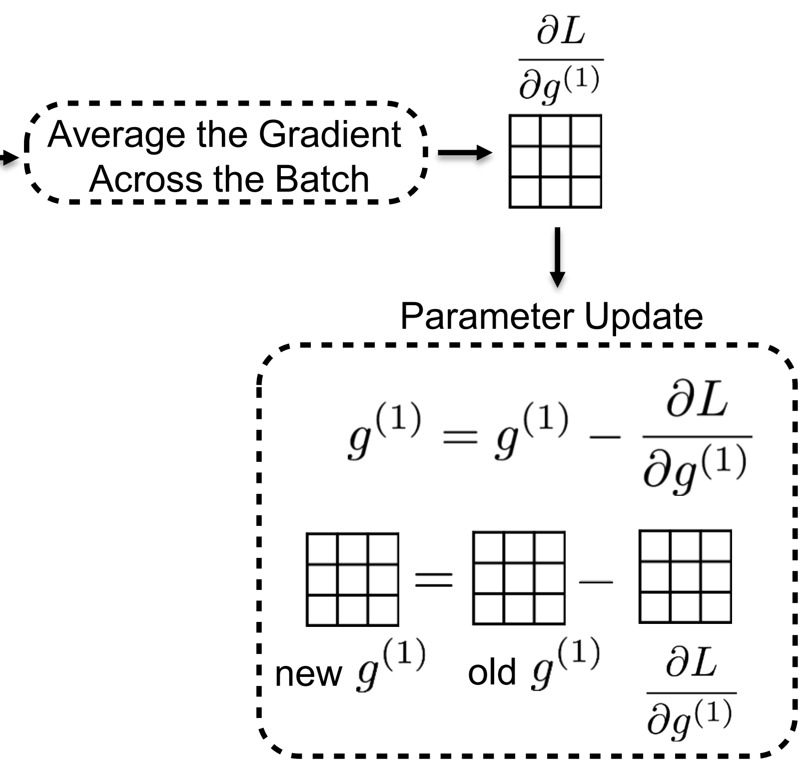




$\odot$ - elementwise multiplication

$$\frac{\partial L}{\partial g_{ij}^{(l)}} = \sum_{y=0}^{H} \sum_{x=0}^{W} \frac{\partial L}{\partial z_{yx}^{(l+1)}} f(z_{(y-i)(x-j)}^{(l)})$$

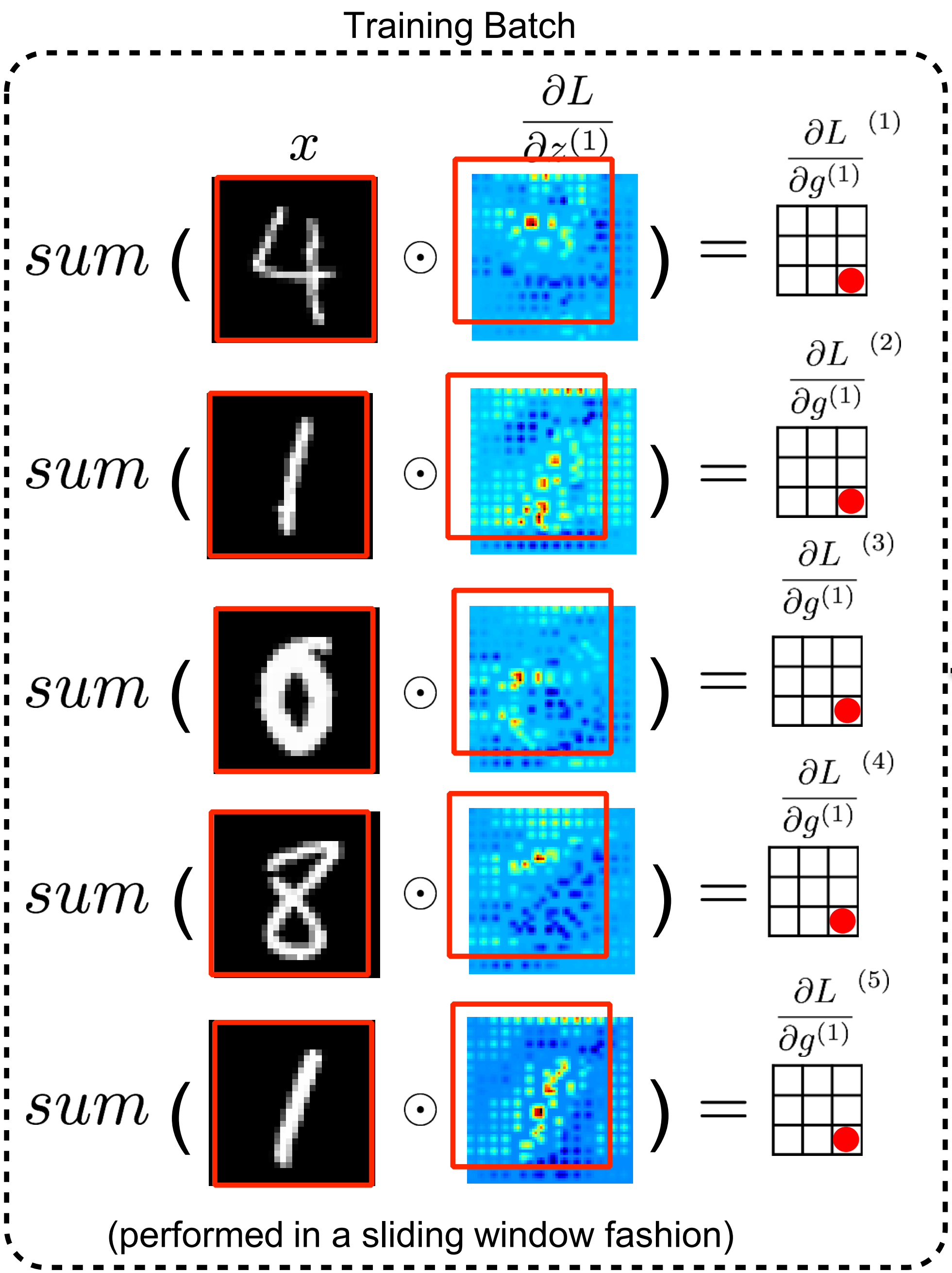


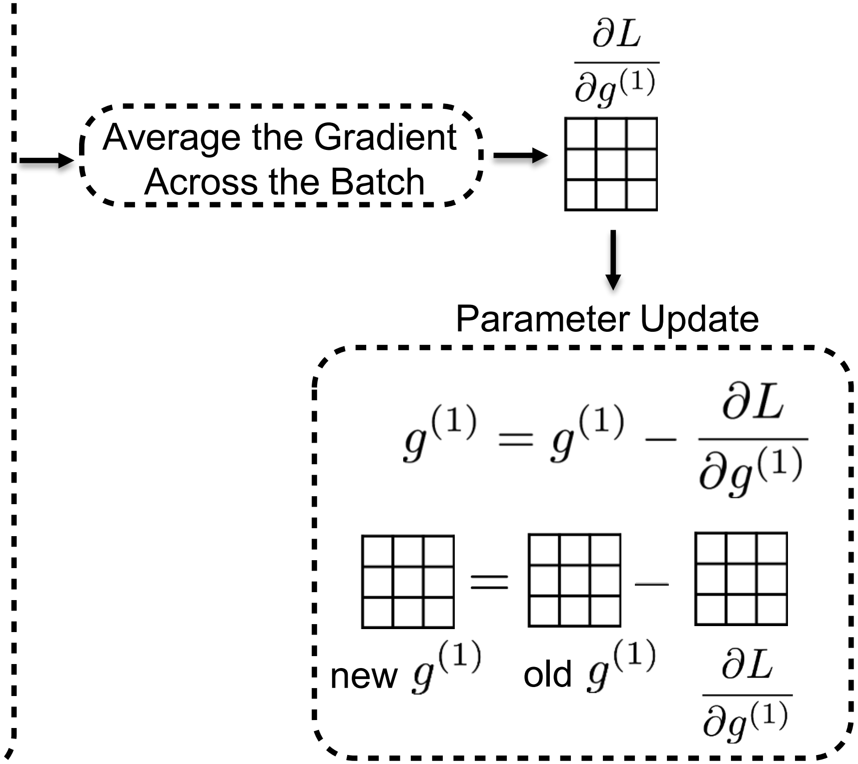


$$g^{(1)} = g^{(1)} - \frac{\partial L}{\partial g^{(1)}}$$

$\odot$ - elementwise multiplication

$$\frac{\partial L}{\partial g^{(l)}_{ij}} = \sum_{y=0}^{H}\sum_{x=0}^{W} \frac{\partial L}{\partial z^{(l+1)}_{yx}} f(z^{(l)}_{(y-i)(x-j)})$$





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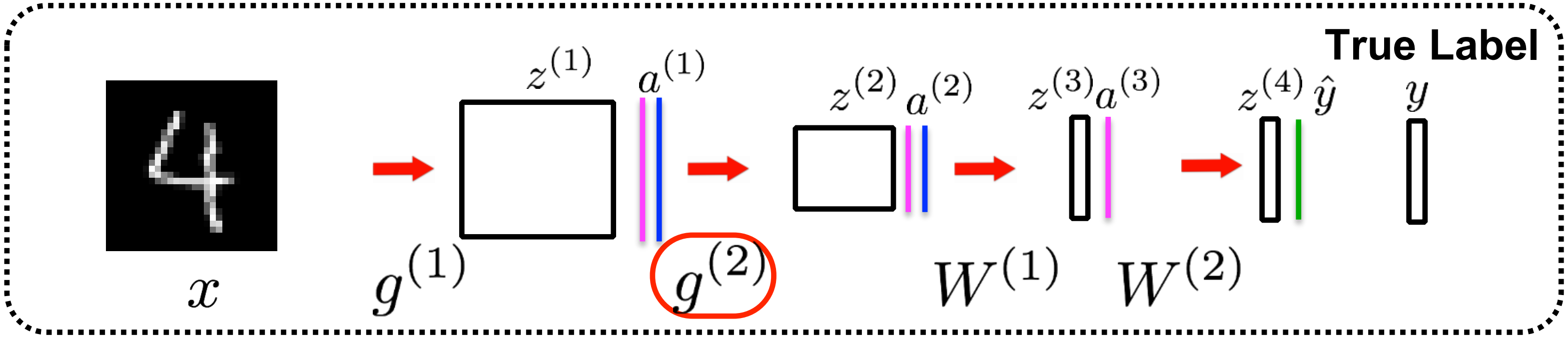
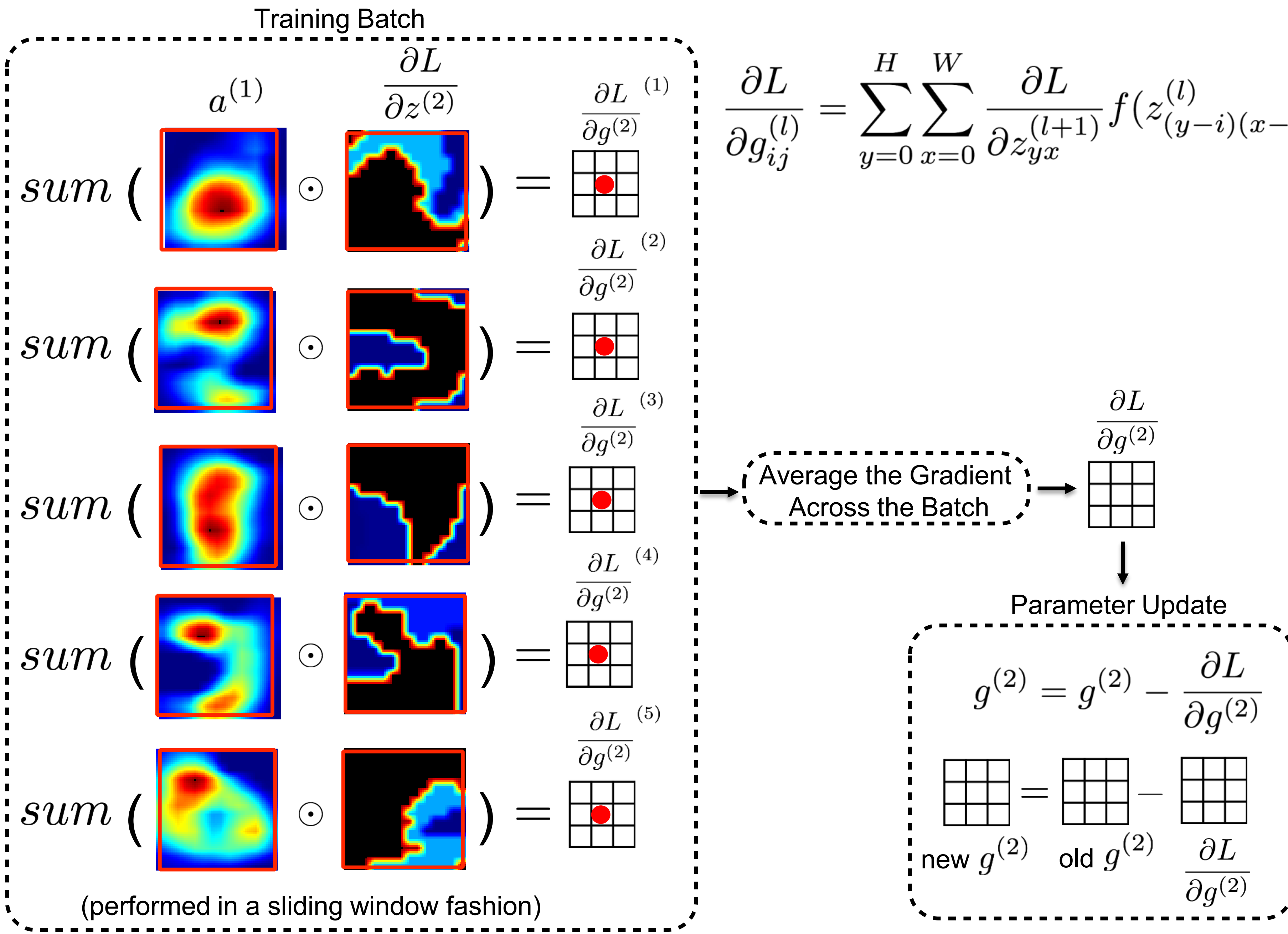


Adjusting the weights:

$$\frac{\partial L}{\partial g_{ij}^{(l)}} = \sum_{y=0}^{H} \sum_{x=0}^{W} \frac{\partial L}{\partial z_{yx}^{(l+1)}} f(z_{(y-i)(x-j)}^{(l)})$$



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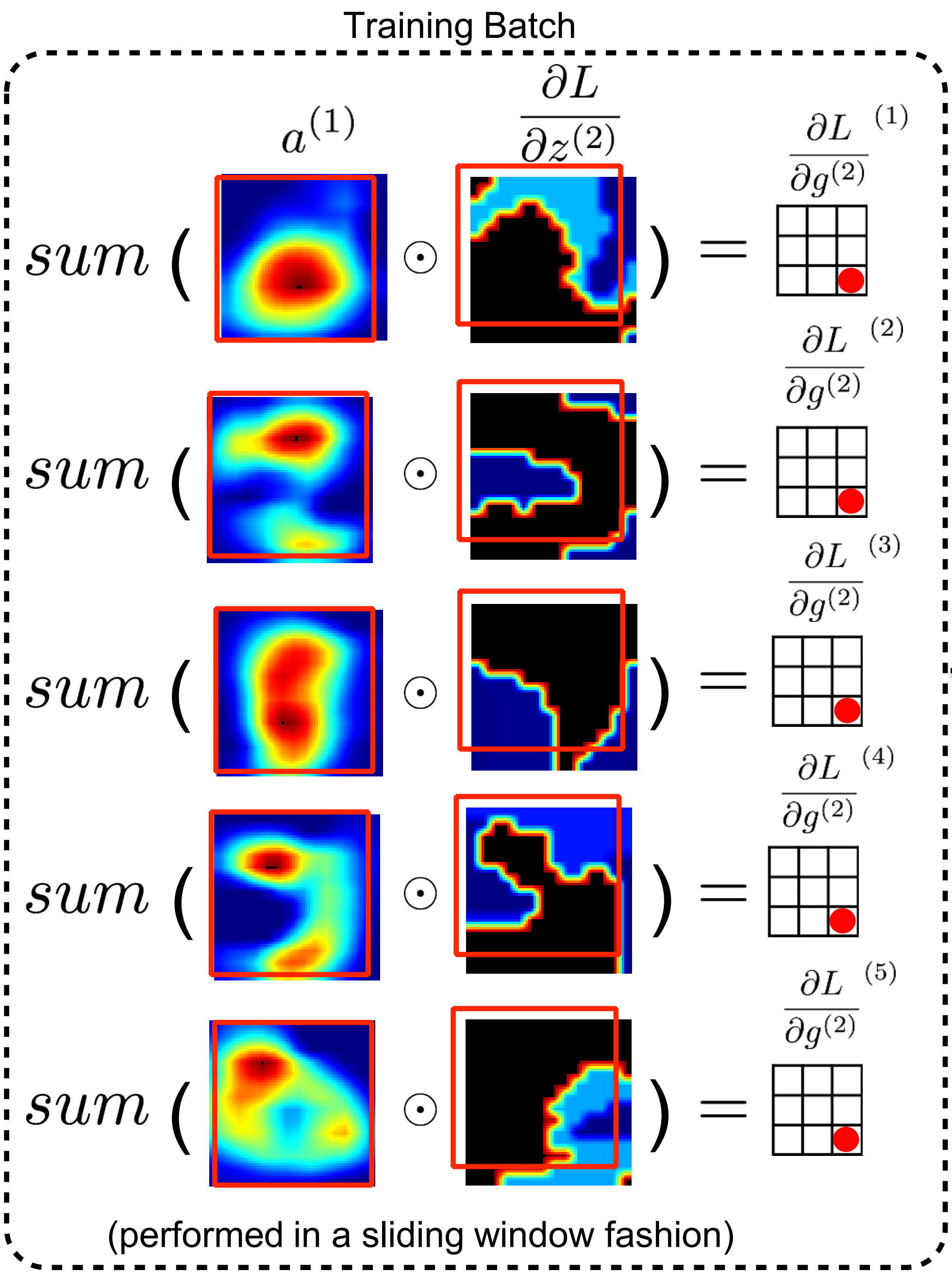

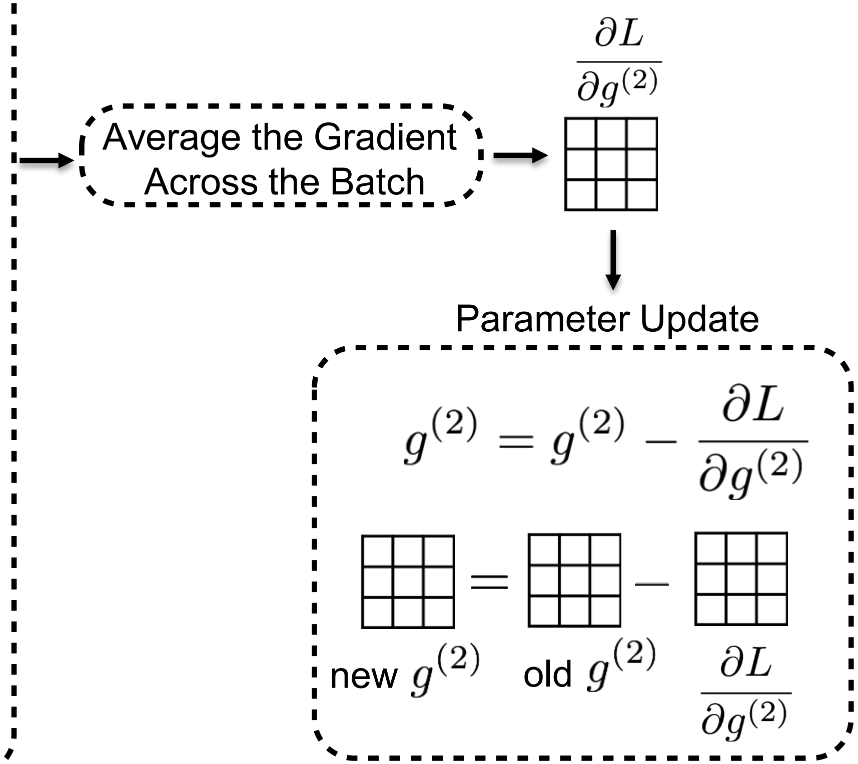

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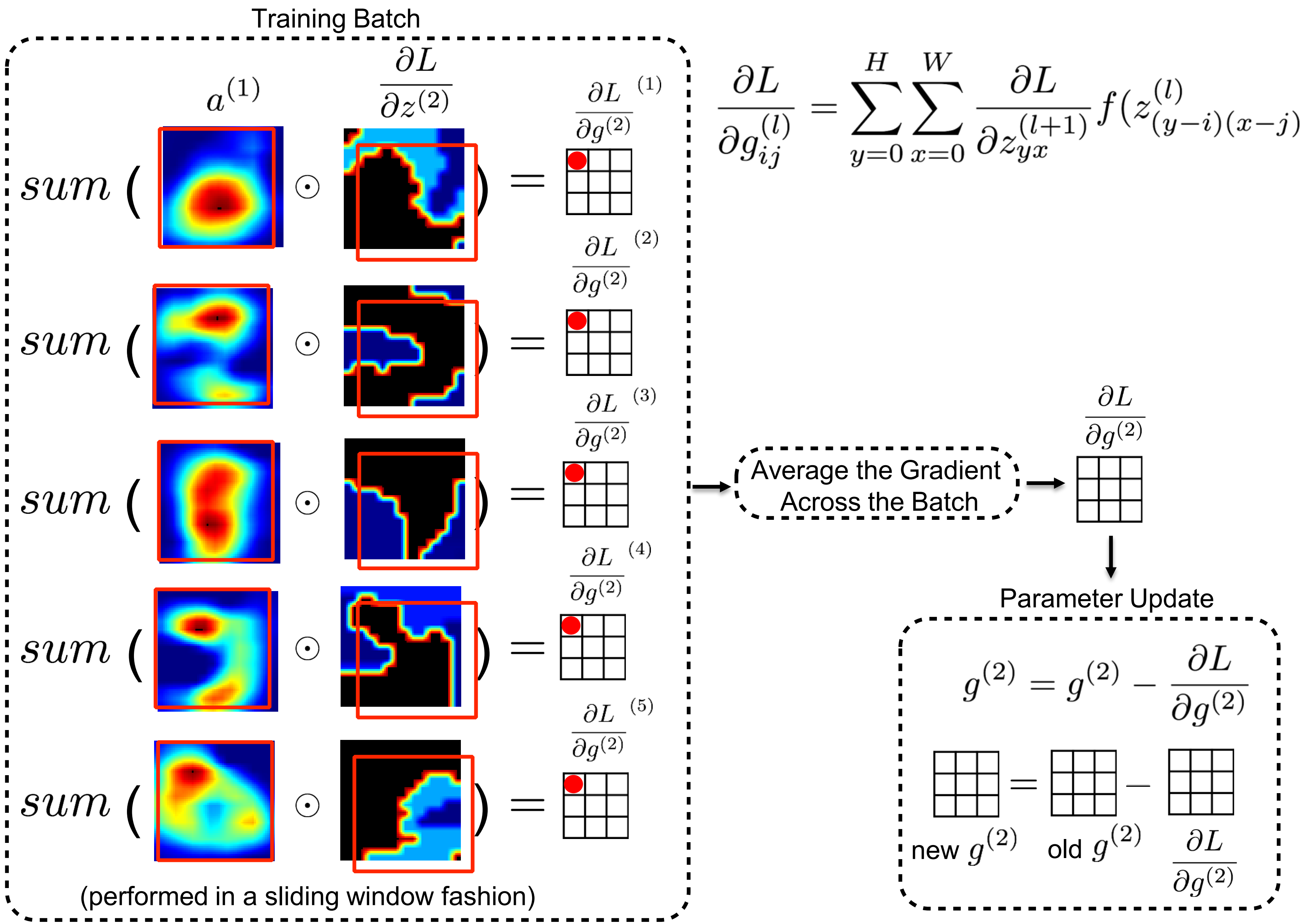

$\odot$ - elementwise multiplication