

Lead Time Variability & Mode Selection

Agenda

- Connections to Inventory Planning ✓
- Transit Time Reliability ✓
- Handling Lead Time Variability ✓
- Mode Selection ✓

Transportation Impact on Inventory

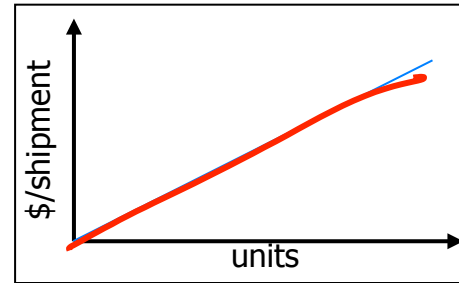
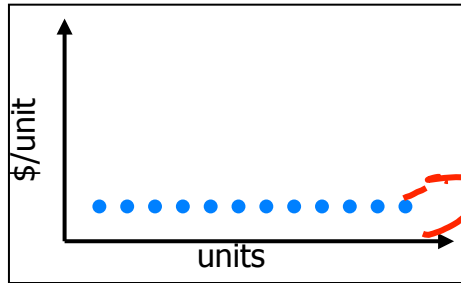
Impact on Inventory

$$TC(Q) = cD + c_t \left(\frac{D}{Q} \right) + c_e \left(\frac{Q}{2} + k\sigma_{DL} + LD \right) + B_{SO} \left(\frac{D}{Q} \right) \Pr[SO]$$

- How does transportation impact our total costs?
 - Cost of transportation
 - ◆ Value & Structure
 - Lead Time
 - ◆ Value & Variability & Schedule
 - Capacity
 - ◆ Limits on Q
 - Miscellaneous Factors
 - ◆ Special Cases

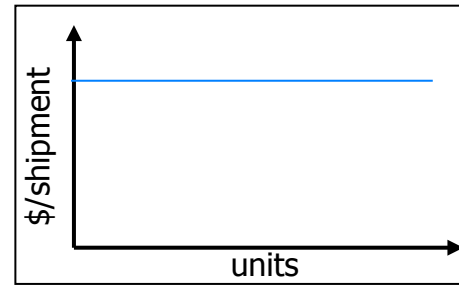
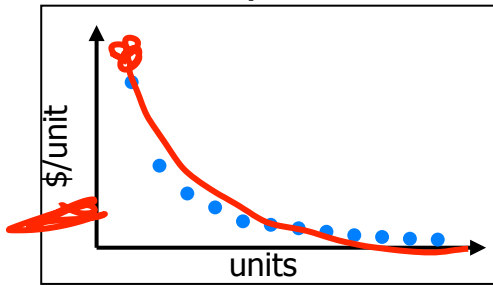
Transportation Cost Functions

Pure Variable Cost / Unit



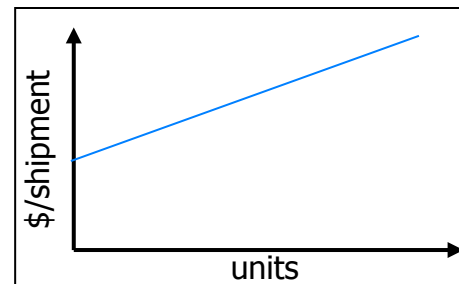
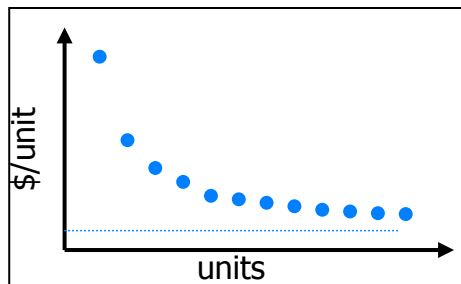
Modify unit cost (c) for Purchase Cost

Pure Fixed Cost / Shipment



Modify fixed order cost (c_t) for Ordering Cost

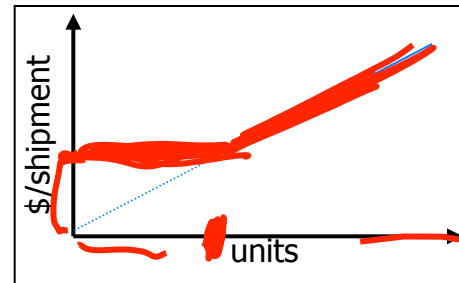
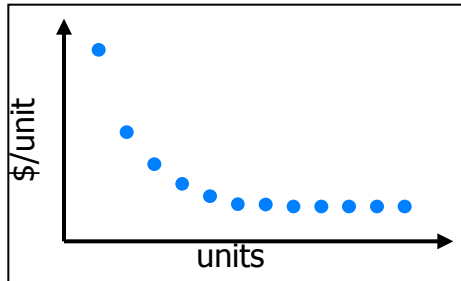
Mixed Variable & Fixed Cost



Modify both c_t and c

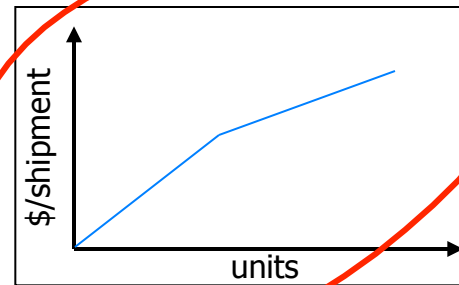
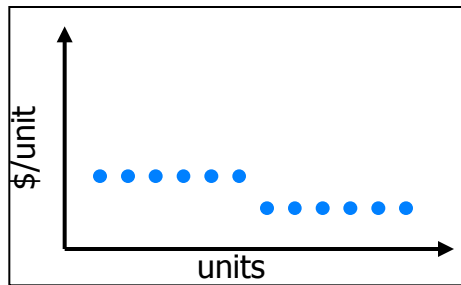
Complex Cost Functions

Variable Cost / Unit with a Minimum



algorithm

Incremental Discounts



algorithm

Note that approach will be similar to quantity discount analysis in deterministic EOQ

Shipping Shoes from Shenzhen II

- The Next Chapter

Shipping Shoes II

How should I ship my shoes from Shenzhen to Kansas City?

- General Information

- Shoes are manufactured, labeled, and packed at plant
- Demand $\sim N(4.5M, 0.54M)$ annual demand
- 3,000 shoe boxes fit into one TEU
- Average cost $\sim \$35$ per pair
- Cost of product in container $\$105,000$
- Average sales price $\sim \$75$ per pair
- Order for shipment cost $\$5000$ per order
- Holding costs are 15%
- Assume 50 weeks/year, 350 days/year
- Assume CSL 95%

Which option provides the lowest logistics cost?

- Transportation Options

Inland Origin: Shenzhen to Ports (\$/container)

- Yantian (\$35, 2 day)
- Hong Kong (\$30, 5 days)

Port to Port: China to US (\$/container)

- CSCL (AAC) Yantian to POLA (\$1100, 20 days)
- CSCL (AAS) Hong Kong to POLA (\$1025, 13 days)
- APL Hong Kong to New York (\$1200, 29 days)

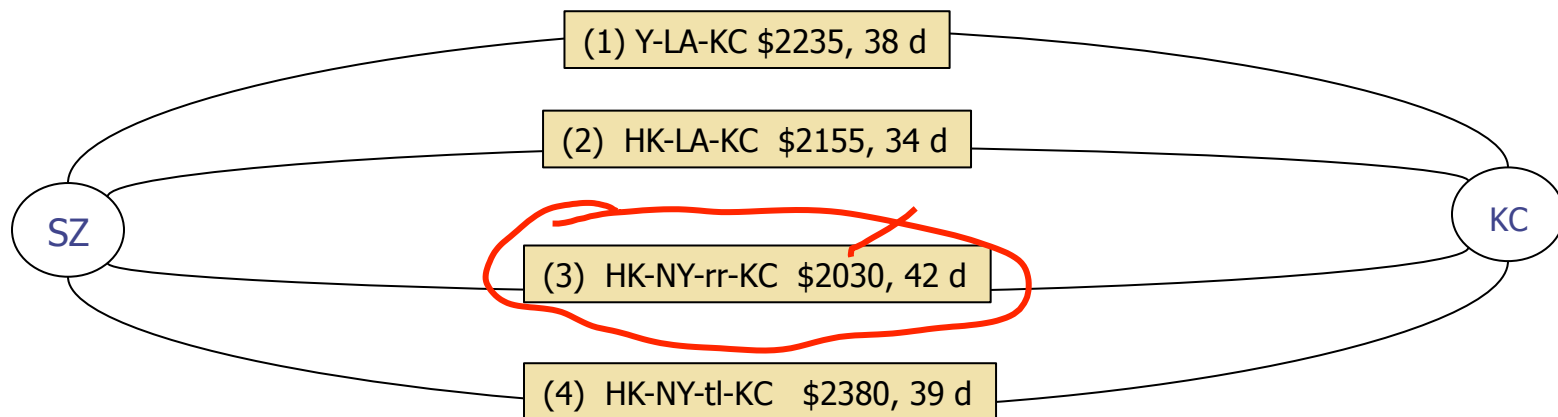
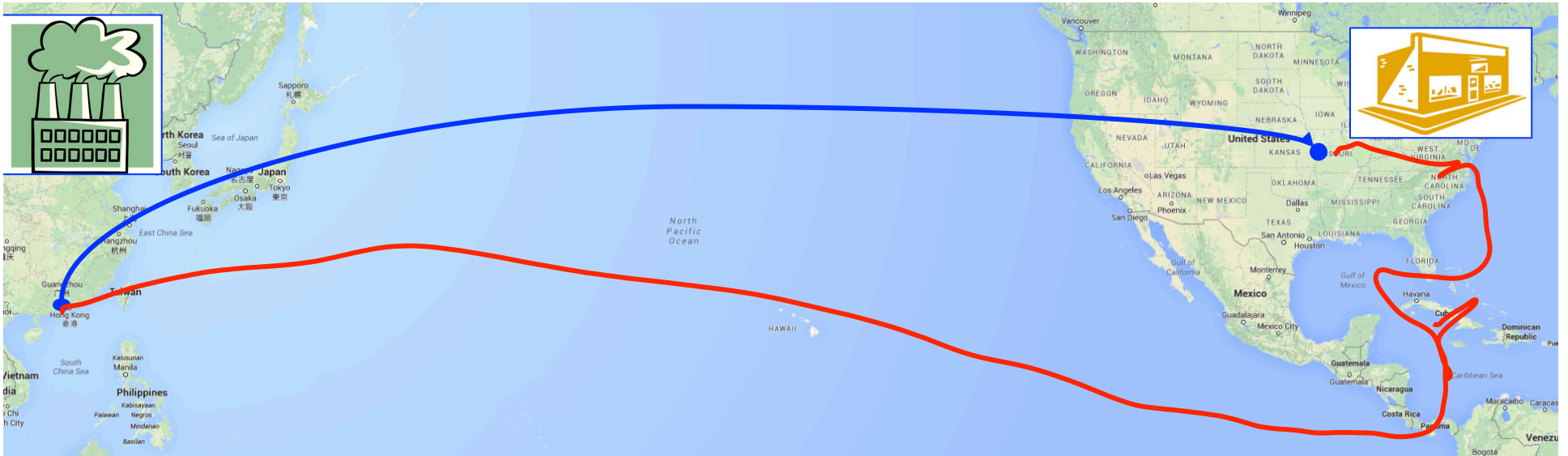
Destination Port: US Ports (\$/container)

- POLA (5 days)
- New York / New Jersey (3 days)

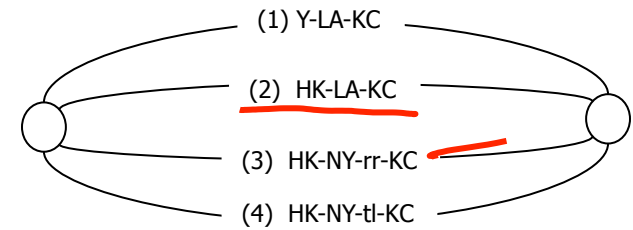
Inland Destination: To Kansas City (\$/container)

- POLA to KC by BNSF (\$1100, 11 days)
- PANYNJ to KC by NS (\$800, 5 days)
- PANYNJ to KC by HJBT Truckload (\$1150, 2 days)

Shipping Shoes



Shipping Shoes Part 2.



$$TC(Q) = cD + c_t \left(\frac{D}{Q} \right) + c_e \left(\frac{Q}{2} + k\sigma_{DL} + LD \right)$$

$k=1.64$

$\sigma_{DL} = (180)\sqrt{(LT/350)}$ shoes

| Path | L (days) | c_{trans} (\$/cnt) | c (\$/cnt) | c_t (\$/ord) | c_e (\$/cnt/yr) | D (cnt/yr) | Q (cnt/ord) | σ_{DL} (cnt) |
|------|----------|----------------------|------------|----------------|-------------------|------------|-------------|---------------------|
| 1 | 38 | \$2,235 | \$107,235 | \$5,000 | \$16,085 | 1,500 | 30 | 59.3 |
| 2 | 34 | \$2,155 | \$107,155 | \$5,000 | \$16,073 | 1,500 | 30 | 56.1 |
| 3 | 42 | \$2,030 | \$107,030 | \$5,000 | \$16,055 | 1,500 | 30 | 62.4 |
| 4 | 39 | \$2,380 | \$107,380 | \$5,000 | \$16,107 | 1,500 | 30 | 60.1 |

| Path | Purchase Cost (\$M) | Ordering Cost (\$K) | Cycle Stock Cost (\$K) | Safety Stock Cost (\$M) | Pipeline Inventory (\$M) | Total Cost (\$M) | Logistics Cost Per Shoe |
|------|---------------------|---------------------|------------------------|-------------------------|--------------------------|------------------|-------------------------|
| 1 | \$160.8 | \$250 | \$241 | \$1.57 | \$2.62 | \$165.5 | \$1.77 |
| 2 | \$160.7 | \$250 | \$241 | \$1.48 | \$2.34 | \$165.0 | \$1.67 |
| 3 | \$160.5 | \$250 | \$241 | \$1.65 | \$2.89 | \$165.5 | \$1.78 |
| 4 | \$161.1 | \$250 | \$242 | \$1.59 | \$2.69 | \$165.9 | \$1.86 |

Lowest **total** cost path is (2) at \$2155 /container = \$1.67 / pair of shoes

Transit Time Reliability

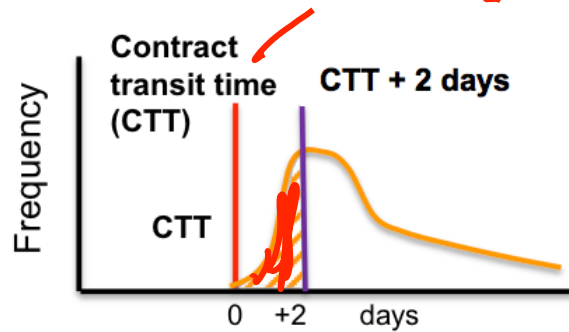
Lead / Transit Time Reliability

- Key Questions:
 - What is the definition of reliability within a firm?
 - What are the sources of unreliability/variability?
 - How can the current situation be improved?
- Two Dimensions of Reliability
 - Credibility
 - ◆ Did the carrier reserve slots as agreed to? (Rejections / Bumping)
 - ◆ Did the carrier stop at all ports agreed to? (Skipping)
 - ◆ Did the carrier load all containers committed? (Cut & Run)
 - Schedule Consistency
 - ◆ How close was the carrier's performance to their quoted schedule?
 - ◆ How consistent was the carrier's actual transit time?

Material adapted from Arntzen, B. (2011) "Global Ocean Transportation Project," Internal MIT Center for Transportation & Logistics (CTL) Report.

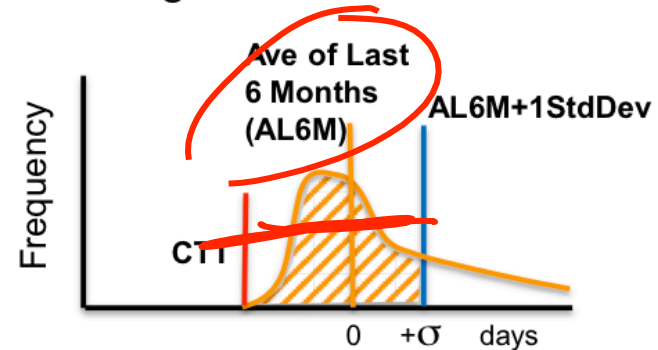
Definitions of Schedule Consistency

Compare actual transit time to the contract.



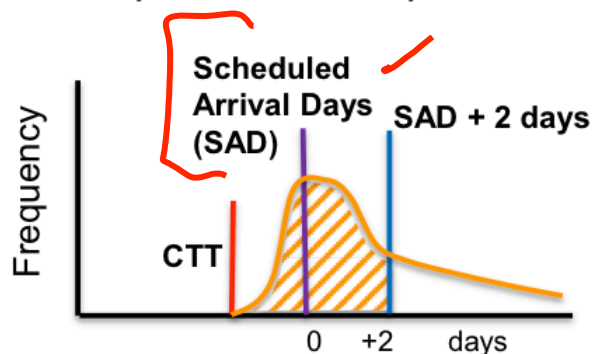
Reliability = % less than [CTT+2]

Compare actual transit time to the average of the last 6 months.



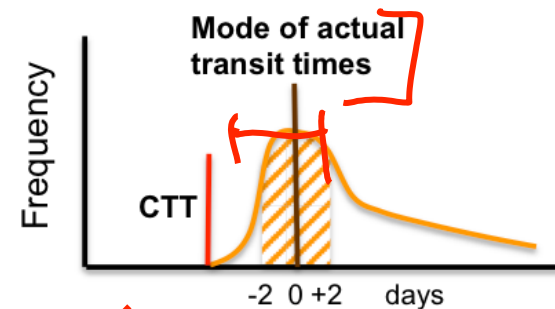
Reliability = % less than [AL6M+1StdDev]

Compare actual transit time to the published ship schedule.



Reliability = % less than [SAD+2]

Measure the “tightness” of the distribution of transit times.

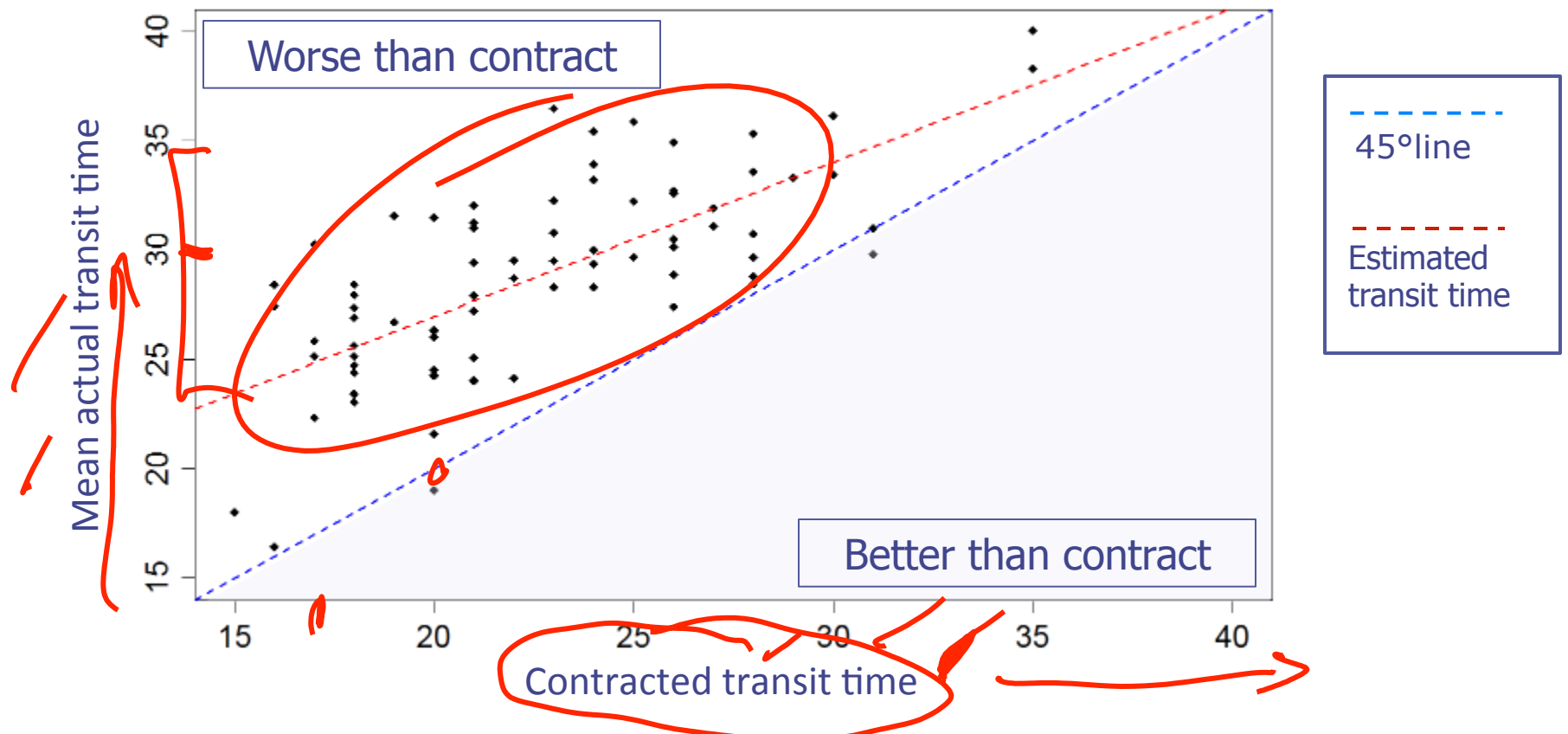


Reliability = % within 2 days of the mode

Material adapted from Arntzen, B. (2011) “Global Ocean Transportation Project,” Internal MIT Center for Transportation & Logistics (CTL) Report.

Three Observations from Practice

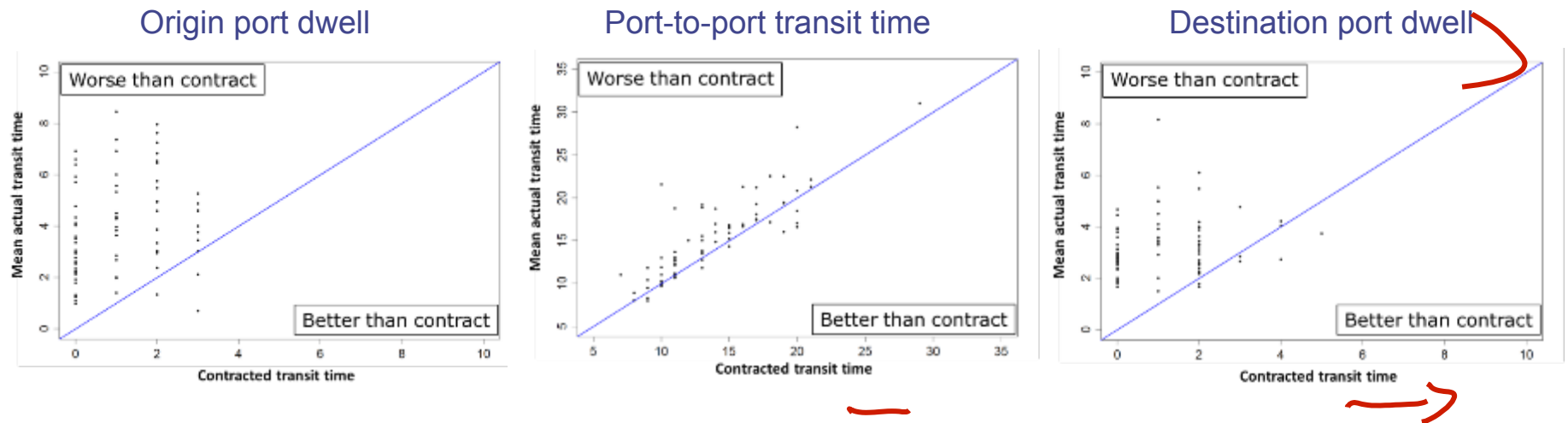
Observation 1: Contract reliability in procurement and operations do not always match



Material adapted from Caplice, C and Kalkanci, B. (2011) "Managing Global Supply Chains: Building end-to-end Reliability," Internal MIT Center for Transportation & Logistics (CTL) Report.

Three Observations from Practice

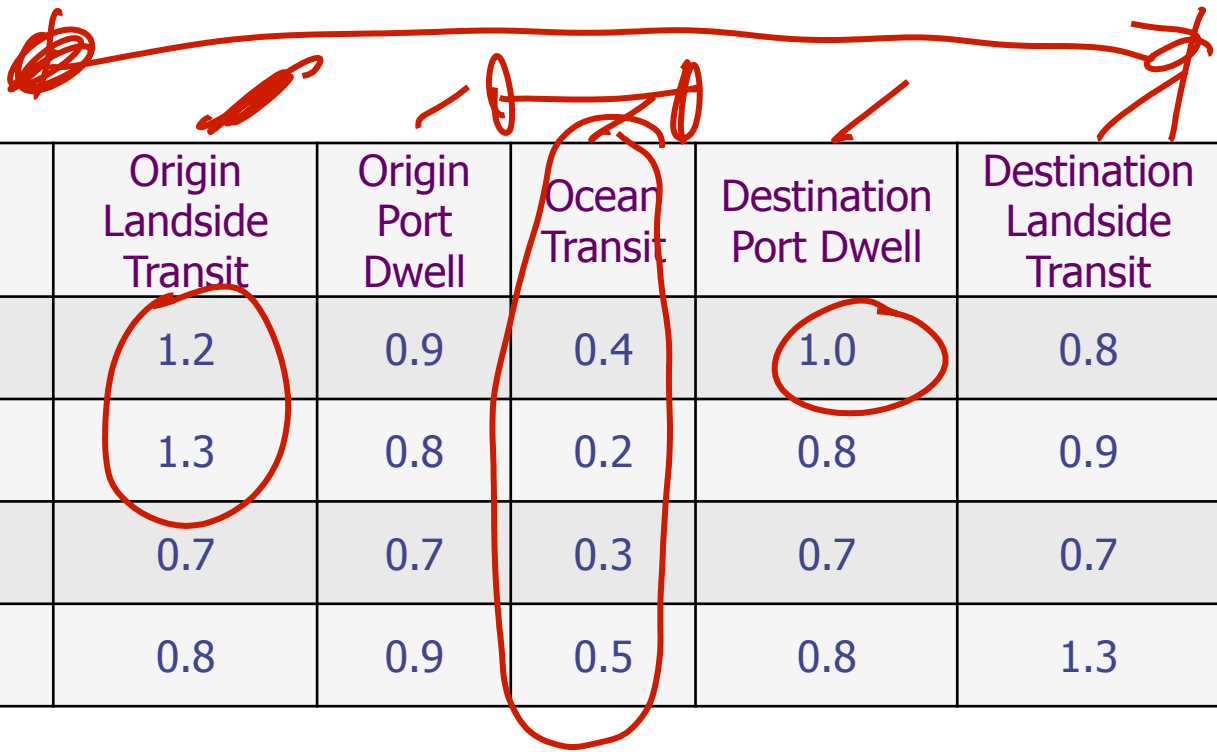
Observation 2: Contract reliability differs dramatically across different route segments



While accurate estimates of the port-to-port transit times exist, there is only limited information on port dwell times.

Three Observations from Practice

Observation 3: Most transit variability occurs in inland transportation and at the ports.

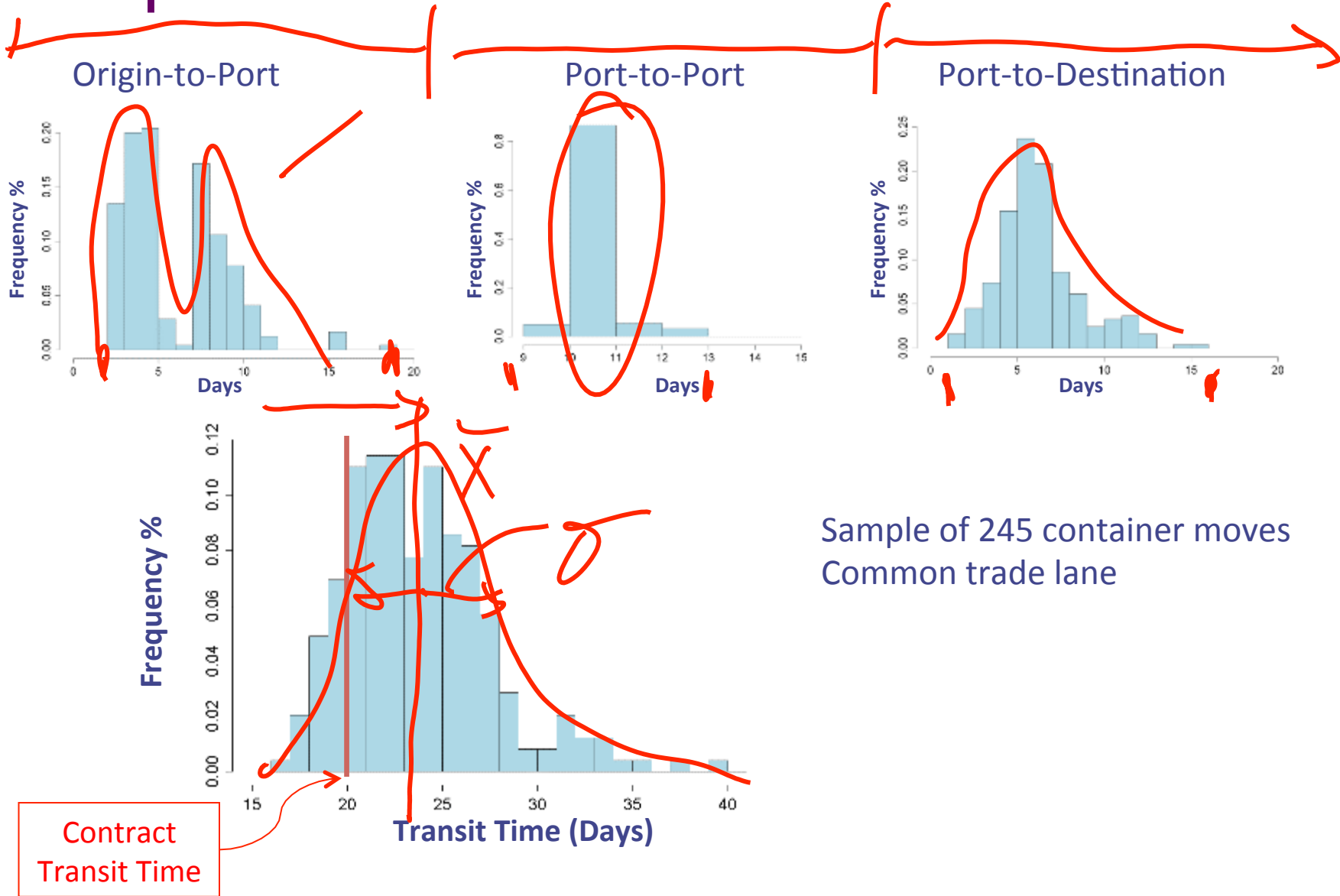


| | Origin Landside Transit | Origin Port Dwell | Ocean Transit | Destination Port Dwell | Destination Landside Transit |
|--------------------------------|-------------------------|-------------------|---------------|------------------------|------------------------------|
| Asia to North America | 1.2 | 0.9 | 0.4 | 1.0 | 0.8 |
| South America to North America | 1.3 | 0.8 | 0.2 | 0.8 | 0.9 |
| Europe to North America | 0.7 | 0.7 | 0.3 | 0.7 | 0.7 |
| North America to Europe | 0.8 | 0.9 | 0.5 | 0.8 | 1.3 |

Coefficient of Variation of Time for Each Segment when $CV = \sigma/\mu$.

Lead Time Variability

Sample of Transit Time Distribution



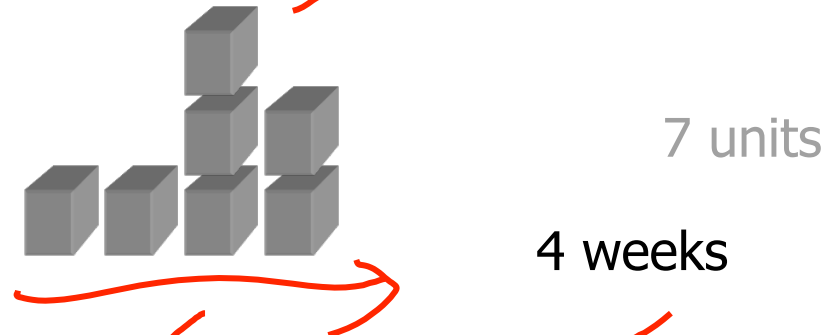
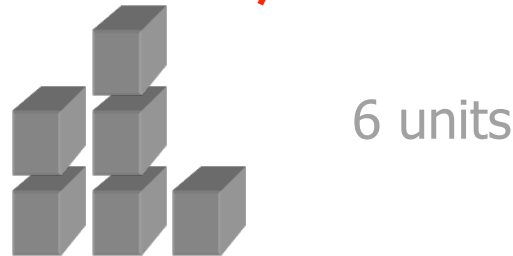
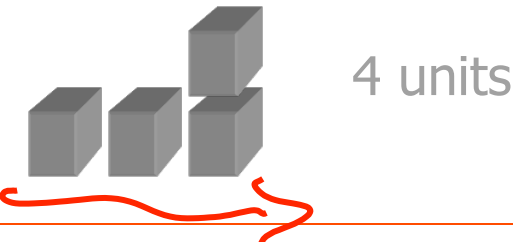
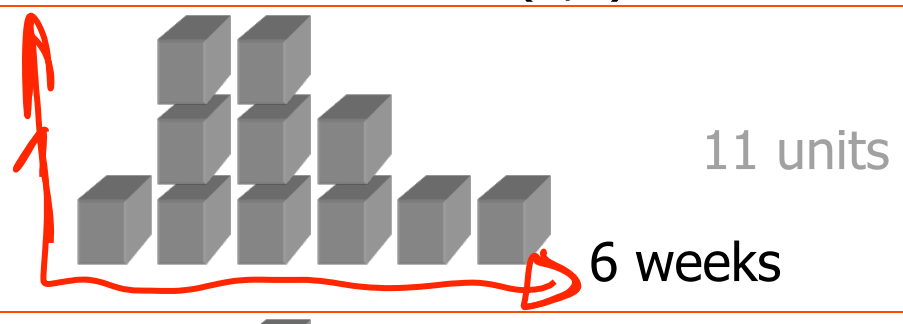
Sample of 245 container moves
Common trade lane

Lead Time Variability Impact

Weekly Demand $\sim U(1,3)$ ✓
 Lead Time 3 weeks



Weekly Demand $\sim U(1,3)$ ✓
 Lead Time $\sim U(3,6)$ weeks ✓



This is an example of Random Sums of Random Variables ✓

Random Sums of Random Variables

- Let
 - N = is a random variable assuming positive integer values 1, 2, 3....
 - X_i = independent random variables so that $E[X_i]=E[X]$
 - S = sum of X_i from $i=1$ to N

• Then

$$E[S] = E\left[\sum_{i=1}^N X_i\right] = E[N]E[X]$$

$$Var[S] = Var\left[\sum_{i=1}^N X_i\right] = E[N]Var[X] + (E[X])^2 Var[N]$$

*unit²
Time*

• Simple Example

- N has a mean of 28 and a standard deviation of 7
- X has a mean of 180 and standard deviation of 68

• What is the mean, variance, and standard deviation of S ?

- $E[S] = \mu_s = (28)(180) = 5040$
- $Var[S] = \sigma_s^2 = (28)(68)^2 + (180)^2(7)^2 = 129472 + 1587600 = 1,717,072$
- $StdDev[S] = \sigma_s = \sqrt{1,717,072} = 1310$

Full proof and discussion can be found at S. K. Ross, Introduction to Probability Models, 11th Edition, Academic Press, 2014, Chapter 3.

Lead Time Variability

- Sometimes referred to as Hadley-Whitin equation
 - Lead Time and Demand are independent RVs
 - μ_D = Expected demand (items) during one time period
 - σ_D = Standard deviation of demand (items) during one time period
 - μ_L = Expected number of time periods for lead time
 - σ_L = Standard deviation of time periods for lead time
 - μ_{DL} = Expected demand (items) over lead time
 - σ_{DL} = Standard deviation of demand (items) over lead time

Unitless multiplier of the base time period!

$$\mu_{DL} = \mu_L \mu_D \qquad \sigma_{DL} = \sqrt{\mu_L \sigma_D^2 + (\mu_D)^2 \sigma_L^2}$$

- **Transportation Example**

- Suppose that lead time is 12 days on average with a standard deviation of 3 days. The daily demand for an item is 100 units with a standard deviation of 22.
- What is my expected demand over lead time as well as standard deviation of demand over lead time.
 - ◆ $\mu_{DL} = (12)(100) = 1200$
 - ◆ $\sigma_{DL} = \sqrt{[(12)(22)^2 + (100)^2(3)^2]} = \sqrt{[5808 + 90000]} = 309.5 \sim 310$

I can now find set an inventory performance metric using this demand distribution!

Shipping Shoes from Shenzhen III

– The Final Chapter

Shipping Shoes III

How should I ship my shoes from Shenzhen to Kansas City?

- General Information

- Shoes are manufactured, labeled, and packed at plant
- Demand $\sim N(4.5M, 0.54M)$ annual demand
- 3,000 shoe boxes fit into one TEU
- Average cost $\sim \$35$ per pair
- Cost of product in container \$105,000
- Average sales price $\sim \$75$ per pair
- Order for shipment cost \$5000 per order
- Holding costs are 15%
- Assume 50 weeks/year, 350 days/year
- Assume CSL 95%

Which option provides the lowest logistics cost?

- Transportation Options

Inland Origin: Shenzhen to Ports (\$/cnt, μ_L, σ_L)

- Yantian (\$35, 2 days, 1 day)

- Hong Kong (\$30, 5 days, 5 days)

Port to Port: China to US (\$/cnt, μ_L, σ_L)

- CSCL (AAC) Yantian to POLA (\$1100, 20 days, 2 days)

- CSCL (AAS) Hong Kong to POLA (\$1025, 13 days, 13 days)

- APL Hong Kong to New York (\$1200, 29 days, 3 days)

Destination Port: US Ports (\$/cnt, μ_L, σ_L)

- POLA (\$0, 5 days, 3 days)

- New York / New Jersey (\$0, 3 days, 1 day)

Inland Destination: To Kansas City (\$/cnt, μ_L, σ_L)

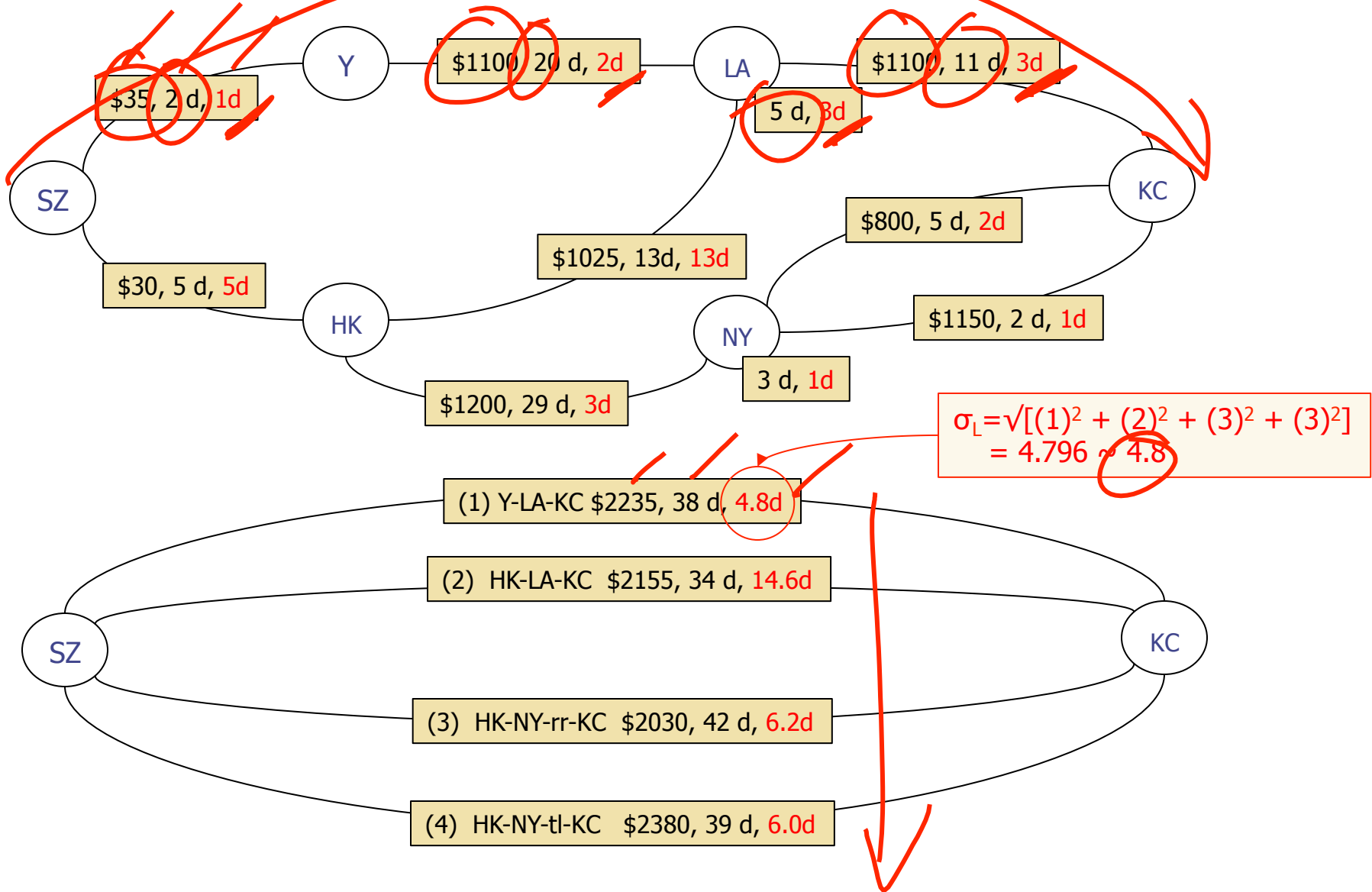
- POLA to KC by BNSF (\$1100, 11 days, 3 days)

- PANYNJ to KC by NS (\$800, 5 days, 2 days)

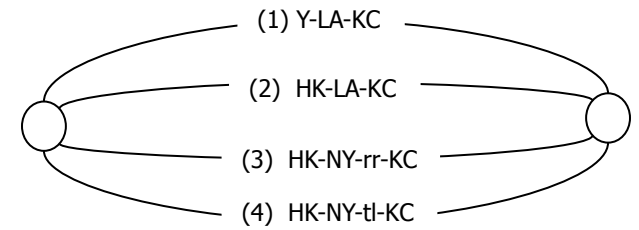
- PANYNJ to KC by HJBT Truckload (\$1150, 2 days, 1 days)

Shipping Shoes III

\$/cnt, μ_L , σ_L



Shipping Shoes III



$$TC(Q) = \cancel{c_t D} + c_t \left(\frac{D}{Q} \right) + c_e \left(\frac{Q}{2} + k\sigma_{DL} + LD \right)$$

Only safety stock changes!

| Path | μ_L | σ_L | μ_D | σ_D | μ_{DL} | σ_{DL} | New SS (\$M) | Old SS (\$M) |
|------|---------|------------|---------|------------|------------|---------------|--------------|--------------|
| 1 | 38 | 4.80 | 4.29 | 9.62 | 162.86 | 62.78 | \$1.66 | \$1.57 |
| 2 | 34 | 14.60 | 4.29 | 9.62 | 145.71 | 84.04 | \$2.22 | \$1.48 |
| 3 | 42 | 6.20 | 4.29 | 9.62 | 180.00 | 67.78 | \$1.79 | \$1.65 |
| 4 | 39 | 6.00 | 4.29 | 9.62 | 167.14 | 65.36 | \$1.73 | \$1.59 |

$$\mu_D = (1500 \text{ cnt/year}) / (350 \text{ days/year})$$

$$\sigma_D = (180 \text{ cnt/year}) / \sqrt{(350 \text{ days/year})}$$

$$\mu_{DL} = \mu_L \mu_D$$

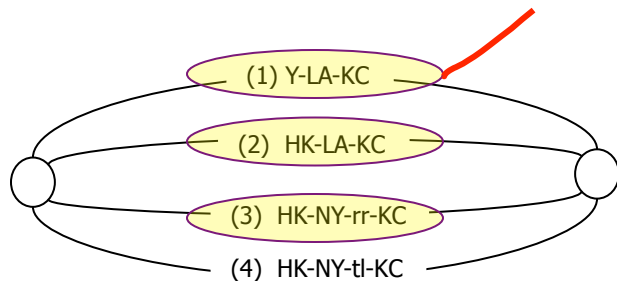
$$\sigma_{DL} = \sqrt{\mu_L \sigma_D^2 + (\mu_D)^2 \sigma_L^2}$$

Shipping Shoes III

$$TC(Q) = cD + c_t \left(\frac{D}{Q} \right) + c_e \left(\frac{Q}{2} + k\sigma_{DL} + LD \right)$$

| Path | Purchase Cost (\$M) | Ordering Cost (\$K) | Cycle Stock Cost (\$K) | Safety Stock Cost (\$M) | Pipeline Inventory (\$M) | Total Cost (\$M) | Logistics Cost Per Shoe |
|------|---------------------|---------------------|------------------------|-------------------------|--------------------------|------------------|-------------------------|
| 1 | \$160.8 | \$250 | \$241 | \$1.66 | \$2.62 | \$165.6 | \$1.79 |
| 2 | \$160.7 | \$250 | \$241 | \$2.22 | \$2.34 | \$165.8 | \$1.83 |
| 3 | \$160.5 | \$250 | \$241 | \$1.79 | \$2.89 | \$165.7 | \$1.82 |
| 4 | \$161.1 | \$250 | \$242 | \$1.73 | \$2.69 | \$166.0 | \$1.89 |

Lowest **total** cost path is (1) at \$2235 /container = \$1.79 / pair of shoes



#3 - Lowest transportation cost route @ ~\$0.68 \$/shoe

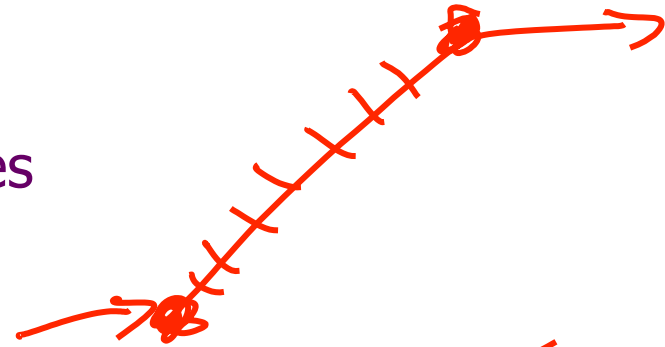
#2 - Lowest logistics cost route @ ~\$1.67 \$/shoe, not considering variability of transit time

#1 - Lowest logistics cost route @ ~\$1.79 \$/shoe, considering variability of transit time

Mode Selection

Mode Selection

- Criteria for selection between modes
 - Feasible choices:
 - ◆ By geography
 - Global: Air versus Ocean
 - Surface: Trucking (TL, LTL, parcel) vs. Rail vs. Intermodal vs. Barge
 - ◆ By required speed
 - >500 miles in 1 day – Air
 - <500 miles in 1 day – TL
 - ◆ By shipment size (weight/density/cube, etc.)
 - High weight, cube items cannot be moved by air
 - Large oversized shipments might be restricted to rail or barge
 - ◆ By other restrictions
 - Nuclear or hazardous materials (HazMat)
 - Product characteristics
 - Trade-offs within the set of feasible choices:
 - ◆ Cost
 - ◆ Time (mean transit time, variability of transit time, frequency)
 - ◆ Capacity
 - ◆ Loss and Damage



Mode Choice Example

- You are in charge of transportation planning for a manufacturer. One of the lanes you are managing brings raw material from a supplier into your plant. Your plant requires about $\sim N(3000, 750)$ pounds of the product per day. The product is valued at \$20 per lb with 20% annual holding cost. You assume a CSL of 95% and 250 working days per year. You take ownership of the product at the origin.
- You have two options for this inbound movement.
 - **Truckload** – Transit time is 3 days on average with a standard deviation of 0.5 days and it costs \$1800 per truckload (capacity of 40,000 lbs)
 - **Intermodal** – Transit time is 6 days on average with a standard deviation of 2 days and it costs \$1400 per container (capacity of 40,000 lbs)
- Questions:
 - Your company's policy is to always "weigh out" your shipments. That is, always ship in full truckload or container quantities. Following this policy, what mode should you select?

Solution: Mode Choice

$$TC(Q) = cD + c_t \left(\frac{D}{Q} \right) + c_e \left(\frac{Q}{2} + k\sigma_{DL} + LD \right)$$

$c = \$ 20$ per lb
 $h = 20\%$ per year
 $c_e = 20(0.20) = 4$ \$/yr
 $\mu_D = 3000$ lbs/day
 $\sigma_D = 750$ lbs/day
 $k = 1.64$

| | TL | IM | |
|----------------------------------|------|------|---------|
| Lead Time (μ_L) | 3 | 6 | days |
| Std Dev Lead Time (σ_L) | 0.5 | 2 | days |
| Cost (c_t) | 1800 | 1400 | \$/load |

$$\mu_{DL} = \mu_L \mu_D$$

$$\sigma_{DL} = \sqrt{\mu_L \sigma_D^2 + (\mu_D)^2 \sigma_L^2}$$

| | TL | IM |
|---|------|-------|
| Average Demand over Lead Time (μ_{DL}) | 9000 | 18000 |
| Std Dev Demand over Lead Time (σ_{DL}) | 1984 | 6275 |

| | TL | IM |
|------------------------------------|------------------|------------------|
| Capacity (Q) | 40000 | 40000 |
| Number of loads/year ($N = D/Q$) | 18.75 | 18.75 |
| Annual Ordering Cost | \$33,750 | \$26,250 |
| Annual Cycle Stock Cost | \$80,000 | \$80,000 |
| Annual Safety Stock Cost | \$13,017 | \$41,164 |
| Annual Pipeline Inventory Cost | \$36,000 | \$72,000 |
| Total Annual Logistics Cost | \$162,767 | \$219,414 |

Select TL for this lane.

TL saves > \$56k /yr, but note – transport costs are higher but are trumped by safety stock and pipeline inventory.

Does the "weigh-out" policy make sense?

Solution: Mode Choice

$$TC(Q) = cD + c_t \left(\frac{D}{Q} \right) + c_e \left(\frac{Q}{2} + k\sigma_{DL} + LD \right)$$

$c = \$ 20$ per lb
 $h = 20\%$ per year
 $c_e = 20(0.20) = 4$ \$/yr
 $\mu_D = 3000$ lbs/day
 $\sigma_D = 750$ lbs/day
 $k = 1.64$

| | TL | IM | |
|----------------------------------|------|------|---------|
| Lead Time (μ_L) | 3 | 6 | days |
| Std Dev Lead Time (σ_L) | 0.5 | 2 | days |
| Cost (c_t) | 1800 | 1400 | \$/load |

$$Q^* = \sqrt{\frac{2c_t D}{c_e}}$$

| | TL | IM |
|---|------|-------|
| Average Demand over Lead Time (μ_{DL}) | 9000 | 18000 |
| Std Dev Demand over Lead Time (σ_{DL}) | 1984 | 6275 |

| | TL | IM |
|------------------------------------|-------|-------|
| Capacity (Q) | 25981 | 22913 |
| Number of loads/year ($N = D/Q$) | 28.87 | 32.73 |

| | | |
|------------------------------------|------------------|------------------|
| Annual Ordering Cost | \$51,962 | \$45,826 |
| Annual Cycle Stock Cost | \$51,962 | \$45,826 |
| Annual Safety Stock Cost | \$13,017 | \$41,164 |
| Annual Pipeline Inventory Cost | \$36,000 | \$72,000 |
| Total Annual Logistics Cost | \$152,940 | \$204,815 |

Still use TL

Shipping below max weight saves ~ \$10k per year! Why?

When would IM make sense for this lane?

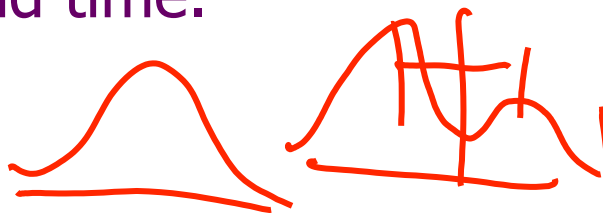
- Lower value ($c \leq 0.73$ \$/lb)
- Better service ($\mu_L = 5, \sigma_L = 1,$ & $c \leq 2.67$ \$/lb)
- Lower IM rate ($c_t \leq \$263$)

Key Points from Lesson

Key Points

$$TC(Q) = cD + c_t \left(\frac{D}{Q} \right) + c_e \left(\frac{Q}{2} + k\sigma_{DL} + LD \right)$$

- Mode/route/carrier selection is a trade-off between
 - Transportation costs ✓
 - Inventory costs (cycle, safety, pipeline) ✓
 - Level of service
- Need to consider more than just direct transport cost
- Lead time impacts safety stock levels and variability impacts it even more so!
- Be careful about shape of distribution for demand over lead time.



$$\mu_{DL} = \mu_L \mu_D$$

$$\sigma_{DL} = \sqrt{\mu_L \sigma_D^2 + (\mu_D)^2 \sigma_L^2}$$

CTL.SC1x -Supply Chain & Logistics Fundamentals
Questions, Comments, Suggestions?
Use the Discussion!



"Wilson – pondering the Hadley-Whitin Equation "
Yankee Golden Retriever Rescued Dog
(www.ygrr.org)



MIT Center for
Transportation & Logistics

caplice@mit.edu