Inventory Models for Special Cases: A & C Items and Challenges
# Inventory Management by Segment

<table>
<thead>
<tr>
<th></th>
<th>A Items</th>
<th>B Items</th>
<th>C Items</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Type of records</strong></td>
<td>Extensive, Transactional</td>
<td>Moderate</td>
<td>None – use a rule</td>
</tr>
<tr>
<td><strong>Level of Management Reporting</strong></td>
<td>Frequent (Monthly or more)</td>
<td>Infrequently - Aggregated</td>
<td>Only as Aggregate</td>
</tr>
<tr>
<td><strong>Interaction w/ Demand</strong></td>
<td>Direct Input</td>
<td>Modified Forecast (promotions etc.)</td>
<td>Simple Forecast at best</td>
</tr>
<tr>
<td></td>
<td>High Data Integrity</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Manipulate (pricing etc.)</td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Interaction w/ Supply</strong></td>
<td>Actively Manage</td>
<td>Manage by Exception</td>
<td>None</td>
</tr>
<tr>
<td><strong>Initial Deployment</strong></td>
<td>Minimize exposure (high v)</td>
<td>Steady State</td>
<td>Steady State</td>
</tr>
<tr>
<td><strong>Frequency of Policy Review</strong></td>
<td>Very Frequent (monthly or more)</td>
<td>Moderate (Annually/Event Based)</td>
<td>Very Infrequent</td>
</tr>
<tr>
<td><strong>Importance of Parameter Precision</strong></td>
<td>Very High – accuracy worthwhile</td>
<td>Moderate – rounding &amp; approximation is ok</td>
<td>Very Low</td>
</tr>
<tr>
<td><strong>Shortage Strategy</strong></td>
<td>Actively manage (confront)</td>
<td>Set service levels &amp; manage by exception</td>
<td>Set &amp; forget service levels</td>
</tr>
<tr>
<td><strong>Demand Distribution</strong></td>
<td>Consider alternatives to Normal as situation fits</td>
<td>Normal</td>
<td>N/A</td>
</tr>
</tbody>
</table>

**ACTIVE**

**AUTOMATIC**

**PASSIVE**
**Inventory Policies By Segment**

- No hard and fast rules, but some rules of thumb

<table>
<thead>
<tr>
<th>Type of Item,</th>
<th>Continuous Review</th>
<th>Periodic Review</th>
</tr>
</thead>
<tbody>
<tr>
<td>A Items</td>
<td>$(s, S)$</td>
<td>$(R, s, S)$</td>
</tr>
<tr>
<td>B Items</td>
<td>$(s, Q)$</td>
<td>$(R, S)$</td>
</tr>
</tbody>
</table>
Agenda

• Class A Policies
  ▪ Fast & Slow Moving Items
  ▪ Poisson Distributions

• Class C Policies

• Real-World Implications
Inventory Policies for A Items
Managing Class A Inventory

• When does it make sense to spend more time?
  ■ Tradeoff between complexity and ‘other’ costs
  ■ Is the savings worth the extra effort?

• Adding precision
  ■ Finding ‘optimal’ parameters
  ■ Using more complex policies

\[
TC = cD + \frac{D}{Q} \left( \frac{Q}{2} + k\sigma_{DL} \right) + B_1 \left( \frac{D}{Q} \right) P[SO]
\]

\[
TC = cD + \frac{D}{Q} \left( \frac{Q}{2} + k\sigma_{DL} \right) + c_s \left( \frac{D}{Q} \right) \sigma_{DL} G(k)
\]
Managing Class A Inventory

• Two Types of Class A items:
  ■ Fast moving but cheap (large D small c → Q>1)
  ■ Slow moving but expensive (large c small D → Q=1)

• Impacts the probability distribution used
  ■ Fast Movers - Normal or Lognormal Distribution
    ♦ Good enough for B items
    ♦ OK for A items if $\mu_{DL}$ or $\mu_{DL+R} \geq 10$
  ■ Slow Movers – Poisson Distribution
    ♦ More complicated to handle
    ♦ Ok for A items if $\mu_{DL}$ or $\mu_{DL+R} < 10$
Fast Moving A Items
Fast Moving A Items

Order-Point, Order-Up-To-Level (s, S)
- Policy: **Order (S-IP) if IP ≤ s**
- Min-Max system
- Continuous Review

**Note on Undershoots:**
- Number of units of IP below reorder point, s, at time the order is placed, s-IP
- Only matters if demand is non-unit sized transactions
- If demand is always in units then (s, Q) = (s, S) where Q = S - s

![Graph showing Inventory Position over Time with s and S levels](image)

**Notation**
- s = Reorder Point
- S = Order-up-to Level
- L = Replenishment Lead Time
- Q = Order Quantity
- R = Review Period
- IOH = Inventory on Hand
- IP = Inventory Position = (IOH) + (Inventory On Order) – (Backorders)
Fast Moving A Items

- Suppose we have a Cost per Stock Out Event or $B_1$

\[
TRC = c_t \left( \frac{D}{Q} \right) + c_e \left( \frac{Q}{2} + k\sigma_{DL} \right) + B_1 \left( \frac{D}{Q} \right) P\left[ x > k \right]
\]

- How did we set (s, Q) policy for B items?
- Sequentially!
  - Set $Q = EOQ$
  - Found $k$ that minimizes TRC

- Is it worth looking for better parameters?
  \[
  \begin{align*}
  Q^* &= \sqrt{\frac{2c_t D}{c_e}} \\
  k^* &= \sqrt{2 \ln \left( \frac{DB_1}{\sqrt{2\pi Qc_e \sigma_{DL}}} \right)}
  \end{align*}
  \]
Fast Moving A Items

\[
TRC = c_t \left( \frac{D}{Q} \right) + c_e \left( \frac{Q}{2} + k\sigma_{DL} \right) + B_1 \left( \frac{D}{Q} \right) P[x > k]
\]

- Finding Better Parameters
  - Solve for \( k^* \) and \( Q^* \) simultaneously
  - Take partial differentials \( \text{wrt} \ Q \) and \( k \)
  - End up with two equations

\[
Q^* = EOQ \sqrt{1 + \frac{B_1 P[x > k]}{c_t}}
\]

\[
k^* = \sqrt{2 \ln \left( \frac{DB_1}{\sqrt{2\pi Qc_e\sigma_{DL}}} \right)}
\]

- How do we solve it?
  - Iteratively solve the two equations
  - Stop when \( Q^* \) and \( k^* \) converge within acceptable range
Slow Moving A Items
Slow Moving A Items

- Normal distribution may not make sense – why?
- Poisson distribution
  - Probability of $x$ events occurring within a time period
  - Mean = Variance = $\lambda$

\[
\begin{align*}
p[x_0] &= \text{Prob}[x = x_0] = \frac{e^{-\lambda} \lambda^{x_0}}{x_0!} \quad \text{for } x_0 = 0, 1, 2, \ldots \\
F[x_0] &= \text{Prob}[x \leq x_0] = \sum_{x=0}^{x_0} \frac{e^{-\lambda} \lambda^x}{x!}
\end{align*}
\]

In Spreadsheets:
\[
\begin{align*}
p(x_0) &= \text{POISSON}(x_0, \lambda, 0) \\
F(x_0) &= \text{POISSON}(x_0, \lambda, 1)
\end{align*}
\]
Example

- **Problem:**
  - Suppose that you want to set up a (s, Q) policy for an A item. Demand over lead time is Poisson distributed with a mean of 2.6 and you have already determined Q*=6 units. What reorder point would you use if you wanted to achieve a CSL of 95%?

- **Solution**
  - We want to find:
    
    \[ F[x_0] = \sum_{x=0}^{x_0} \frac{e^{-\lambda} \lambda^x}{x!} \geq 0.95 \]
    
    - Simply build a table with pdf and cdf
    - Select s where F[x] ≥ CSL

- **But what is the expected IFR?**
  - \[ IFR = 1 - \frac{E[US]}{Q} \]

<table>
<thead>
<tr>
<th>Demand</th>
<th>p[x]</th>
<th>F[x]</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>7%</td>
<td>7%</td>
</tr>
<tr>
<td>1</td>
<td>19%</td>
<td>27%</td>
</tr>
<tr>
<td>2</td>
<td>25%</td>
<td>52%</td>
</tr>
<tr>
<td>3</td>
<td>22%</td>
<td>74%</td>
</tr>
<tr>
<td>4</td>
<td>14%</td>
<td>88%</td>
</tr>
<tr>
<td>5</td>
<td>7%</td>
<td>95%</td>
</tr>
<tr>
<td>6</td>
<td>3%</td>
<td>98%</td>
</tr>
<tr>
<td>7</td>
<td>1%</td>
<td>99%</td>
</tr>
<tr>
<td>8</td>
<td>0%</td>
<td>100%</td>
</tr>
</tbody>
</table>
Loss Function for Discrete Function

- For any discrete function we can find the loss function, $L[X_i]$, for each value of $X$ given the cumulative probability $F[X_i]$.
- Start with first value
  - $L[X_1] = \text{mean} - X_1$
  - $L[X_2] = L[X_1] - (X_2 - X_1)(1-F[X_1])$
  - $L[X_3] = L[X_2] - (X_3 - X_2)(1-F[X_2])$
  - ..... 
  - $L[X_i] = L[X_{i-1}] - (X_i - X_{i-1})(1-F[X_{i-1}])$
- For our problem:
  - $L[X_1] = L[0] = 2.60 - 0 = 2.60$
  - $L[X_2] = L[1] = 2.60 - (1 - 0)(1 - .074) = 1.67$
  - etc.

Method adapted from Cachon & Terwiesch (2005), Matching Supply & Demand

<table>
<thead>
<tr>
<th>i</th>
<th>Demand (Xi)</th>
<th>p[x]</th>
<th>F[x]</th>
<th>L[x]</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0</td>
<td>7.4%</td>
<td>7.4%</td>
<td>2.60</td>
</tr>
<tr>
<td>2</td>
<td>1</td>
<td>19.3%</td>
<td>26.7%</td>
<td>1.67</td>
</tr>
<tr>
<td>3</td>
<td>2</td>
<td>25.1%</td>
<td>51.8%</td>
<td>0.94</td>
</tr>
<tr>
<td>4</td>
<td>3</td>
<td>21.8%</td>
<td>73.6%</td>
<td>0.46</td>
</tr>
<tr>
<td>5</td>
<td>4</td>
<td>14.1%</td>
<td>87.7%</td>
<td>0.20</td>
</tr>
<tr>
<td>6</td>
<td>5</td>
<td>7.4%</td>
<td>95.1%</td>
<td>0.07</td>
</tr>
<tr>
<td>7</td>
<td>6</td>
<td>3.2%</td>
<td>98.3%</td>
<td>0.02</td>
</tr>
<tr>
<td>8</td>
<td>7</td>
<td>1.2%</td>
<td>99.5%</td>
<td>0.01</td>
</tr>
<tr>
<td>9</td>
<td>8</td>
<td>0.4%</td>
<td>99.9%</td>
<td>0.00</td>
</tr>
</tbody>
</table>

Loss Function for $\sim P(\lambda=2.6)$

At $s=5$, $E[US] = 0.07$
IFR=1-(0.07/6) = 98.8%
What $s$ for IFR=80% since $E[US]=Q(1-\text{IFR}) = 1.2$ then select $s=2$
Managing Class C Inventories
Managing Class C Inventories

• What are C items?
  ■ Typically low cD values
  ■ Large number, low total value items
  ■ Need to consider implicit & explicit costs

• Objective: minimize management attention
  ■ Regardless of policy, savings not significant
  ■ Design simple rules to follow
  ■ Explore opportunities for disposing of inventory

Material adopted from Silver, Pyke, & Peterson (1999), Inventory Management and Production Planning
Simple Reorder Rules

• Set Common Reorder Quantities
  ■ Assume common $c_t$ and $h$ values
  ■ Find $D_i c_i$ values for ordering frequencies
  ■ Example:
    ♦ Select between monthly, quarterly, semi-annual, or annual so that $w_1=1$, $w_2=3$, $w_3=6$, $w_4=12$

\[
\begin{align*}
\frac{c_t D_i}{Q_{i1}} + \frac{(c_i h Q_{i1})}{2} &= \frac{c_t D_i}{Q_{i2}} + \frac{(c_i h Q_{i2})}{2} \\
12c_t D_i / D_i w_1 + c_i h D_i w_1 / 24 &= 12c_t D_i / D_i w_2 + c_i h D_i w_2 / 24 \\
(c_i h D_i / 24)(w_1 - w_2) &= (12c_t)(1/w_2 - 1/w_1) \\
D_i c_i &= [(24)(12c_t)/(h(w_1-w_2))](1/w_2 - 1/w_1) \\
D_i c_i &= 288c_t / (h w_1 w_2)
\end{align*}
\]

Rule if $D_i c_i \geq 96(c_t / h)$ then order Monthly
Else: if $D_i c_i \geq 16(c_t / h)$ then order Quarterly
Else: if $D_i c_i \geq 4(c_t / h)$ then order Semi-Annually
Else: Order Annually
Disposing of Excess Inventory

• Why does excess inventory occur?
  ■ SKU portfolios tend to grow
  ■ Poor forecasts - Shorter lifecycles

• Which items to dispose?
  ■ Look at DOS (days of supply) for each item = IOH/D
  ■ Consider getting rid of items that have DOS > x years

• What actions to take?
  ■ Convert to other uses
  ■ Ship to more desired location
  ■ Mark down price
  ■ Auction
Real-World Challenges
Sadly the world is not so simple

- Reality is often ugly!
  - Models are not used exactly as in textbooks
  - Data is not always available or correct
  - Technology matters
  - Business processes matter even more

- Inventory policies try to answer three questions:
  - How often should I check my inventory?
  - How do I know if I should order more?
  - How much to order?

- All inventory models use two key numbers
  - Inventory Position
  - Order Point

Material adopted from Blanco, E. E. (2005), MIT Course Notes, ESD.260 - Logistics Systems
Inventory Position – how much do I have?

**Data Collection?**
- Number of item-location combinations: 
  \((10^3 \text{ locations})(10^{4-5} \text{ SKUs}) \approx 10^8\!\)!
- Database processing power: 
  @ \(\sim 10^3\) transactions/second \(> 24\) hrs
- Business process cycles:
  - hourly / 3-4x daily / daily / weekly

**Inventory Position (IP)**
\(=\) **Inventory On Hand (IOH)**

**When is an item “On-Order”?**
- Order has been generated by the system?
- Order has been transmitted to the supplier?
- Order has been accepted by the supplier?
- Order has been shipped by the supplier?

**Inventory On Order (IOO)**

**Backorders & Commitments**

**What is the lead time?**
Orders have expected arrival dates, but
- Is it updated? By whom?
- How about partial orders?
- How about multiple vendors?

**Data Integrity?**
- Wrong/missing product codes
- “Fat Finger” data entry
- Scanner/reader problems
- Shrinkage & Returns

**Challenges?**
- Cancellation policy?
- “Phantom Orders”?
- Ordering grace periods
Order Point – when should I place an order?

Order Point (s) = Expected Demand over Lead and Review Time (µ_{DL+R})

\[ \text{Order Point (s)} = \mu_{DL+R} + k \sigma_{DL+R} \]

- Are forecasts accurate?
- Demand vs. Sales?
- Promotions?
- Trends or Seasonality?
- How was forecast error estimated?
- Initial vs. final forecasts?
- Level of detail (SKU-location/aggregate)
- Includes lead time variability?
- What demand PDF was assumed?
- Who defines/owns service levels?
- How often are they updated?
- How is this entered into the ERP?
- Who collects lead times?
- How often are they updated?
Other Challenges

- Inventory decisions @ item-location level
  - Local “Optimal” ≠ System Optimal
  - Are items really independent?

- Technology
  - Homegrown (Legacy) vs. ERP vs Niche systems
  - Parameter configuration & installation (Daunting!)
  - Integration with forecasting systems (data level)
  - Implicit assumptions and parameter updates

- Demand Forecasting
  - Assumed stationary demand
  - Recalculate parameters on a regular basis (s[t], S[t], Q[t])
  - Need to deal with SKU life-cycle
Key Points from Lesson
Key Points from Lesson

• Manage Inventory by Segment
  ■ Class A items ➔ Active
  ■ Class B items ➔ Automatic
  ■ Class C (and lower) items ➔ Passive

• Real-World Challenges
  ■ Data availability and integrity
  ■ Technology capabilities and limitations
  ■ Avoid relying on “magic tools” that no one understands
  ■ Be cognizant of business process and rules
Questions, Comments, Suggestions?
Use the Discussion!

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