One to Many Distribution
How can I distribute products?

**One-to-One** – direct or point to point movements from origin to destination

**One-to-Many** – multi-stop moves from a single origin to many destinations

**Many-to-Many** – moving from multiple origins to multiple destinations usually with a hub or terminal
Example: OfficeMin

- Your firm delivers office supplies to firms within the I-95 highway loop around Boston from your distribution center located in Newton.
- You want to estimate:
  1. Expected cost per day,
  2. Expected truck fleet size, and

- What information do I need?
- What methodology should I use?
Defining Delivery Districts
One to Many System

Single Distribution Center:
- Products originate from one origin
- Products are demanded at many destinations
- All destinations are within a specified Service Region
- Ignore inventory (same day delivery)

Assumptions:
- Vehicles are homogenous
- Same capacity, $Q_{MAX}$
- Fleet size is constant

One to Many System

Finding the estimated total distance:
- Divide the Service Region into Delivery Districts
- Estimate the distance required to service each district

![Diagram of Delivery Districts and Service Region]
One to Many System

Route to serve a specific district:
- Line haul from origin to the 1st customer in the district
- Local delivery from 1st to last customer in the district
- Back haul (empty) from the last customer to the origin

\[ d_{TOUR} \approx 2d_{LineHaul} + d_{Local} \]

- \( d_{LineHaul} = \) Distance from origin to center of gravity (centroid) of delivery district
- \( d_{Local} = \) Local delivery between customers in one district

How do we estimate distances?
1. Point to Point
2. Routing or within a Tour
Estimating Point to Point Distances
Distance Estimation: Point to Point

- Why bother?
- How to do it?
  - Depends on the topography of the underlying region
    - Euclidean Space: \[ d_{A-B} = \sqrt{(x_A-x_B)^2+(y_A-y_B)^2} \]
    - Grid: \[ d_{A-B} = |x_A-x_B|+|y_A-y_B| \]
    - Random Network: different approach
Distance Estimation: Point to Point

- For Random (real) Networks use: \( D_{A-B} = k_{CF} d_{A-B} \)
- Find \( d_{A-B} \) - the “as crow flies” distance.
  - Euclidean: for really short distances
    \[ d_{A-B} = \sqrt{(x_A-x_B)^2 + (y_A-y_B)^2} \]
  - Great Circle: for locations within the same hemisphere
    \[ d_{A-B} = 3959 \times \arccos(\sin[\text{LAT}_A]\sin[\text{LAT}_B] + \cos[\text{LAT}_A] \cos[\text{LAT}_B] \cos[\text{LONG}_A - \text{LONG}_B]) \]
    Where:
    - \( \text{LAT}_i \) = Latitude of point \( i \) in radians
    - \( \text{LONG}_i \) = Longitude of point \( i \) in radians
    - Radians = (Angle in Degrees)(\( \pi/180^o \))
- Apply an appropriate circuity factor (\( k_{CF} \))
  - How do you get this value?
  - What do you think the ranges are?
  - What are some cautions for this approach?
### Selected Values of $k_{CF}$

<table>
<thead>
<tr>
<th>Country</th>
<th>$k_{CF}$</th>
<th>StdDev</th>
</tr>
</thead>
<tbody>
<tr>
<td>Argentina</td>
<td>1.22</td>
<td>0.15</td>
</tr>
<tr>
<td>Australia</td>
<td>1.28</td>
<td>0.17</td>
</tr>
<tr>
<td>Belarus</td>
<td>1.12</td>
<td>0.05</td>
</tr>
<tr>
<td>Brazil</td>
<td>1.23</td>
<td>0.11</td>
</tr>
<tr>
<td>Canada</td>
<td>1.30</td>
<td>0.10</td>
</tr>
<tr>
<td>China</td>
<td>1.33</td>
<td>0.34</td>
</tr>
<tr>
<td>Egypt</td>
<td>2.10</td>
<td>1.96</td>
</tr>
<tr>
<td>Europe</td>
<td>1.46</td>
<td>0.58</td>
</tr>
<tr>
<td>England</td>
<td>1.40</td>
<td>0.66</td>
</tr>
<tr>
<td>France</td>
<td>1.65</td>
<td>0.46</td>
</tr>
<tr>
<td>Germany</td>
<td>1.32</td>
<td>0.95</td>
</tr>
<tr>
<td>Italy</td>
<td>1.18</td>
<td>0.10</td>
</tr>
<tr>
<td>Spain</td>
<td>1.58</td>
<td>0.80</td>
</tr>
<tr>
<td>Hungary</td>
<td>1.35</td>
<td>0.25</td>
</tr>
<tr>
<td>India</td>
<td>1.31</td>
<td>0.21</td>
</tr>
<tr>
<td>Indonesia</td>
<td>1.43</td>
<td>0.34</td>
</tr>
<tr>
<td>Japan</td>
<td>1.41</td>
<td>0.15</td>
</tr>
<tr>
<td>Mexico</td>
<td>1.46</td>
<td>0.43</td>
</tr>
<tr>
<td>New Zealand</td>
<td>2.05</td>
<td>1.63</td>
</tr>
<tr>
<td>Poland</td>
<td>1.21</td>
<td>0.09</td>
</tr>
<tr>
<td>Russia</td>
<td>1.37</td>
<td>0.26</td>
</tr>
<tr>
<td>Saudi Arabia</td>
<td>1.34</td>
<td>0.19</td>
</tr>
<tr>
<td>South Africa</td>
<td>1.23</td>
<td>0.12</td>
</tr>
<tr>
<td>Thailand</td>
<td>1.42</td>
<td>0.44</td>
</tr>
<tr>
<td>Turkey</td>
<td>1.36</td>
<td>0.34</td>
</tr>
<tr>
<td>Ukraine</td>
<td>1.29</td>
<td>0.12</td>
</tr>
<tr>
<td>United States</td>
<td>1.20</td>
<td>0.17</td>
</tr>
<tr>
<td>Alaska</td>
<td>1.79</td>
<td>0.87</td>
</tr>
<tr>
<td>US East</td>
<td>1.20</td>
<td>0.16</td>
</tr>
<tr>
<td>US West</td>
<td>1.21</td>
<td>0.17</td>
</tr>
</tbody>
</table>

Estimating Route Distances
Distance Estimation: Routing

- Traveling Salesman Problem
  - Starting from an origin, find the minimum distance required to visit each destination once and only once and return to origin.
  - The expected TSP distance, $d_{\text{TSP}}$, is proportional to $\sqrt{(nA)}$ where $n=$ number of stops and $A=$area of district
  - The factor ($k_{\text{TSP}}$) is a function of the topology

\[
d_{\text{TSP}} = k_{\text{TSP}} \sqrt{(nA)}
\]
One to Many System

- What can we say about the expected TSP distance to cover \( n \) stops in district with an area of \( A \)?

- A good approximation, assuming a "fairly compact and fairly convex" area, is:

\[
E[d_{TSP}] \approx k_{TSP} \sqrt{nA} = k_{TSP} \sqrt{n \left( \frac{n}{\delta} \right)} = \frac{k_{TSP} n}{\sqrt{\delta}}
\]

- What values of \( k_{TSP} \) should we use?
  - Lots of research on this for \( L_1 \) and \( L_2 \) networks - depends on district shape, approach to routing, etc.
  - Euclidean (\( L_2 \)) Networks
    - \( k_{TSP} = 0.57 \) to 0.99 depending on clustering & size of \( N \) (MAPE~4%, MPE~1%)
    - \( k_{TSP} = 0.765 \) commonly used
  - Grid (\( L_1 \)) Networks
    - \( k_{TSP} = 0.97 \) to 1.15 depending on clustering and partitioning of district

References:
Estimating Tour Distances
Estimating Tour Distance

- Finding the total distance traveled on all tours, where:
  - \( l \) = number of tours
  - \( c \) = number of customer stops per tour and
  - \( n \) = total number of stops = \( c \times l \)

\[
E\left[ d_{TOUR} \right] = 2d_{LineHaul} + \frac{ck_{TSP}}{\sqrt{\delta}}
\]

\[
E\left[ d_{AllTours} \right] = lE\left[ d_{TOUR} \right] = 2ld_{LineHaul} + \frac{nk_{TSP}}{\sqrt{\delta}}
\]

- Minimize number of tours by maximizing vehicle capacity

\[
l = \left\lceil \frac{D}{Q_{MAX}} \right\rceil^+
\]

\[
E\left[ d_{AllTours} \right] = 2\left\lceil \frac{D}{Q_{MAX}} \right\rceil^+ d_{LineHaul} + \frac{nk_{TSP}}{\sqrt{\delta}}
\]

\([x]^+ = \text{lowest integer value} > x.\]

This is a step function

Estimate this with continuous function:

\( E([x]^+) \sim E(x) + \frac{1}{2} \)
Continuous Approximation

In this example, $Q_{\text{MAX}} = 20$. The number of tours, $l$, would be $[D/Q_{\text{MAX}}]^+$ which is a step function. Step functions are not continuous – let’s create a continuous approximation of this function that we can use.
One to Many System

• So that expected distance for all tours becomes:

\[
E \left[ d_{\text{AllTours}} \right] = 2 \left[ \frac{E[D]}{Q_{\text{MAX}}} \right]^+ d_{\text{LineHaul}} + \frac{E[n]k_{TSP}}{\sqrt{\delta}} = 2 \left[ \frac{E[D]}{Q_{\text{MAX}}} + \frac{1}{2} \right] d_{\text{LineHaul}} + \frac{E[n]k_{TSP}}{\sqrt{\delta}}
\]

• Note that if each delivery district has a different density, then:

\[
E \left[ d_{\text{AllTours}} \right] = 2 \sum_i \left[ \frac{E[D_i]}{Q_{\text{MAX}}} + \frac{1}{2} \right] d_{\text{LineHaul}_i} + k_{TSP} \sum_i \frac{E[n_i]}{\sqrt{\delta_i}}
\]
Putting it all together

For identical districts, the approximate transportation cost to deliver to each customer becomes:

\[
\text{TransportCost} = c_s \left[ E[n] + \frac{E[D]}{Q_{MAX}} + \frac{1}{2} \right] + c_d \left( 2 \frac{E[D]}{Q_{MAX}} + \frac{1}{2} \right) d_{\text{LineHaul}} + E[n] k_{TSP} + c_{vs} E[D]
\]

- \( E[n] \): Expected number of stops in district
- \( E[D] \): Expected demand in district
- \( Q_{MAX} \): Capacity of each truck
- \( c_s \): Cost per stop ($/stop)
- \( c_d \): Cost per distance ($/mile)
- \( c_{vs} \): Cost per unit per stop ($/item-stop)
- \( \delta \): Density (# stops/Area)
- \( k_{TSP} \): TSP network factor (unitless)
- \( d_{\text{TSP}} \): Traveling Salesman Distance
- \( d_{\text{stop}} \): Average distance per stop
Solution OfficeMin
OfficeMin Problem

- You deliver office supplies to firms within the I95 loop around Boston from your DC in Newton. This region is about 8 miles by 14 miles.
- You expect ~ 100 customer orders per day – for about 1 to 2 pallets of product each. Local vans can handle 5 pallets at most.
- You estimate it costs about $10 per stop (to load or unload), about $5 per pallet to deliver to end customer, and about $1 a mile for driving.
- What is the expected daily transportation cost?
OfficeMin

TransportCost = \( c_s \left[ E[n] + \frac{E[D]}{Q_{MAX}} + \frac{1}{2} \right] + c_d \left[ 2 \left( \frac{E[D]}{Q_{MAX}} + \frac{1}{2} \right) \right] d_{LineHaul} + \frac{E[n]k_{TSP}}{\sqrt{\delta}} + c_{vs} E[D] \)

- What do we know?
  - \( c_s = 10 \$/stop \)
  - \( c_d = 1 \$/mile \)
  - \( c_{vs} = 5 \$/pallet \)
  - \( E[n] = 100 \)
  - \( E[D] = 150 \)
  - \( Q_{MAX} = 5 \) pallets

- What do we need to find?
  - \( k = 1.15 \) (estimate)
  - \( \delta = \frac{100}{(8)(14)} = 0.89 \sim 1 \)
  - \( d_{LineHaul} = ?? \)
TransportCost = $cs\left[ E[n] + \frac{E[D]}{Q_{MAX}} + \frac{1}{2}\right] + c_d\left(2\frac{E[D]}{Q_{MAX}} + \frac{1}{2}\right) d_{LineHaul} + \frac{E[n]k_{TSP}}{\sqrt{\delta}} + c_{vs} E[D]$

Estimated number of tours per day:
\[ l = \frac{E[D]}{Q_{MAX}} + \frac{1}{2} = \frac{150}{5} + .5 = 30.5 \]

Estimated stop (load/unload) cost per day:
\[ c_s\left[ E[n] + E[l] \right] = 10(100 + 30.5) = $1305 \]

Estimated distance (driving) cost per day:
\[ c_d\left(2E[l]d_{LineHaul} - \frac{E[n]k_{TSP}}{\sqrt{\delta}}\right) = 1\left(2(30.5)(5) + \frac{100(1.15)}{\sqrt{1}}\right) = 305 + 115 = $420 \]

Estimated stop-pallet costs per day:
\[ c_{vs} E[D] = 5(150) = $750 \]

Estimated total daily cost ~ $2400 to $2500
Estimating Fleet Size

[OPTIONAL – MORE ADVANCED]
Estimating the Fleet Size

- Find minimum number of vehicles required based on the amount of required work time each day where
  - \( M \) = minimum number of vehicles needed in fleet
  - \( t_w \) = available worktime for each vehicle per period
  - \( W \) = required amount of work time each day
  - \( s \) = average vehicle speed
  - \( l \) = number of shipments per period
  - \( t_l \) = loading time per shipment
  - \( t_s \) = unloading time per stop

\[
W = \frac{2E[l]d_{LineHaul} + \frac{E[n]k_{TSP}}{\sqrt{\delta}}}{s} + E[l]t_l + E[n]t_s
\]

\[
W = \left( \frac{2d_{LineHaul}}{s} + t_l \right)E[l] + E[n]\left( \frac{k_{TSP}}{s\sqrt{\delta}} + t_s \right)
\]

\[
Mt_w \geq W
\]
Fleet Size

Note that $W$ is a linear combination of two random variables, $n$ and $D$. But, they are not independent, in fact, $D = nD_c$ where $D_c$ is the number of pallets per customer.

$$W = \left(\frac{2d_{\text{LineHaul}}}{s} + t_l\right)\left[\frac{D}{Q_{\text{MAX}}} + \left(\frac{1}{2}\right)\right] + n\left(\frac{k_{\text{TSP}}}{s\sqrt{\delta}} + t_s\right)$$

$$a = \left(\frac{2d_{\text{LineHaul}}}{s} + t_l\right)\left[\frac{1}{Q_{\text{MAX}}}\right] = 0.144 \text{ hrs}$$

$$b = \left(\frac{k_{\text{TSP}}}{s\sqrt{\delta}} + t_s\right) = 0.525 \text{ hrs}$$

$$c = \frac{1}{2}\left(\frac{2d_{\text{LineHaul}}}{s} + t_l\right) = 0.361 \text{ hrs}$$

$$E[D_c] = 1.5$$

$$\Var[D_c] = \frac{\sum_{i=1}^{n}(x_i - \bar{x})^2}{n} = 0.25$$

Setting $X = aD_c + b$, $W$ is now a function of random variables, $X$ and $n$.

$$W = aD + bn + c$$

$$W = aD_c n + bn + c$$

$$W = (aD_c + b)n + c$$

$$W = Xn + c$$

$$E[W] = E[X]E[n] + c$$

$$\Var[W] = E[n]\Var[X] + E[X]^2\Var[n]$$

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$c_s$</td>
<td>10 $/stop</td>
</tr>
<tr>
<td>$c_d$</td>
<td>1 $/mile</td>
</tr>
<tr>
<td>$c_v$</td>
<td>5 $/pallet</td>
</tr>
<tr>
<td>$E[n]$</td>
<td>100</td>
</tr>
<tr>
<td>$E[D]$</td>
<td>150</td>
</tr>
<tr>
<td>$k$</td>
<td>1.15</td>
</tr>
<tr>
<td>$\delta$</td>
<td>1</td>
</tr>
<tr>
<td>$Q_{\text{MAX}}$</td>
<td>5 pallets</td>
</tr>
<tr>
<td>$d_{\text{linehaul}}$</td>
<td>5</td>
</tr>
<tr>
<td>$t_w$</td>
<td>10 hrs</td>
</tr>
<tr>
<td>$s$</td>
<td>45 mph</td>
</tr>
<tr>
<td>$t_l$</td>
<td>0.5 hr</td>
</tr>
<tr>
<td>$t_s$</td>
<td>0.5 hr</td>
</tr>
</tbody>
</table>

CTL.SC1x - Supply Chain and Logistics Fundamentals  Lesson: One to Many Distribution
Fleet Size

\[ W = \left( \frac{2d_{\text{LineHaul}}}{s} + t_l \right) \left[ \frac{D}{Q_{\text{MAX}}} + \frac{1}{2} \right] + n \left( k_{\text{TSP}} + t_s \right) \]

\[ W = (aD_c + b)n + c = \left( 0.144D_c + 0.525 \right)n + 0.361 = Xn + c \]

\[ E[X] = aE[D_c] + b = 0.144(1.5) + 0.525 = 0.741 \quad E[n] = 100 \text{ customers} \]

\[ \text{Var}[X] = a^2\text{Var}[D_c] = 0.021(0.25) = 0.00525 \quad \text{Var}[n] = 400 \text{ (assume } \sigma_n = 20) \]

\[ E[W] = E[X]E[n] + c \]

\[ E[W] = (0.741)(100) + 0.361 = 74.46 \text{ hr} \]

\[ \text{Var}[W] = E[n]\text{Var}[X] + E[X]^2\text{Var}[n] \]

\[ \text{Var}[W] = (100)(0.00525) + (0.741)^2(400) = 220 \]

\( c_s = 10 \text{$/stop} \quad \quad E[n] = 100 \quad \quad k = 1.15 \)

\( c_d = 1 \text{$/mile} \quad \quad E[D] = 150 \quad \quad \delta = 1 \)

\( c_{vS} = 5 \text{$/pallet} \quad \quad Q_{\text{MAX}} = 5 \text{ pallets} \quad \quad D_{\text{linehaul}} = 5 \)

\( t_w = 10 \text{ hrs} \quad \quad s = 45 \text{ mph} \quad \quad t_l = 0.5 \text{ hr} \)

\( t_s = 0.5 \text{ hr} \quad \quad \quad \quad \quad \quad a = 0.144 \text{hrs} \quad \quad \quad \quad \quad b = 0.525 \text{ hrs} \)

\( \quad \quad \quad \quad \quad c = 0.361 \text{ hrs} \quad \quad \quad \quad \quad E[D_c] = 1.5 \text{ pallets} \quad \quad \quad \quad \quad \text{Var}[D_c] = 0.25 \)

Distribution of required daily work hours:

\( \mu_W \sim 75 \text{ hrs} \quad \quad \quad \quad \quad \sigma_W \sim 15 \text{ hrs} \)
Fleet Size

• Daily distribution of required work time \( \sim N(75, 15) \)
• Set the fleet size \( (M) \) to match our level of risk – how?

• Select a cycle service level (CSL) equal to \( P[W < Mt_w] \)
  - Set \( M = (\mu_W + k_{CSL} \sigma_W) / t_w \)
  - \( M(80\%) = (75 \text{ hrs} + 0.84(15 \text{ hrs})) / (10 \text{ hrs/veh}) = 8.00 \approx 8 \)
  - \( M(90\%) = (75 \text{ hrs} + 1.28(15 \text{ hrs})) / (10 \text{ hrs/veh}) = 9.42 \approx 10 \)
  - \( M(95\%) = (75 \text{ hrs} + 1.64(15 \text{ hrs})) / (10 \text{ hrs/veh}) = 9.96 \approx 10 \)
  - \( M(99\%) = (75 \text{ hrs} + 2.33(15 \text{ hrs})) / (10 \text{ hrs/veh}) = 10.99 \approx 11 \)

• Using very few, very rough estimates of input values, we can get a feel for the trade-offs between costs and service.
• Approximations can be used for sensitivity analysis.

Note: This is not the TSP k!

Key Take Aways
Key Take Aways

• Many forms of product flow: One-to-One, One-to-Many, Many-to-One, and Many-to-Many

• Approximations are good “first steps”
  - Require minimal data
  - Allow for fast sensitivity analysis
  - Enables quick scoping of the solution space

• Optimal methods usually require tremendous amounts of detailed information

• Planning problems usually have lots of uncertainty – the actual conditions are unknown

• Before deciding to spend the time and energy to find an optimal solution, it is helpful to see if it is worth it.
Questions, Comments, Suggestions? Use the Discussion!

“Dexter – continuously approximating” Yankee Golden Retriever Rescued Dog (www.ygrr.org)

caplice@mit.edu