

Key Concepts: Week 7 Lesson 2:

Probabilistic Inventory Models II

Learning Objectives

- Able to establish a periodic review, Order Up To (S,R) Replenishment Policy using any of the four performance metrics
- Understand relationships between the performance metrics (CSL, IFR, CSOE, and CIS) and be able to calculate the implicit values.
- Understand the trade-off between lead time and replenishment time in Period Review Policies
- Able to use the inventory models to make trade-offs and estimate impacts of policy changes

Lesson Summary:

In this lesson, we examined the trade-offs between the different performance metrics (both cost and service based). We demonstrated that once one of the metrics is determined (or explicitly set) then the other three are implicitly set. Because they all lead to the establishment of a safety factor, k , they are dependent on each other. This means that once you have set the safety stock, regardless of the method, you can calculate the expected performance implied by the remaining three metrics.

We also introduced the Order-Up To Periodic Review Policy, (R,S). We demonstrated that the same methods of determining the four performance metrics in the (S,Q) model can be used here, with minor modifications. Periodic Review policies are very popular because they fit the regular pattern of work where ordering might occur only once a week or once every two weeks. The lead-time and the review period are related and can be traded-off to achieve certain goals.

Notation:

B_1 :	Cost associated with a stockout event
c :	Purchase cost (\$/unit)
c_t :	Ordering Costs (\$/order)
c_e :	Excess holding Costs (\$/unit/time); Equal to ch
c_s :	Shortage costs (\$/unit)
c_g :	One Time Good Deal Purchase Price (\$/unit)
D :	Average Demand (units/time)
h :	Carrying or holding cost (\$/inventory \$/time)
L :	Replenishment Lead Time (time)
Q :	Replenishment Order Quantity (units/order)
T :	Order Cycle Time (time/order)
μ_{DL} :	Expected Demand over Lead Time (units/time)
σ_{DL} :	Standard Deviation of Demand over Lead Time (units/time)
μ_{DL+R} :	Expected Demand over Lead Time plus Review Period (units/time)
σ_{DL+R} :	Standard Deviation of Demand over Lead Time plus Review Period (units/time)
k :	Safety Factor
s :	Reorder Point (units)
S :	Order up to Point (units)
R :	Review Period (time)
N :	Orders per Time or $1/T$ (order/time)
IP :	Inventory Position = Inventory on Hand + Inventory on Order – Backorders
IOH :	Inventory on Hand (units)
IOO :	Inventory on Order (units)
IFR :	Item Fill Rate (%)
CSL :	Cycle Service Level (%)
$CSOE$:	Cost of Stock Out Event (\$ / event)
CIS :	Cost per Item Short
$E[US]$:	Expected Units Short (units)
$G(k)$:	Unit Normal Loss Function

Formulas:

Inventory Performance Metrics

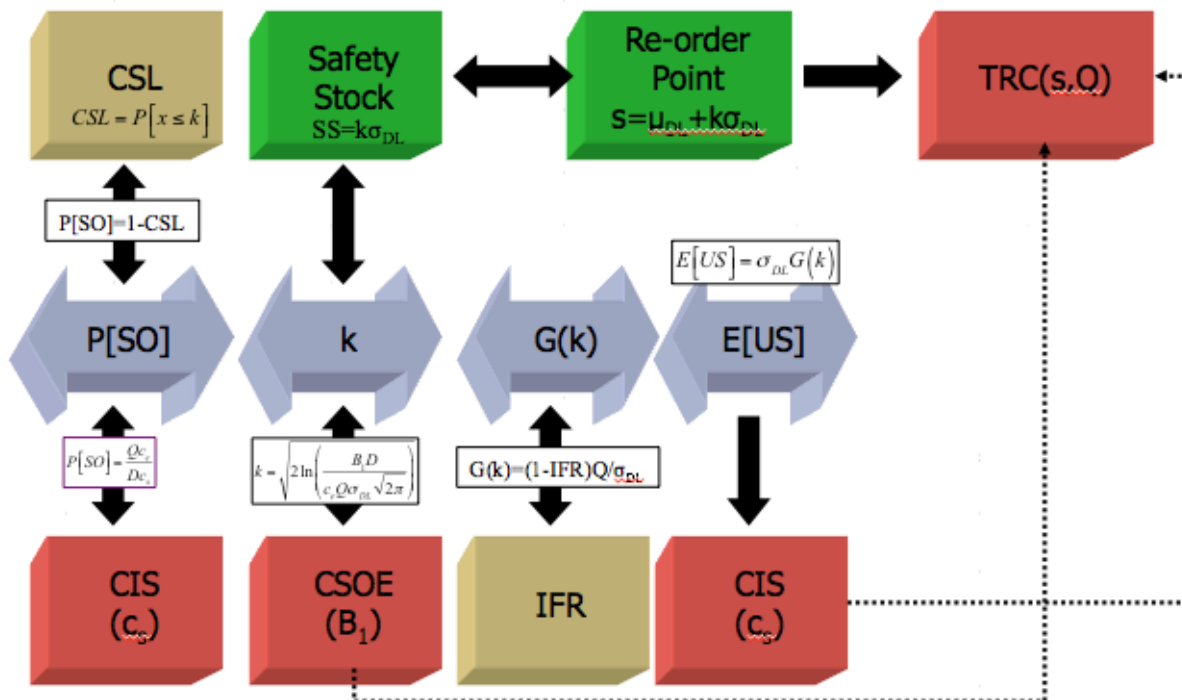
We learned in the last lesson that safety stock is determined by the safety factor, k , so that: $s = \mu_{DL} + k\sigma_{DL}$ and the expected cost of safety stock $= c_e k \sigma_{DL}$.

We learned two ways to calculate k : Service based or Cost based metrics:

- Service Based Metrics—set k to meet expected level of service
 - Cycle Service Level ($CSL = P[x \leq k]$)
 - Item Fill Rate ($IFR = 1 - \frac{\sigma_{DL} G[k]}{Q}$)
- Cost Based Metrics—find k that minimizes total costs
 - Cost per Stockout Event ($E[CSOE] = (B_1)P[x \geq k] \left(\frac{D}{Q}\right)$)
 - Cost per Items Short ($E[CIS] = c_s \sigma_{DL} G(k) \left(\frac{D}{Q}\right)$)

Safety Stock Logic – relationship between performance metrics

The relationship between the four metrics (2 cost and 2 service based) is shown in the flowchart below. Once one metric (CSL, IFR, CSOE, or CIS) is explicitly set, then the other three metrics are implicitly determined.



Periodic Review Policy (R,S)

This is also known as the Order Up To policy and is essentially a two-bin system. The policy is “Order Up To S^* units every R time periods”. This means, the order quantity will be $S^* - IP$. The order up to point, S^* is the sum of the expected demand over the lead-time and the replenishment time plus the RMSE of the forecast error over lead plus replenishment time multiplied by some safety factor k .

- Order Up To Point: $S = \mu_{DL+R} + k\sigma_{DL+R}$

Periodic (R,S) versus Continuous (s, Q) Review

- There is a convenient transformation of (s, Q) to (R, S)
 - (s,Q) = Continuous, order Q when $IP \leq s$
 - (R, S) = Periodic, order up to S every R time periods
- Allows for the use of all previous (s, Q) decision rules
 - Reorder point, s, for continuous becomes Order Up To point, S, for periodic system
 - Q for continuous becomes $D \cdot R$ for periodic
 - L for a continuous becomes $R+L$ for periodic
- Approach
 - Make transformations
 - Solve for (s, Q) using transformations
 - Determine final policy such that $S = x_{DL+R} + k\sigma_{DL+R}$

(s, Q)		(R, S)
s	$\leftrightarrow$	S
Q	$\leftrightarrow$	$D \cdot R$
L	$\leftrightarrow$	$R+L$

Relationship Between L & R

The lead-time, L, and the review period, R, both influence the total costs. Note that the average inventory costs for a (R,S) system is $= c_e \left[\frac{DR}{2} + k\sigma_{DL+R} + LD \right]$. This implies that increasing Lead Time, L, will increase Safety Stock non-linearly and Pipeline Inventory linearly while increasing the Review Period, R will increase the Safety Stock non-linearly and the Cycle Stock linearly.

Additional References:

Base stock and continuous inventory models are covered in Nahmias Chpt 5 and Silver, Pyke & Peterson Chpt 7, and Ballou Chpt 9.