

Key Concepts: Week 5 Lesson 2: Economic Order Quantity (EOQ)

Learning Objectives

- Able to estimate the Economic Order Quantity (EOQ) and to determine when it is appropriate to use
- Understand strengths (robust, simple) and weaknesses (strong assumptions) of EOQ model
- Able to estimate sensitivity of EOQ to underlying changes in the input data and understanding of its underlying robustness

Lesson Summary:

The Economic Order Quantity or EOQ is the most influential and widely used (and sometimes mis-used!) inventory model in existence. While very simple, it provides deep and useful insights. Essentially, the EOQ is a trade-off between fixed (ordering) and variable (holding) costs. It is often called Lot-Sizing as well. The minimum of the Total Cost equation (when assuming demand is uniform and deterministic) is the EOQ or Q^* . The Inventory Replenishment Policy becomes “Order Q^* every T^* time periods” which under our assumptions is the same as “Order Q^* when $IP=0$ ”.

Like Wikipedia, the EOQ is a GREAT place to start, but not necessarily a great place to finish. It is a great first estimate because it is exceptionally robust. For example, a 50% increase in Q over the optimal quantity (Q^*) only increase the TRC by ~ 8%!

While very insightful, the EOQ model should be used with caution as it has restrictive assumptions (uniform and deterministic demand). It can be safely used for items with relatively stable demand and is a good first-cut “back of the envelope” calculation in most situations. It is helpful to develop insights in understanding the trade-offs involved with taking certain managerial actions, such as lowering the ordering costs, lowering the purchase price, changing the holding costs, etc.

Key Concepts:

EOQ Model

- Assumptions
 - Demand is uniform and deterministic

- Lead time is instantaneous (0) – although this is not restrictive at all since the lead time, L, does not influence the Order Size, Q.
- Total amount ordered is received
- Inventory Replenishment Policy
 - Order Q* units every T* time periods
 - Order Q* units when inventory on hand (IOH) is zero
- Essentially, the Q* is the Cycle Stock for each replenishment cycle. It is the expected demand for that amount of time between order deliveries.

Notation:

c:	Purchase cost (\$/unit)
c_t :	Ordering Costs (\$/order)
c_e :	Excess holding Costs (\$/unit/time); equal to ch
c_s :	Shortage costs (\$/unit)
D:	Demand (units/time)
D_A :	Actual Demand (units/time)
D_F :	Forecasted Demand (units/time)
h:	Carrying or holding cost (\$/inventory \$/time)
Q:	Replenishment Order Quantity (units/order)
Q*:	Optimal Order Quantity Under EOQ (units/order)
Q^*_A :	Optimal Order Quantity with Actual Demand (units/order)
Q^*_F :	Optimal Order Quantity with Forecasted Demand (units/order)
T:	Order Cycle Time (time/order)
T*:	Optimal Time between Replenishments (time/order)
N:	Orders per Time or $1/T$ (order/time)
TRC(Q):	Total Relevant Cost (\$/time)
TC(Q):	Total Cost (\$/time)

Formulas:

Total Costs: $TC = \text{Purchase} + \text{Order} + \text{Holding} + \text{Shortage}$

This is the generic total cost equation. The specific form of the different elements depends on the assumptions made concerning the demand, the shortage types, etc.

$$TC(Q) = cD + c_t \left(\frac{D}{Q} \right) + c_e \left(\frac{Q}{2} \right) + c_s E[\text{Units Short}]$$

Total Relevant Costs: $TRC = \text{Order} + \text{Holding}$

The purchasing cost and the shortage costs are not relevant for the EOQ because the purchase price does not change the optimal order quantity (Q^*) and since we have deterministic demand, we will not stock out.

$$TRC(Q) = c_t \left(\frac{D}{Q} \right) + c_e \left(\frac{Q}{2} \right)$$

Optimal Order Quantity (Q^*)

Recall, this is simply the First Order condition of the TRC equation – where it is a global minimum.

$$Q^* = \sqrt{\frac{2c_t D}{c_e}}$$

Optimal Time between Replenishments

Recall that $T^* = Q^*/D$. That is, the time between orders is the optimal order size divided by the annual demand. Similarly, the number of replenishments per year is simply $N^* = 1/T^* = D/Q^*$. Plugging in the actual Q^* gives you the formula below.

$$T^* = \sqrt{\frac{2c_t}{Dc_e}}$$

Optimal Total Relevant Costs

Plugging the Q^* back into the TRC equation and simplifying gives you the formula below.

$$TRC(Q^*) = \sqrt{2c_t c_e D}$$

Optimal Total Costs

Adding the purchase cost to the $TRC(Q^*)$ costs gives you the $TC(Q^*)$. We still assume no stock out costs.

$$TC(Q^*) = cD + \sqrt{2c_t c_e D}$$

Sensitivity Analysis

The EOQ is very robust. The following formulas provide simple ways of calculating the impact of using a non-optimal Q, an incorrect annual Demand D, or a non-optimal time interval, T.

EOQ Sensitivity with Respect to Order Quantity

The equation below calculates the percent difference in total relevant costs to optimal, when using a non-optimal order quantity (Q):

$$\frac{TRC(Q)}{TRC(Q^*)} = \left(\frac{1}{2}\right) \left(\frac{Q^*}{Q} + \frac{Q}{Q^*}\right)$$

EOQ Sensitivity with Respect to Demand

The equation below calculates the percent difference in total relevant costs to optimal, when using assuming an incorrect annual demand (D_F) when in fact the actual annual demand is D_A :

$$\frac{TRC(Q_F^*)}{TRC(Q_A^*)} = \left(\frac{1}{2}\right) \left(\sqrt{\frac{D_A}{D_F}} + \sqrt{\frac{D_F}{D_A}}\right)$$

EOQ Sensitivity with Respect to Time Interval between Orders

The equation below calculates the percent difference in total relevant costs to optimal, when using a non-optimal replenishment time interval (T). This will become very important when finding realistic replenishment intervals. The Power of Two Policy shows that ordering in increments of 2^k time periods, we will stay within 6% of the optimal solution. For example, if the base time period is 1 week, then the Power of Two Policy would suggest ordering every week (2^0) or every 2 weeks (2^1) or every four weeks (2^2) or every 8 weeks (2^3) etc. Select the interval closest to one of these increments.

$$\frac{TRC(T)}{TRC(T^*)} = \left(\frac{1}{2}\right) \left(\frac{T}{T^*} + \frac{T^*}{T}\right)$$

Additional References:

There are more books that cover the basics of inventory management than there are grains of sand on the beach! Inventory management is also usually covered in Operations Management and Industrial Engineering texts as well. A word of warning, though. Every textbook uses different notation for the same concepts. Get used to it. Always be sure to understand what the nomenclature means so that you do not get confused.

I will make references to our core texts we are using in this course but will add some additional texts as they fit the topics. Inventory is introduced in Nahmias Chpt 4 and Silver, Pyke & Peterson Chpt 5, and Ballou Chpt 9.