Key Concepts: Week 3 Lesson 1: Exponential Smoothing

Learning Objectives

- Understand how exponential smoothing treats old and new information differently
- Understand how changing the alpha or beta smoothing factors influences the forecasts
- Able to apply the techniques to generate forecasts in spreadsheets

Summary of Lesson

Exponential smoothing, as opposed to the three other time series models we have discussed (Cumulative, Naïve, and Moving Average), treats data differently depending on its age. The idea is that the value of data degrades over time so that newer observations of demand are weighted more heavily than older observations. The weights decrease exponentially as they age. Exponential models simply blend the value of new and old information. We have students create forecast models in spreadsheets so they understand the mechanics of the model and hopefully develop a sense of how the parameters influence the forecast.

The alpha factor (ranging between 0 and 1) determines the weighting for the newest information versus the older information. The "α" value indicates the value of "new" information versus "old" information:

- As $\alpha \rightarrow 1$, the forecast becomes more nervous, volatile and naïve
- As $\alpha \rightarrow 0$, the forecast becomes more calm, staid and cumulative
- α can range from $0 \leq \alpha \leq 1$, but in practice, we typically see $0 \leq \alpha \leq 0.3$

The most basic exponential model, or Simple Exponential model, assumes stationary demand. Holt's Model is a modified version of exponential smoothing that also accounts for trend in addition to level. A new smoothing parameter, β, is introduced. It operates in the same way as the α.

We can also use exponential smoothing to dampen trend models to account for the fact that trends usually do not remain unchanged indefinitely as well as for creating a more stable estimate of the forecast errors.

Key Concepts:

Notation:

x_t: Actual demand in period t

$\hat{x}_{t,t+1}$: Forecast for time t+1 made during time t

α: Exponential smoothing factor for level ($0 \le \alpha \le 1$)

β: Exponential smoothing factor for trend ($0 \le \beta \le 1$)

ϕ: Exponential smoothing factor for dampening ($0 \le \phi \le 1$)

ω: Mean Square Error trending factor ($0.01 \le \omega \le 0.1$)

Forecasting Models:

Simple Exponential Smoothing Model (Level Only) – This model is used for stationary demand. The "new" information is simply the latest observation. The "old" information is the most recent forecast since it encapsulates the older information.

$$\hat{x}_{t,t+1} = \alpha x_t + (1-\alpha)\hat{x}_{t-1,t}$$

Exponential Smoothing for Level & Trend – also known as Holt's Method, this assumes a linear trend. The forecast for time t+τ made at time t is shown below. It is a combination of the latest estimates of the level and trend. For the level, the new information is the latest observation and the old information is the most recent forecast for that period – that is, the last period's estimate of level plus the last period's estimate of trend. For the trend, the new information is the difference between the most recent estimate of the level minus the second most recent estimate of the level. The old information is simply the last period's estimate of the trend.

$$\hat{x}_{t,t+\tau} = \hat{a}_t + \tau\hat{b}_t$$

$$\hat{a}_t = \alpha x_t + (1-\alpha)\left(\hat{a}_{t-1} + \hat{b}_{t-1}\right)$$

$$\hat{b}_t = \beta(\hat{a}_t - \hat{a}_{t-1}) + (1-\beta)\hat{b}_{t-1}$$

Damped Trend Model with Level and Trend – We can use exponential smoothing to dampen a linear trend to better reflect the tapering effect of trends in practice.

$$\hat{x}_{t,t+\tau} = \hat{a}_t + \sum_{i=1}^{\tau} \varphi^i \hat{b}_t$$

$$\hat{a}_t = \alpha x_t + (1-\alpha)(\hat{a}_{t-1} + \varphi\hat{b}_{t-1})$$

$$\hat{b}_t = \beta(\hat{a}_t - \hat{a}_{t-1}) + (1-\beta)\varphi\hat{b}_{t-1}$$

Mean Square Error Estimate – We can also use exponential smoothing to provide a more robust or stable value for the mean square error of the forecast.

$$MSE_t = \omega(x_t - \hat{x}_{t-1,t})^2 + (1-\omega)MSE_{t-1}$$

Additional References:

Exponential Smoothing is discussed in these texts:

- Spyros Makridakis ;; Steven C. Wheelwright ; Rob J. Hyndman., 1998, "Forecasting : methods and applications", Wiley, New York. ISBN 9780471532330
- Rob J Hyndman, George Athanasopoulos., 2014 "Forecasting : principles and practice," OTexts, ISBN 0987507109.

Within the texts mentioned earlier: Silver, Pyke, and Peterson Chapter 4; Chopra & Meindl Chapter 7; Nahmias Chapter 2.