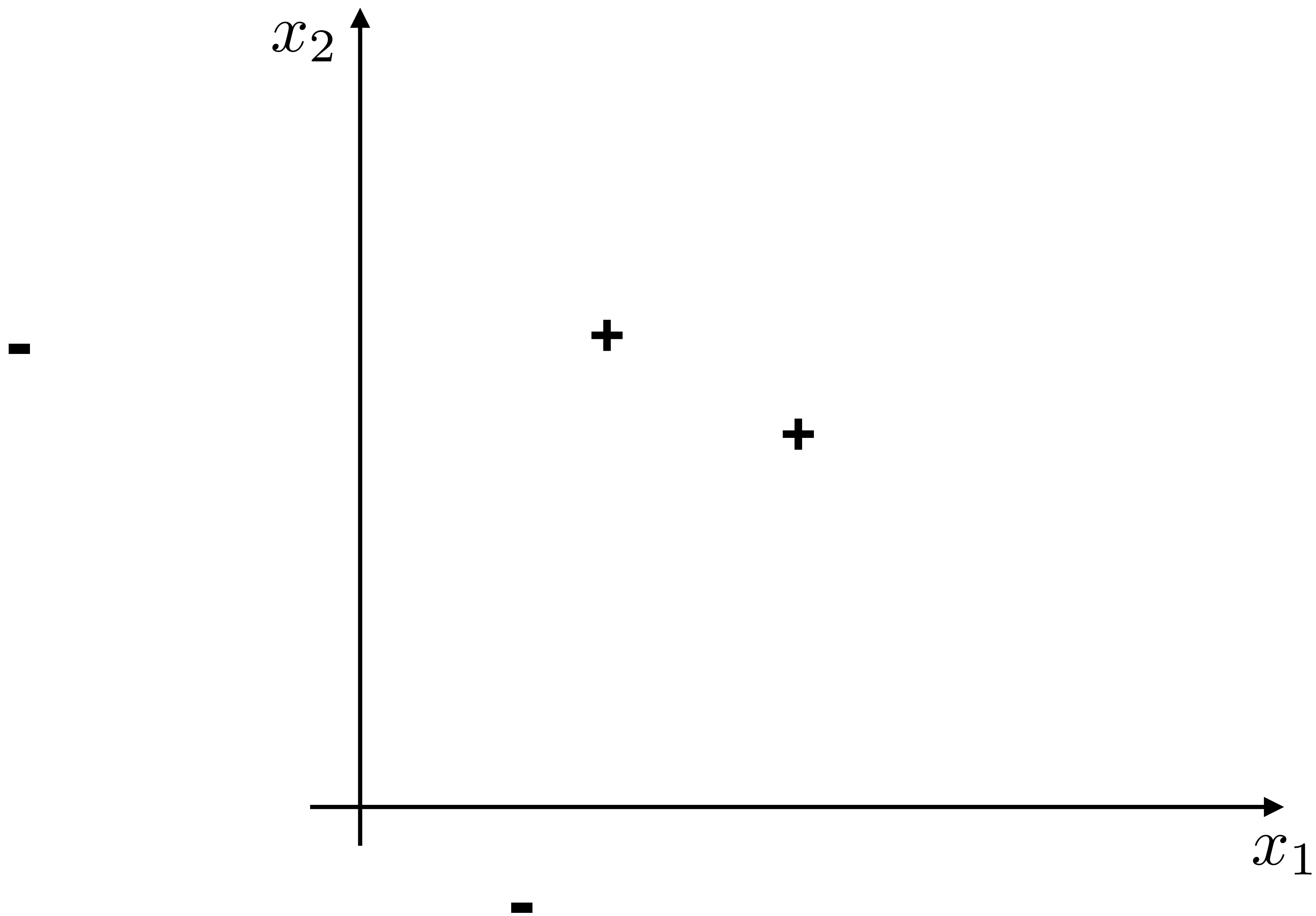
Machine Learning
Lecture 2

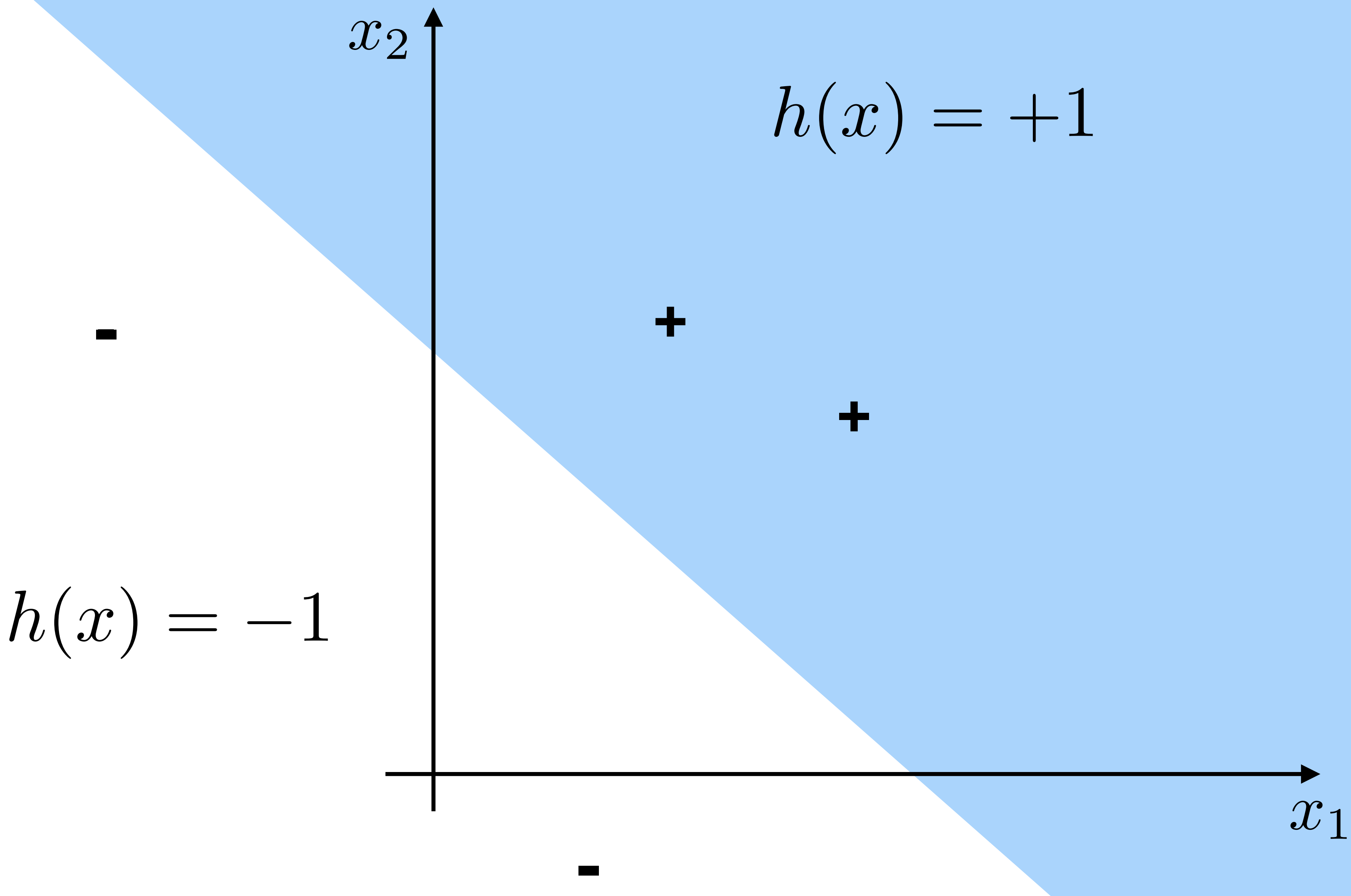
Review of basic concepts

- Feature vectors, labels
- Training set
- Classifier
- Training error
- Test error
- Set of classifiers

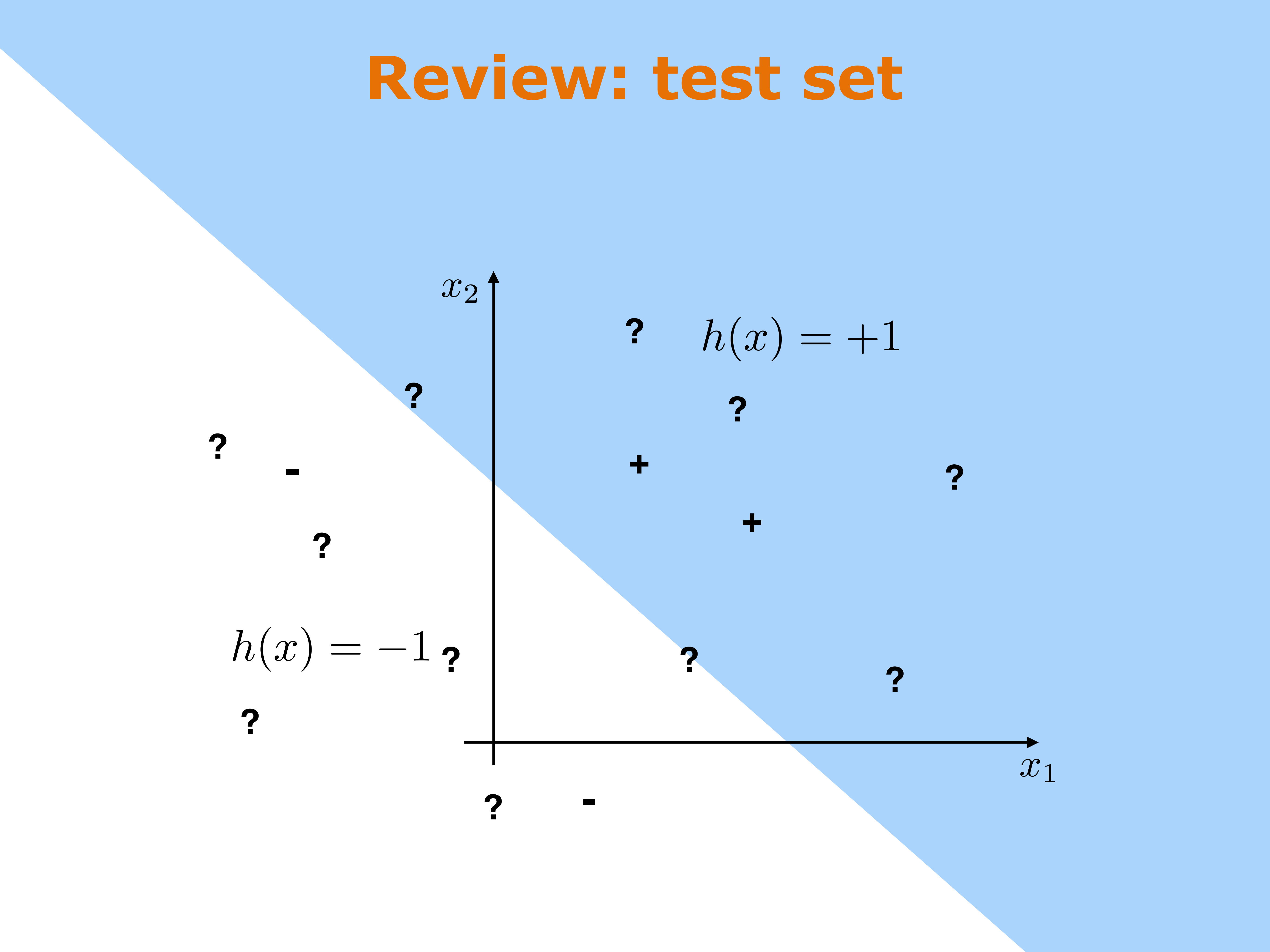
Review: training set



Review: a classifier



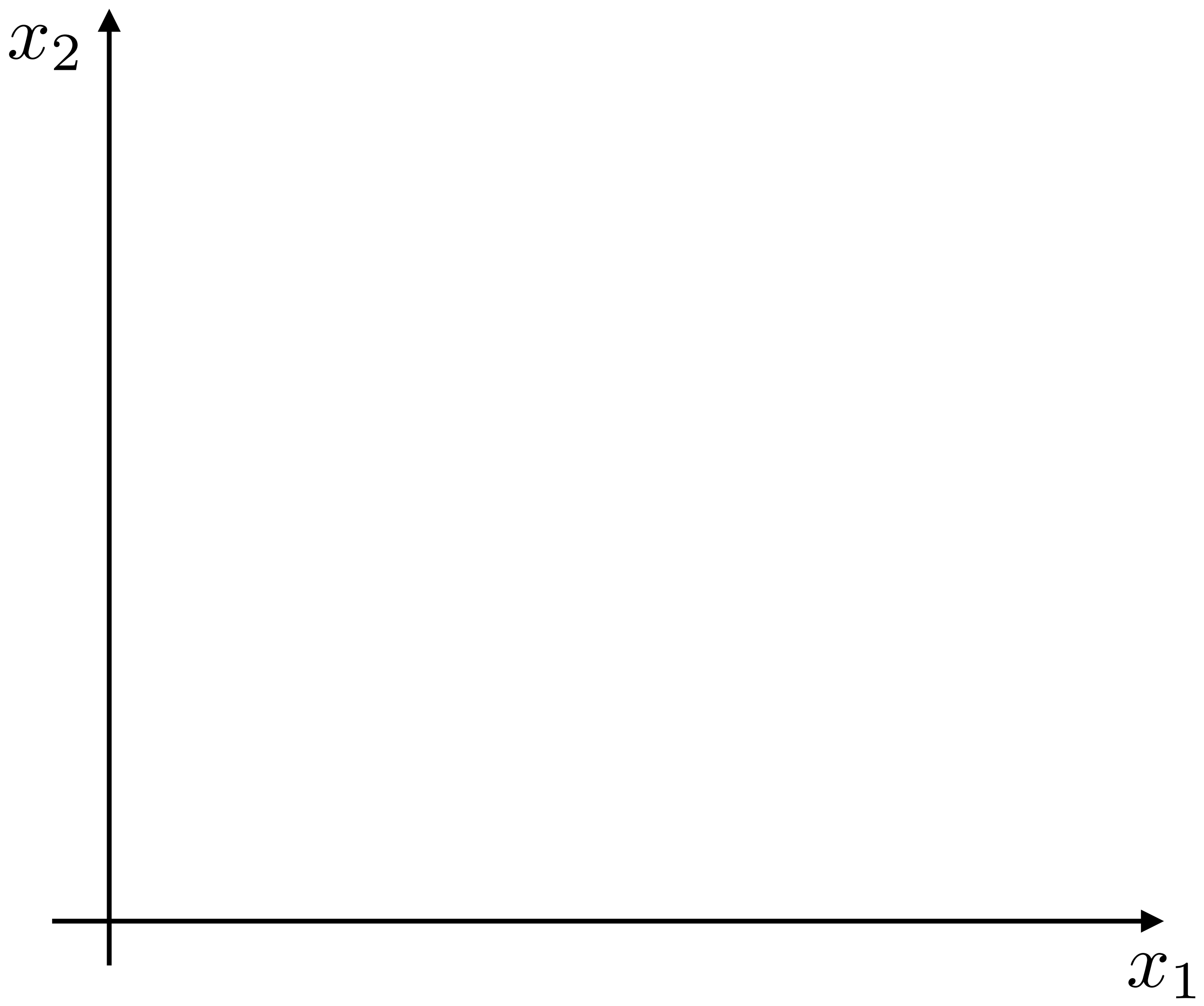
Review: test set



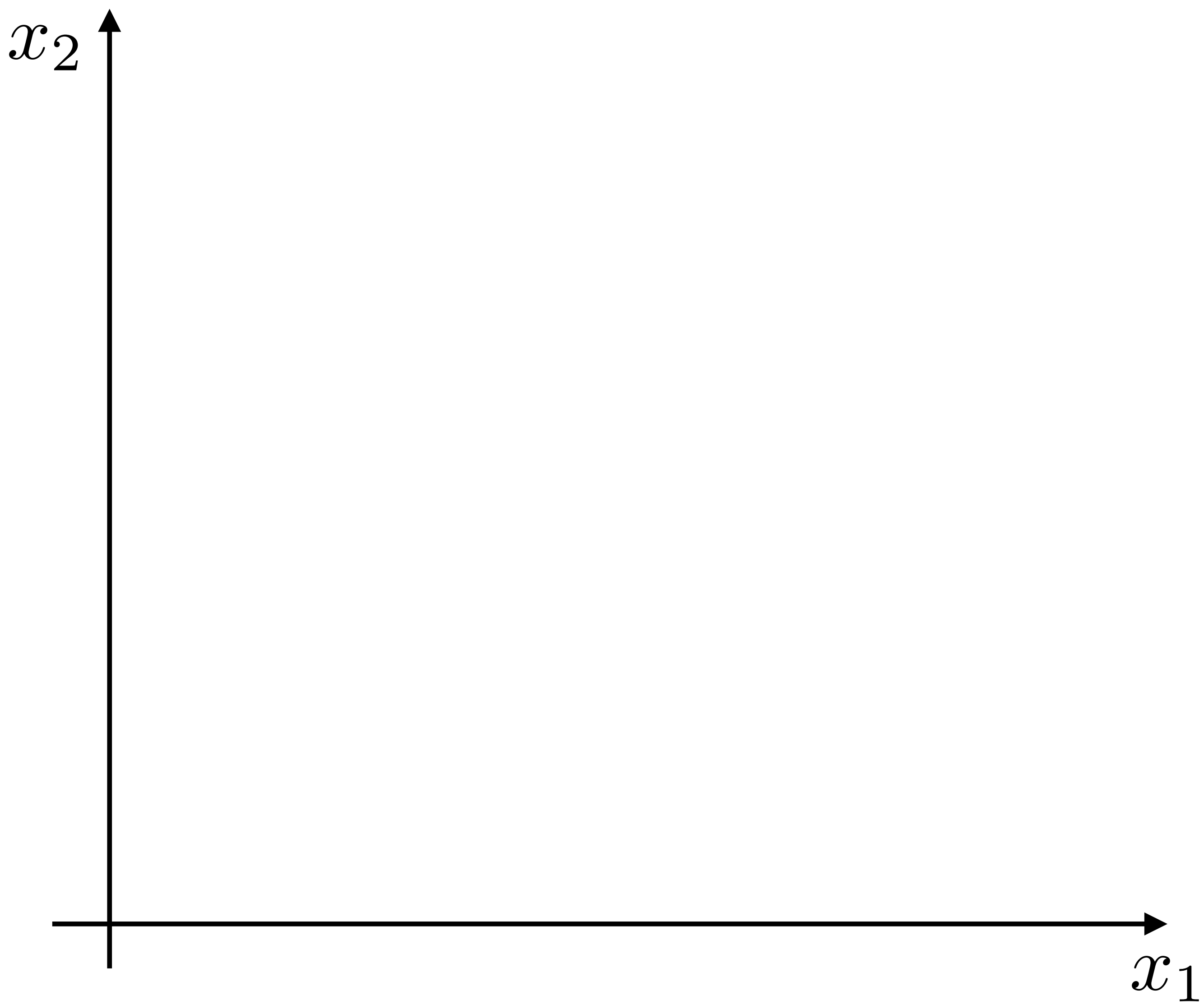
This lecture

- The set of linear classifiers
- Linear separation
- Perceptron algorithm

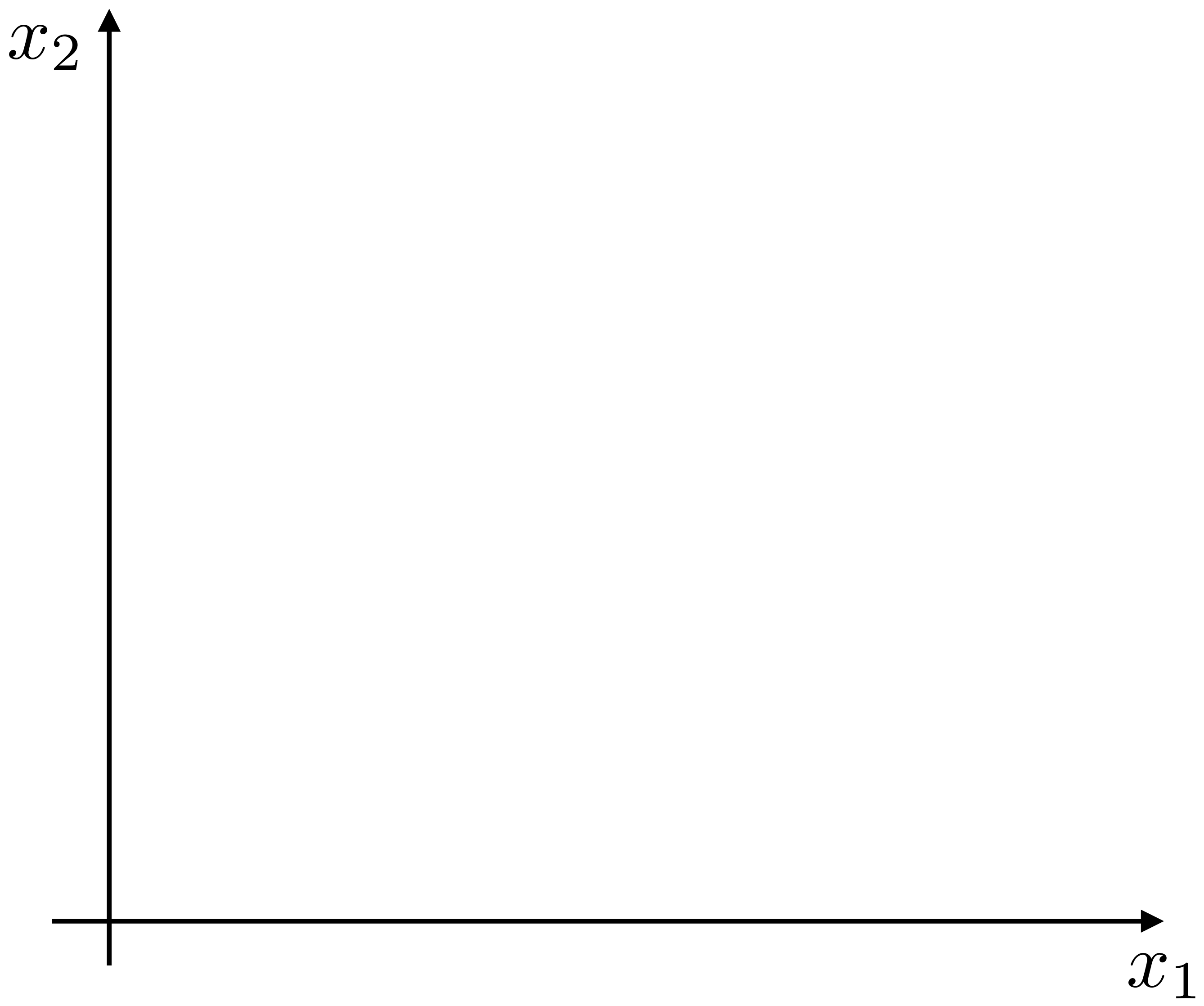
Linear classifiers



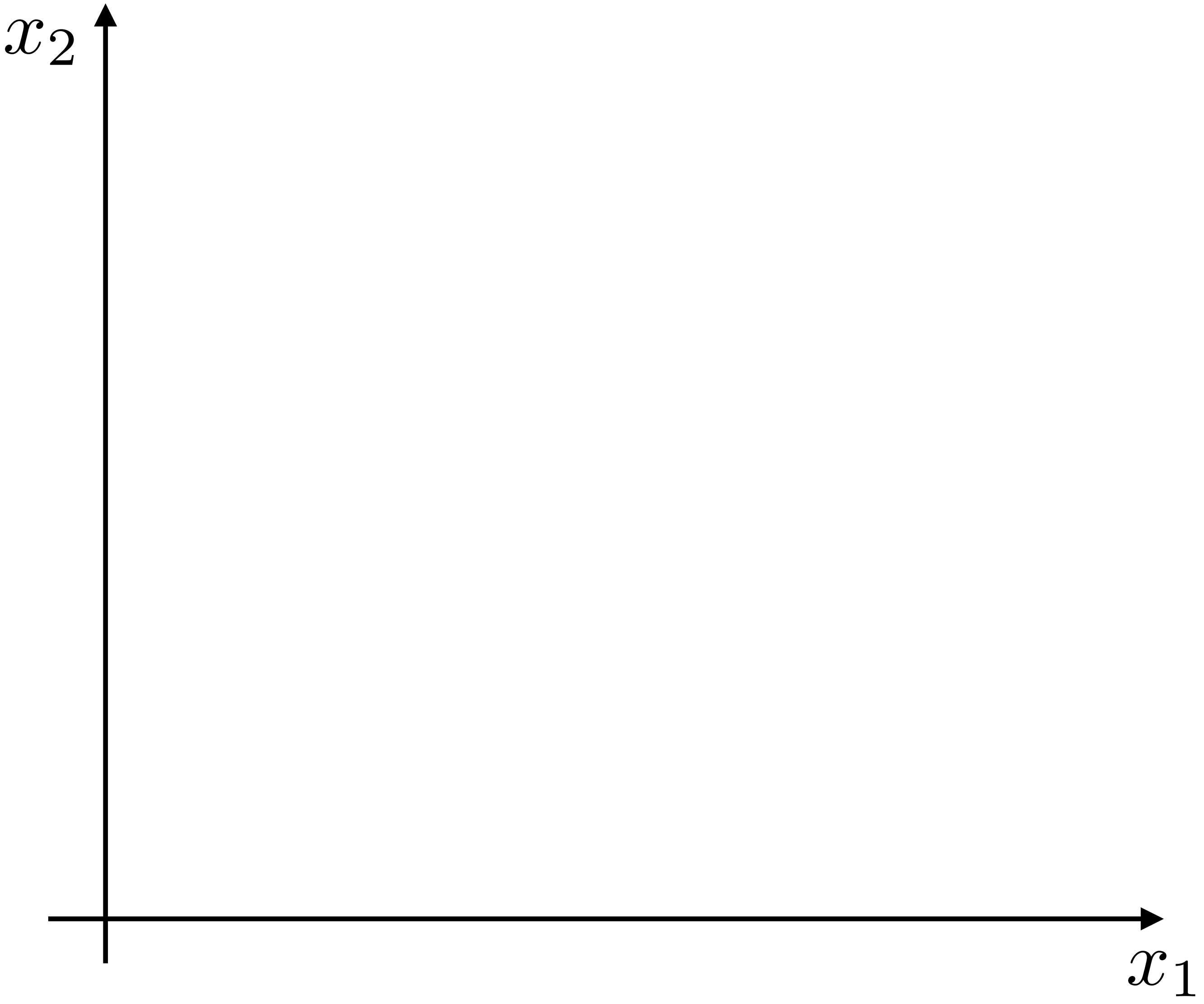
Linear classifiers through origin



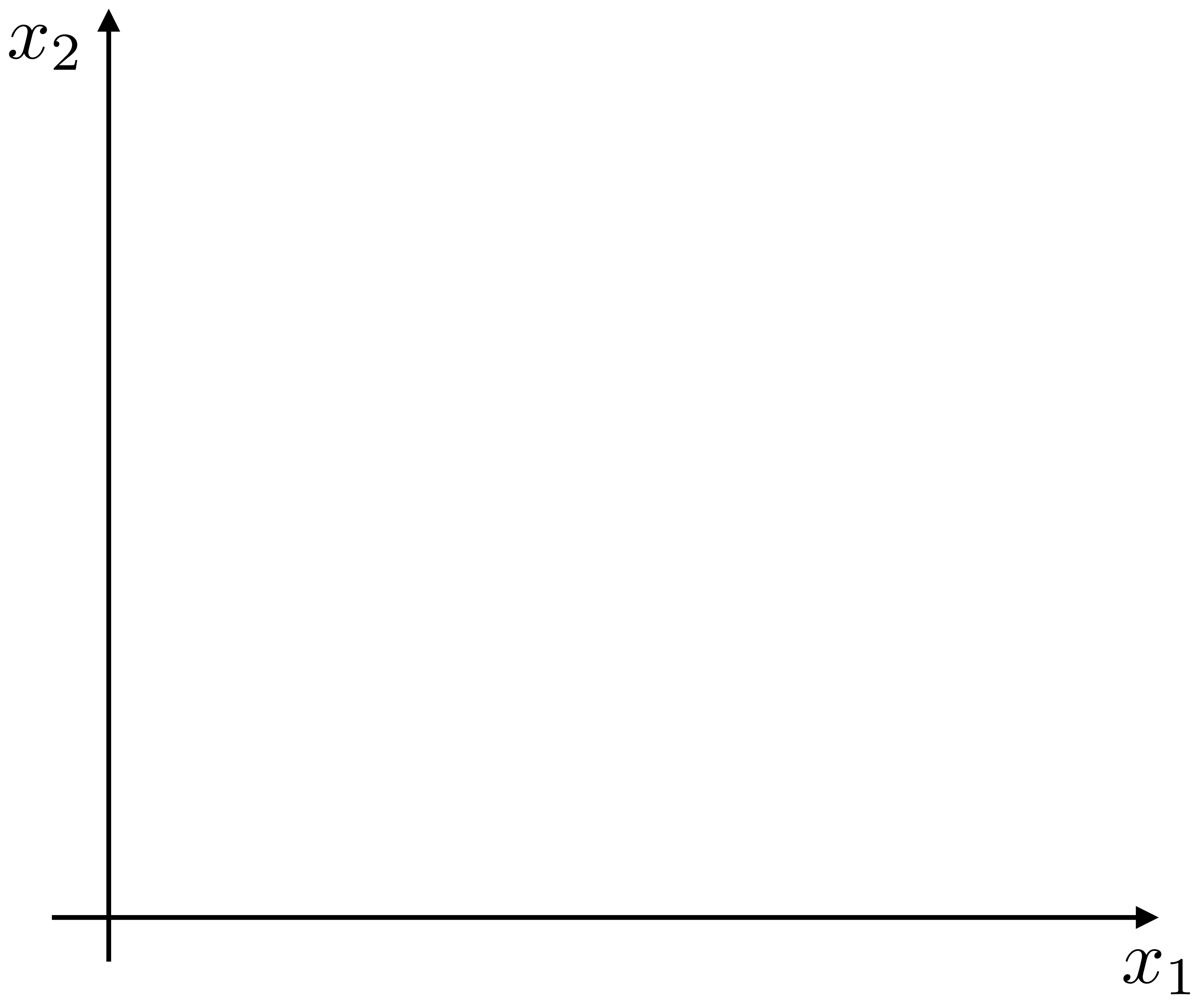
Linear classifiers



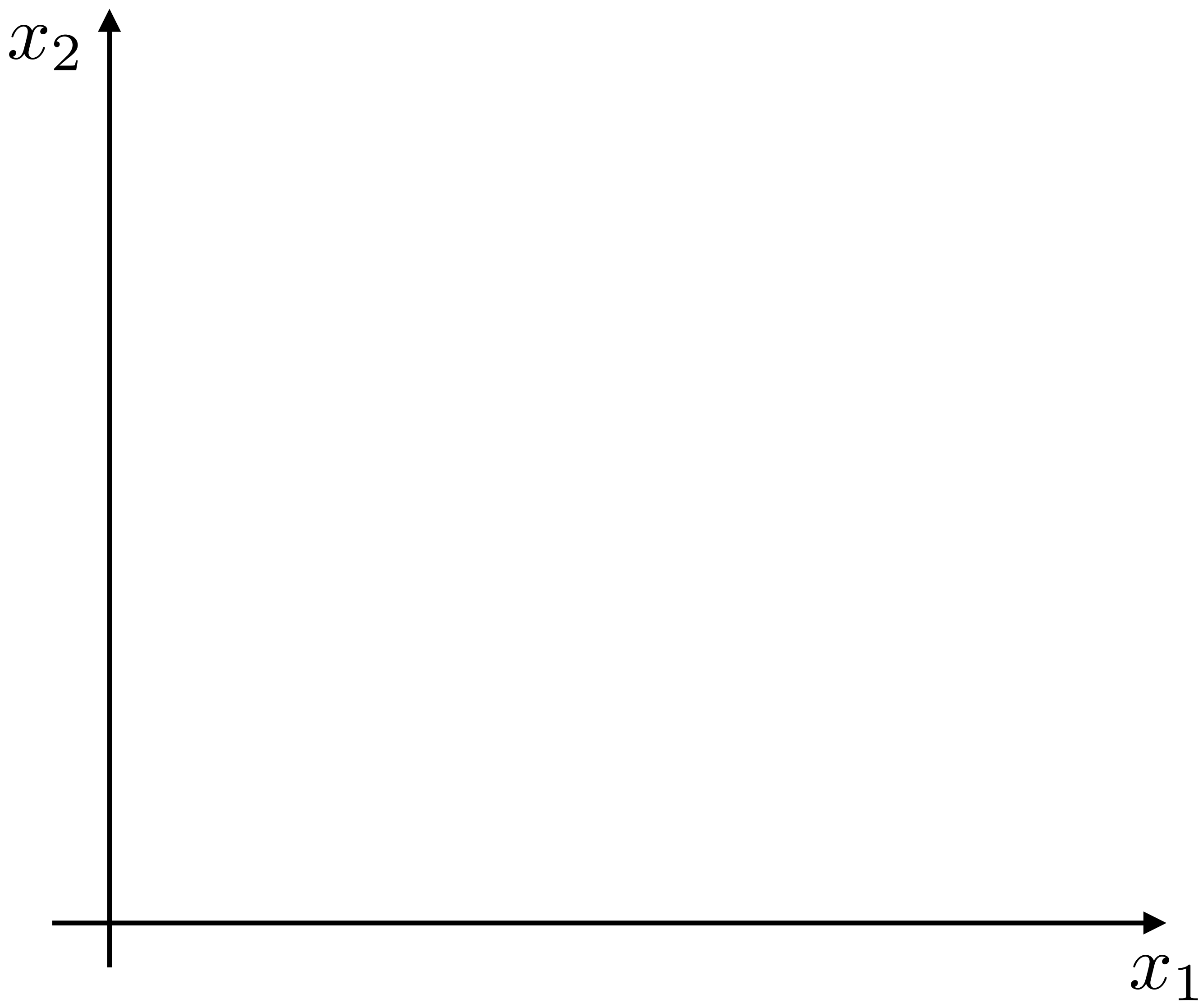
Linear separation: ex



Linear separation: ex



Linear separation: ex



Linear separation

Definition:
Training examples $S_n = \{(x^{(i)}, y^{(i)}\}), i = 1, \ldots, n\}$ are *linearly separable* if there exists a parameter vector $\hat{\theta}$ and offset parameter $\hat{\theta}_0$ such that $y^{(i)}(\hat{\theta} \cdot x^{(i)} + \hat{\theta}_0) > 0$ for all $i = 1, ..., n$.

Learning linear classifiers

- Training error for a linear classifier (through origin)

Learning linear classifiers

- Training error for a linear classifier

Learning algorithm: perceptron

$\theta = 0 \text{ (vector)}$

$$\begin{aligned} &\textbf{if} \quad y^{(i)}(\theta \cdot x^{(i)}) \leq 0 \textbf{ then} \\ &\qquad \theta = \theta + y^{(i)} x^{(i)} \end{aligned}$$

Learning algorithm: perceptron

$\theta = 0$ (vector)

$$
\begin{aligned}
&\textbf{for } i = 1, \ldots, n \textbf{ do} \\
&\quad \textbf{if } \; y^{(i)}(\theta \cdot x^{(i)}) \leq 0 \textbf{ then} \\
&\quad\quad \theta = \theta + y^{(i)} x^{(i)}
\end{aligned}
$$

Learning algorithm: perceptron

procedure $\textsc{Perceptron}(\{(x^{(i)}, y^{(i)}), i = 1, \ldots, n\}, T)$

$\theta = 0$ (vector)

for $t = 1, \ldots, T$ **do**

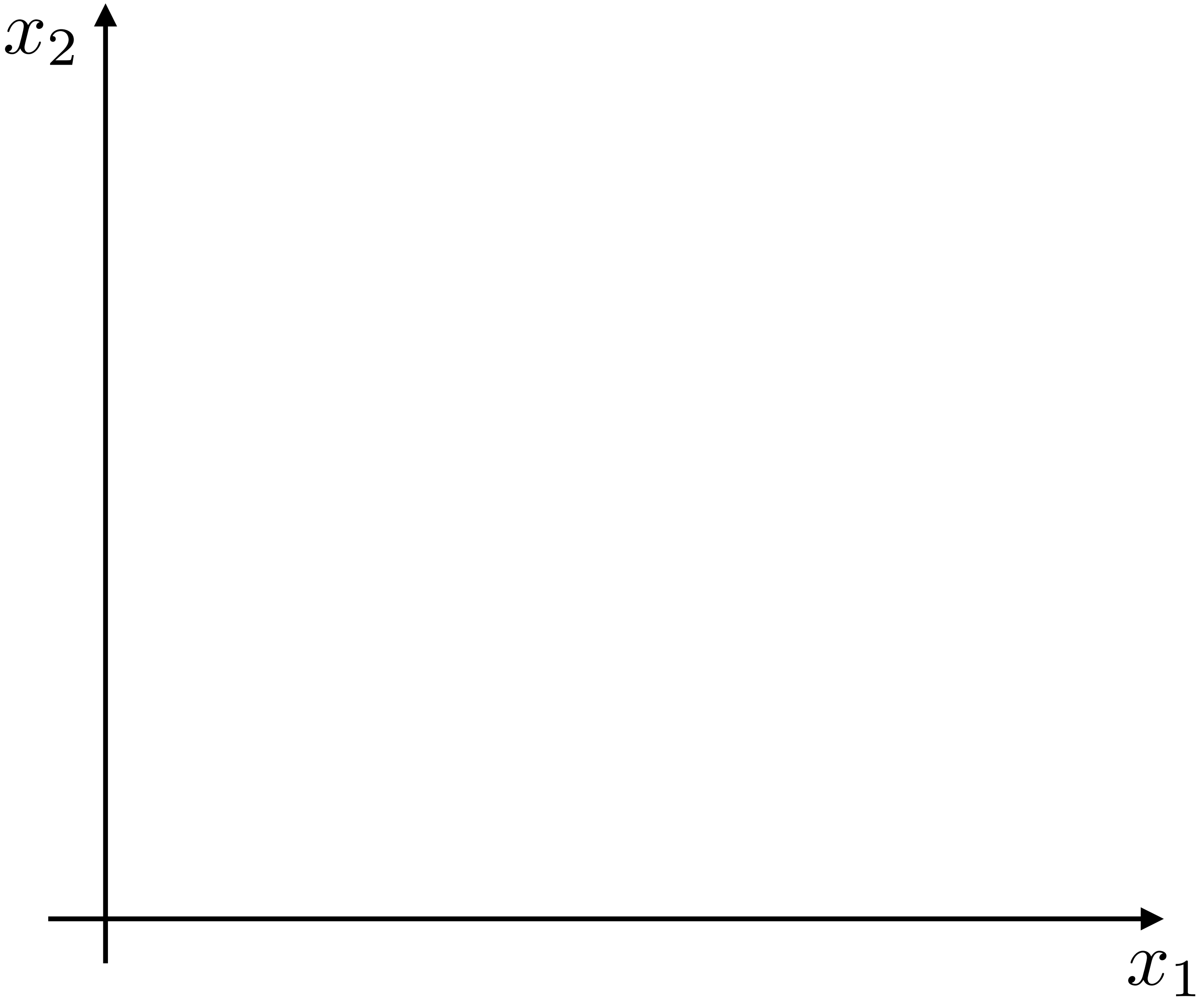
  for $i = 1, \ldots, n$ **do**

    if $y^{(i)}(\theta \cdot x^{(i)}) \leq 0$ **then**

      $\theta = \theta + y^{(i)} x^{(i)}$

return θ

Perceptron algorithm: ex



Perceptron (with offset)

procedure PERCEPTRON($\{(x^{(i)}, y^{(i)}), i = 1, \ldots, n\}, T$)
    $\theta = 0$ (vector), $\theta_0 = 0$ (scalar)
    for $t = 1, \ldots, T$ **do**
        for $i = 1, \ldots, n$ **do**
            if $y^{(i)}(\theta \cdot x^{(i)} + \theta_0) \leq 0$ **then**
                $\theta = \theta + y^{(i)} x^{(i)}$
                $\theta_0 = \theta_0 + y^{(i)}$
    return θ, θ_0

Key things to understand

- Parametric families (sets) of classifiers
- The set of linear classifiers
- Linear separation
- Perceptron algorithm