Data Analysis: Statistical Modeling and Computation in Applications

Time Series Analysis: Autocorrelation and Stationarity

Overview

- What are time series?
- Statistics on time series
- Stationarity
- Transformations
- Autocorrelation

Time series

- **Time series:** collection of observations $x_1, \ldots, x_n$ indexed by time (fixed time intervals)

Time series

- **Time series:** collection of observations $x_1, \ldots, x_n$ indexed by time (fixed time intervals)
- realizations of a collection of random variables $X_1, \ldots, X_n$ indexed by time

Where do time series come up?

Time series

- **Time series:** collection of observations $x_1, \ldots, x_n$ indexed by time (fixed time intervals)
- realizations of a collection of random variables $X_1, \ldots, X_n$ indexed by time

Where do time series come up?

- economics, finance

Time series

- **Time series:** collection of observations $x_1, \ldots, x_n$ indexed by time (fixed time intervals)
- realizations of a collection of random variables $X_1, \ldots, X_n$ indexed by time

Where do time series come up?

- economics, finance
- social sciences

Time series

- **Time series:** collection of observations $x_1, \dots, x_n$ indexed by time (fixed time intervals)
- realizations of a collection of random variables $X_1, \dots, X_n$ indexed by time

Where do time series come up?

- economics, finance
- social sciences
- epidemiology

Time series

- **Time series:** collection of observations $x_1, \ldots, x_n$ indexed by time (fixed time intervals)
- realizations of a collection of random variables $X_1, \ldots, X_n$ indexed by time

Where do time series come up?

- economics, finance
- social sciences
- epidemiology
- fMRI / neuroscience

Time series

- **Time series:** collection of observations $x_1, \ldots, x_n$ indexed by time (fixed time intervals)
- realizations of a collection of random variables $X_1, \ldots, X_n$ indexed by time

Where do time series come up?

- economics, finance
- social sciences
- epidemiology
- fMRI / neuroscience
- environmental sciences

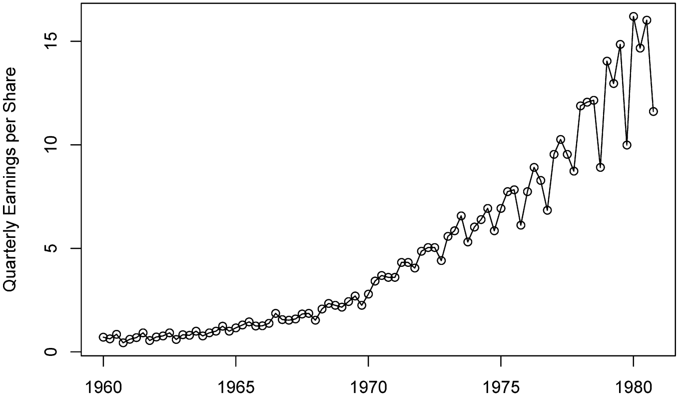
Time series

- **Time series:** collection of observations $x_1, \ldots, x_n$ indexed by time (fixed time intervals)
- realizations of a collection of random variables $X_1, \ldots, X_n$ indexed by time

Where do time series come up?

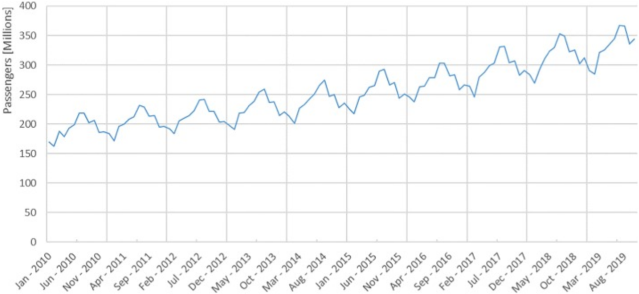
- economics, finance
- social sciences
- epidemiology
- fMRI / neuroscience
- environmental sciences
- speech analysis . . .

Examples of Time series: Johnson & Johnson quarterly earnings per share



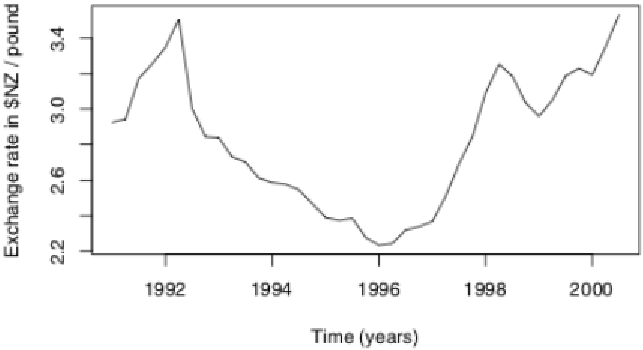
source: R.H. Shumway, D.S. Stoffer. Time Series Analysis and its Applications, with Examples in R. Springer, 2011.

Examples of Time series: global air traffic passengers



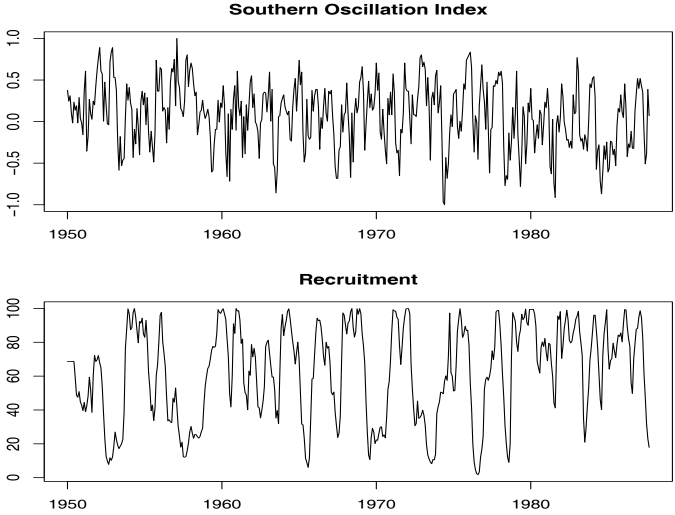
source: S. M. Iacus, F. Natale, C. Santamaria, S. Spyratos, M. Vespe. Estimating and projecting air passenger traffic during the COVID-19 coronavirus outbreak and its socio-economic impact. *Safety Science* vol 129, 2020.

Examples of Time series: Exchange rates GBP to NZ dollar



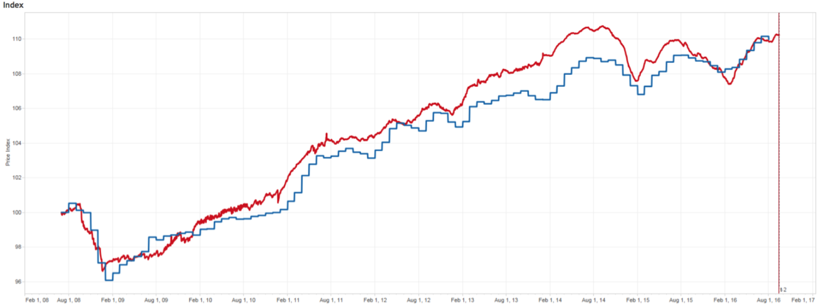
source: P. S.P. Cowpertwait, A. V. Metcalfe. Introductory Time Series with R. Springer, 2009.

Examples of Time series: Air pressure & fish population



source: R.H. Shumway, D.S. Stoffer. Time Series Analysis and its Applications, with Examples in R. Springer, 2011.

Examples of Time series: Price Index USA



source: Roberto Rigobon

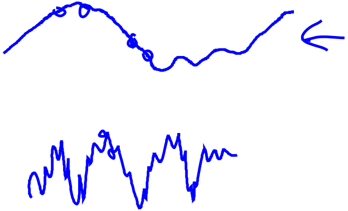
Differences bteween the time series

Differences bteween the time series

- "components" $X_t = T_t + S_t + Z_t$

Differences bteween the time series

- "components"
- smoothness: correlation in time



Objectives of Time Series Analysis

Time series: collection of random variables $X_0, \ldots, X_n$ indexed in time

- Descriptive analysis (visualization, components, dependencies)

Objectives of Time Series Analysis

Time series: collection of random variables $X_0, \ldots, X_n$ indexed in time

- Descriptive analysis (visualization, components, dependencies)
- Modeling

Objectives of Time Series Analysis

Time series: collection of random variables $X_0, \ldots, X_n$ indexed in time

- Descriptive analysis (visualization, components, dependencies)
- Modeling
- Forecasting

Objectives of Time Series Analysis

Time series: collection of random variables $X_0, \ldots, X_n$ indexed in time

- Descriptive analysis (visualization, components, dependencies)
- Modeling
- Forecasting
- Time Series Regression

$$O_t = \beta_1 T_{t-k} + \beta_2 S_{t-k'}$$

Objectives of Time Series Analysis

Time series: collection of random variables $X_0, \ldots, X_n$ indexed in time

- Descriptive analysis (visualization, components, dependencies)
- Modeling
- Forecasting
- Time Series Regression
- Control
- ...

Overview

- What are time series?
- Statistics on time series
- Stationarity
- Transformations towards stationarity
- Autocorrelation

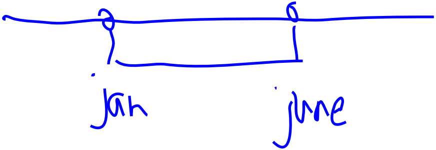
Some Basic Statistics

- **Marginal mean** per time step: $\mu_X(t) = \mathbb{E}[X_t]$.
- **Marginal variance:** $\mathrm{var}(X_t) = \mathbb{E}[(X_t - \mu_X(t))^2]$

- **Marginal mean** per time step: $\mu_X(t) = \mathbb{E}[X_t]$.
- **Marginal variance:** $\mathrm{var}(X_t) = \mathbb{E}[(X_t - \mu_X(t))^2]$

- **Autocovariance** $\gamma_X(s,t)$:

$$\gamma_X(s,t) = \mathrm{cov}(X_s, X_t) = \mathbb{E}[(X_s - \mu_X(s))(X_t - \mu_X(t))]$$



Some Basic Statistics

- **Marginal mean** per time step: $\mu_X(t) = \mathbb{E}[X_t]$.
- **Marginal variance:** $\mathrm{var}(X_t) = \mathbb{E}[(X_t - \mu_X(t))^2] = \gamma(t,t)$
- **Autocovariance** $\gamma_X(s,t)$:

$$\gamma_X(s,t) = \mathrm{cov}(X_s, X_t) = \mathbb{E}[(X_s - \mu_X(s))(X_t - \mu_X(t))]$$

Can we estimate these?

Stationarity

We will need to compute statistics, fit models, etc.
important concept for estimation, forecasting, . . . : stationarity

Stationarity

We will need to compute statistics, fit models, etc.
important concept for estimation, forecasting, . . . : stationarity

- **weak stationarity:** mean/variance same for all X_t:
  - $\mu_X(t) = \mu_X$, $\text{var}_X(t) = \sigma_X^2$

We will need to compute statistics, fit models, etc.
important concept for estimation, forecasting, . . . : stationarity

- **weak stationarity:** mean/variance same for all X_t:
  - $\mu_X(t) = \mu_X$, $\text{var}_X(t) = \sigma_X^2$
  - $\text{cov}(X_s, X_t) = \gamma_X(s-t)$ covariance only a function of gap

jan july

6

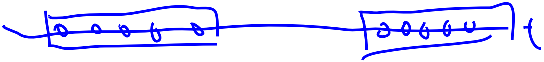
cov(jan, july) = cov(feb, aug) $\neq$ cov(jan, feb) = cov(feb, march)

Stationarity

We will need to compute statistics, fit models, etc.
important concept for estimation, forecasting, . . . : stationarity

- **weak stationarity:** mean/variance same for all X_t:
  - $\mu_X(t) = \mu_X$, $\text{var}_X(t) = \sigma_X^2$
  - $\text{cov}(X_s, X_t) = \gamma_X(s-t)$ covariance only a function of gap

- **strong stationarity:** distribution of $X_t, \ldots X_{t+n}$ same as distribution of $X_{t+h}, \ldots X_{t+n+h}$ for any t, n, h.

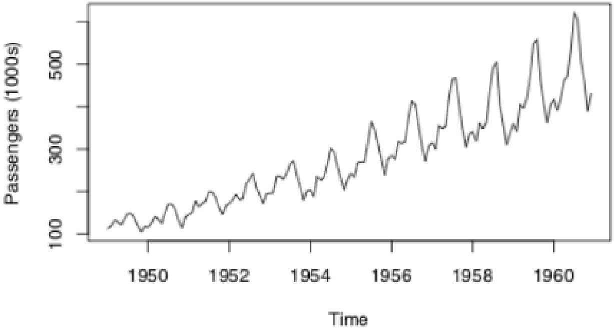


Stationarity

We will need to compute statistics, fit models, etc.
important concept for estimation, forecasting, ...: stationarity

- **weak stationarity:** mean/variance same for all X_t:
  - $\mu_X(t) = \mu_X$, $\mathrm{var}_X(t) = \sigma_X^2$
  - $\mathrm{cov}(X_s, X_t) = \gamma_X(s - t)$ covariance only a function of gap
- **strong stationarity:** distribution of $X_t, \ldots X_{t+n}$ same as distribution of $X_{t+h}, \ldots X_{t+n+h}$ for any t, n, h.
- stationarity allows estimation!
  Typically: transform series to be stationary, then estimate.

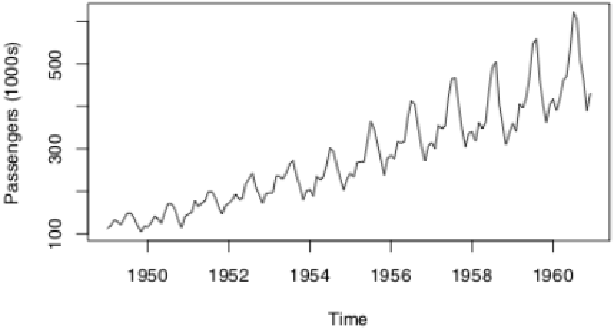
Stationary?



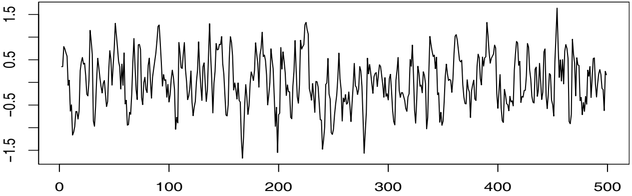
$\mu_t(X_t)$

$= \mu_X$

Stationary?



Passengers (1000s)
100
300
500
1950
1952
1954
1956
1958
1960
Time



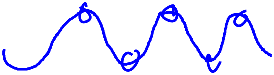
1.5
0.5
−0.5
−1.5
0
100
200
300
400
500

Evidence for Non-Stationarity

weak stationarity:

- $\mu_X(t) = \mu_X$, $\mathrm{var}_X(t) = \sigma_X^2$
- $\mathrm{cov}(X_s, X_t) = \gamma_X(s-t)$ covariance only a function of gap

- Trend: non-constant expected value
- Periodical oscillations (seasonal effect)



Evidence for Non-Stationarity

weak stationarity:

- $\mu_X(t) = \mu_X$, $\text{var}_X(t) = \sigma_X^2$
- $\text{cov}(X_s, X_t) = \gamma_X(s-t)$ covariance only a function of gap

- Trend: non-constant expected value
- Periodical oscillations (seasonal effect)
- Non-constant variance

Evidence for Non-Stationarity

weak stationarity:

- $\mu_X(t) = \mu_X$, $\text{var}_X(t) = \sigma_X^2$
- $\text{cov}(X_s, X_t) = \gamma_X(s-t)$ covariance only a function of gap

- Trend: non-constant expected value
- Periodical oscillations (seasonal effect)
- Non-constant variance
- Changes in the dependency structure

Evidence for Non-Stationarity

weak stationarity:

- $\mu_X(t) = \mu_X$, $\text{var}_X(t) = \sigma_X^2$
- $\text{cov}(X_s, X_t) = \gamma_X(s - t)$ covariance only a function of gap

- Trend: non-constant expected value
- Periodical oscillations (seasonal effect)
- Non-constant variance
- Changes in the dependency structure

Detecting Non-Stationarity

- Plot series: trends, seasonal effects, variance

Evidence for Non-Stationarity

weak stationarity:

- $\mu_X(t) = \mu_X$, $\mathrm{var}_X(t) = \sigma_X^2$
- $\mathrm{cov}(X_s, X_t) = \gamma_X(s-t)$ covariance only a function of gap

- Trend: non-constant expected value
- Periodical oscillations (seasonal effect)
- Non-constant variance
- Changes in the dependency structure

Detecting Non-Stationarity

- Plot series: trends, seasonal effects, variance
- Autocovariance:
  trends/seasonal effects, changes in dependency structure

Overview

- What are time series?
- Statistics on time series
- Stationarity
- Transformations towards stationarity
- Autocorrelation

Search for Stationarity

$$X_t = T_t + S_t + \text{remainder}$$

Search for Stationarity

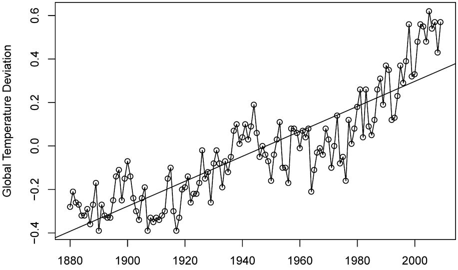
- remove trends and seasonal components: $X_t = T_t + S_t + Y_t$
  - deterministic trend T_t: linear regression
  - deterministic seasonal component S_t
  - remainder: stationary, mean zero

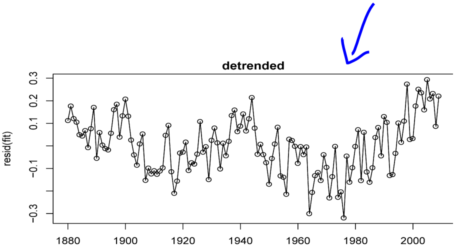
$$\hat{T}_t = \hat{\beta} t + \hat{\beta}_0$$

$$\underline{\underline{X'_t}} = X_t - \hat{T}_t \leftarrow$$

Search for Stationarity

- remove trends and seasonal components: $X_t = T_t + S_t + Y_t$
  - deterministic trend T_t: linear regression
  - deterministic seasonal component S_t
  - remainder: stationary, mean zero





Search for Stationarity

- remove trends and seasonal components: $X_t = T_t + S_t + Y_t$
  - deterministic trend T_t: linear regression
  - deterministic seasonal component S_t
  - remainder: stationary, mean zero

Seasonal patterns

- fit periodic regression model & subtract

$$X'_t = -\hat{S}_t + X_t$$

Seasonal patterns

- fit periodic regression model & subtract
- subtract monthly averages

$\hat{\mu}_{jan}$

$\hat{\mu}_{feb}$

$\vdots$

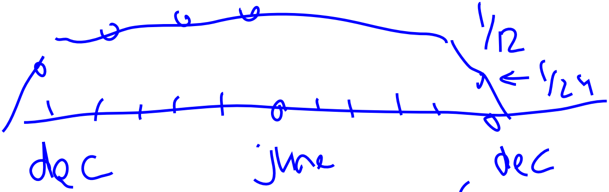
$X'_{jan21} = X_{jan21} - \hat{\mu}_{jan}$

Seasonal patterns

- fit periodic regression model & subtract
- subtract monthly averages
- Fourier analysis (we don't cover this)

Seasonal patterns

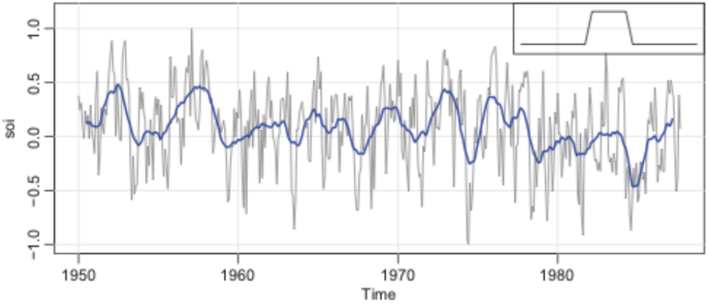
- fit periodic regression model & subtract
- subtract monthly averages
- Fourier analysis (we don't cover this)
- smoothing, e.g. by moving averages: new series $Y_t = \sum_{h=-k}^{k} a_h X_{t+h}$



$$Y_{june20} = \sum_{h=-6}^{6} a_h X_{t+h}$$

Seasonal patterns

- fit periodic regression model & subtract
- subtract monthly averages
- Fourier analysis (we don't cover this)
- smoothing, e.g. by moving averages: new series $Y_t = \sum_{h=-k}^{k} a_h X_{t+h}$

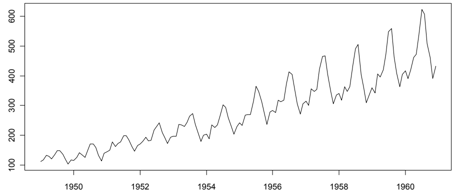


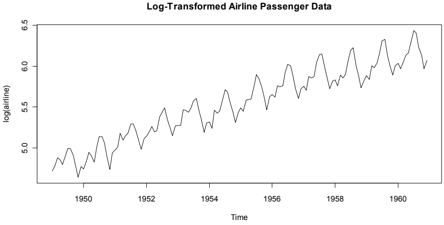
Search for Stationarity

- remove trends and seasonal components: $X_t = T_t + S_t + Z_t$
  - deterministic trend T_t: linear regression
  - deterministic seasonal component S_t
  - remainder: stationary, mean zero
- nonlinear transformations, e.g. $Y_t = \log(X_t - \mu_t)$, $Y_t = \sqrt{X_t - \mu_t}$

Search for Stationarity

- remove trends and seasonal components: $X_t = T_t + S_t + Z_t$
  - deterministic trend T_t: linear regression
  - deterministic seasonal component S_t
  - remainder: stationary, mean zero
- nonlinear transformations, e.g. $Y_t = \log(X_t - \mu_t)$, $Y_t = \sqrt{X_t - \mu_t}$





Search for Stationarity

- remove trends and seasonal components: $X_t = T_t + S_t + Z_t$
  - deterministic trend T_t: linear regression
  - deterministic seasonal component S_t
  - remainder: stationary, mean zero
- nonlinear transformations, e.g. $Y_t = \log(X_t - \mu_t)$, $Y_t = \sqrt{X_t - \mu_t}$
- differentiation

$$Y_t = X_t - X_{t-1}$$

Search for Stationarity

- remove trends and seasonal components: $X_t = T_t + S_t + Z_t$
  - deterministic trend T_t: linear regression
  - deterministic seasonal component S_t
  - remainder: stationary, mean zero
- nonlinear transformations, e.g. $Y_t = \log(X_t - \mu_t)$, $Y_t = \sqrt{X_t - \mu_t}$
- differentiation

$$Y_t = \nabla X_t = X_t - X_{t-1} \qquad \text{removes linear trend}$$

$X_t = \beta t + W_t$ $\qquad$ $Y_t = \beta + W_t - W_{t-1}$

$\mu(x) = \beta$

Search for Stationarity

- remove trends and seasonal components: $X_t = T_t + S_t + Z_t$
  - deterministic trend T_t: linear regression
  - deterministic seasonal component S_t
  - remainder: stationary, mean zero
- nonlinear transformations, e.g. $Y_t = \log(X_t - \mu_t)$, $Y_t = \sqrt{X_t - \mu_t}$
- differentiation

$$\begin{aligned} Y_t = \nabla X_t &= X_t - X_{t-1} \qquad \text{removes linear trend} \\ \nabla^2 X_t &= \nabla X_t - \nabla X_{t-1} \end{aligned}$$

Search for Stationarity

- remove trends and seasonal components: $X_t = T_t + S_t + Z_t$
  - deterministic trend T_t: linear regression
  - deterministic seasonal component S_t
  - remainder: stationary, mean zero
- nonlinear transformations, e.g. $Y_t = \log(X_t - \mu_t)$, $Y_t = \sqrt{X_t - \mu_t}$
- differentiation

$$\begin{aligned} Y_t = \nabla X_t &= X_t - X_{t-1} && \text{removes linear trend} \\ \nabla^2 X_t &= \nabla X_t - \nabla X_{t-1} \\ &= X_t - 2X_{t-1} + X_{t-2} && \text{removes quadratic trend} \end{aligned}$$

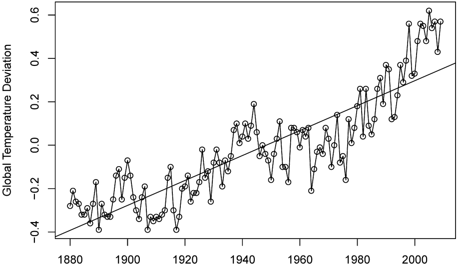
Search for Stationarity

- remove trends and seasonal components: $X_t = T_t + S_t + Z_t$
  - deterministic trend T_t: linear regression
  - deterministic seasonal component S_t
  - remainder: stationary, mean zero
- nonlinear transformations, e.g. $Y_t = \log(X_t - \mu_t)$, $Y_t = \sqrt{X_t - \mu_t}$
- differentiation

$$\begin{aligned} Y_t = \nabla X_t &= X_t - X_{t-1} && \text{removes linear trend} \\ \nabla^2 X_t &= \nabla X_t - \nabla X_{t-1} && \\ &= X_t - 2X_{t-1} + X_{t-2} && \text{removes quadratic trend} \end{aligned}$$

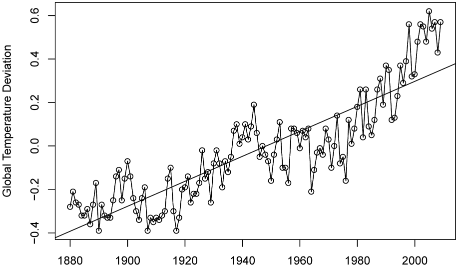
$\{X_t\}_t$ is *integrated of order p*: $\{\nabla^p X_t\}_t$ is stationary

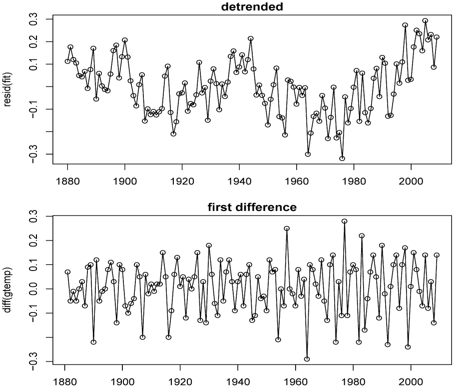
Search for Stationarity: illustration



Search for Stationarity: illustration

$X_t = T_t + S_t + \ldots$





$X_t - X_{t-1}$

Overview

- What are time series?
- Statistics on time series
- Stationarity
- Transformations towards stationarity
- Autocorrelation

Measuring Correlation in Time

- **Marginal mean** per time step: $\mu_X(t) = \mathbb{E}[X_t]$.
- **Marginal variance:** $\mathrm{var}(X_t) = \mathbb{E}[(X_t - \mu_X(t))^2]$

Measuring Correlation in Time

- **Marginal mean** per time step: $\mu_X(t) = \mathbb{E}[X_t]$.
- **Marginal variance:** $\mathrm{var}(X_t) = \mathbb{E}[(X_t - \mu_X(t))^2]$

- **Autocovariance** $\gamma_X(s,t)$:

$$\gamma_X(s,t) = \mathrm{cov}(X_s, X_t) = \mathbb{E}[(X_s - \mu(s))(X_t - \mu(t))]$$

Measuring Correlation in Time

- **Marginal mean** per time step: $\mu_X(t) = \mathbb{E}[X_t]$.
- **Marginal variance:** $\text{var}(X_t) = \mathbb{E}[(X_t - \mu_X(t))^2] = \gamma(t,t)$

- **Autocovariance** $\gamma_X(s,t)$:

$$\gamma_X(s,t) = \text{cov}(X_s, X_t) = \mathbb{E}[(X_s - \mu(s))(X_t - \mu(t))]$$

- **Autocorrelation** (ACF):

$$\rho_X(s,t) = \frac{\gamma_X(s,t)}{\sqrt{\gamma_X(s,s)\gamma_X(t,t)}}$$

Measuring Correlation in Time

- **Marginal mean** per time step: $\mu_X(t) = \mathbb{E}[X_t]$.
- **Marginal variance:** $\mathrm{var}(X_t) = \mathbb{E}[(X_t - \mu_X(t))^2] = \gamma(t,t)$

- **Autocovariance** $\gamma_X(s,t)$:

$$\gamma_X(s,t) = \mathrm{cov}(X_s, X_t) = \mathbb{E}[(X_s - \mu(s))(X_t - \mu(t))]$$

- **Autocorrelation** (ACF):

$$\rho_X(s,t) = \frac{\gamma_X(s,t)}{\sqrt{\gamma_X(s,s)\gamma_X(t,t)}}$$

- Properties of $\gamma(s,t)$:
  - symmetric

Measuring Correlation in Time

- **Marginal mean** per time step: $\mu_X(t) = \mathbb{E}[X_t]$.
- **Marginal variance:** $\mathrm{var}(X_t) = \mathbb{E}[(X_t - \mu_X(t))^2] = \gamma(t,t)$
- **Autocovariance** $\gamma_X(s,t)$:

$$\gamma_X(s,t) = \mathrm{cov}(X_s, X_t) = \mathbb{E}[(X_s - \mu(s))(X_t - \mu(t))]$$

- **Autocorrelation** (ACF):

$$\rho_X(s,t) = \frac{\gamma_X(s,t)}{\sqrt{\gamma_X(s,s)\gamma_X(t,t)}}$$

- Properties of $\gamma(s,t)$:
  - symmetric
  - measures *linear* dependence of X_t, X_s

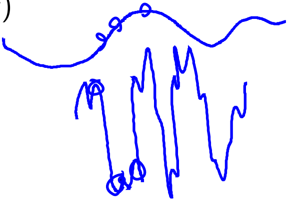
Measuring Correlation in Time

- **Marginal mean** per time step: $\mu_X(t) = \mathbb{E}[X_t]$.
- **Marginal variance:** $\mathrm{var}(X_t) = \mathbb{E}[(X_t - \mu_X(t))^2] = \gamma(t,t)$
- **Autocovariance** $\gamma_X(s,t)$:

$$\gamma_X(s,t) = \mathrm{cov}(X_s, X_t) = \mathbb{E}[(X_s - \mu(s))(X_t - \mu(t))]$$

- **Autocorrelation** (ACF):

$$\rho_X(s,t) = \frac{\gamma_X(s,t)}{\sqrt{\gamma_X(s,s)\gamma_X(t,t)}}$$



- Properties of $\gamma(s,t)$:
  - symmetric
  - measures *linear* dependence of X_t, X_s
  - relates to smoothness

Measuring Correlation in Time

- **Marginal mean** per time step: $\mu_X(t) = \mathbb{E}[X_t]$.
- **Marginal variance:** $\mathrm{var}(X_t) = \mathbb{E}[(X_t - \mu_X(t))^2] = \gamma(t,t)$

- **Autocovariance** $\gamma_X(s,t)$:

$$\gamma_X(s,t) = \mathrm{cov}(X_s, X_t) = \mathbb{E}[(X_s - \mu(s))(X_t - \mu(t))]$$

- **Autocorrelation** (ACF):

$$\rho_X(s,t) = \frac{\gamma_X(s,t)}{\sqrt{\gamma_X(s,s)\gamma_X(t,t)}}$$

- Properties of $\gamma(s,t)$:
  - symmetric
  - measures *linear* dependence of X_t, X_s
  - relates to smoothness
  - for weakly stationary series: $\gamma_X(t, t+h) = \gamma_X(0,h) =: \gamma_X(h)$

6 6

$\mathrm{cov}(X_t, X_{t+6}) = \mathrm{cov}(X_{i'}, X_{i'+6})$

Measuring Correlation in Time

- **Marginal mean** per time step: $\mu_X(t) = \mathbb{E}[X_t]$.
- **Marginal variance:** $\mathrm{var}(X_t) = \mathbb{E}[(X_t - \mu_X(t))^2] = \gamma(t,t)$
- **Autocovariance** $\gamma_X(s,t)$:

$$\gamma_X(s,t) = \mathrm{cov}(X_s, X_t) = \mathbb{E}[(X_s - \mu(s))(X_t - \mu(t))]$$

- **Autocorrelation** (ACF):

$$\rho_X(s,t) = \frac{\gamma_X(s,t)}{\sqrt{\gamma_X(s,s)\gamma_X(t,t)}}$$

- Properties of $\gamma(s,t)$:
  - symmetric
  - measures *linear* dependence of X_t, X_s
  - relates to smoothness
  - for weakly stationary series: $\gamma_X(t, t+h) = \gamma_X(0,h) =: \gamma_X(h)$

Sample estimates for stationary series

- $\hat{\mu}_X = \bar{x} = \frac{1}{n}\sum_{t=1}^{n} x_t$

Sample estimates for stationary series

- $\hat{\mu}_X = \bar{x} = \frac{1}{n}\sum_{t=1}^{n} x_t$
- sample autocovariance:

$$\hat{\gamma}_X(h) = \frac{1}{n}\sum_{t=1}^{n-|h|}(x_t - \bar{x})(x_{t+|h|} - \bar{x}) \quad \text{for } -n < h < n$$

Sample estimates for stationary series

- $\hat{\mu}_X = \bar{x} = \frac{1}{n}\sum_{t=1}^{n} x_t$

- sample autocovariance:

$$\hat{\gamma}_X(h) = \frac{1}{n}\sum_{t=1}^{n-|h|}(x_t - \bar{x})(x_{t+|h|} - \bar{x}) \quad \text{for } -n < h < n$$

- sample autocorrelation:

$$\hat{\rho}_X(h) = \hat{\gamma}_X(h)/\hat{\gamma}_X(0) \qquad \text{for } -n < h < n$$

Sample estimates for stationary series

- $\hat{\mu}_X = \bar{x} = \frac{1}{n}\sum_{t=1}^{n} x_t$ std error: $\text{var}(\bar{x}) = \frac{1}{n}\sum_{h=-n}^{n}(1 - \frac{|h|}{n})\gamma(h)$

estimation error depends on γ. Not iid data!

- sample autocovariance:

$$\hat{\gamma}_X(h) = \frac{1}{n}\sum_{t=1}^{n-|h|}(x_t - \bar{x})(x_{t+|h|} - \bar{x}) \quad \text{for } -n < h < n$$

- sample autocorrelation:

$$\hat{\rho}_X(h) = \hat{\gamma}_X(h)/\hat{\gamma}_X(0) \qquad \text{for } -n < h < n$$

Example

$$-1 \quad 0 \quad +1 \quad 0 \quad -1 \quad 0 \quad +1$$

$$\hat{\mu}_x = \bar{x} = 0$$

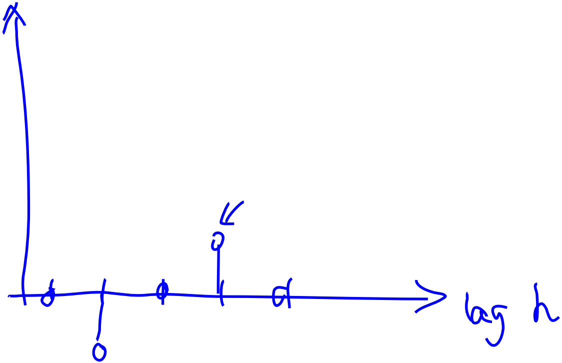
$$\gamma(1) = \frac{1}{7} \cdot 0 = 0$$

$$\gamma(2) = \frac{1}{7} \cdot (-3) = -\frac{3}{7}$$

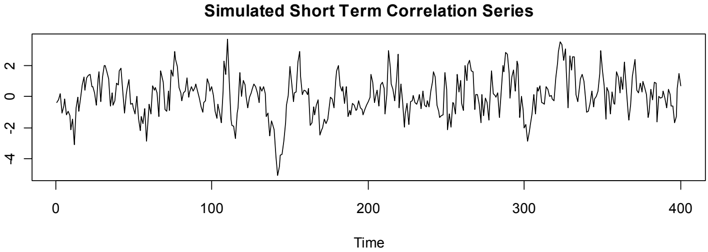
$$\gamma(3) = \frac{1}{7} \cdot 0 = 0$$

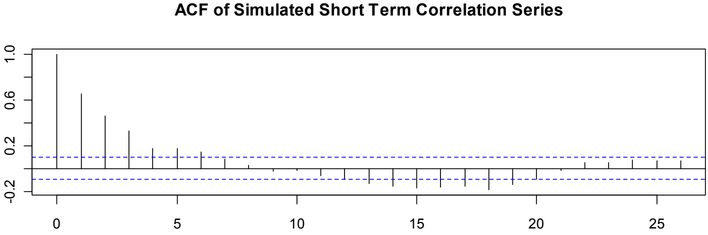
$$\gamma(4) = \frac{1}{7} \cdot 2 = \frac{2}{7}$$

Correlogram

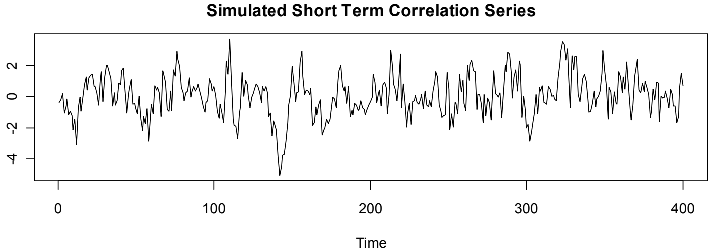


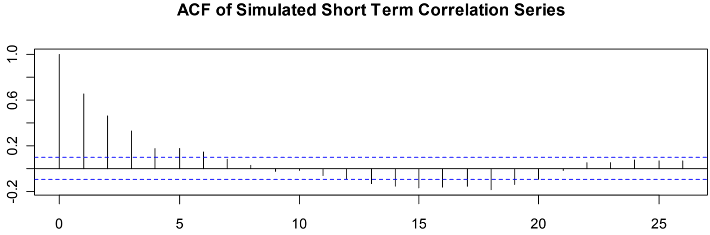
Autocorrelation





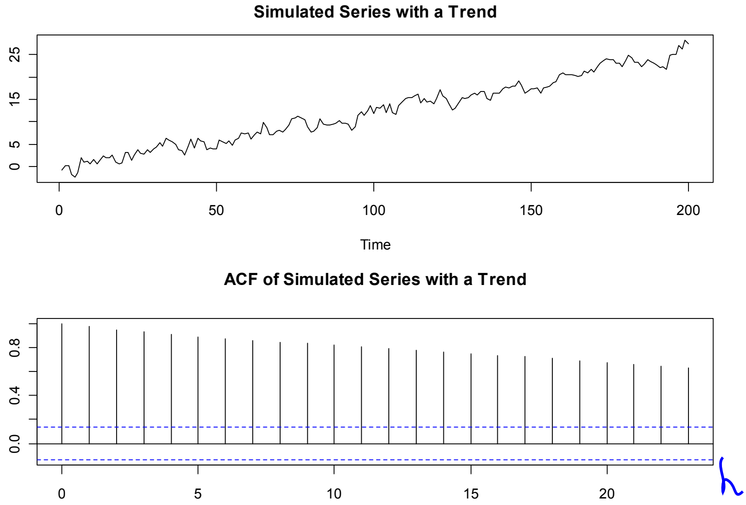
Autocorrelation



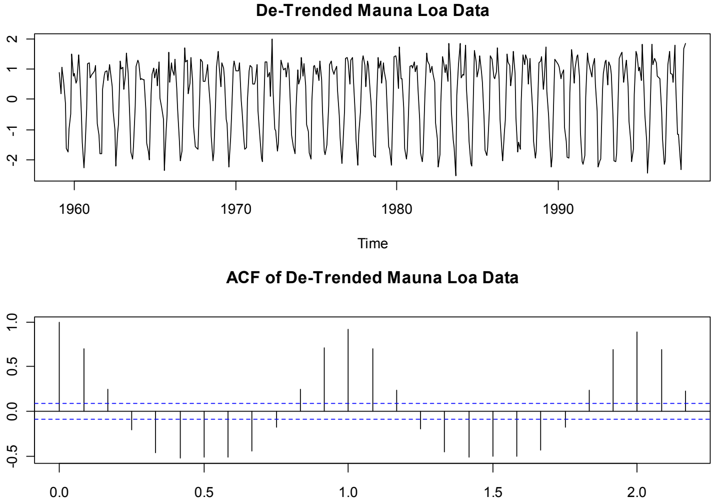


Stationary series often exhibit short-term correlations.

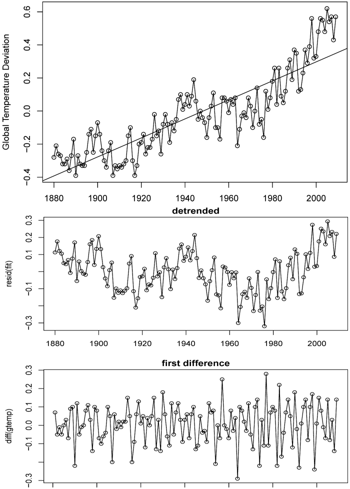
Autocorrelation: with trend

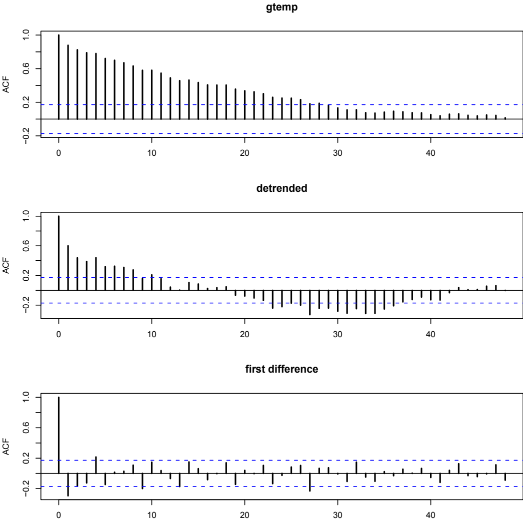


Autocorrelation: with seasonal pattern (CO_2)

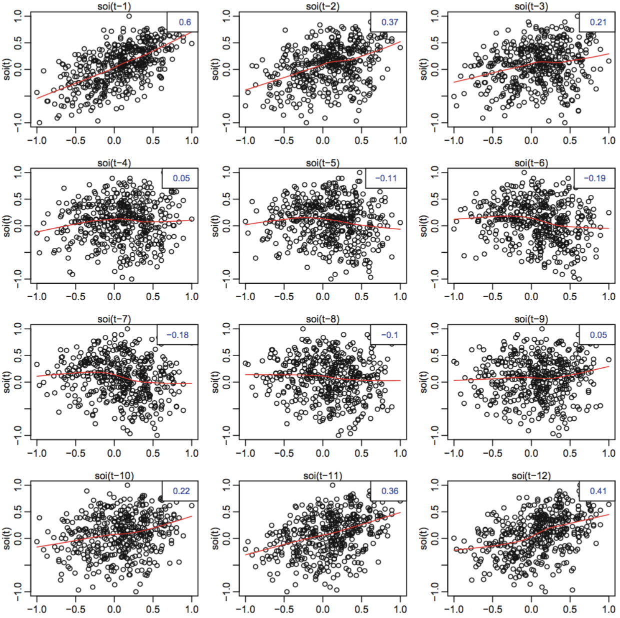


Recall: Search for Stationarity





Scatter plots (SOI)



Summary: Statistics and Stationarity

- collection of random variables indexed by time
- need stationarity to estimate statistics and models
- transformations to achieve weak stationarity
- autocorrelation: important quantity and diagnostic tool

References and Further Reading

- Paul S.P. Cowpertwait and Andrew V. Metcalfe. Introductory Time Series with R. Springer, 2009.
  Chapters 1,2,4
- R.H. Shumway, D.S. Stoffer. Time Series Analysis and its Applications, with Examples in R. Springer, 2011.
  Chapters 1,2.
- R. Carmona. Statistical Analysis of Financial Data in R. Springer, 2014.
  Chapter 6.