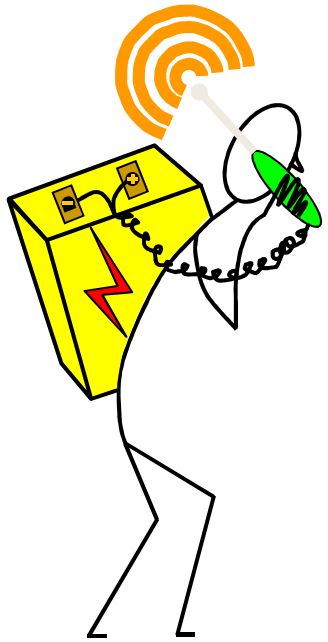
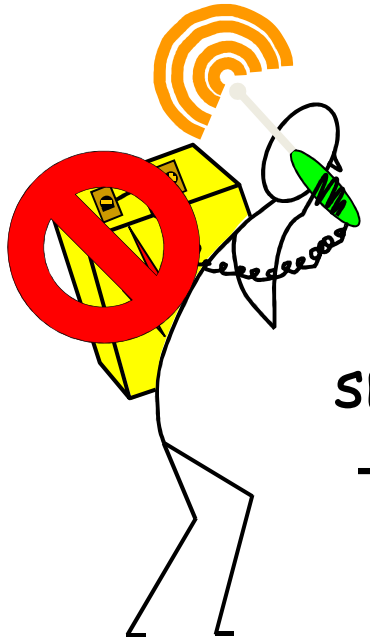
6.002x CIRCUITS AND ELECTRONICS

Energy and Power



Why worry about energy?

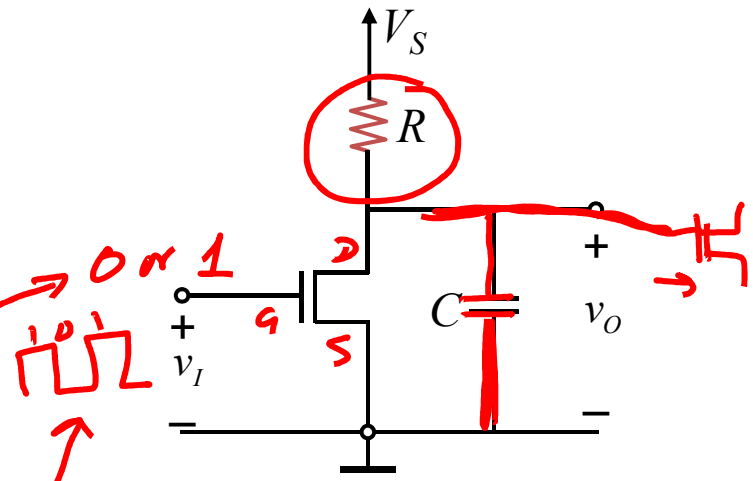


small batteries
→ good

Today:

- How long will the battery last?
  - in standby mode
  - in active use
- Will the chip overheat and self-destruct?
- Did you know, data centers in the US consume as much energy as the entire state of Utah!

Look at energy dissipation in MOSFET gates

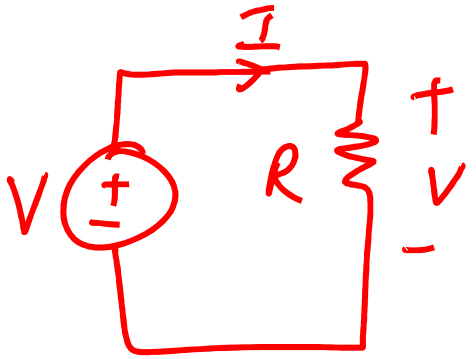


C: wiring capacitance and C_{GS} of following gate

Let us determine
- standby power or static power
- active use power

Let's work out a few related examples first — this will help build intuition

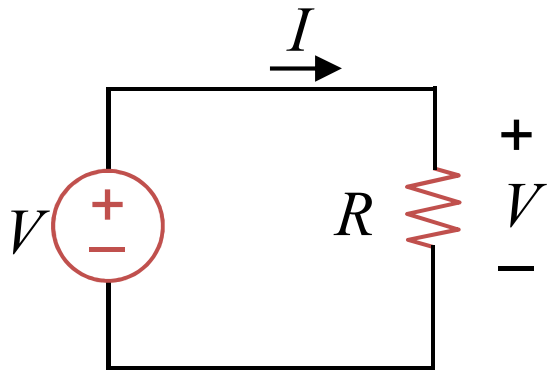
Example 1:



① Power dissipated in the resistor ?

② Energy dissipated in the resistor in time T ?

Example 1:



Power : $P = VI = \frac{V^2}{R}$ watts

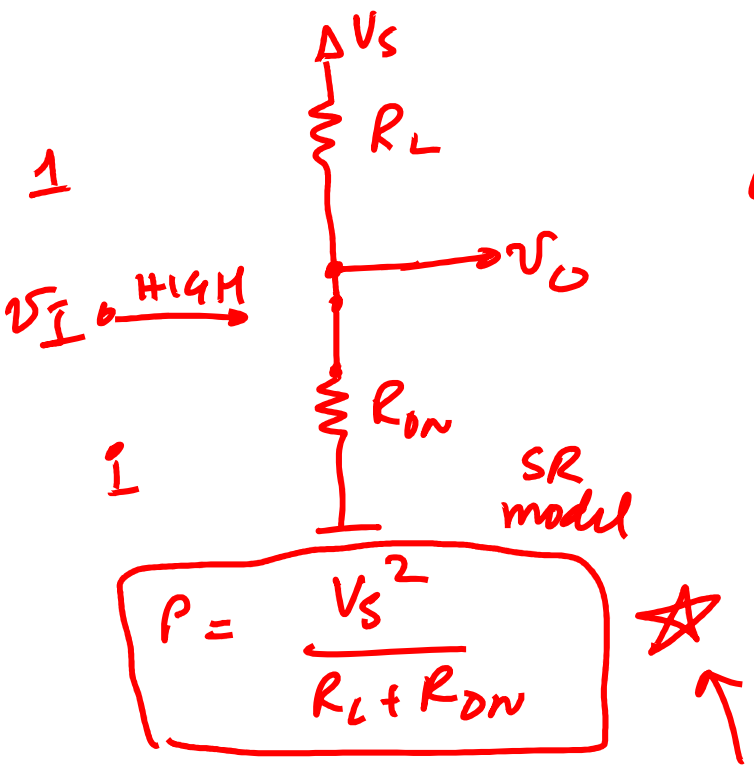
Energy dissipated in time T

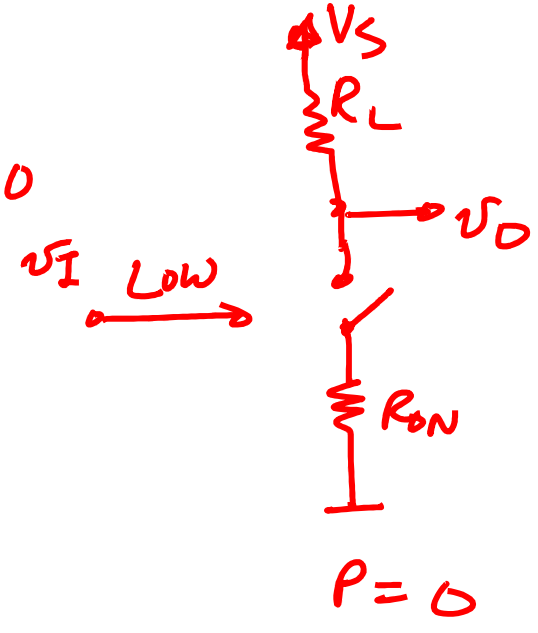
$E = PT = VIT$ → joules

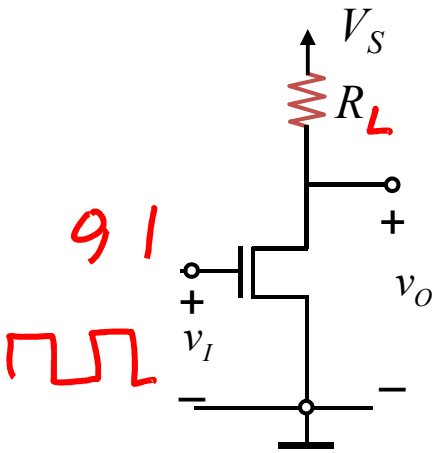
V: volts, I: amps, T: secs

Static power for our gate

I.e., when the gate is not switching

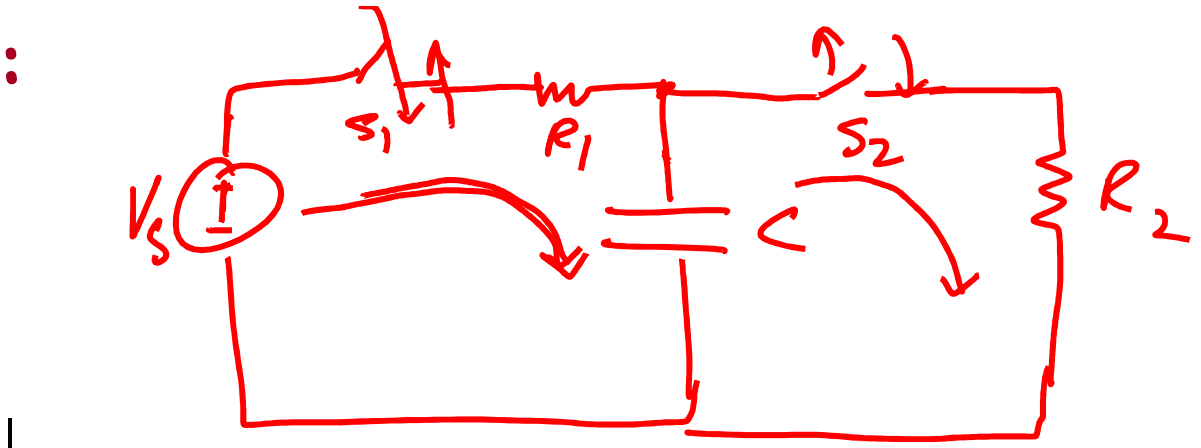


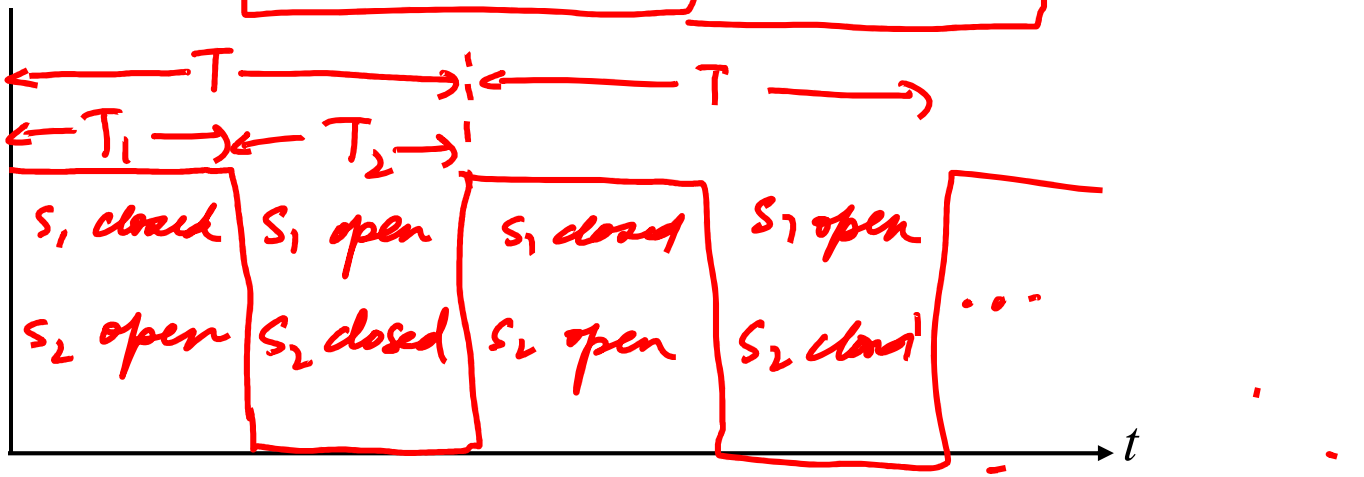




Example 2:

Consider

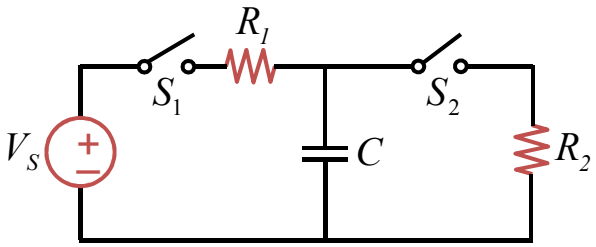


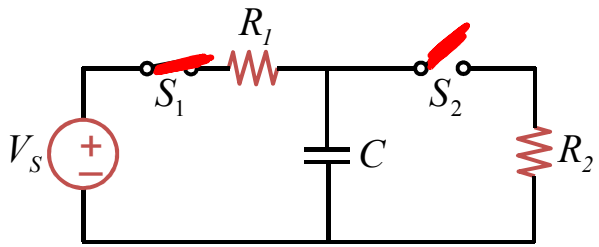


$f = \frac{1}{T}$

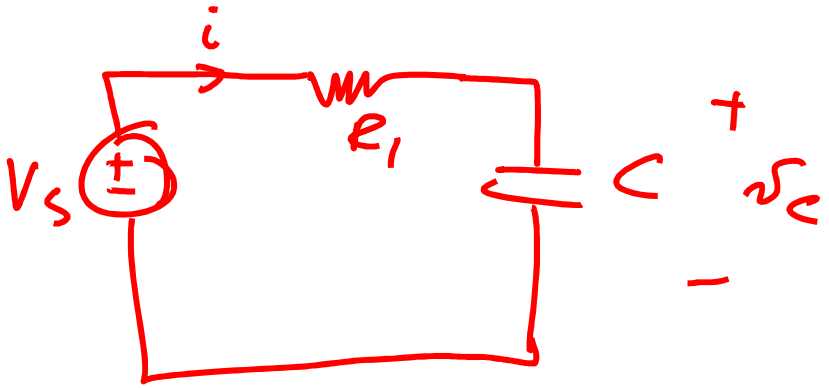
- Energy dissipated in each cycle
- find average power

T_1 : S_1 closed, S_2 open

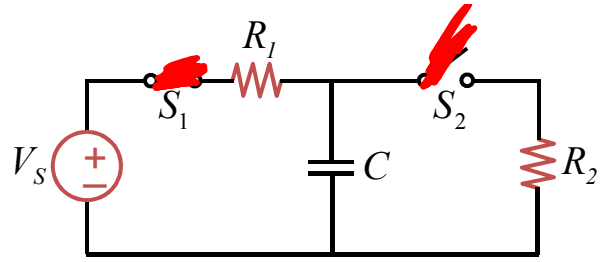


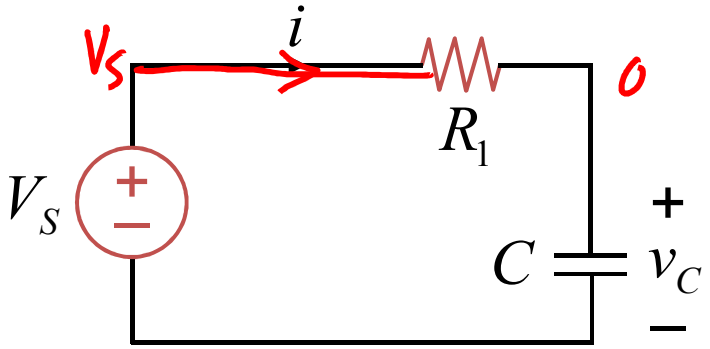


Switches are ideal

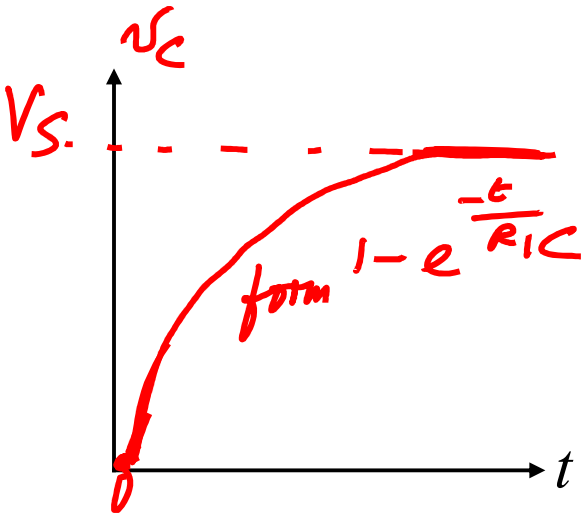


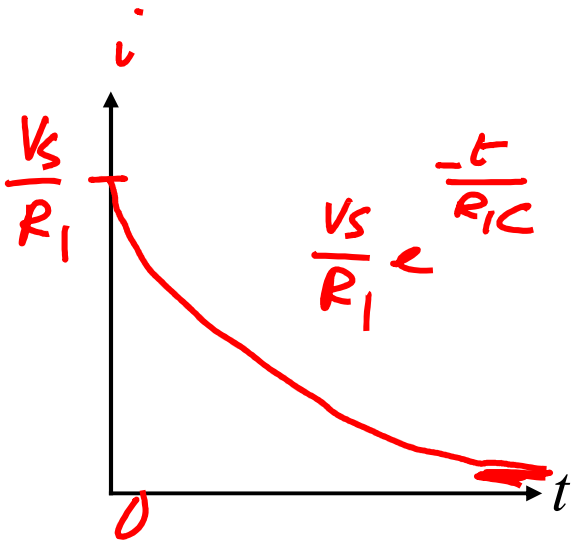
T_1 : S_1 closed, S_2 open



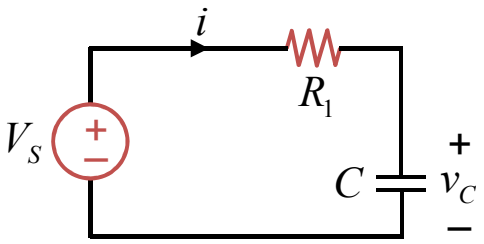


assume $v_C = 0$ at $t = 0$





Total energy provided by source during T_1

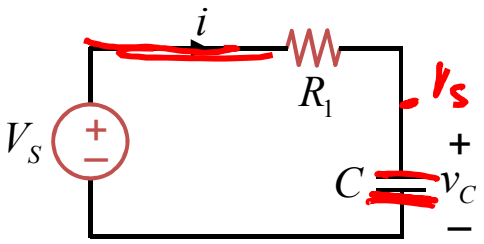


$$E = \int_0^{T_1} V_S \cdot i \, dt$$

energy supplied by source

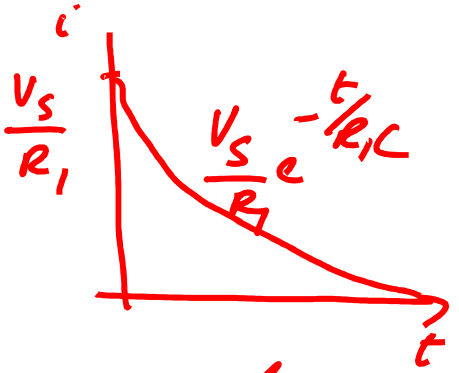
Inst. power $= V_S \cdot i$ being supplied by source

Total energy provided by source during T_1



$$E = \int_0^{T_1} V_S\, i\, dt = \int_0^{T_1} \frac{V_S^2}{R_1} e^{-\frac{t}{R_1 C}} dt = -\frac{V_S^2}{R_1} R_1 C\, e^{-\frac{t}{R_1 C}} \Big|_0^{T_1}$$

$$= CV_S^2 - CV_S^2 e^{-\frac{T_1}{R_1 C}} = CV_S^2\left(1 - e^{-\frac{T_1}{R_1 C}}\right)$$



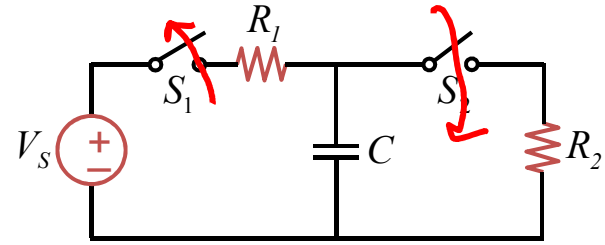
if $T_1 \gg R_1 C$

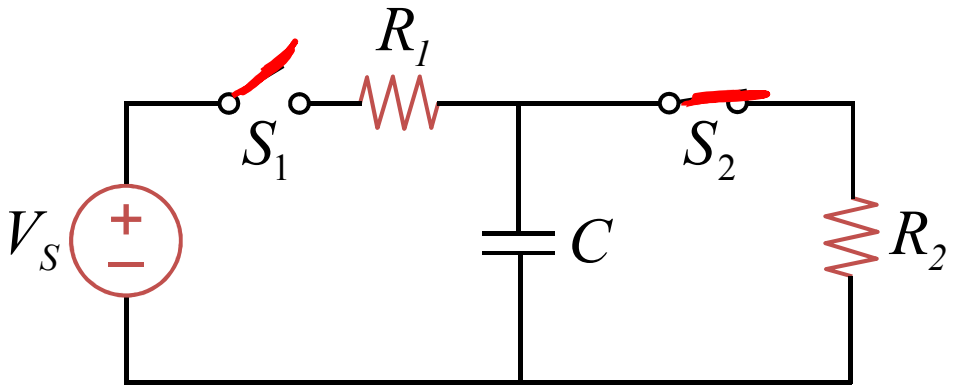
$E \simeq CV_S^2$ → energy supplied by source in T_1

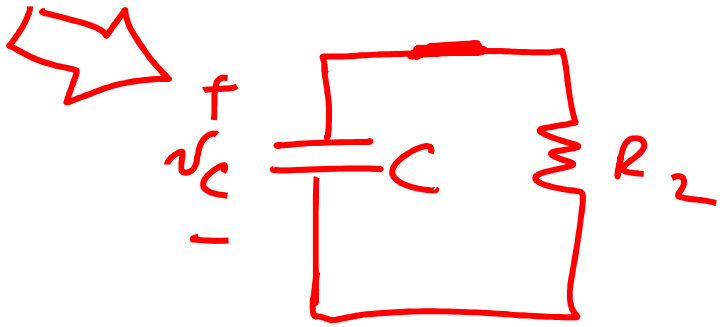
CV_S^2 < $\frac{1}{2}CV_S^2$ stored on capacitor
$\frac{1}{2}CV_S^2$ dissipated in R_1 } Independent of R_1

E_1

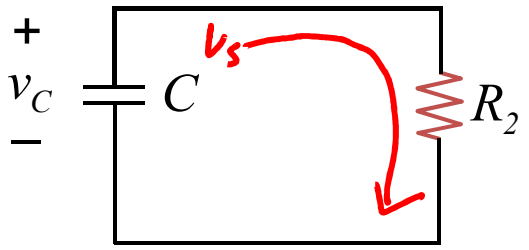
T_2 : S_2 closed, S_1 open







T_2: S_2 closed, S_1 open



Initially, at the start of T_2, $v_C = V_S$ (recall $T_1 >> R_1 C$)

at the end of T_1, $v_C = V_S$

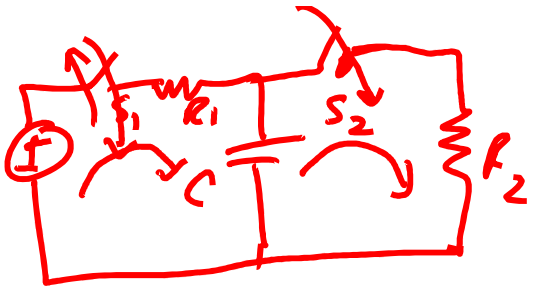
So, energy stored in capacitor $= \frac{1}{2} C V_S^2$

Assume $T_2 >> R_2 C$, so capacitor discharges more or less fully in interval T_2

So, energy dissipated in R_2 during T_2 is $\frac{1}{2} C V_S^2$ $\leftarrow E_2$

E_2 is also independent of R_2!

Putting the two together



Total energy dissipated in each cycle

$$E = \overset{T_1}{E_1} + \overset{T_2}{E_2} = \frac{1}{2}CV_S^2 + \frac{1}{2}CV_S^2 = CV_S^2$$

(charge to V_S, and then discharge fully)

Energy dissipated in charging and discharging C

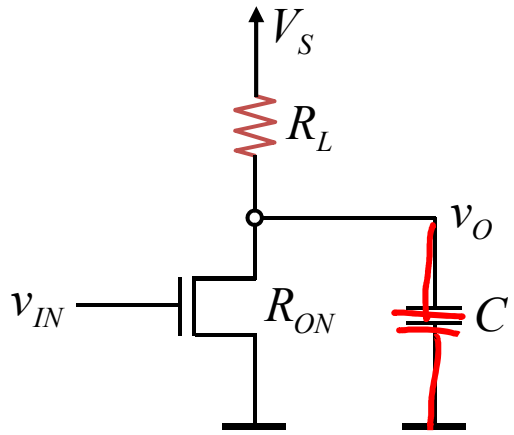
$$\bar{P} = \frac{E}{T} = \frac{CV_S^2}{T_2} = \boxed{CV_S^2 f} \quad ★★$$

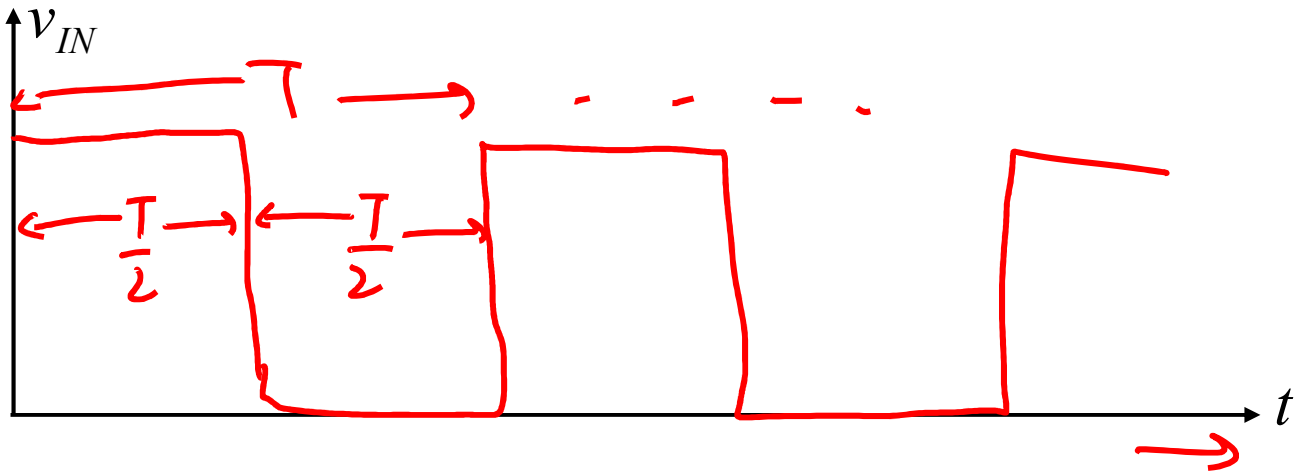
$T = T_1 + T_2$

T_2: $1/f$ is frequency

Back to our inverter —

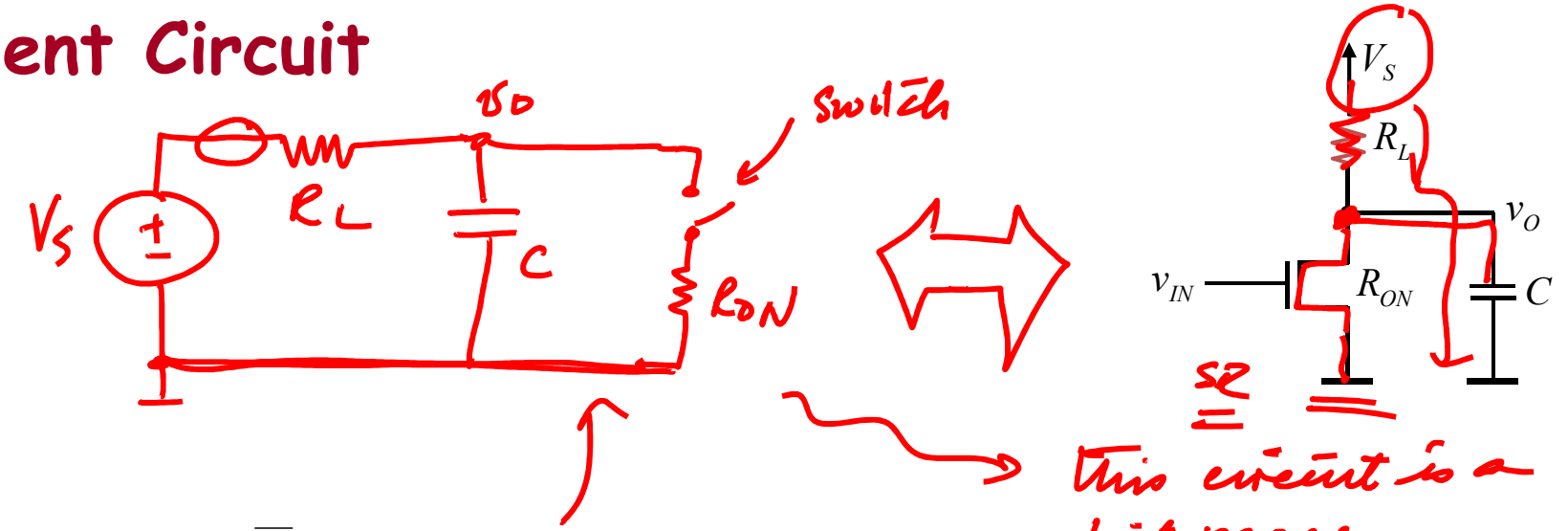
What is $\overline{P}$ for the following input?

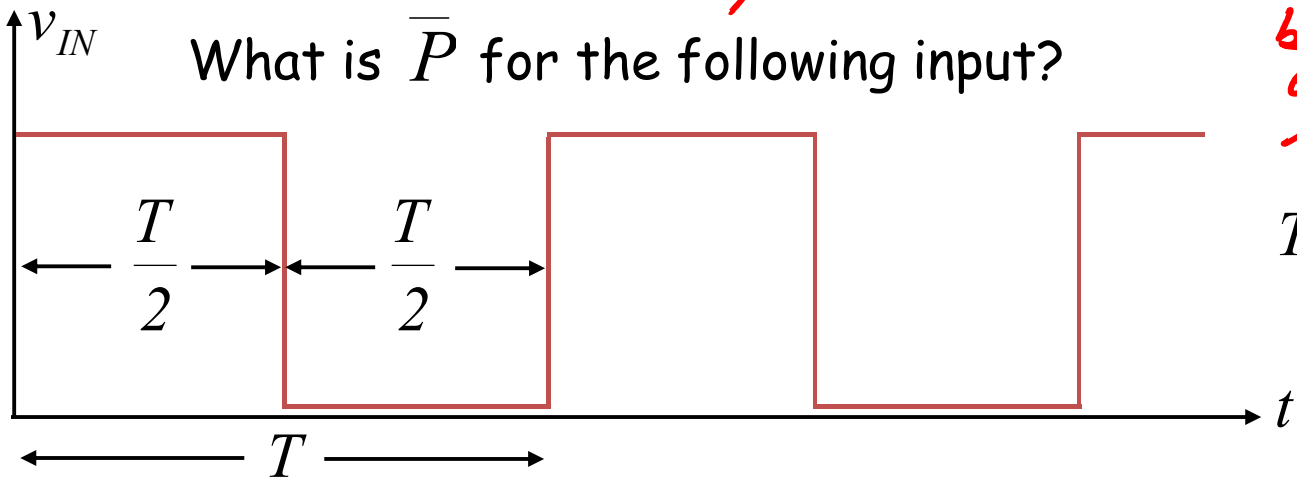




$T = \frac{1}{f}$ frequency

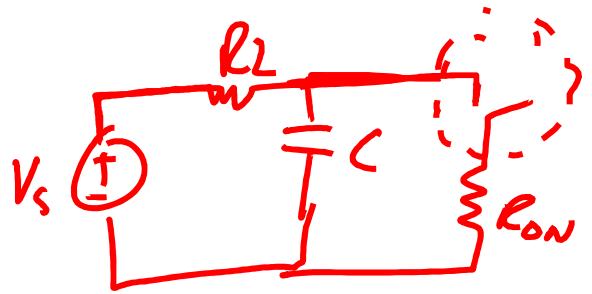
Equivalent Circuit





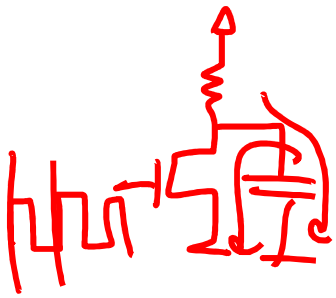
$$T = \frac{1}{f}$$

What is $\overline{P}$ for gate?



we can show (from section 11.3 of A&L) for derivation

$$\overline{P} = \frac{V_S^2}{2(R_L + R_{ON})} + CV_S^2 f \frac{R_L^2}{(R_L + R_{ON})^2}$$



when $R_L >> R_{ON}$

$$\overline{P} = \frac{V_S^2}{2R_L} + CV_S^2 f$$

Remember ★ → $\overline{P}_{STATIC}$ independent of frequency. $\frac{1}{2}$? MOSFET ON for half the time

$\overline{P}_{DYNAMIC}$ relate to switching the capacitor

remember ★★

What is $\overline{P}$ for gate?

Total power for a gate: $\overline{P} = \frac{V_S^{\,2}}{2R_L} + CV_S^{\,2} f$ when $R_L \gg R_{ON}$

$\overline{P}_{STATIC}$ $\overline{P}_{DYNAMIC}$

Expected standby power for a gate $= \overline{P}_{STATIC} = \frac{V_S^2}{2R_L}$

In standby mode, $f = 0$, so only static power

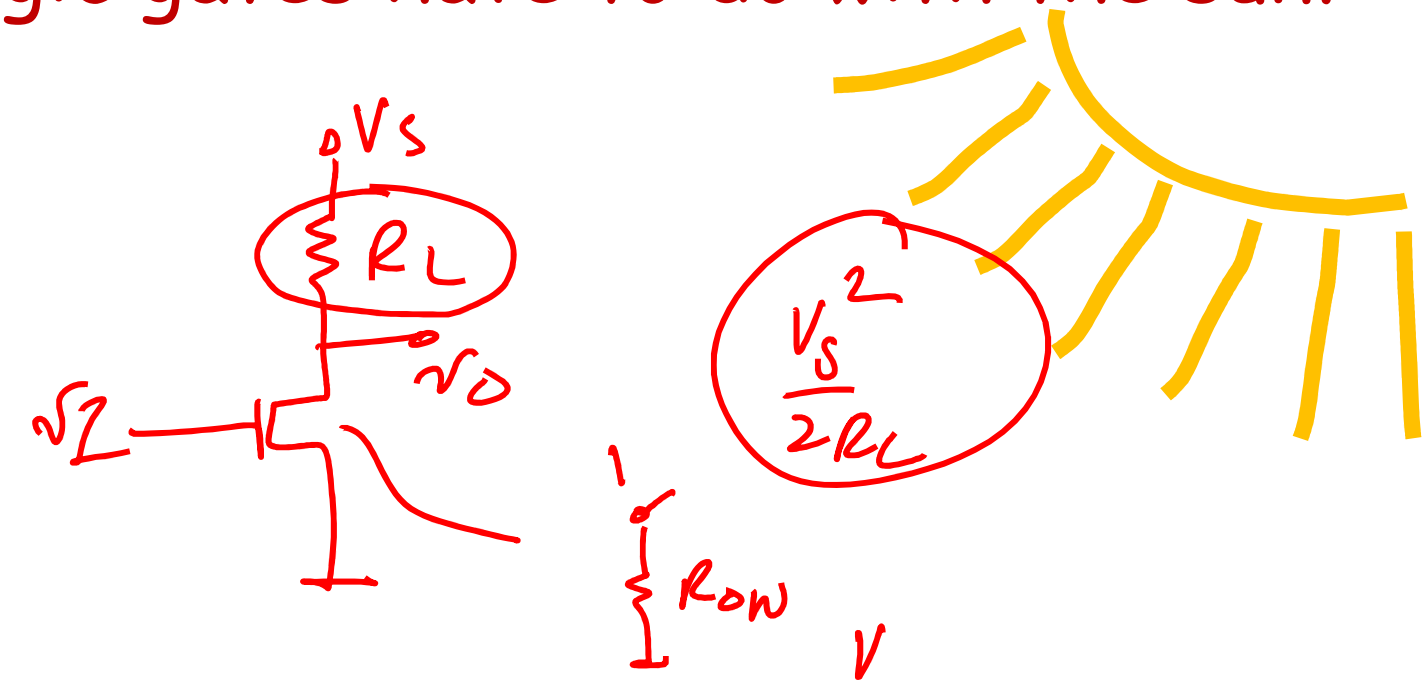
If gate is on, then it burns power $\overline{P} = \frac{V_S^2}{R_L}$

But, if gate is off, it does not burn power.

If the probability that any gate is on is half, the average power per gate is $\overline{P} = \frac{V_S^2}{2R_L}$



Question: what does our previous style of building logic gates have to do with the sun?



Some startling numbers…

a chip with 5×10^6 gates clocking at 3GHz

One gate: $\overline{P} = \frac{V_S^2}{2R_L} + CV_S^2 f$

$C = 1fF$

$R_L = 10k\Omega$

$R_L = 100\times10^6$

$R_L = 10M\Omega$

$V_S = 5V$

$f = 3\times10^9 Hz$

$V_S = 5V$

$\overline{P}_{STATIC}$ $\overline{P}_{DYNAMIC}$

$\overline{P} = 5\times10^6 \left[\frac{25}{2\times10^4} + 10^{-15}\times 25\times 3\times10^9\right]$

$= 5\times10^6 \left[1.25 \text{ milliwatts} + 75 \text{ microwatts}\right]$

$\overline{P}_{STATIC} \approx 6.25kW!$

$P_{DYNAMIC}$ 375 W → bad

$\propto V_S^2$ 5V → 1V → 5W

$\propto f$

disaster!

Needs fundamental change to our design

Move in next sequence

Question: what does our previous style of building logic gates have to do with the sun?

