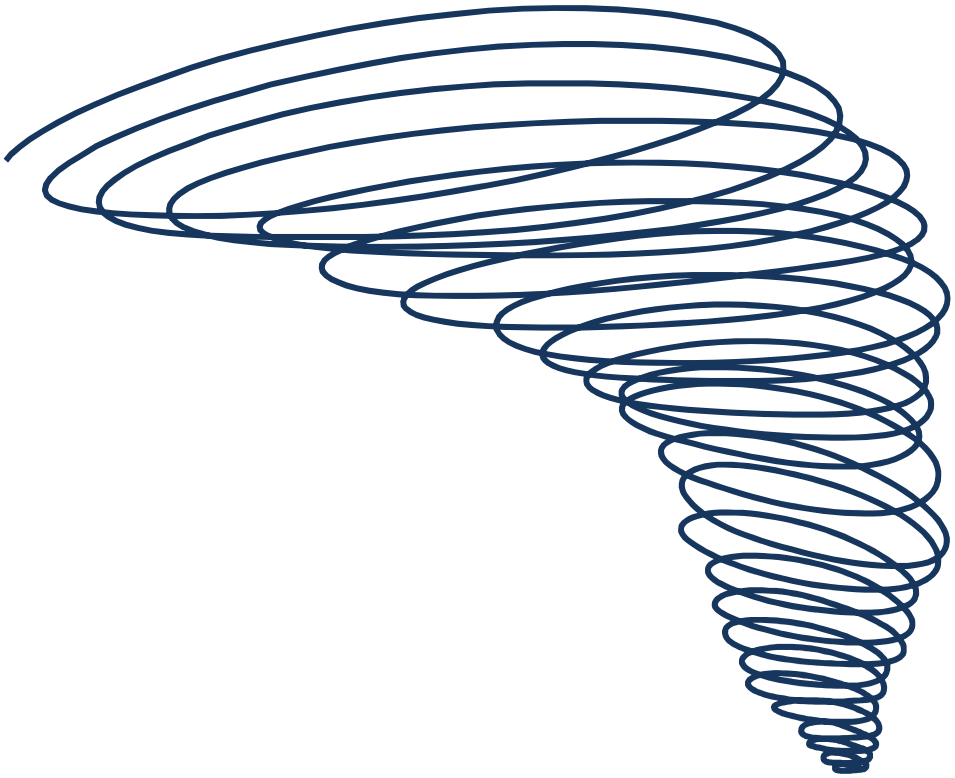
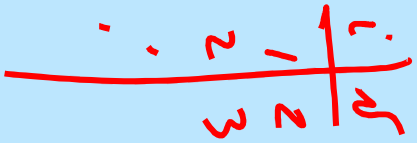
6.002x CIRCUITS AND ELECTRONICS

Op Amps Positive Feedback



Nature as observed in experiments



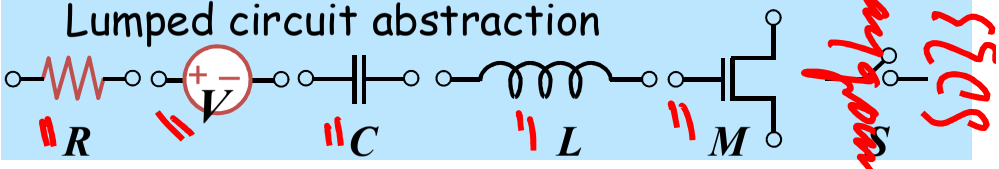
Physics laws or "abstractions"

- Maxwell's
- Ohm's

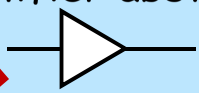
$V = R\,I$

→ abstraction for tables of data

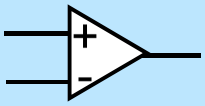
Lumped circuit abstraction



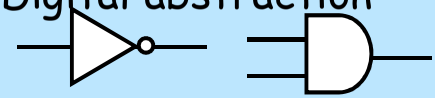
Simple amplifier abstraction



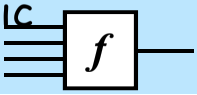
Operational amplifier abstr.



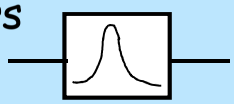
Digital abstraction



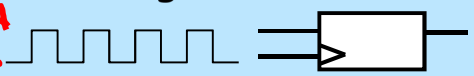
Combinational logic



Filters



Clocked digital abstraction



Instruction set abstraction

Pentium, MIPS 6.004, 6.846

Analog subsystems

Modulators, oscillators, RF amps, power supplies 6.061

Programming languages

Java, C++, Matlab, Python 6.00

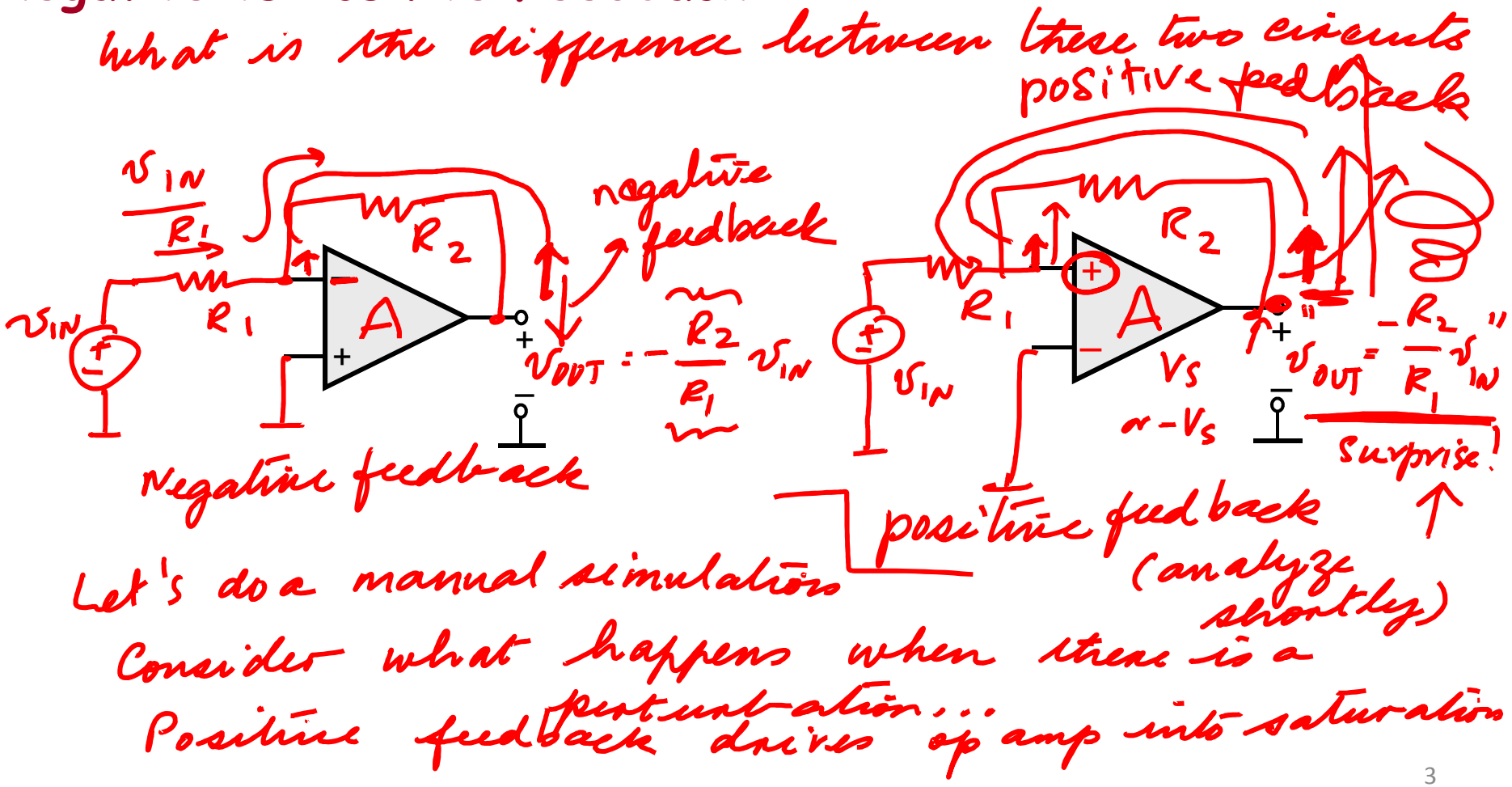
Software systems 6.033

Operating systems, Browsers

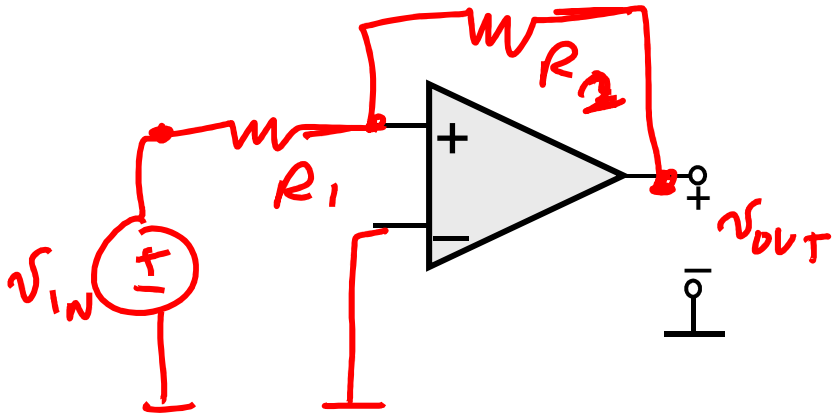
Mice, toasters, sonar, stereos, angry birds, space shuttle, iPAD 6.455 6.172,6.173

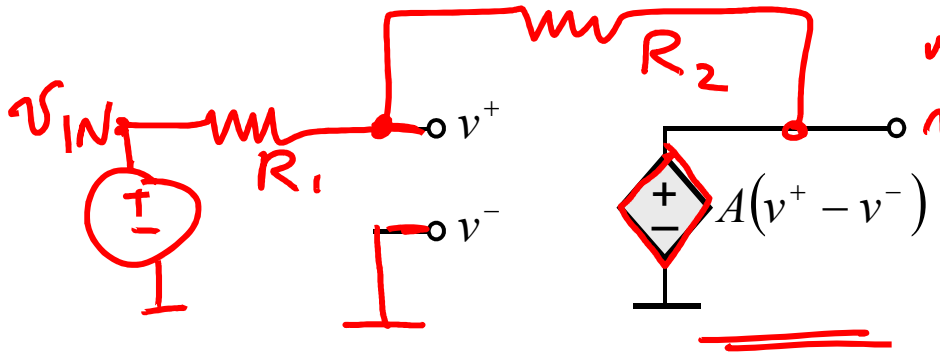
6.002x

Negative vs Positive Feedback



Positive Feedback Ckt

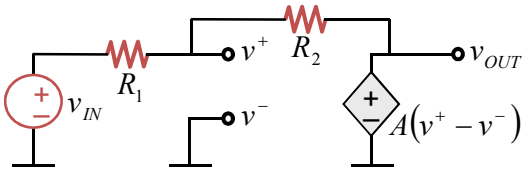




Find v_{OUT} as a function of v_{IN}

"$v_{OUT} = -\frac{R_2}{R_1} \cdot v_{IN}$"

Static Analysis of Positive Feedback Ckt



Node method:

$$\frac{v^+ - v_{IN}}{R_1} + \frac{v^+ - v_{OUT}}{R_2} = 0$$

$$\frac{\frac{v_{OUT}}{A} - v_{IN}}{R_1} + \frac{\frac{v_{OUT}}{A} - v_{OUT}}{R_2} = 0 \qquad * AR_1R_2$$

$$R_2 v_{OUT} - AR_2 v_{IN} + R_1 v_{OUT} - AR_1 v_{OUT} = 0$$

$$v_{OUT}(R_2 + R_1 - AR_1) = v_{IN}(AR_2)$$

$$v_{OUT} = \frac{AR_2}{R_2 + R_1 - AR_1} v_{IN} = \frac{R_2 v_{IN}}{\frac{R_2}{A} + \frac{R_1}{A} - R_1}$$

$A \to \infty$

$$v_{OUT} = -\frac{R_2}{R_1} v_{IN}$$

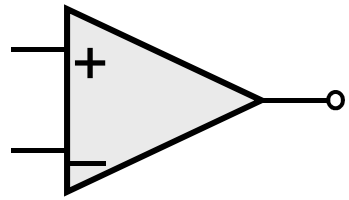
$v^- = 0$

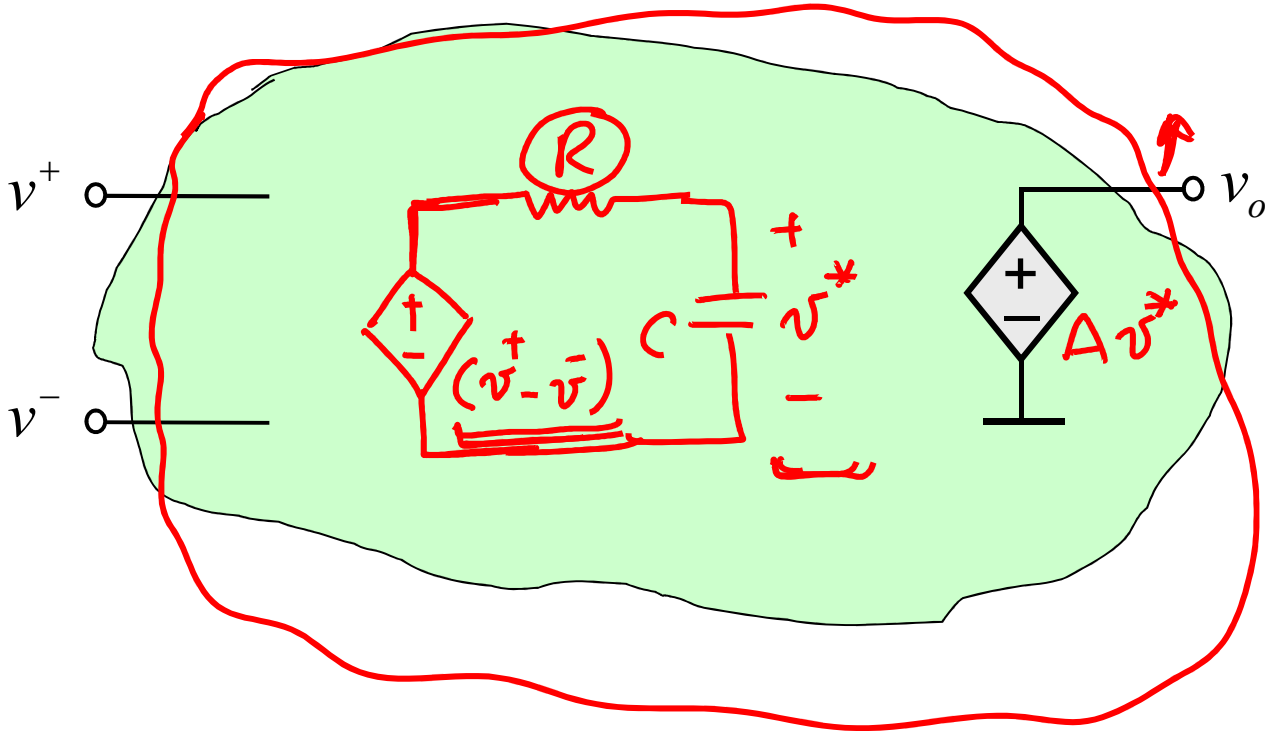
$v_{OUT} = A v^+$

$v^+ = \frac{v_{OUT}}{A}$

Representing dynamics of op amp

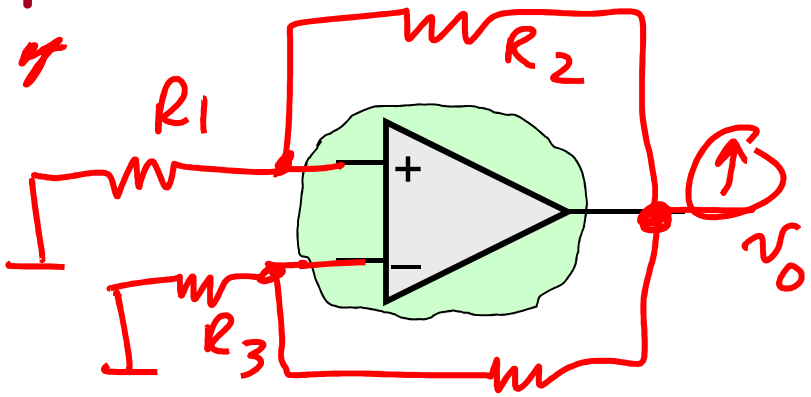
Need a better model to represent temporal behavior of op amp



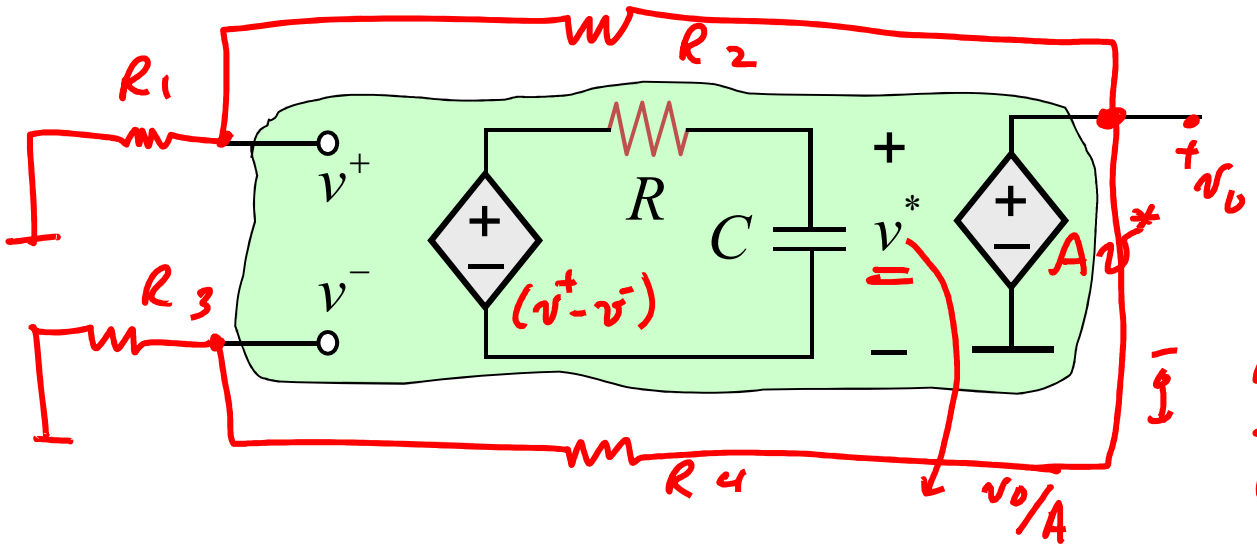


Representing dynamics of op amp...

Let's analyze the dynamics of this circuit to build insight.



Circuit model



at $t=0$, $v_o(0) = K$

e.g. v^* has some noise on it

Let's develop an equation representing the time behavior of v_o

Dynamics of op amp...

node method: v^*

$$v^+ = \frac{v_o R_1}{R_2+R_1} = \overset{+}{\gamma} v_o$$

$$i = C\frac{dv^*}{dt}$$

$$C\frac{dv^*}{dt} + \frac{v^* - (v^+ - v^-)}{R} = 0$$

$$v^* = \frac{v_o}{A} \quad * R$$

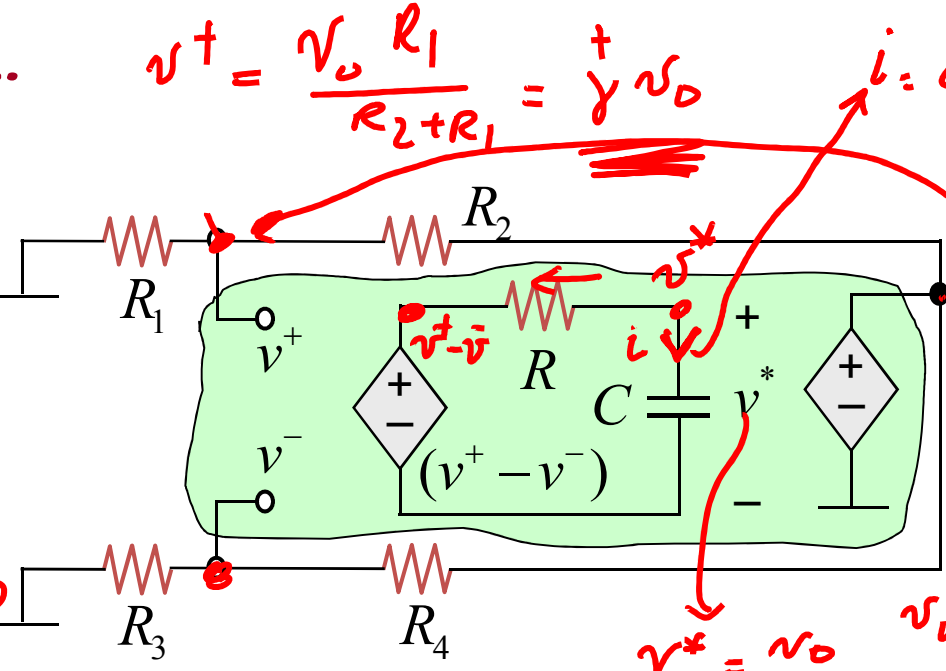
$$\frac{RC}{A}\frac{dv_o}{dt} + \frac{v_o}{A} - (v^+ - v^-) = 0$$

$$\frac{RC}{A}\frac{dv_o}{dt} + \frac{v_o}{A} = (v^+ - v^-)$$

$$= \overset{+}{\gamma} v_o - \overset{-}{\gamma} v_o$$

$$= (\overset{+}{\gamma} - \overset{-}{\gamma}) v_o$$

$$v^- = v_o \frac{R_3}{R_3+R_4} = \overset{-}{\gamma} v_o$$



$$v^* = \frac{v_o}{A}$$

$$v_o = A v^*$$

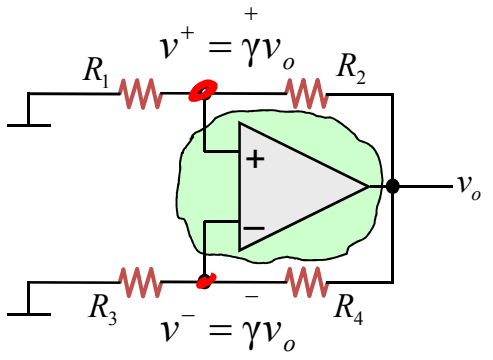
$v_o(0) = K$

small perturbation

$\overset{+}{\gamma}$: fraction of v_o fed back to +ve input

$\overset{-}{\gamma}$: " " " " " " -ve "

Dynamics of op amp... continued



$$\frac{RC}{A}\frac{dv_o}{dt} + \frac{v_o}{A} = (\gamma - \gamma)v_o$$

$$\frac{dv_o}{dt} + \left[\frac{1}{RC} + \frac{A}{RC}(\bar{\gamma} - \overset{+}{\gamma})\right]v_o = 0$$

* A/RC

neglect

$$\frac{dv_o}{dt} + \frac{A}{RC}(\bar{\gamma} - \overset{+}{\gamma})v_o = 0$$

$\frac{RC}{A}$ time^{-1}

$$T = \frac{RC}{A(\bar{\gamma} - \overset{+}{\gamma})}$$

assume

$v_o(0) = K$

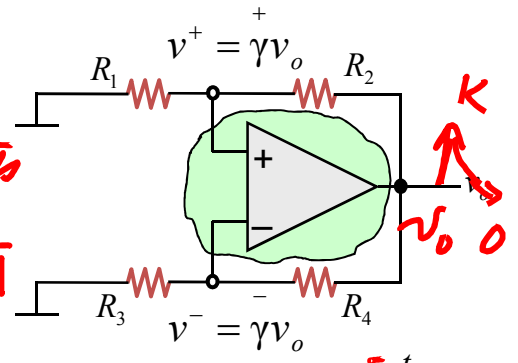
small perturbation

or $\quad \frac{dv_o}{dt} + \frac{v_o}{T} = 0$

Solution is $\quad v_o = K e^{-\frac{t}{T}}$

when $t = 0$, $v_o = K$

Response to small disturbance of K to v_o (noise)



$v_o = K e^{-t/T}$

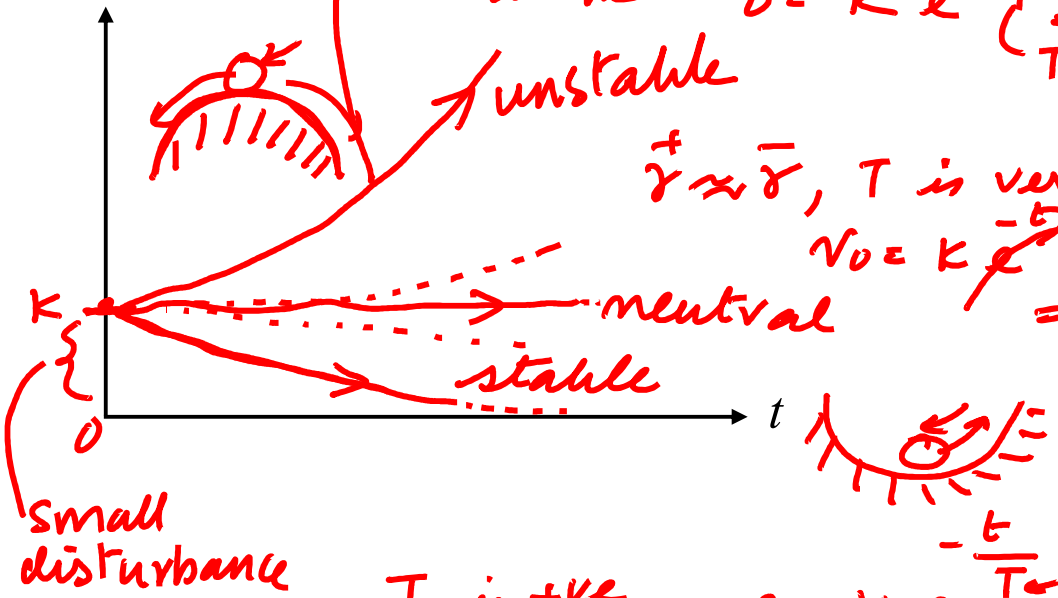
If $\bar{\gamma} - \gamma^+$ i.e. +ve feedback dominates

T is -ve $v_o = K e^{-\frac{t}{T}} = K e^{\frac{+t}{|T|}}$ T-ve

unstable

$\bar{\gamma} \approx \gamma^+$, T is very large

$v_o = K e^{-\frac{t}{T}} \approx 1 \Rightarrow v_o \approx K$



T is +ve when $\bar{\gamma} > \gamma^+$ $v_o = K e^{-\frac{t}{T}}$ +ve

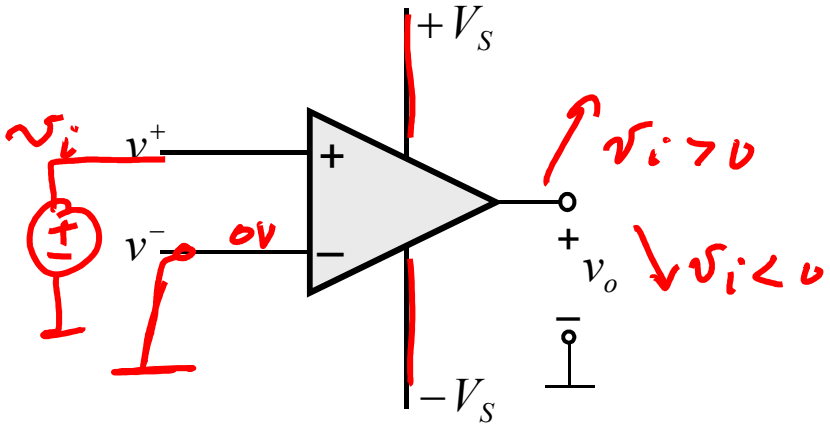
$v_o = K e^{-\frac{t}{T}}$

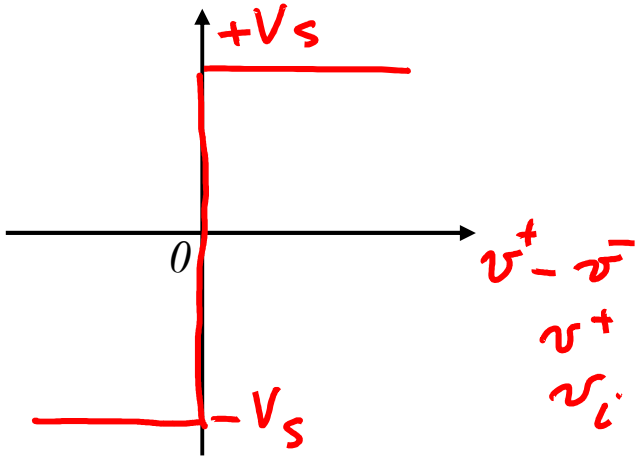
RC

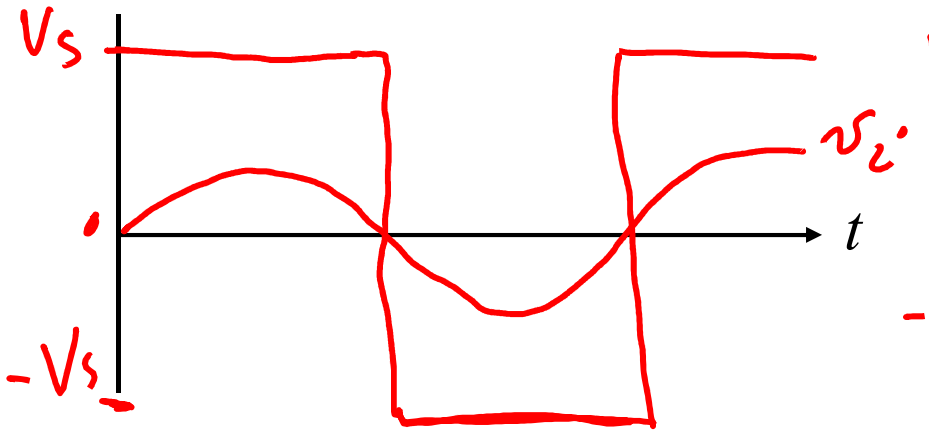
$A(\bar{\gamma} - \gamma^+)$

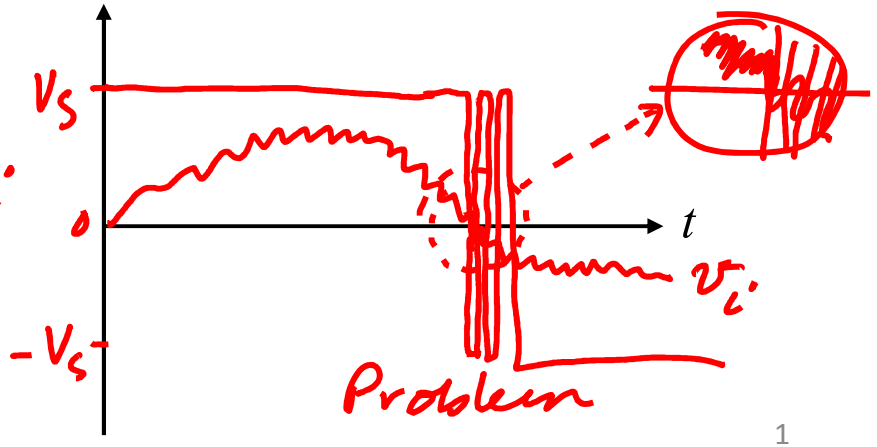
Now, let's build some useful circuits with positive feedback

One use for instability:
Build on the basic op amp as a comparator

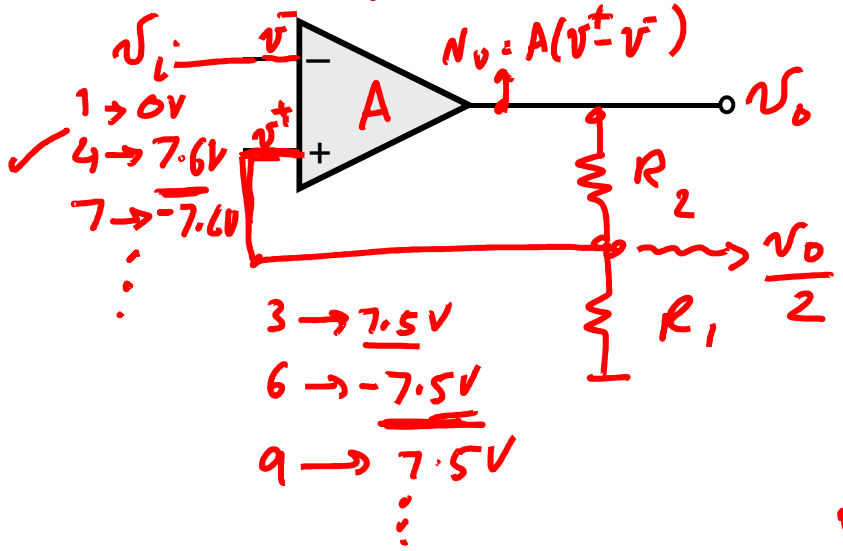








Now, use positive feedback



1 → 0V
4 → 7.6V
7 → -7.6V

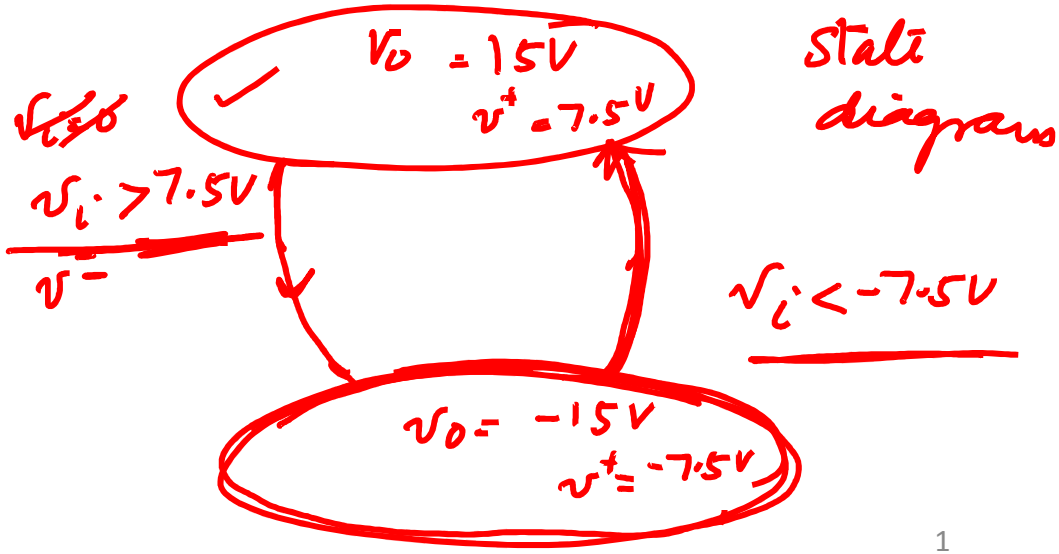
2 → 15V
5 → -15V
8 → +15V

3 → 7.5V
6 → -7.5V
9 → 7.5V

$v^+ = \frac{v_o R_1}{R_1 + R_2} = \frac{v_o}{2}$ if $R_1 = R_2$

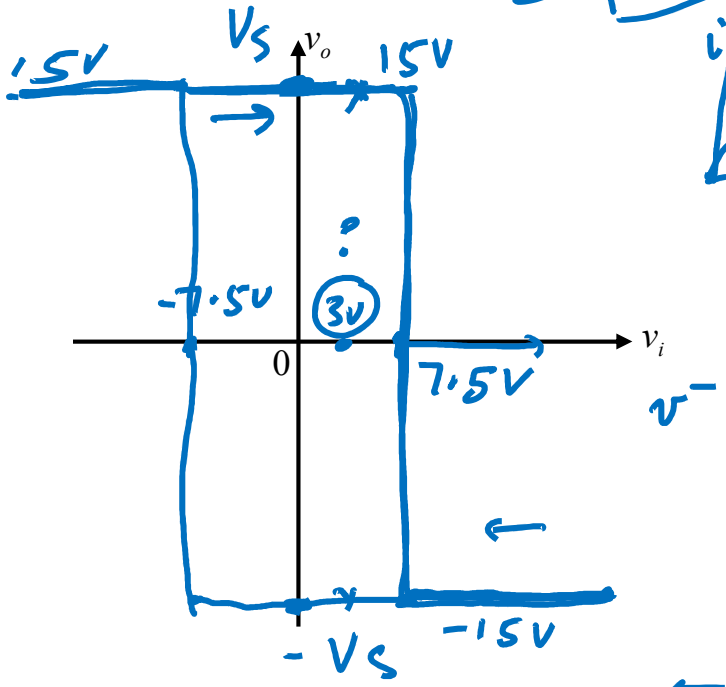
e.g: $R_1 = R_2$

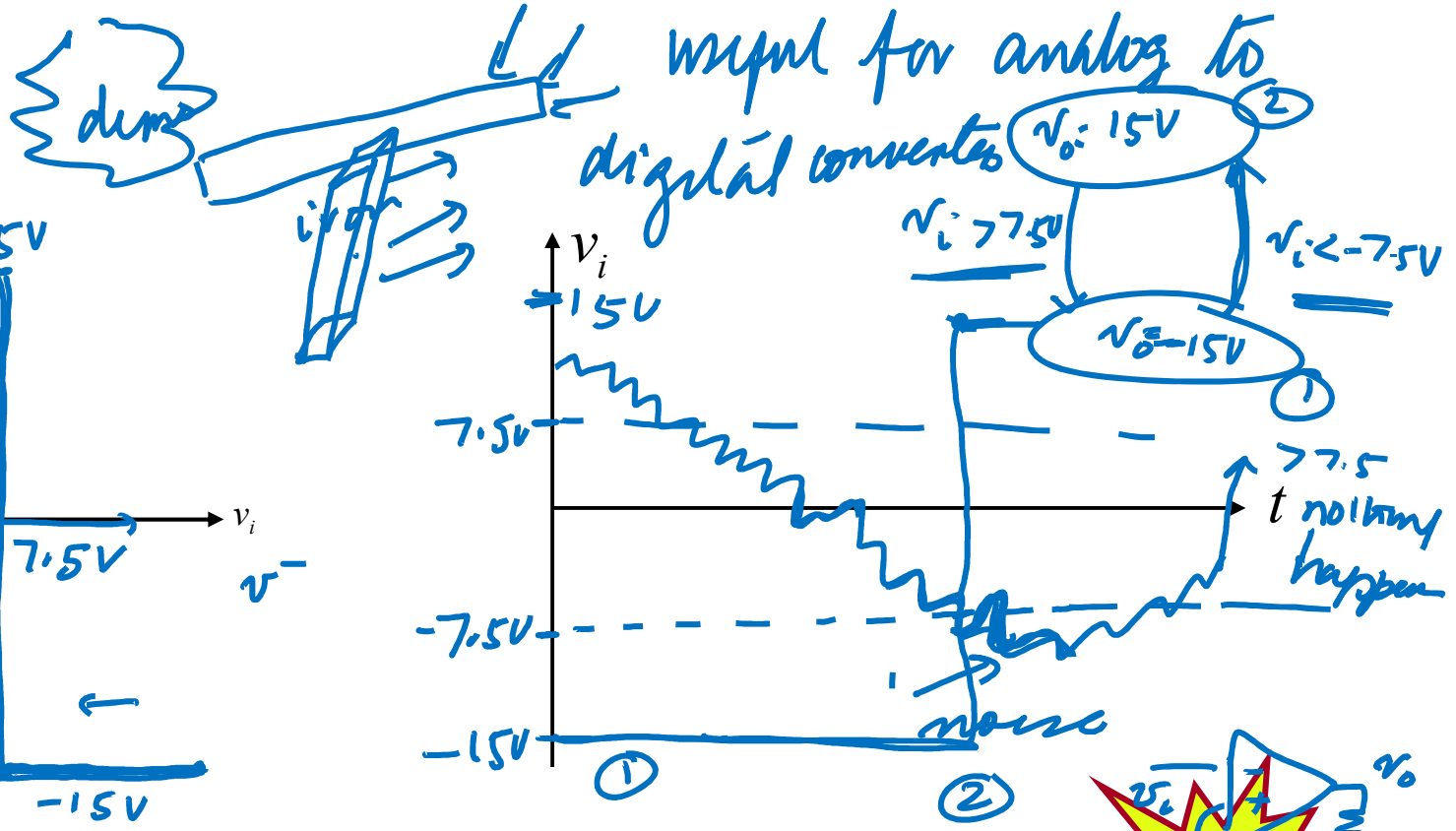
$V_S = 15^- V$



state diagram

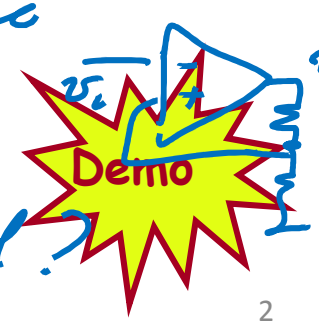
Hysteresis



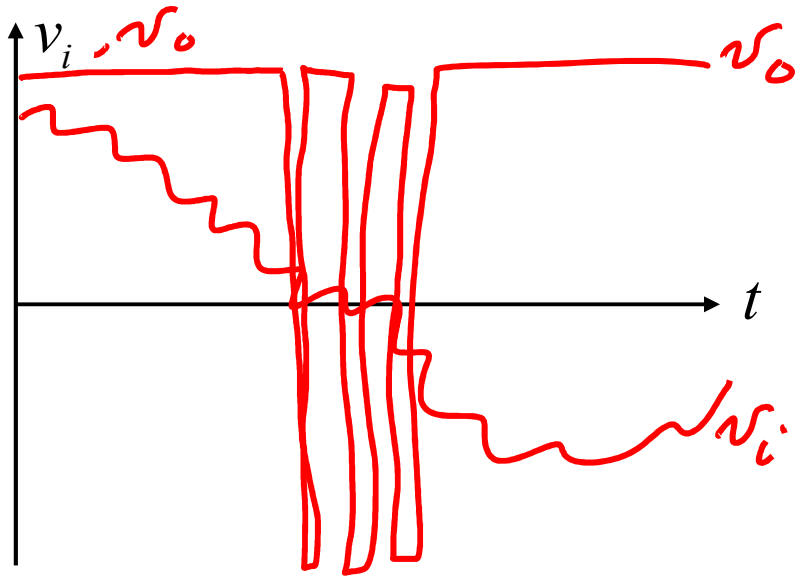


memory property!

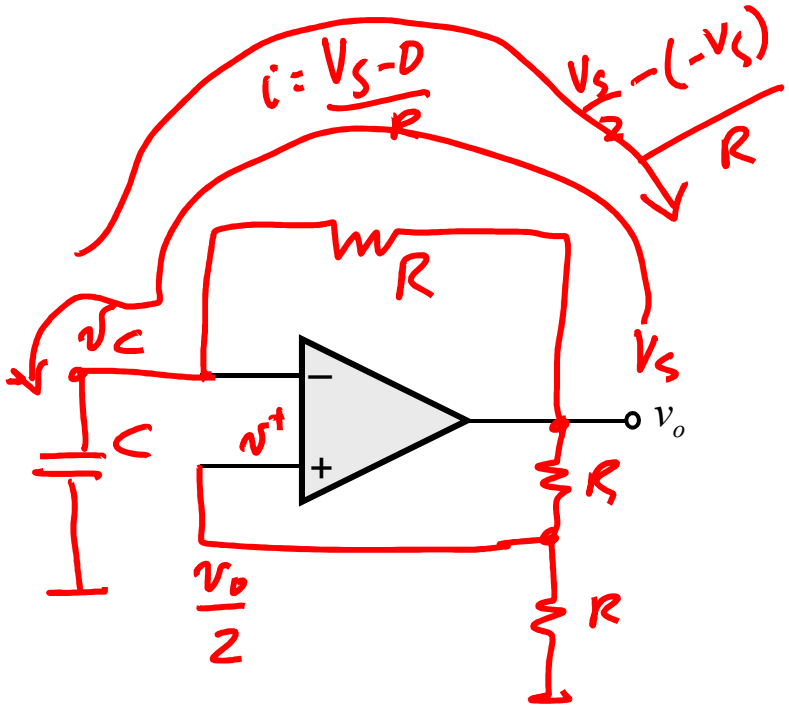
why is hysteresis useful?

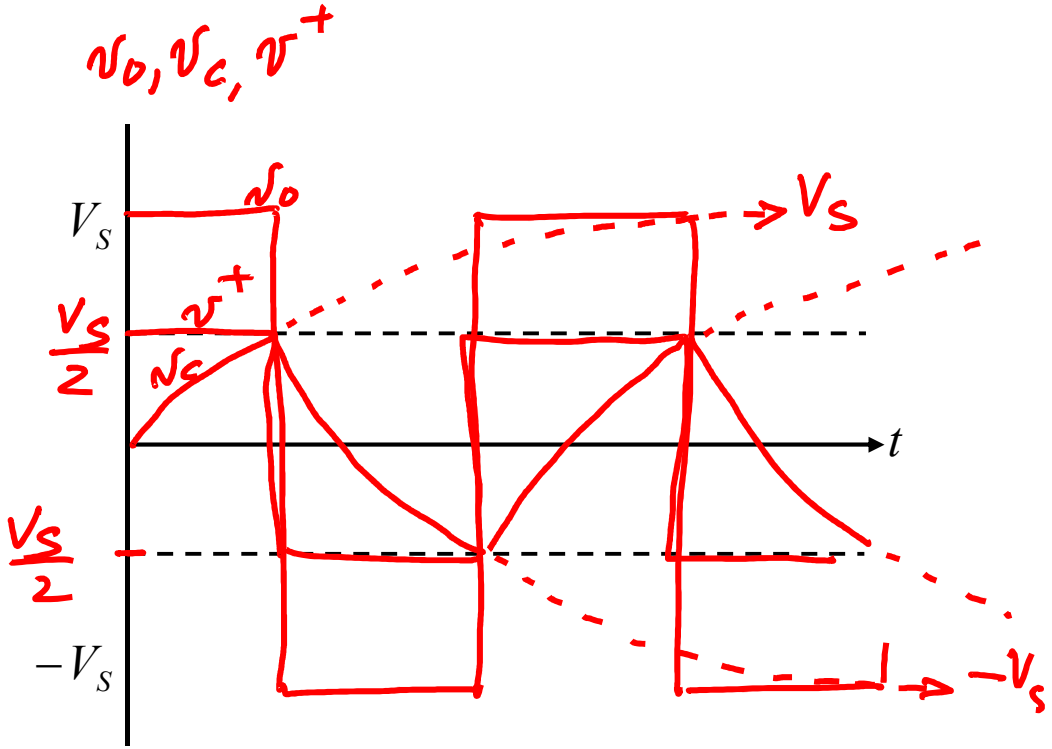


Without hysteresis



Oscillator — can create a clock

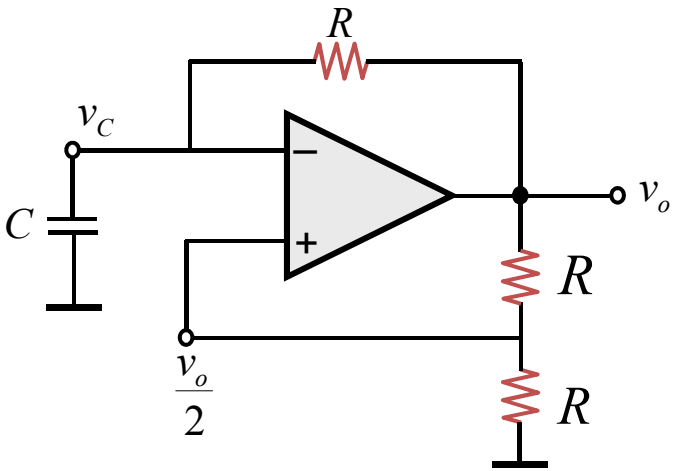


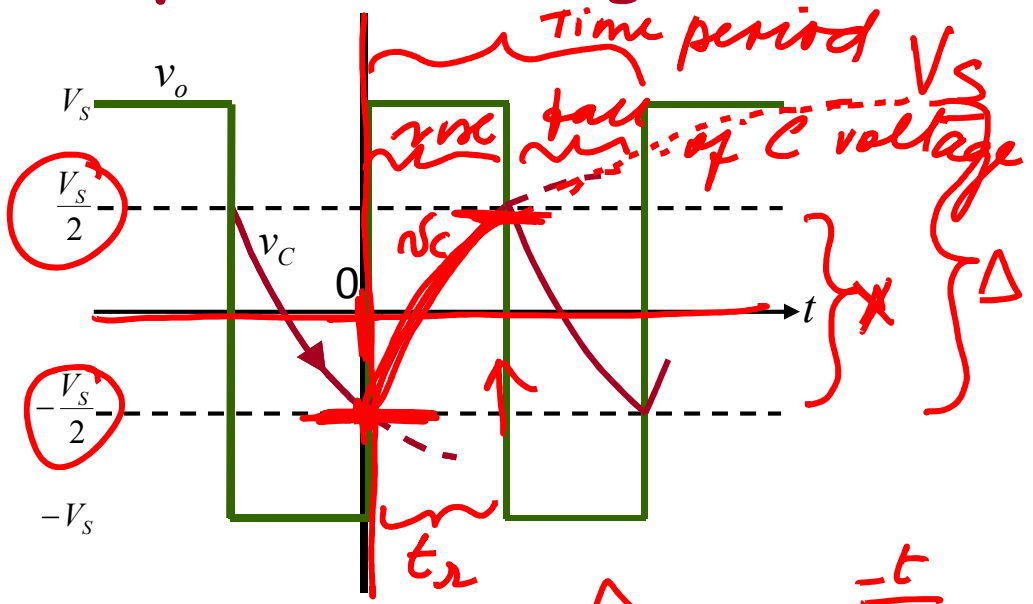


assume $v_c(t=0) = 0$

$v_o(t=0) = V_S$

Oscillator — frequency - capacitor voltage rise time





Time period

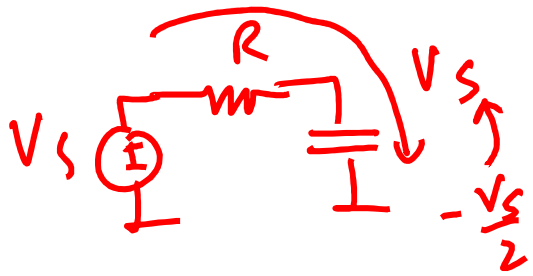
V_S

Δ

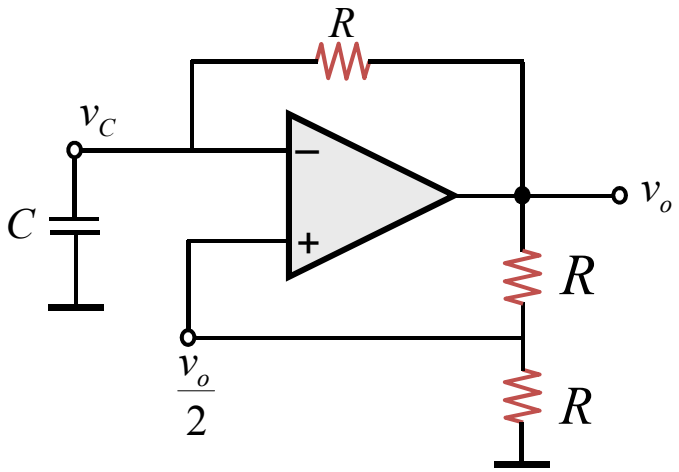
$\rightarrow v_C$ = Initial + Change $(1 - e^{\frac{-t}{RC}})$

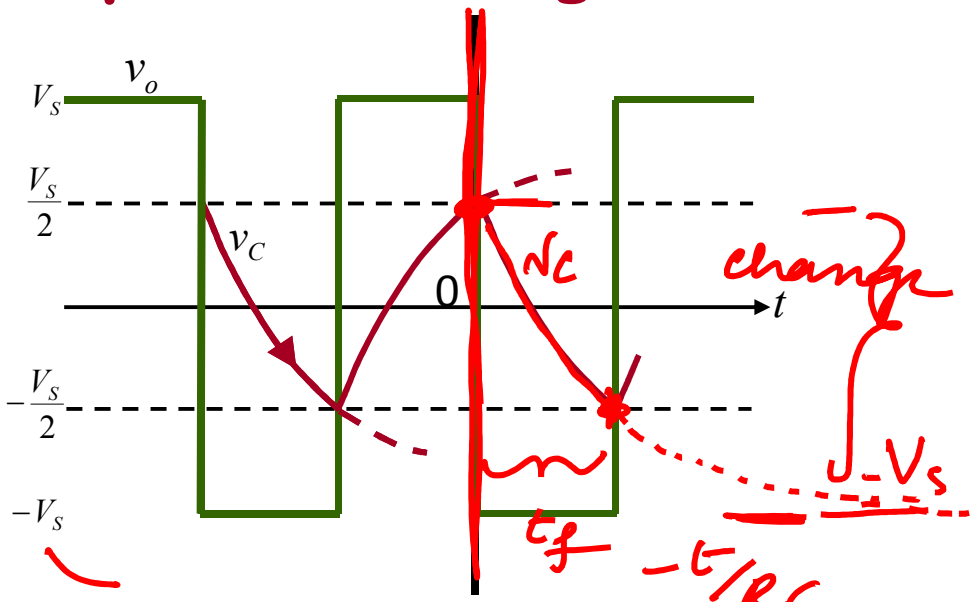
$-\frac{V_S}{2}$ $\quad V_S - (-\frac{V_S}{2}) = \frac{3V_S}{2}$

$\frac{V_S}{2} = -\frac{V_S}{2} + \frac{3V_S}{2}(1 - e^{\frac{-t_r}{RC}})$



Oscillator — frequency - capacitor voltage fall time



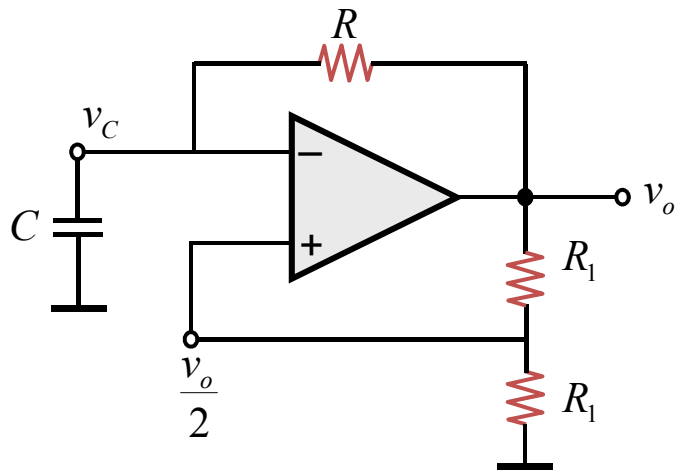


v_C = final + change e

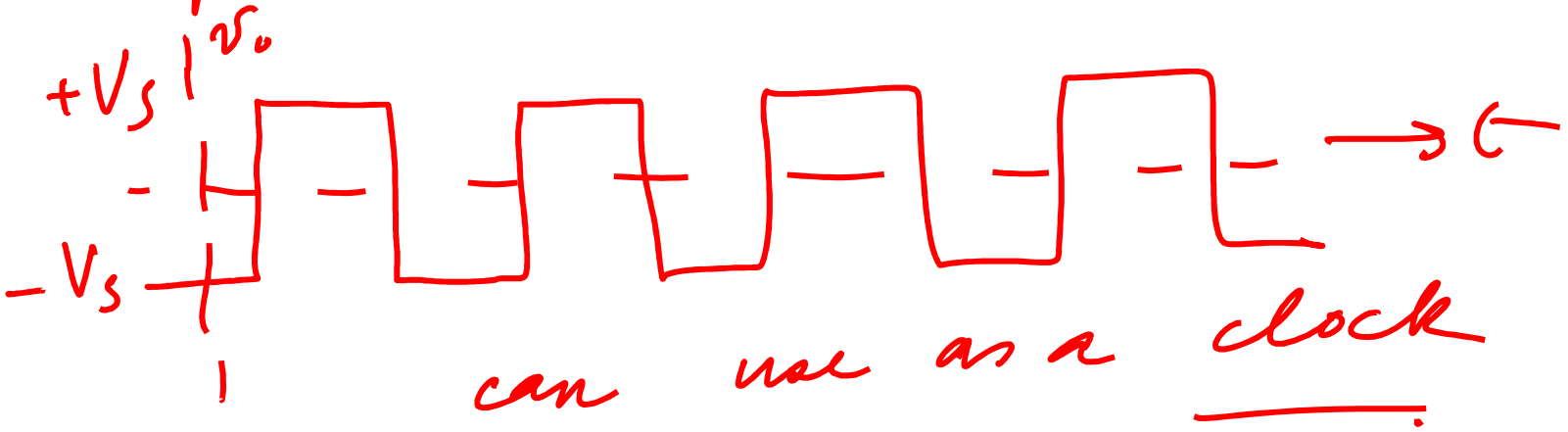
$-V_S$ ↓ $\frac{V_S}{2} - (-V_S) = \frac{3V_S}{2}$

solve to get $t_f \rightarrow -\frac{V_S}{2} = -V_S + \frac{3V_S}{2} e^{-\frac{t_f}{RC}}$

Clocks in Digital Systems

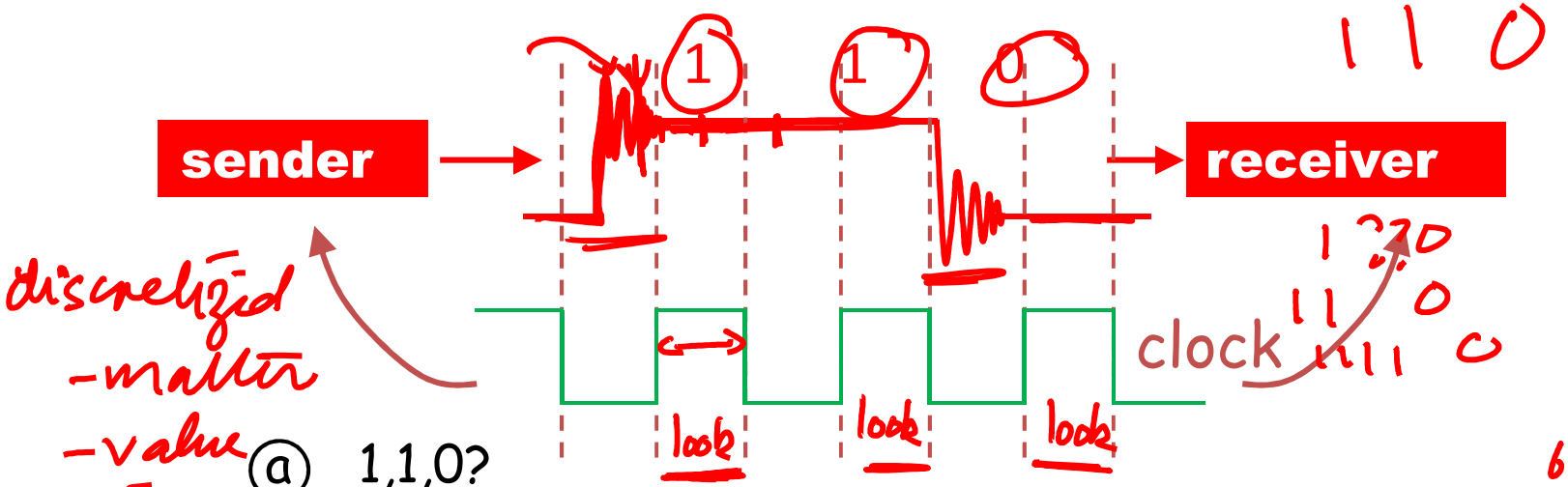


we have built an oscillator using the op amp



can use as a clock.

Why Use a Clock in Digital Systems?



discretized
- matter
- value
- time

ⓐ 1,1,0?

ⓑ When is the signal valid?

common timebase -- when to "look" at a signal
(e.g. whenever the clock is high)

→ Discretization of time – one bit of information associated with an interval of time (cycle)

(See page 555 of A & L)

6.004x