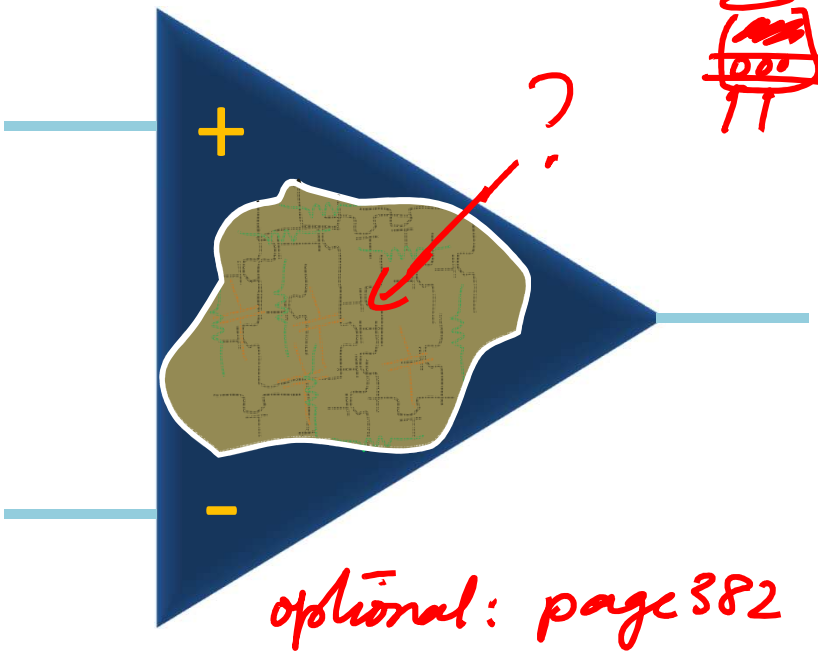
6.002x CIRCUITS AND ELECTRONICS



The Operational Amplifier Abstraction

op Amp



optional: page 382

Chapter 15 of the textbook

Two big aha moments in this sequence

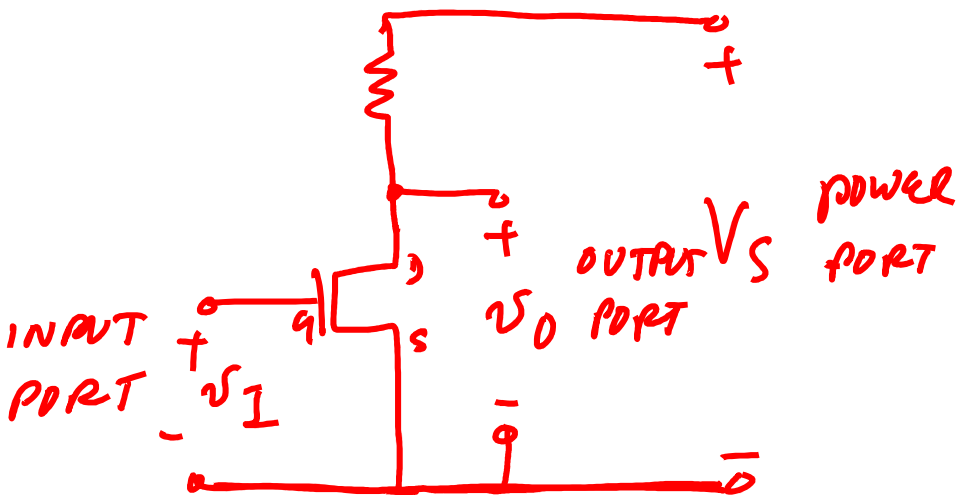
- Power of abstraction (gate level abstr)
- Feedback



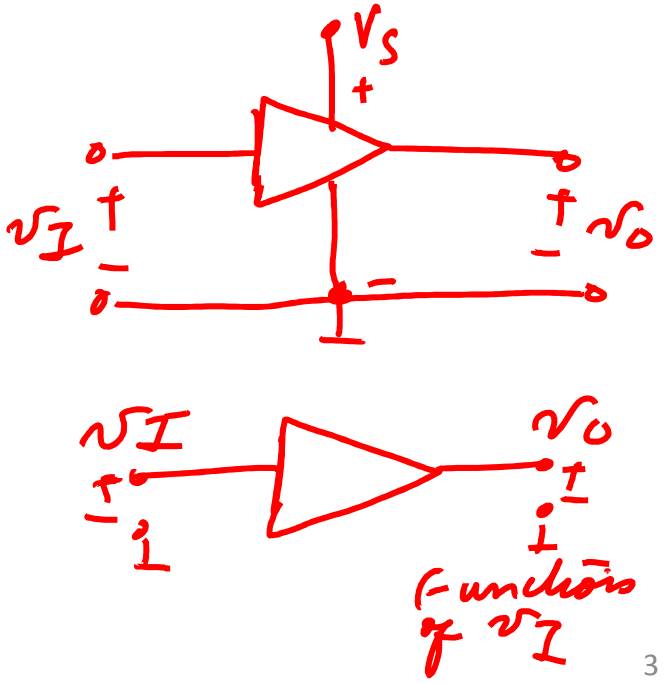
- LMI
- Static discipline for noise immunity in digital design
- Small signal method
- Impedance method

Review our Previous Amplifier Abstraction

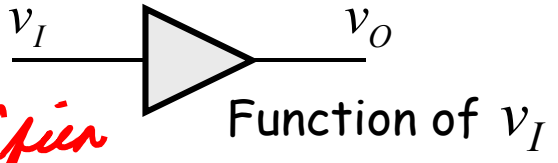
- MOSFET amplifier — 3 ports



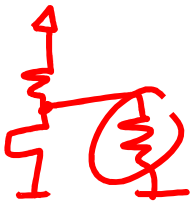
- Amplifier abstraction



Amplifier Building Block

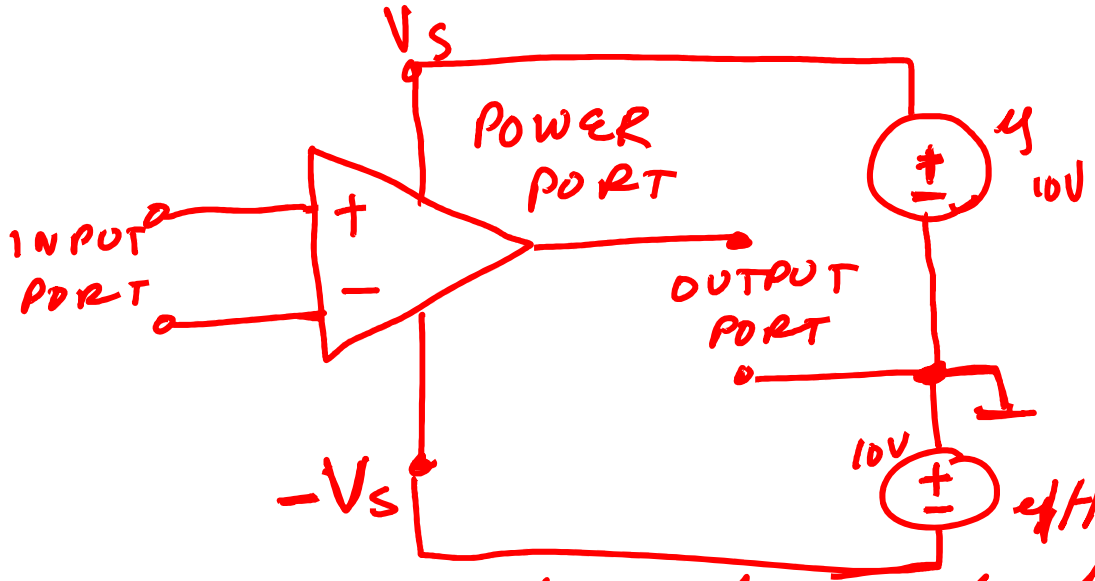


- We could use abstract amplifier as a building block for more complex circuits
- But our MOSFET amplifier is actually not a great abstract amplifier block because we need to be careful about what we connect to it, since its behavior changes
- Today - introduce a more powerful amplifier abstraction

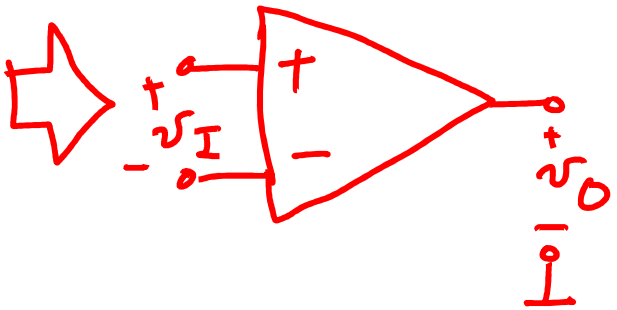


load would change small signal gain!

Operational Amplifier Op Amp



MORE ABSTRACT REPRESENTATION

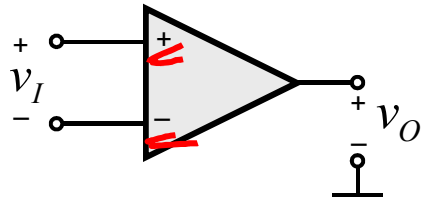


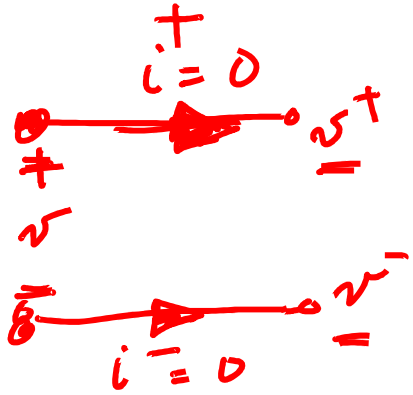
Learn to work with abstractions without really knowing what's inside

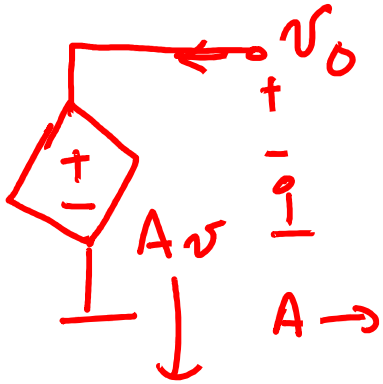
INPUT: NOT single ended

optional: page 382 of A&L – see how to build a differential amp which is a key component of the op amp

Circuit model of Op Amp (ideal):







$v = v^+ - v^-$

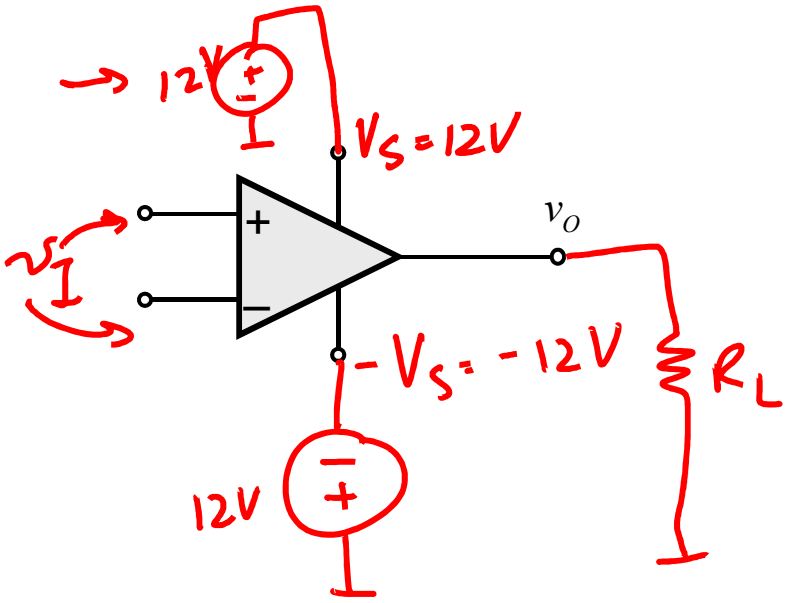
$v_O = A(v^+ - v^-)$

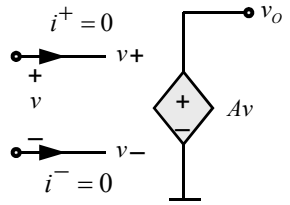
That is:

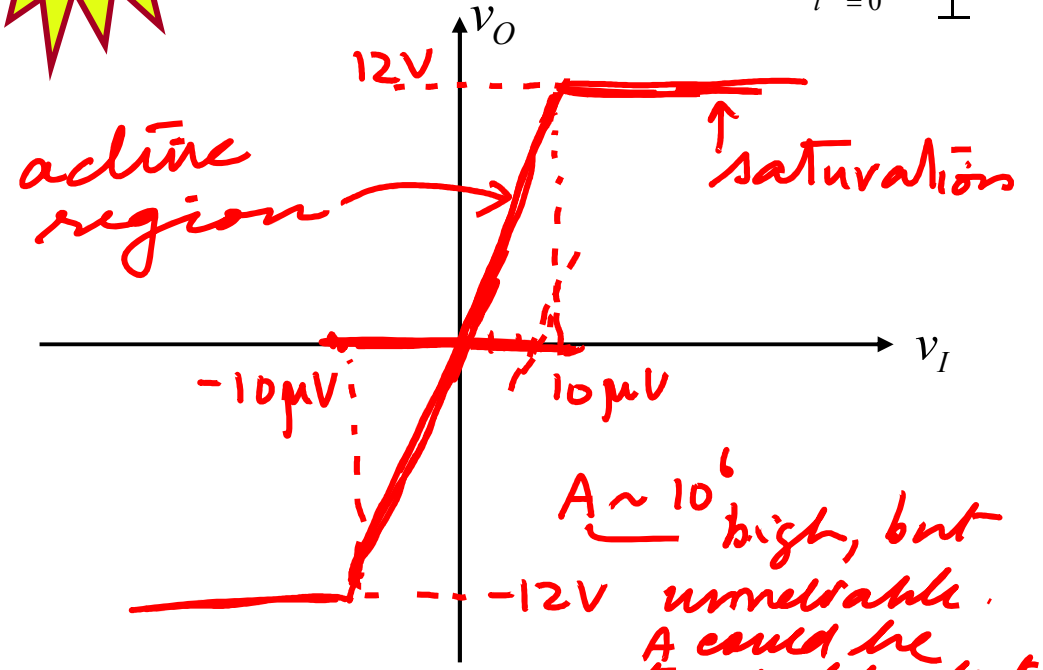
- 0 output resistance
- ∞ input resistance
- "A" virtually ∞
- No saturation (i.e. v_O can take on any value)

Using it…

Demo



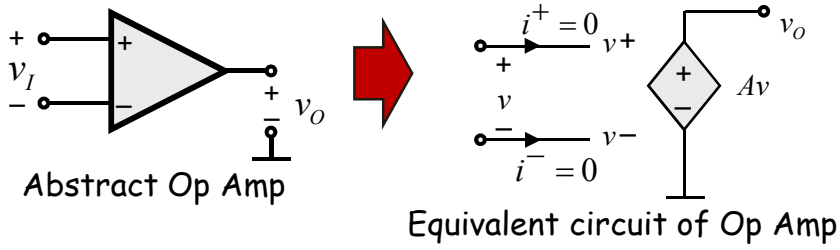




$A \sim 10^6$ high, but unreliable. A could be temp. dependent.

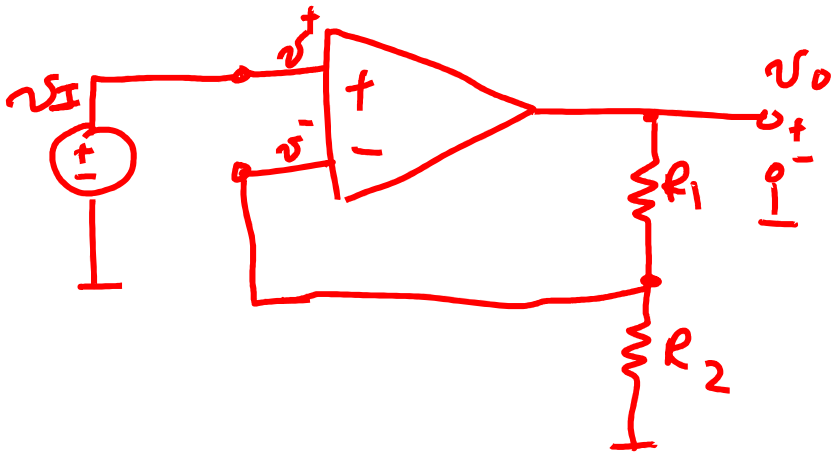
Note: Watch out for possible confusion with MOSFET saturation! This saturation is different.

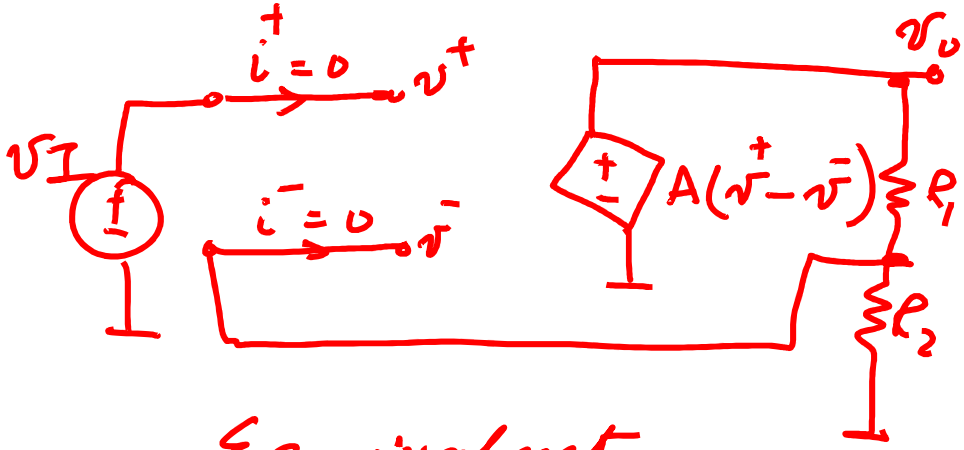
Let us build a circuit: noninverting amplifier



Abstract Op Amp

Equivalent circuit of Op Amp

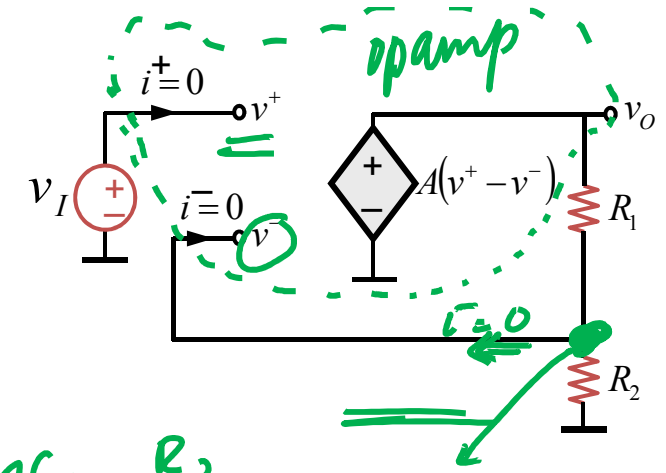




Equivalent circuit model

Let us analyze the circuit:

Find v_O in terms of v_I



Generally a good starting point is to write

$$v_O = A(v^+ - v^-)$$

$v^+ \rightarrow v_I$

$v^- \rightarrow v_O \cdot \frac{R_2}{R_1 + R_2}$

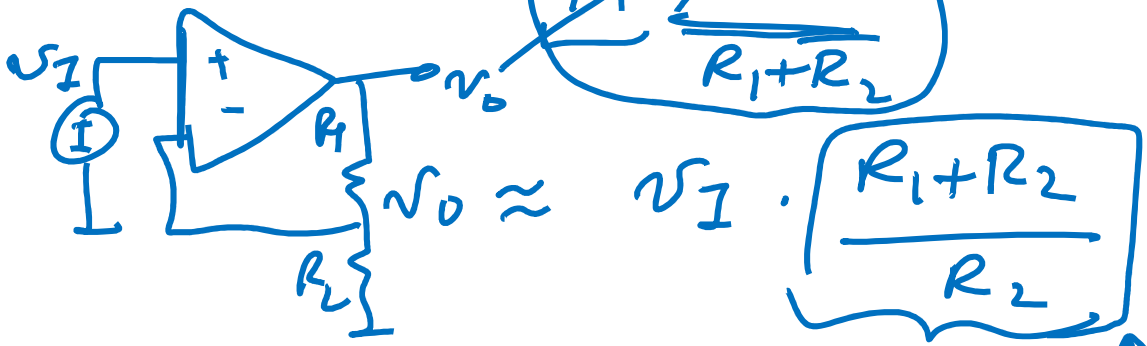
$$v_O = A\left(v_I - v_O \frac{R_2}{R_1 + R_2}\right)$$

$$v_O\left(1 + \frac{A R_2}{R_1 + R_2}\right) = A v_I$$

$$v_O = \frac{A v_I}{1 + \frac{A R_2}{R_1 + R_2}}$$

What happens when "A" is very large?

Let's see… When A is large



$$v_o = \frac{Av_I}{1 + \frac{AR_2}{R_1+R_2}}$$

$$v_o \approx v_I \cdot \frac{R_1+R_2}{R_2}$$

A is gone! gain ← external component

Gain:
- determined by resistor ratio
- insensitive to A, temperature, fab variation…

Suppose $A = 10^6$, $R_1 = 9R$, $R_2 = R$

$$v_o = \frac{10^6 v_I}{1 + \frac{10^6 R}{9R + R}} = \frac{10^6 v_I}{1 + \frac{10^6}{10}} \approx 10 \cdot v_I$$

Demo

Why did this (temp insensitivity) happen? Our next Aha!

Insight:

$v_O = v_I \frac{R+R}{R}$

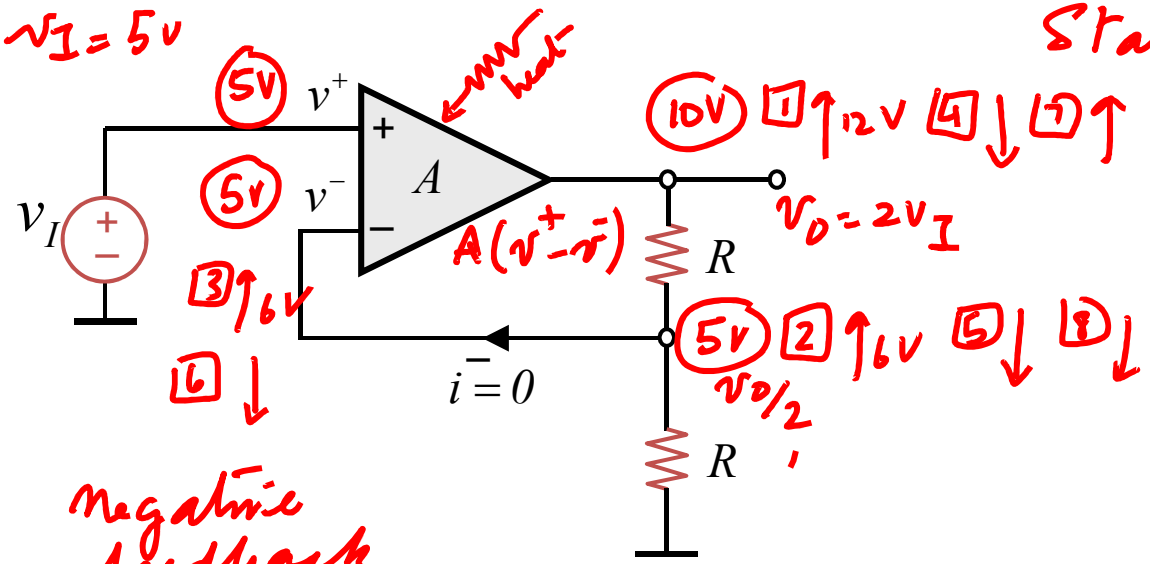
$= 2v_I$

Suppose I perturb the circuit.... (eg. force v_O momentarily to 12V somehow)

Stable point when $v^+ \approx v^-$

opamp tries to maintain this under

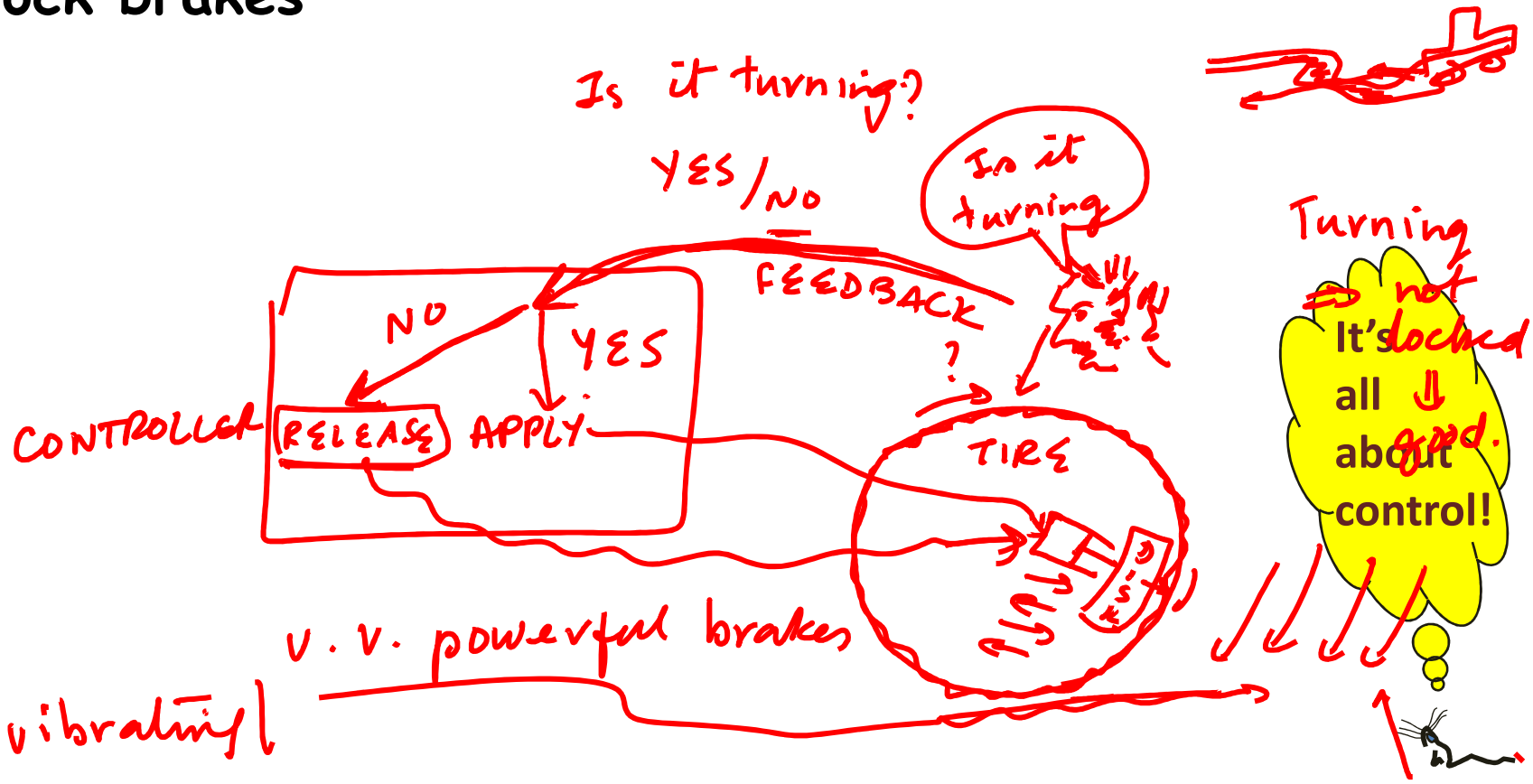
Key: negative feedback: portion of the output fed back to the −ve input



e.g. car antilock brakes

Question: How to control a high-strung device?

Antilock brakes

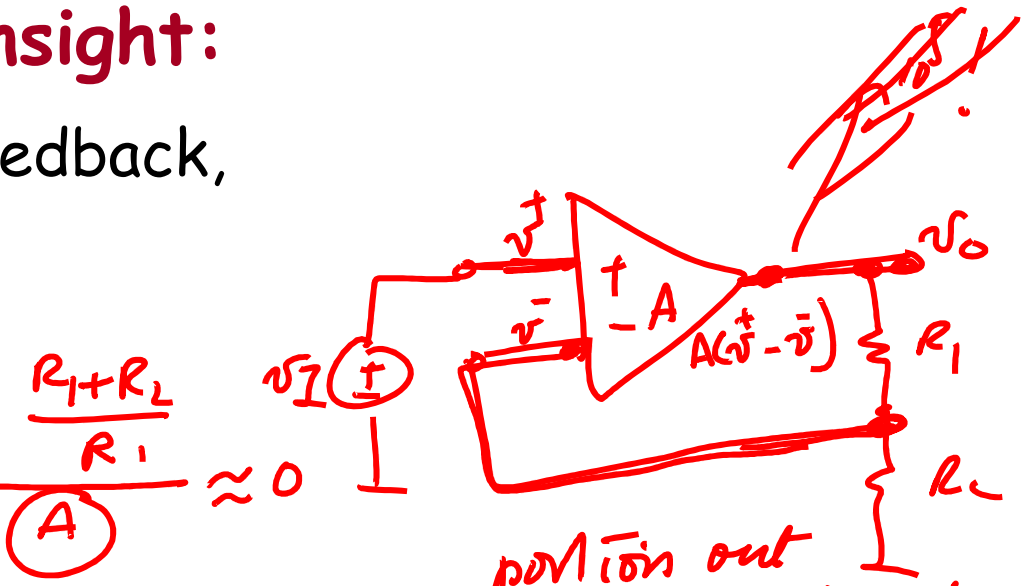


Another cool op amp insight:

Observe, under negative feedback,

$$v_O = A(v^+ - v^-)$$

$$v^+ - v^- = \frac{v_O}{A} = \frac{v_I \frac{R_1+R_2}{R_1}}{A} \approx 0$$



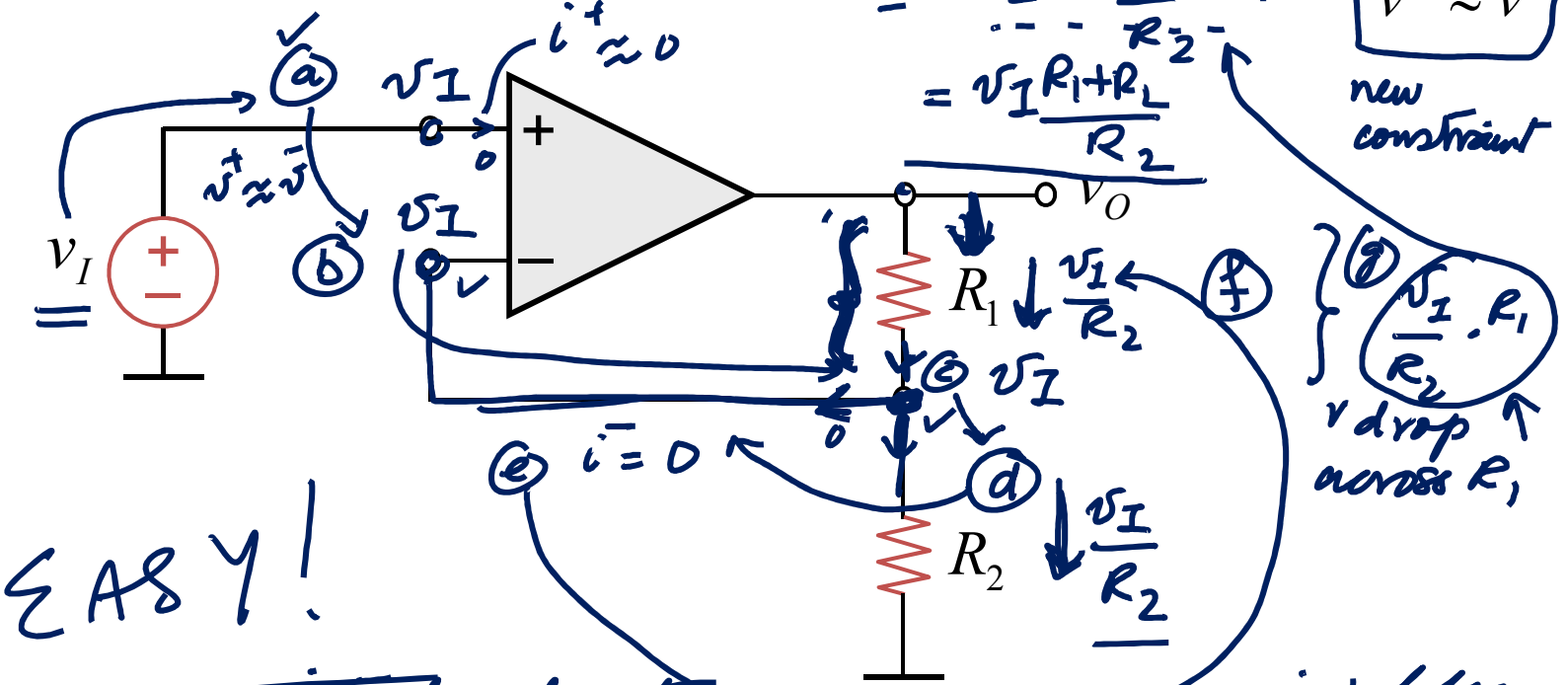
$v^+ - v^- \to 0$

virtual short → $v^+ \approx v^-$ ← assume this!

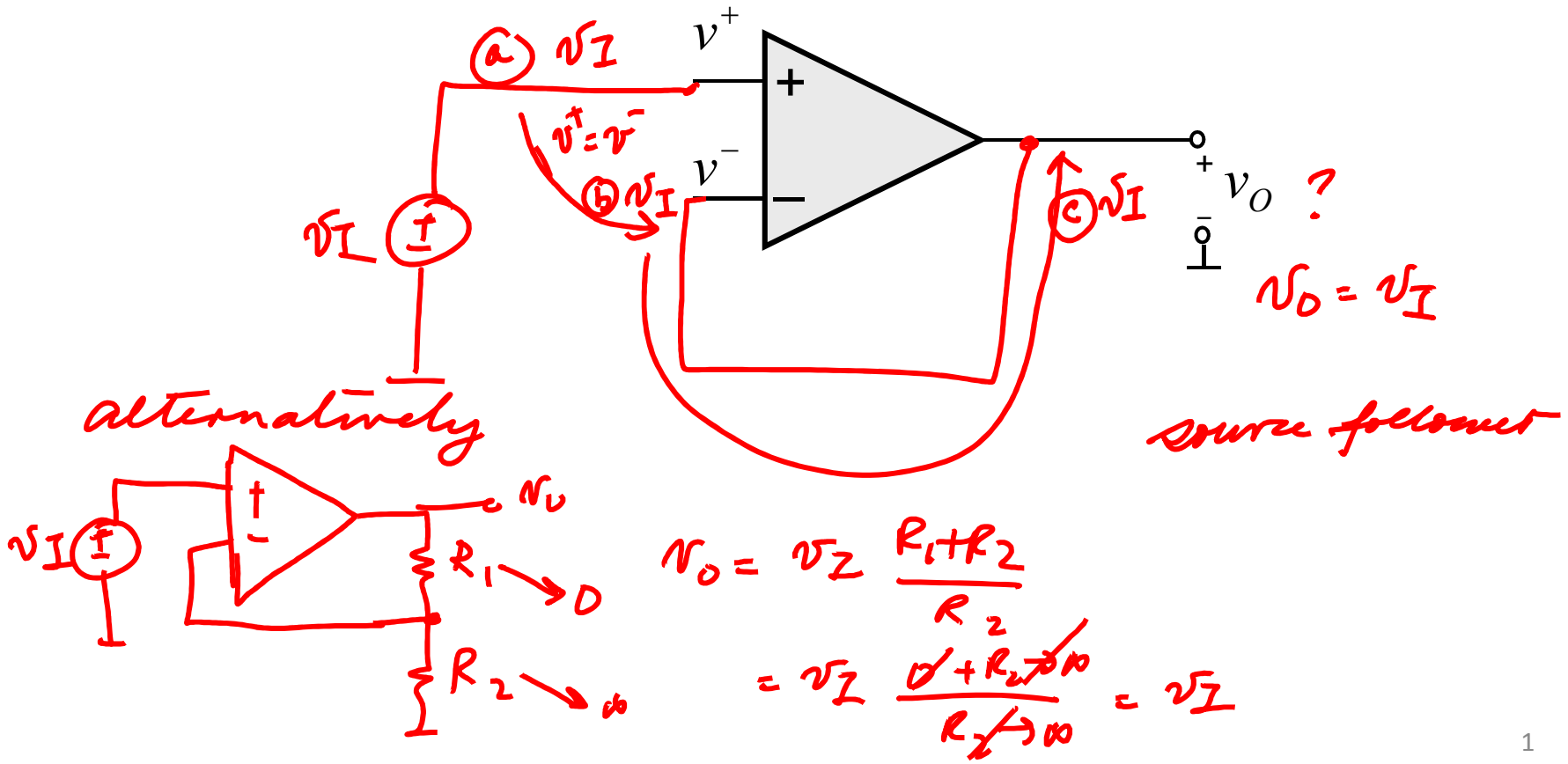
We also know $i^+ = 0$, $i^- = 0$

→ yields an easier analysis method (under negative feedback). Called virtual short method, or virtual ground method

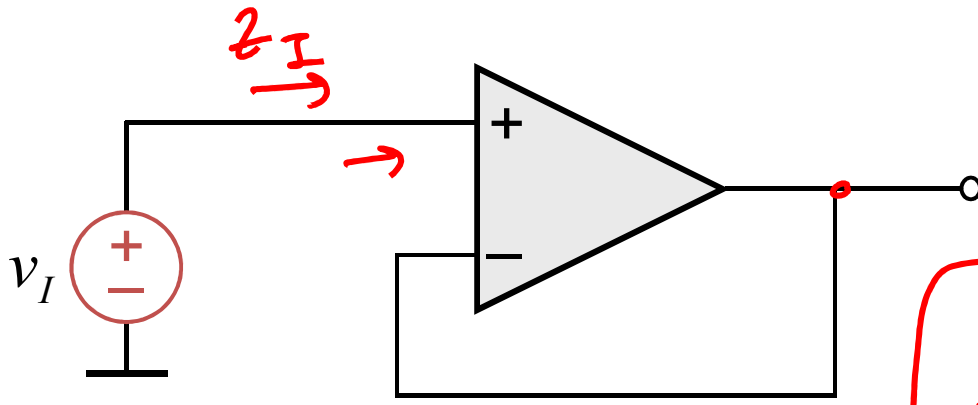
Insightful analysis method under negative feedback – the virtual short method



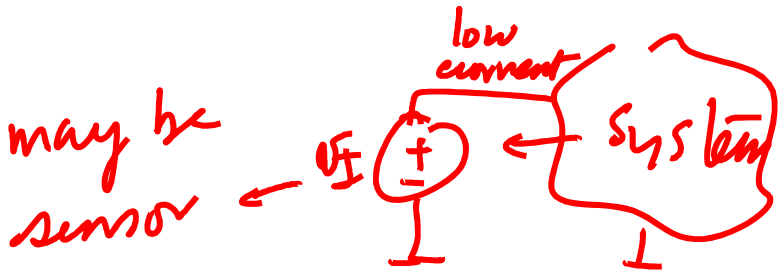
Question: What does this circuit do?



Why is this circuit useful?



$$v_O \approx v_I$$



Buffer

Voltage gain = 1

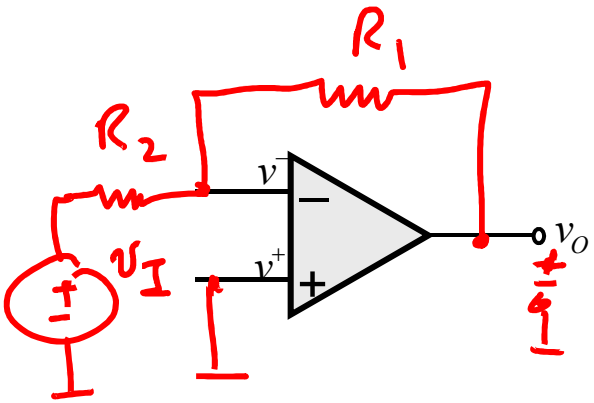
Input impedance = ∞

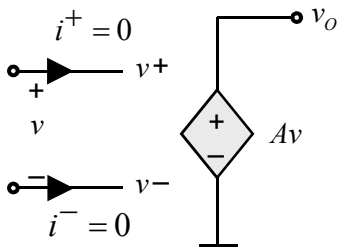
Output impedance = 0

Current gain = ∞

Power gain = ∞

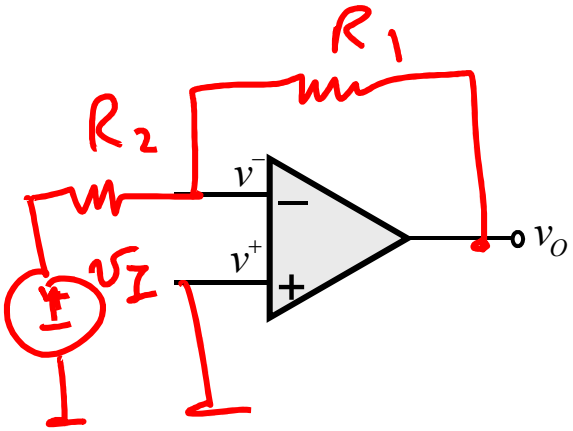
Inverting amplifier

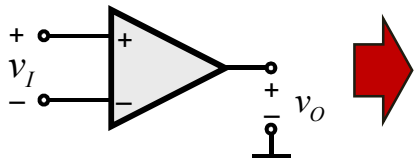


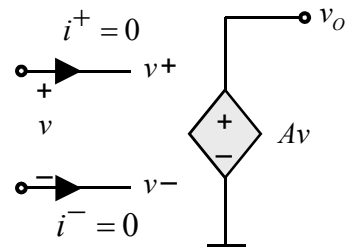


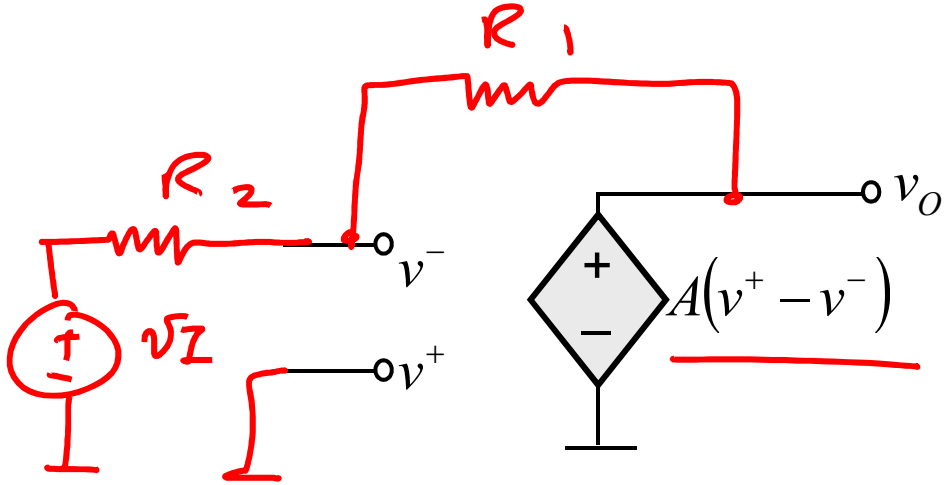
analyze this ckt < op amp eqt ckt
virtual short method

Inverting amplifier





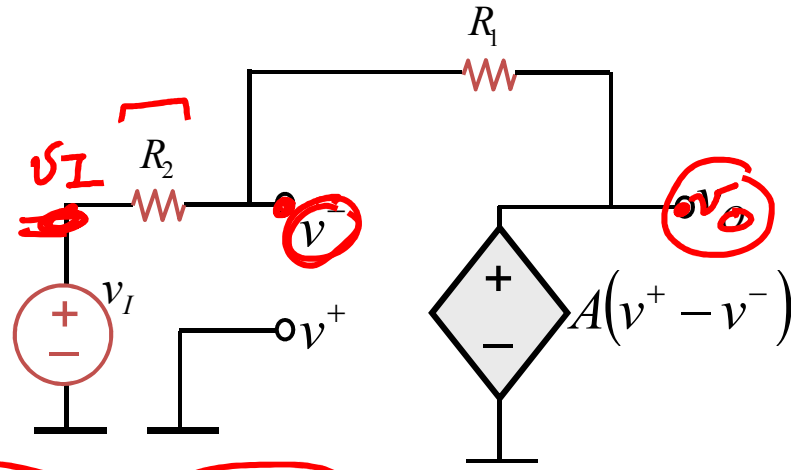




Equivalent circuit model

Let us analyze the circuit:

Find v_O in terms of v_I



start by $v_O = A(v^+ - v^-)$

$v^+ = 0$

$v^- = v_I \frac{R_1}{R_1+R_2} + v_O \frac{R_2}{R_1+R_2}$

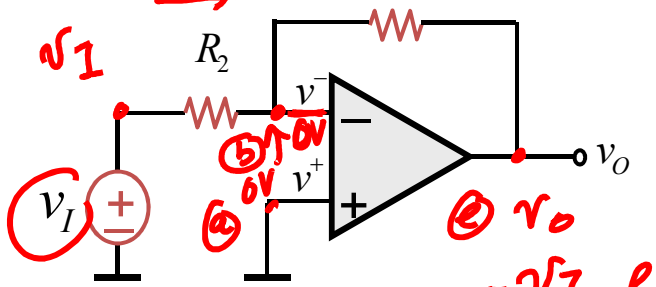
$= A\left[0 - v_I \frac{R_1}{R_1+R_2} - v_O \frac{R_2}{R_1+R_2}\right]$

$v_O\left[1 + \frac{A R_2}{R_1+R_2}\right] = -A v_I \frac{R_1}{R_1+R_2}$

$v_O = \frac{-A v_I R_1/(R_1+R_2)}{1 + AR_2/(R_1+R_2)} = \frac{-A v_I R_1}{R_1+R_2+AR_2}$

"A" large $\Rightarrow$ $v_O \simeq -\frac{v_I R_1}{R_2}$

Inverting Amplifier analysis using virtual short



$v^+ \approx v^-$

$i^+ \approx 0$

$i^- \approx 0$

$v_O = 0V - \text{drop across } R_1$

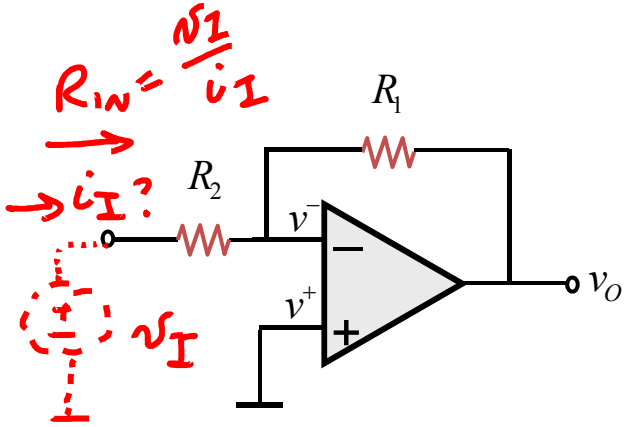
$v_O = -v_I \frac{R_1}{R_2}$

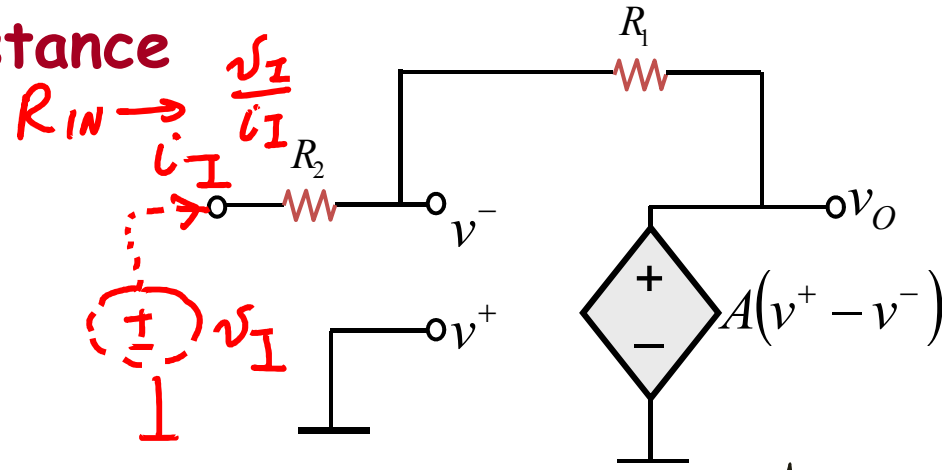
aside

$R_1 = 1\Omega$

$v_O = -2V$

Inverting amplifier input resistance



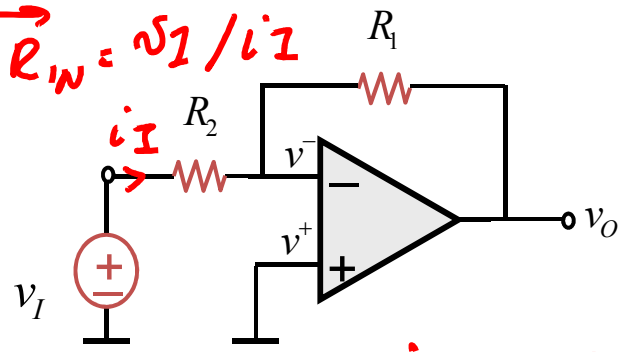


$$v_O = \frac{-Av_I R_1}{R_1 + R_2 + AR_2}$$

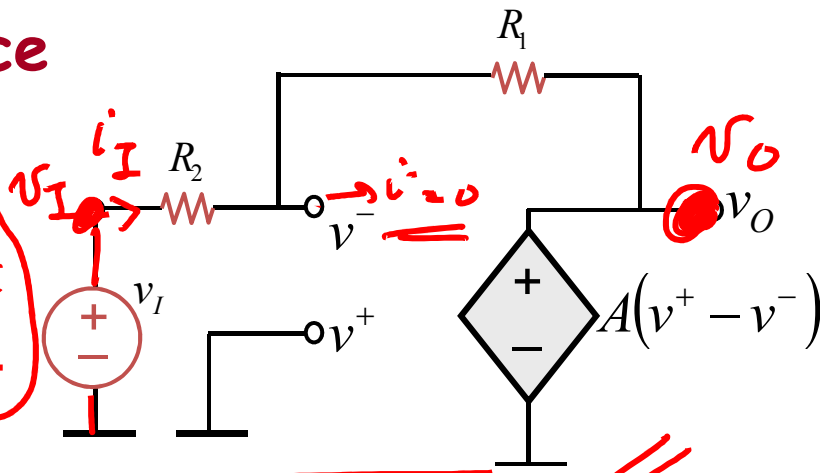
IDEA: apply a test v_I at the input, and measure the resulting current i_I. Then $R_{IN} = \frac{v_I}{i_I}$

Inverting amplifier input resistance

$R_{IN} = v_I / i_I$



Hard way!



$R_{IN} = \frac{v_I}{i_I}$

$i_I = \frac{v_I - v_O}{R_1 + R_2}$

$v_O = \frac{-A v_I R_1}{R_1 + R_2 + A R_2}$

$i_I = \left(v_I + \frac{A v_I R_1}{R_1 + R_2 + A R_2} \right) \frac{1}{R_1 + R_2}$

v_a — R — v_b $\quad i = \frac{v_a - v_b}{R}$

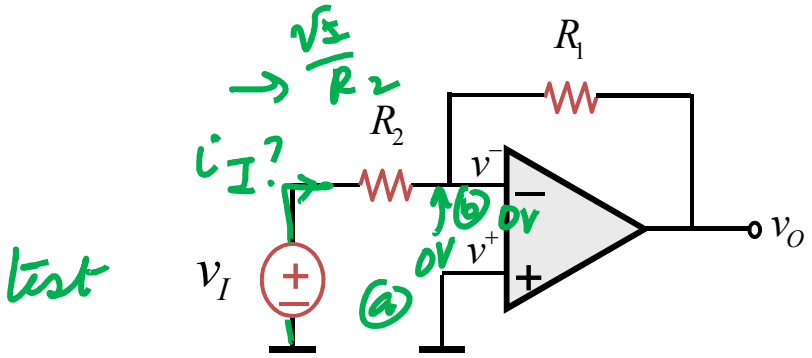
$G_{IN} = \frac{1}{R_{IN}} = \frac{i_I}{v_I} = \left(1 + \frac{A R_1}{R_1 + R_2 + A R_2} \right) \frac{1}{R_1 + R_2} \approx \left(\frac{R_2 + R_1}{R_2} \right) \frac{1}{R_1 + R_2} = \frac{1}{R_2}$

A is huge?

$R_{IN} = R_2$

Inverting amplifier input resistance using virtual short method

Easy way!



$$R_{IN} = \frac{v_I}{i_I}$$

$$R_{IN} = \frac{v_I}{\left(\frac{v_I}{R_2}\right)}$$

$$\boxed{R_{IN} = R_2}$$

Virtual short trick
↳ anointed as our next aha moment!