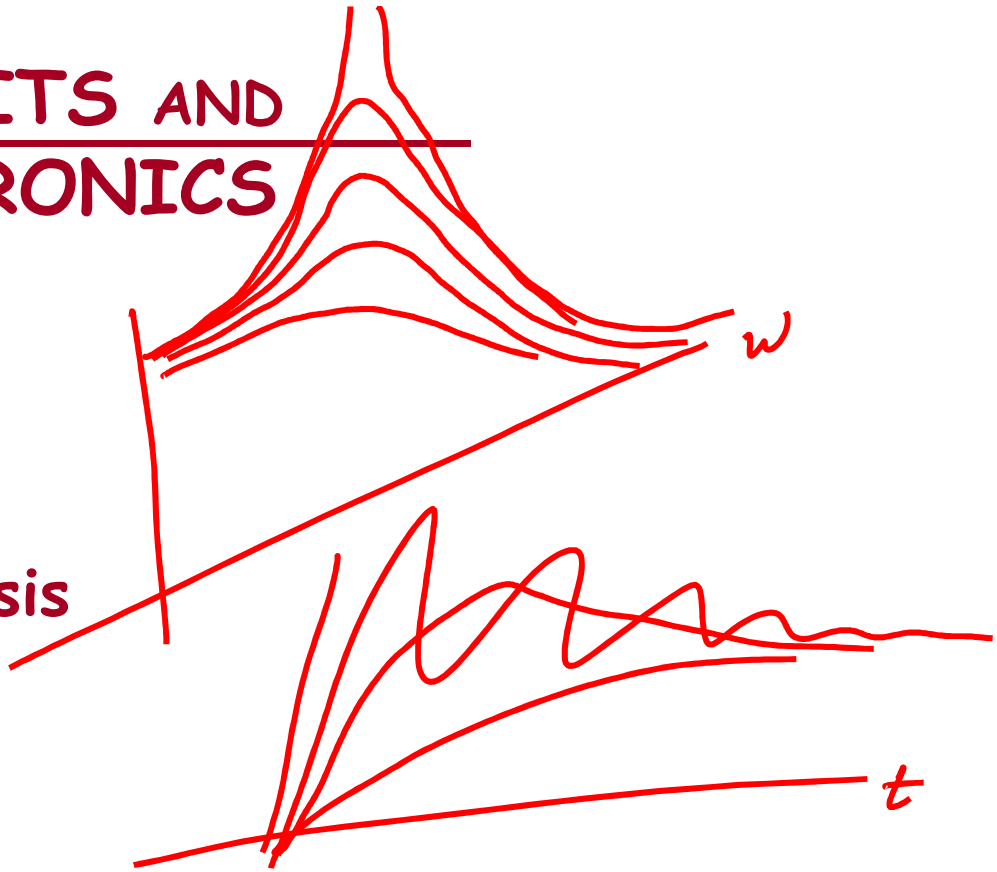
6.002x CIRCUITS AND ELECTRONICS

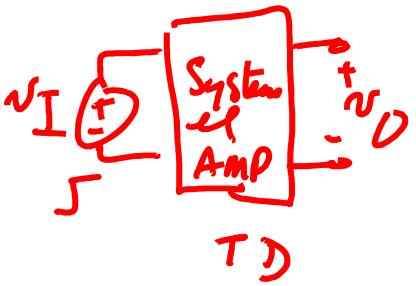
Time Domain versus Frequency Domain Analysis

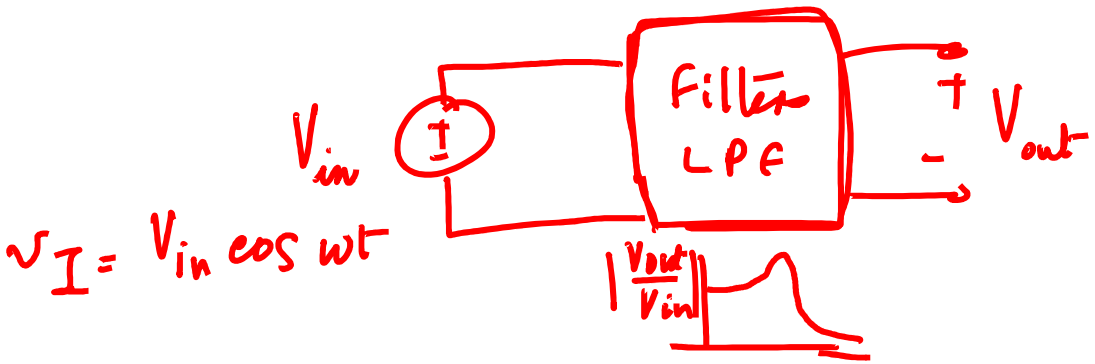


Reading: Section 13.6.3 and Pages 819 – 823 of A & L.

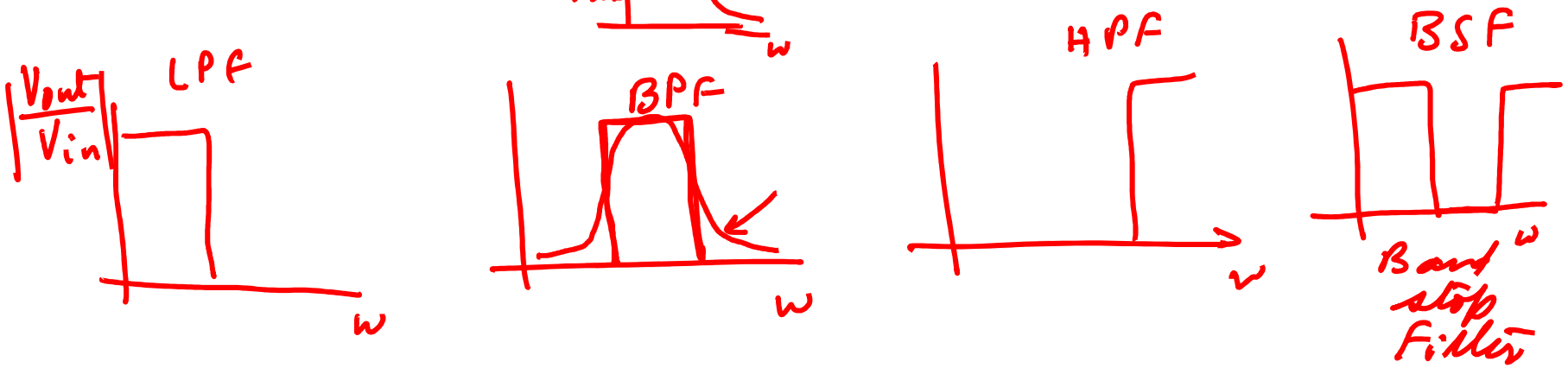
Review

Filter view of the world



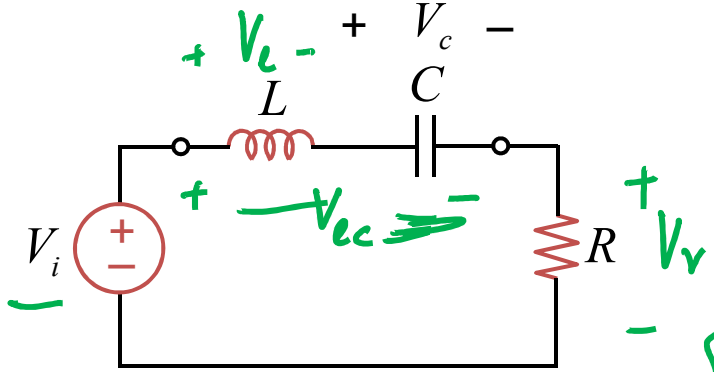


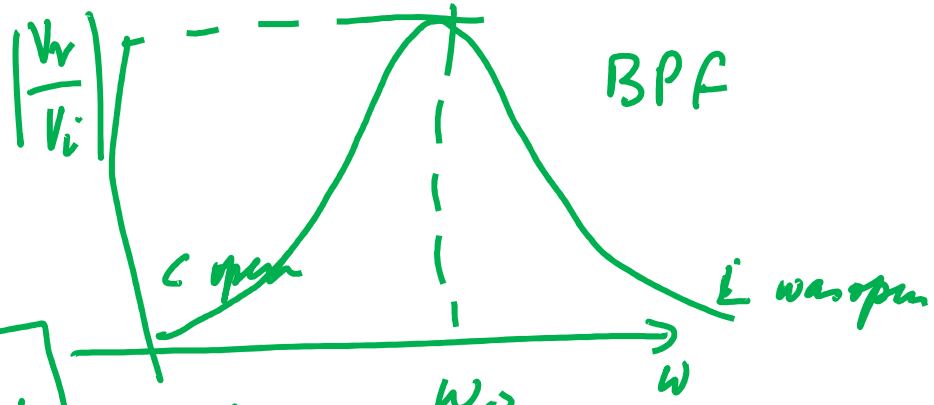
$v_{OUT} = |V_{out}|\cos(\omega t + \angle V_{out})$

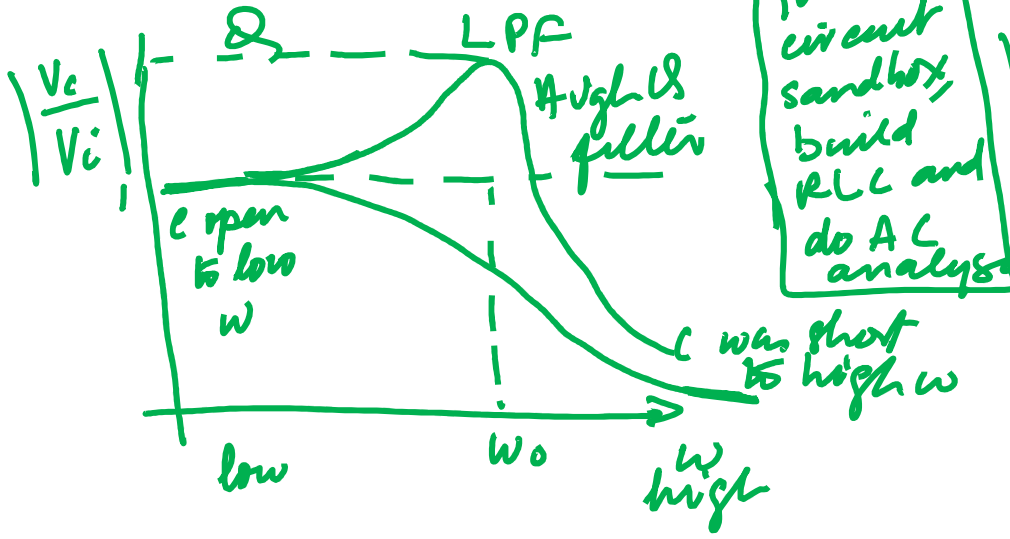


Review

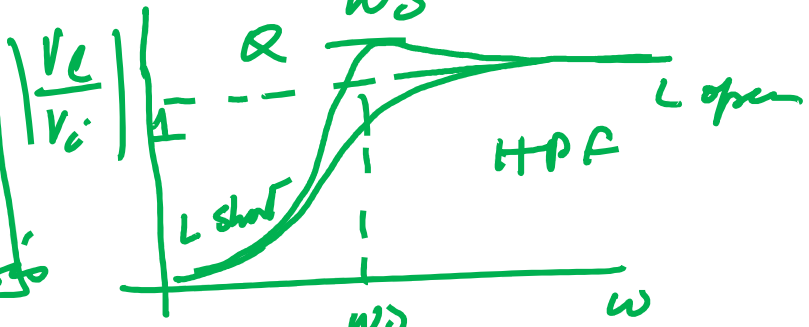
One Circuit, Different Filters

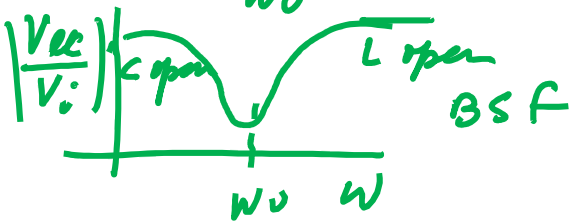




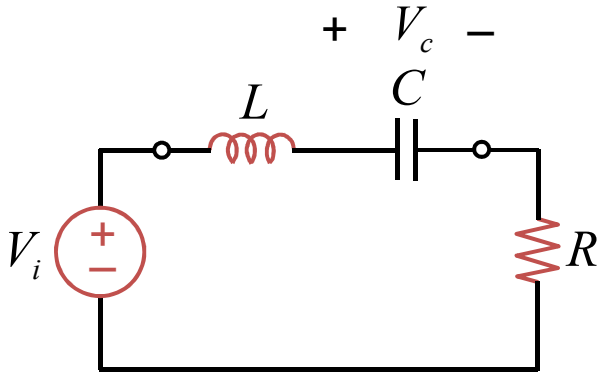


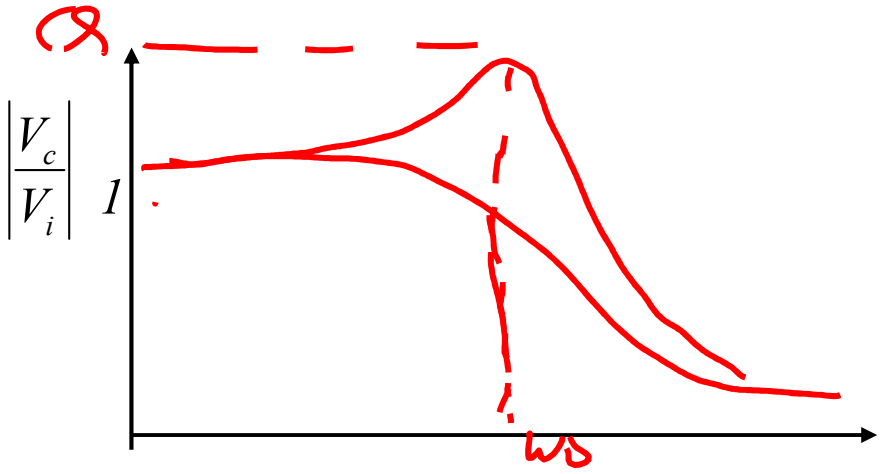
Go to circuit sandbox, build RLC and do AC analysis





Review Q indicates peakiness





$$\frac{V_c}{V_i} = \frac{\frac{1}{sC}}{sL + \frac{1}{sC} + R} = \frac{1}{s^2LC + 1 + sRC} = \frac{\omega_0^2}{s^2 + s\frac{R}{L} + \omega_0^2}$$

$\omega_0 = \frac{1}{\sqrt{LC}}$

for $\omega = \omega_0$, recalling $s = j\omega$

$$\left|\frac{V_c}{V_i}\right| = Q$$

$$Q = \frac{\omega_0}{2\zeta}$$

$$2\zeta = \frac{R}{L}$$

$$\omega_0 = \frac{1}{\sqrt{LC}}$$

s^2 → $j^2\omega_0^2$ → $\omega = \omega_0$ → ω_0^2

Mapping back to time domain is easy

$$V_c\left(s^2 + 2\alpha s + \omega_o^2\right) = \omega_o^2 V_i$$

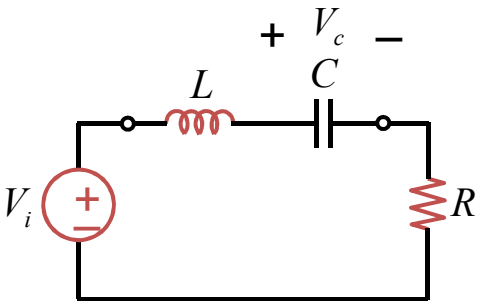
$$\frac{d^2 v_c}{dt^2} + 2\alpha \frac{dv_c}{dt} + \omega_o^2 v_c = \omega_o^2 V_I u(t)$$

For example, step input

$$s \longrightarrow \frac{d}{dt}$$

$$Q = \frac{\omega_o}{2\alpha} = \frac{\omega_o}{R/L}$$

Let's use Q to look at Time domain (TD) and freq domain (FD) behavior



$$\frac{V_c}{V_i} = \frac{\omega_o^2}{s^2 + 2\alpha s + \omega_o^2}$$

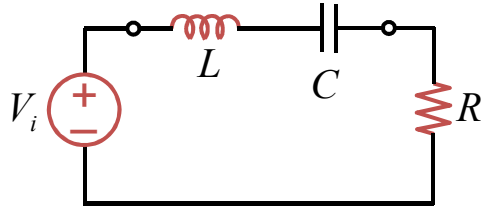
$s = j\omega$

e^{st}

$\frac{d}{dt} \rightarrow s$

Let's use Q to compare time domain and frequency domain

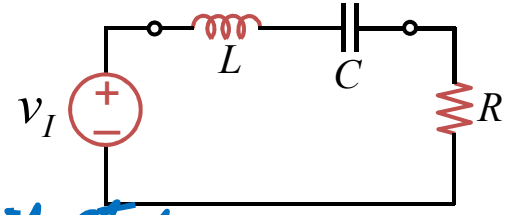
Frequency domain FD



Sinusoid

$v_I = V_i \cos \omega t$

Time domain TD



Unit step

$v_I = u(t)$

$Q = \frac{\omega_0}{2\alpha}$

$\omega_0 = \frac{1}{\sqrt{LC}}$

$2\alpha = \frac{R}{L}$

3 cases:

low Q

→ $\alpha > \omega_0$, or $Q = \frac{\omega_0}{2\alpha} < \frac{1}{2}$ → overdamped

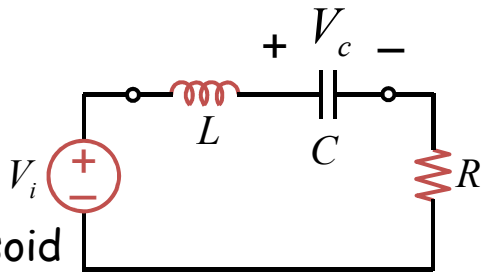
$\alpha < \omega_0$, or $Q = \frac{\omega_0}{2\alpha} > \frac{1}{2}$ → underdamped

high Q

$\alpha = \omega_0$, $Q = \frac{1}{2}$ → critically damped

Overdamped case: $\alpha > \omega_o$ Or $Q = \frac{\omega_o}{2\alpha} < \frac{1}{2}$

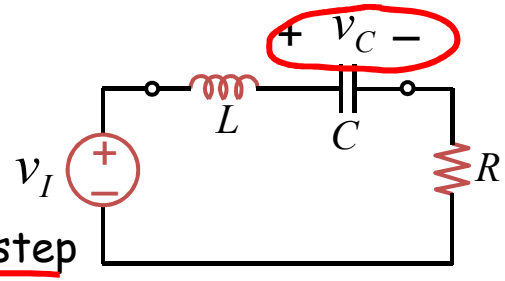
Frequency domain



Sinusoid

$v_I = V_i \cos \omega t$

Time domain



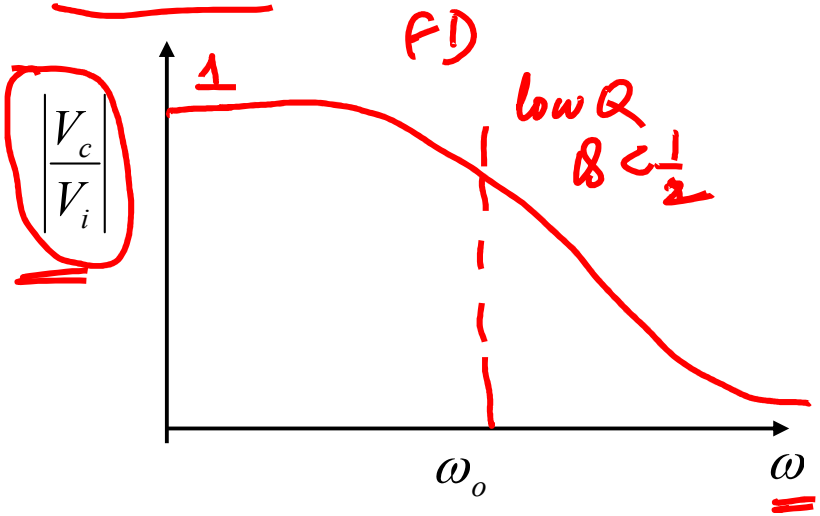
Unit step

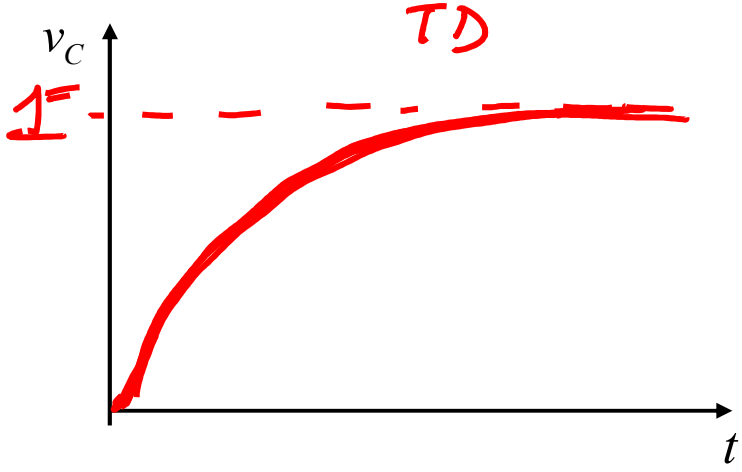
$v_I = u(t)$

$$Q = \frac{\omega_o}{2\alpha}$$

$$\omega_o = \frac{1}{\sqrt{LC}}$$

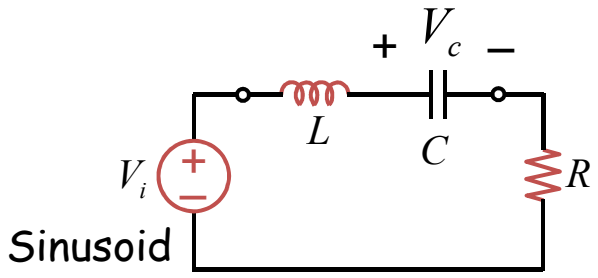
$$2\alpha = \frac{R}{L}$$





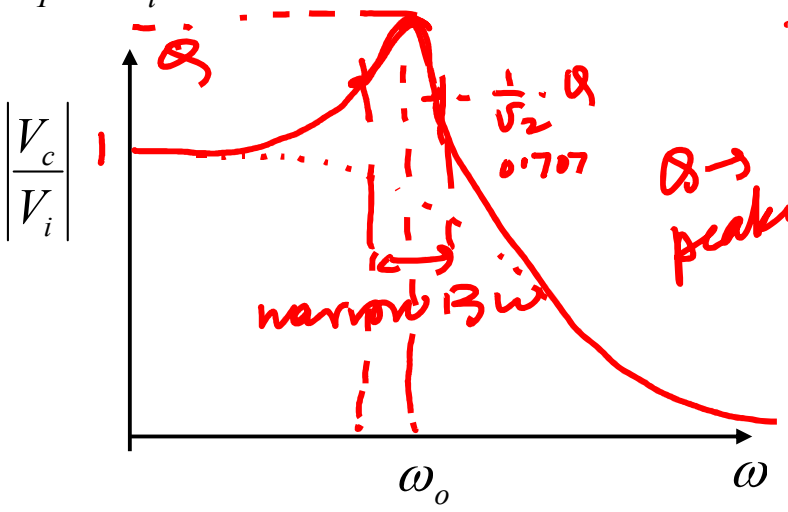
Underdamped case: $\alpha < \omega_o$ Or $Q = \frac{\omega_o}{2\alpha} > \frac{1}{2}$

Frequency domain

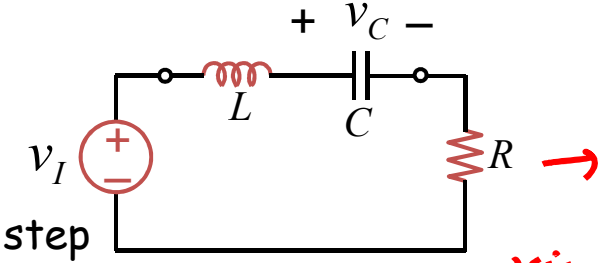


Sinusoid

$v_I = V_i \cos \omega t$

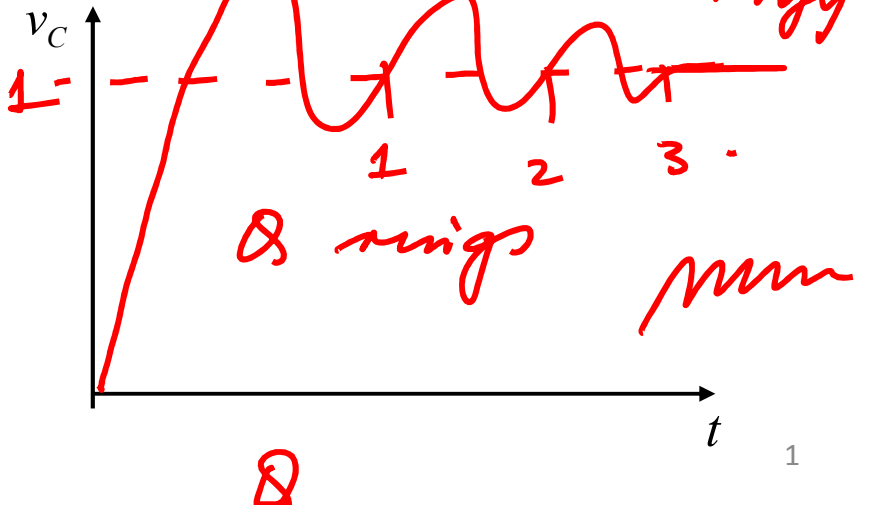


Time domain



Unit step

$v_I = u(t)$



Q

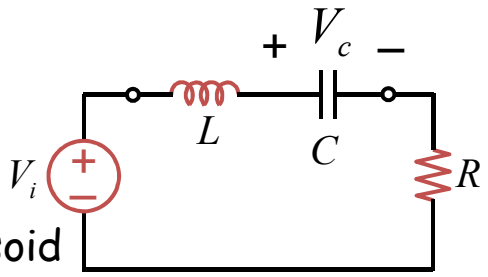
$$Q = \frac{\omega_o}{2\alpha}$$

$$\omega_o = \frac{1}{\sqrt{LC}}$$

$$2\alpha = \frac{R}{L}$$

Critically damped case: $\alpha = \omega_o$ Or $Q = \frac{\omega_o}{2\alpha} = \frac{1}{2}$

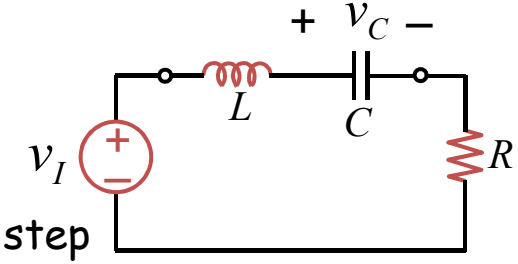
Frequency domain



Sinusoid

$v_I = V_i \cos \omega t$

Time domain



Unit step

$v_I = u(t)$

$$Q = \frac{\omega_o}{2\alpha}$$

$$\omega_o = \frac{1}{\sqrt{LC}}$$

$$2\alpha = \frac{R}{L}$$

