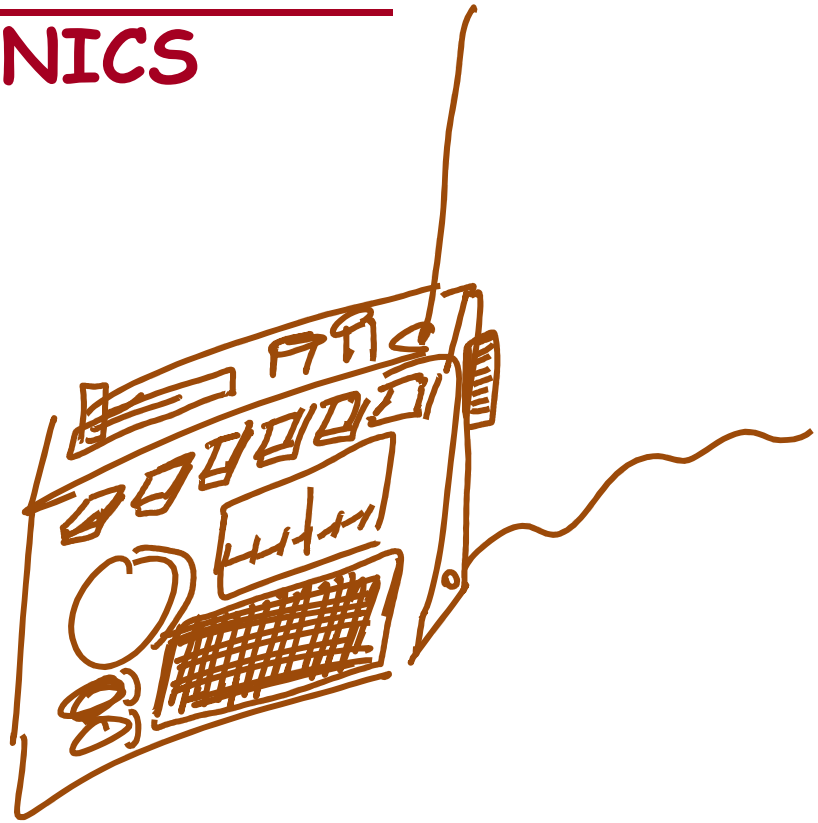
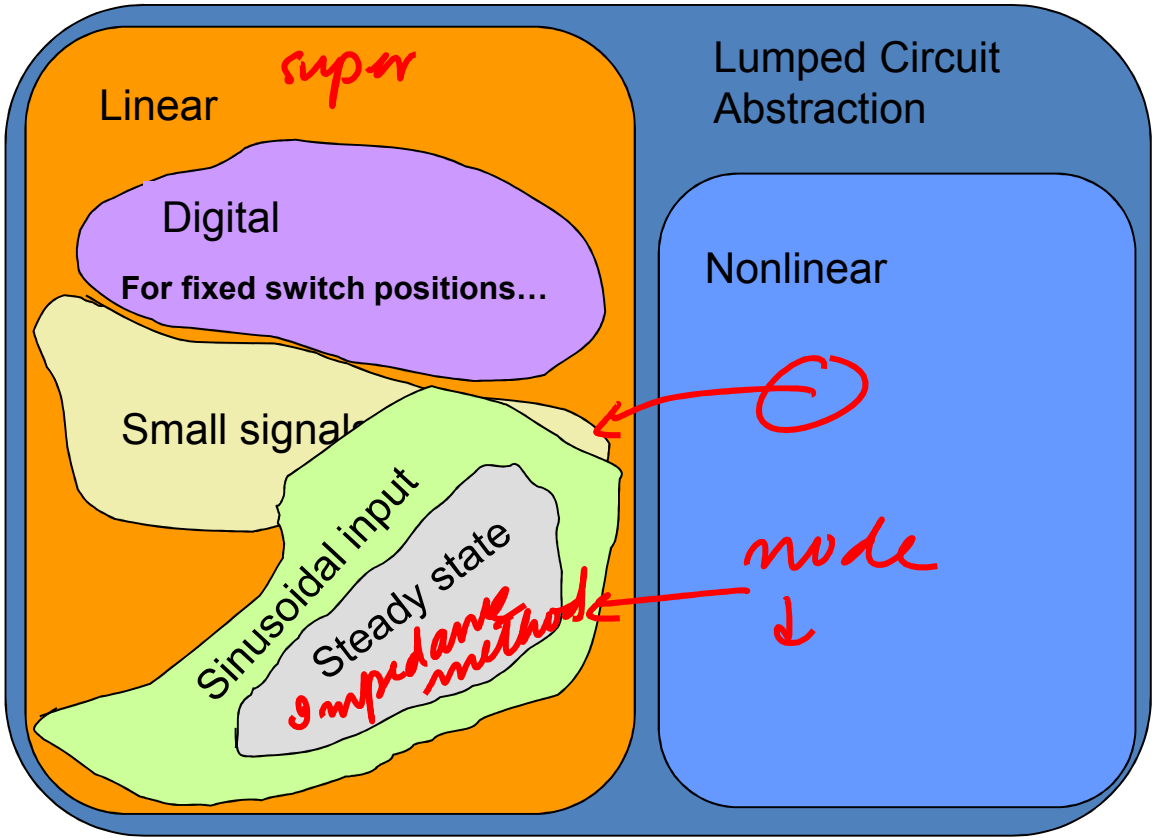
6.002x CIRCUITS AND ELECTRONICS

Filters

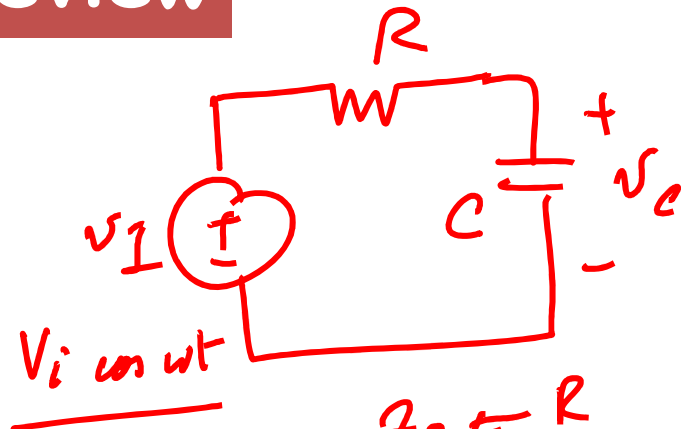


Reading: Section 13.5, 13.6, 14.3 from A & L.

Review


Linear
super
Lumped Circuit Abstraction
Digital
For fixed switch positions…
Nonlinear
Small signal
Sinusoidal input
Steady state
impedance method
node
EECS
playground

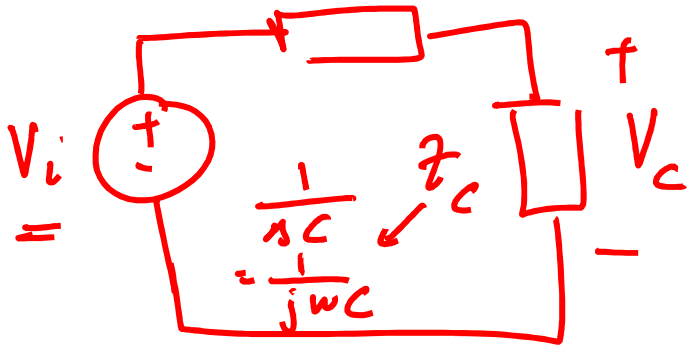
Review



$$V_c = \frac{Z_c}{Z_c + Z_R} V_i$$

$$\frac{V_c}{V_i} = \frac{\frac{1}{j\omega C}}{\frac{1}{j\omega C} + R} = \frac{1}{1 + j\omega RC}$$

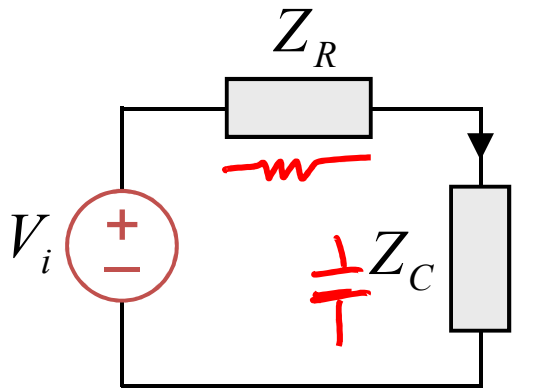
Transfer function



V_c ✓

$$v_c = |V_c| \cos \omega t + \phi \quad \angle V_c$$

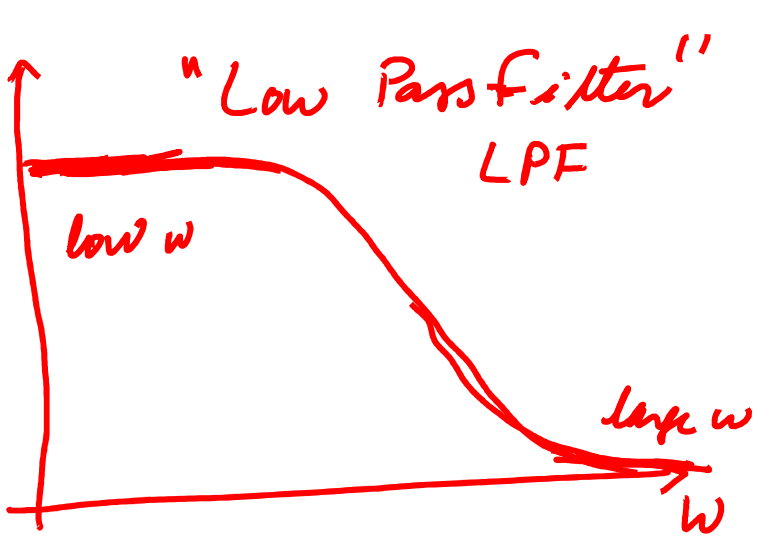
Review A Filter



small w open

large w short

$|H(w)| = \left|\frac{V_c}{V_i}\right|$



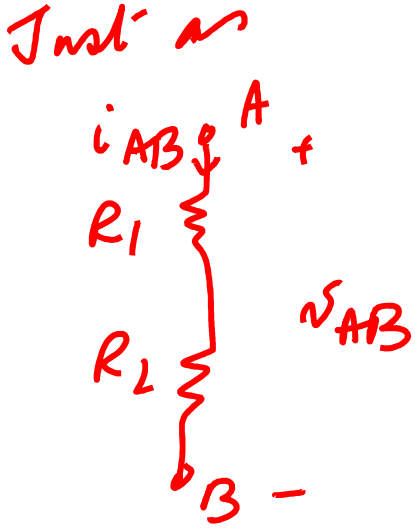
$$\frac{V_c}{V_i} = \frac{Z_c}{Z_c + Z_R} = \frac{1}{1 + jwRC}$$

low w

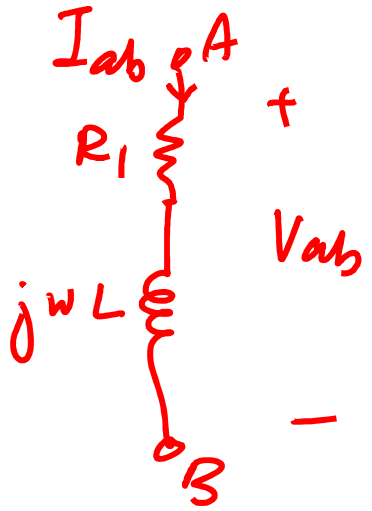
high w

This response is related to the amplifier demo in S19V2 which showed that the response was attenuated as w ↑

Review Impedances analyze like resistances



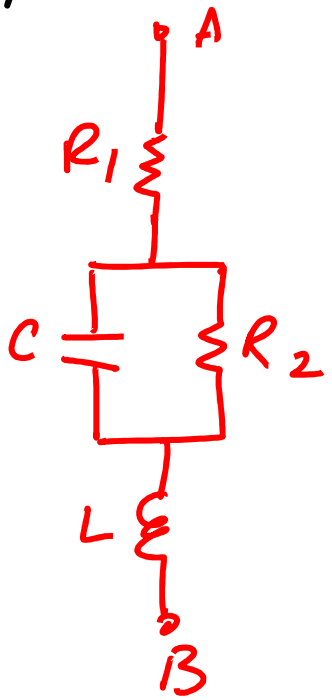
$$R_{AB} = \frac{v_{AB}}{i_{AB}} = R_1 + R_2$$



$$Z_{AB} = \frac{V_{ab}}{I_{ab}} = R_1 + j\omega L$$

Quick Review of Impedances

Similarly



$$Z_{AB} = Z_{R_1} + Z_C \,||\, Z_{R_2} + Z_L$$

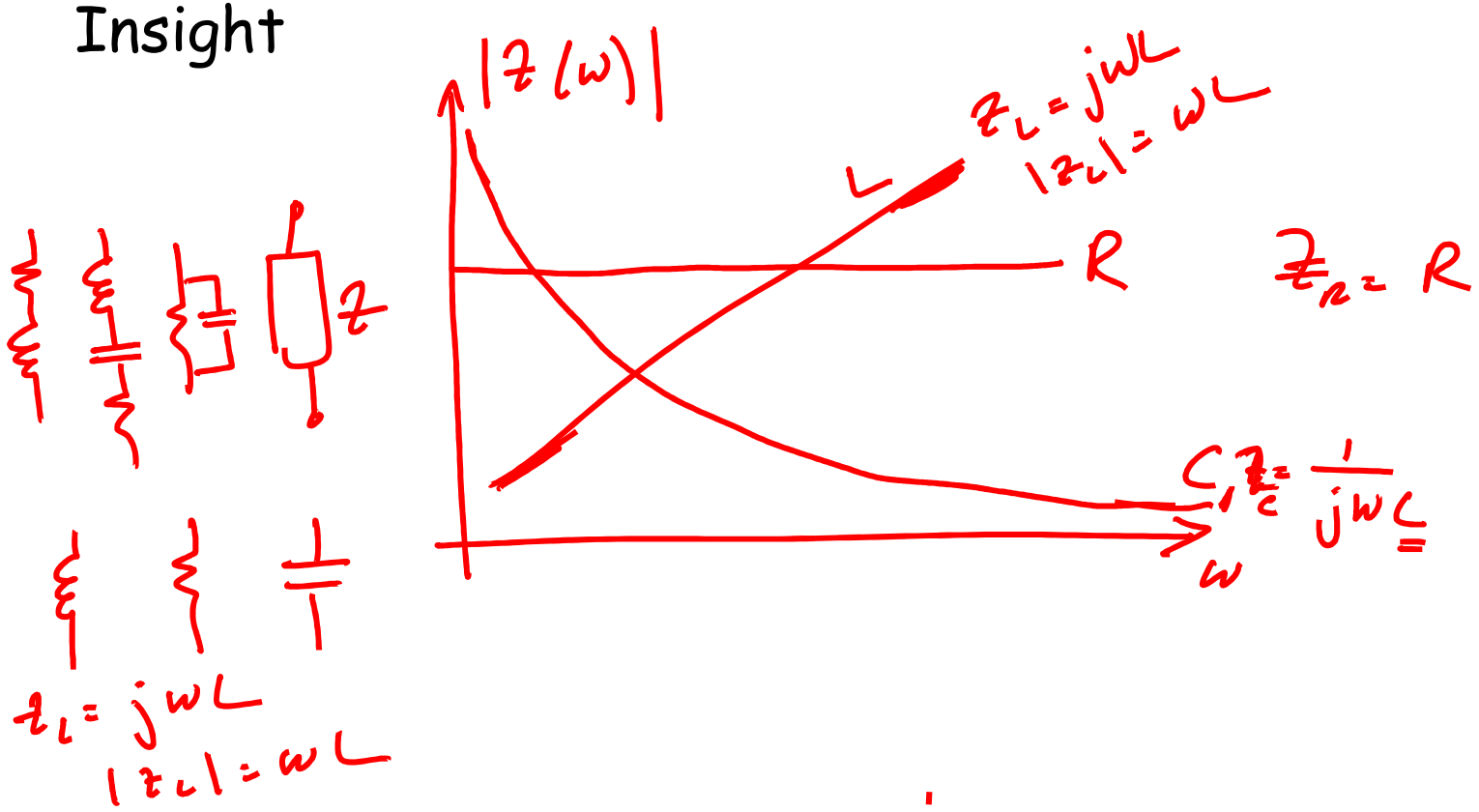
$$= R_1 + \frac{1}{jwC} \,||\, R_2 + jwL$$

$$= R_1 + \frac{\frac{1}{jwC} \cdot R_2}{\frac{1}{jwC} + R_2} + jwL$$

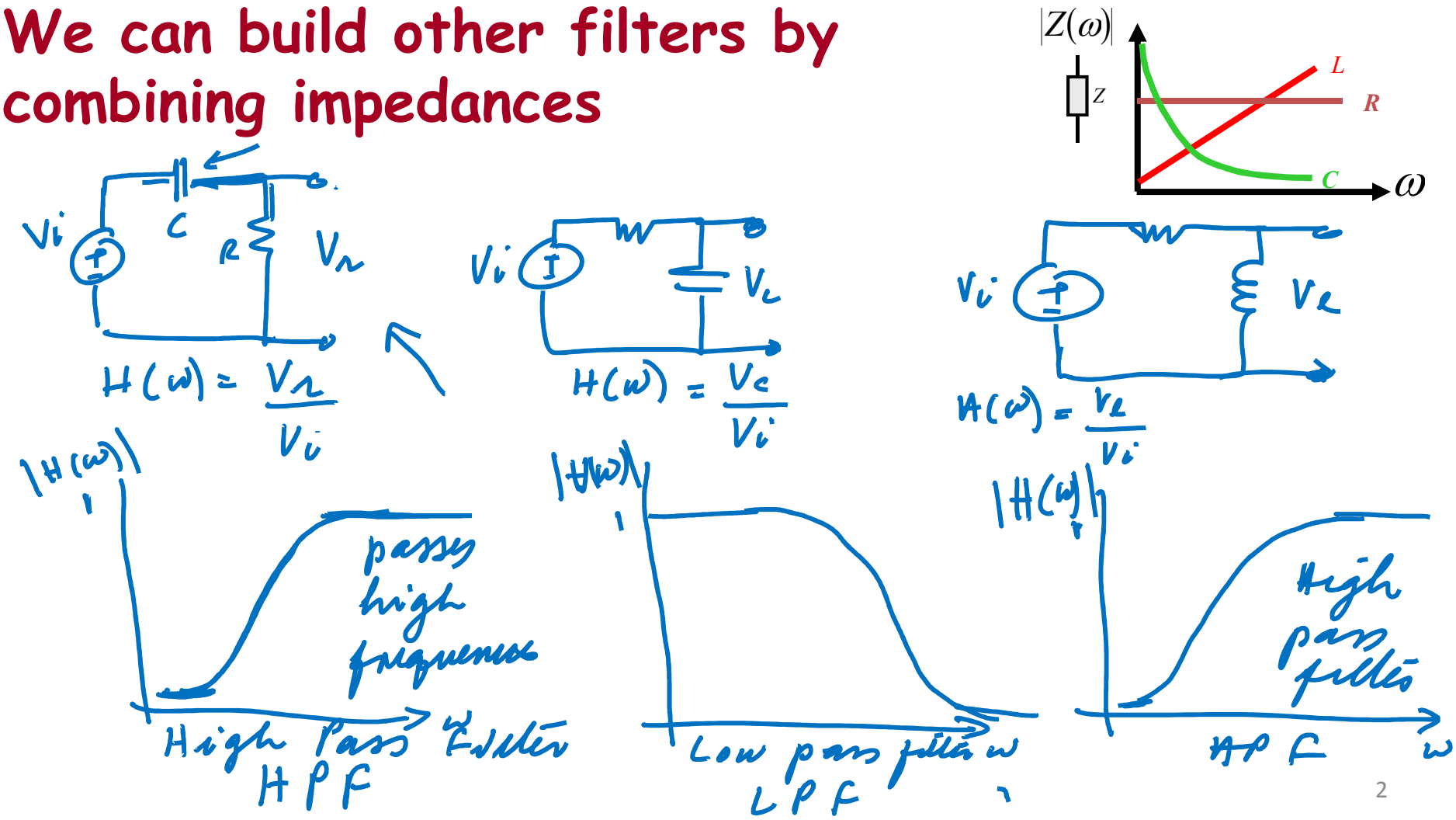
$$= R_1 + \frac{R_2}{1 + jwR_2C} + jwL$$

We can build other filters by combining impedances

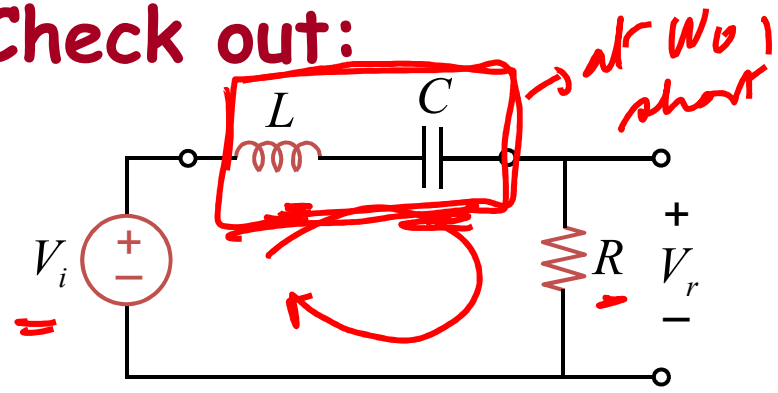
Insight



We can build other filters by combining impedances



Check out:



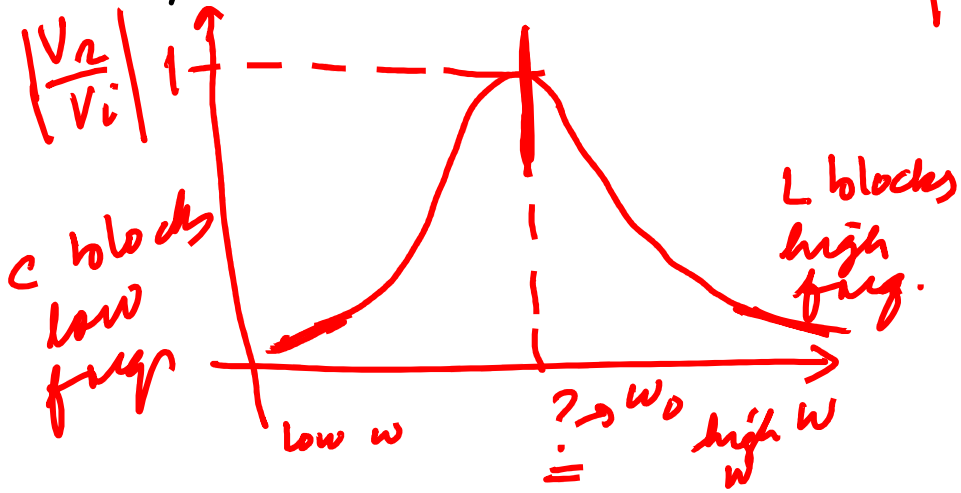
at ω_0 short

$$\frac{V_r}{V_i} = \frac{R}{j\omega L + \frac{1}{j\omega C} + R} = \frac{j\omega CR}{1 - \omega^2 LC + j\omega CR}$$

$\frac{V_r}{V_i}$

$$\left|\frac{V_r}{V_i}\right| = \frac{\omega RC}{\sqrt{(1-\omega^2 LC)^2 + (\omega RC)^2}}$$

Intuitively:



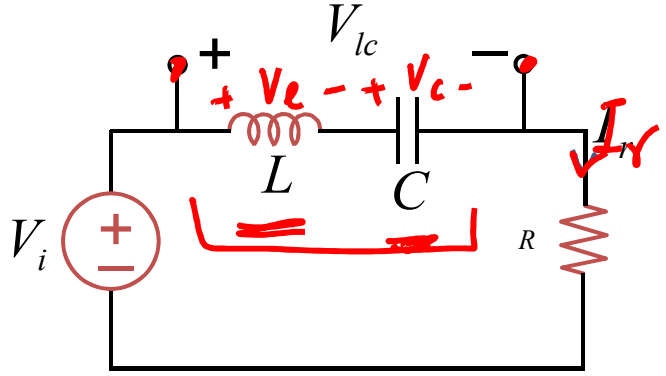
At resonance, $\omega = \omega_0 = \frac{1}{\sqrt{LC}}$

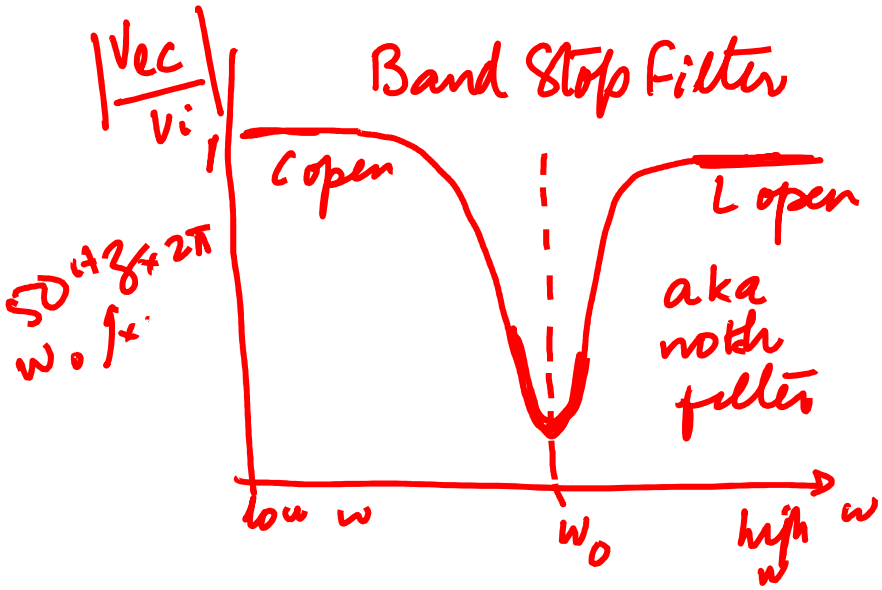
$\left|\frac{V_r}{V_i}\right| = 1$

$Z_L + Z_C = 0$ at $\omega = \omega_0$

(so V_i just drops across R!

What about:





Look at the impedance of series L, C

$$z_{LC} = j\omega L + \frac{1}{j\omega C} = j\omega L - \frac{j}{\omega C}$$

subst $\omega = \omega_0 = \frac{1}{\sqrt{LC}}$

$$z_{LC} = j\omega_0 L - \frac{j}{\omega_0 C} = j\frac{1}{\sqrt{LC}}\sqrt{L} - \frac{j}{\frac{1}{\sqrt{LC}}\cdot\sqrt{LC}} =$$

$$= j\sqrt{\frac{L}{C}} - j\sqrt{\frac{L}{C}} = 0$$

z_{LC} → impedance is 0 at ω_0

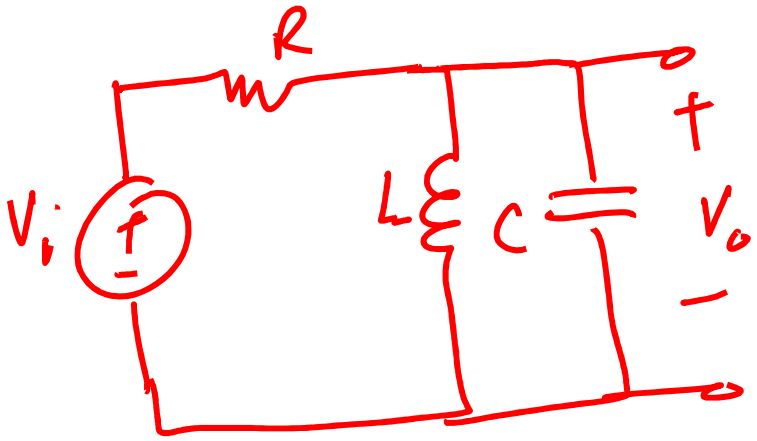
so v drop is 0

$$V_{LC} = V_L + V_C$$

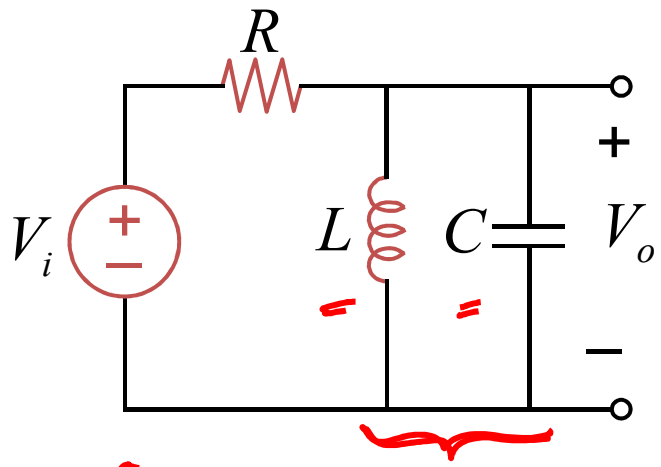
$$= I_r\, j\sqrt{\frac{L}{C}} - I_r\, j\sqrt{\frac{L}{C}} = 0$$

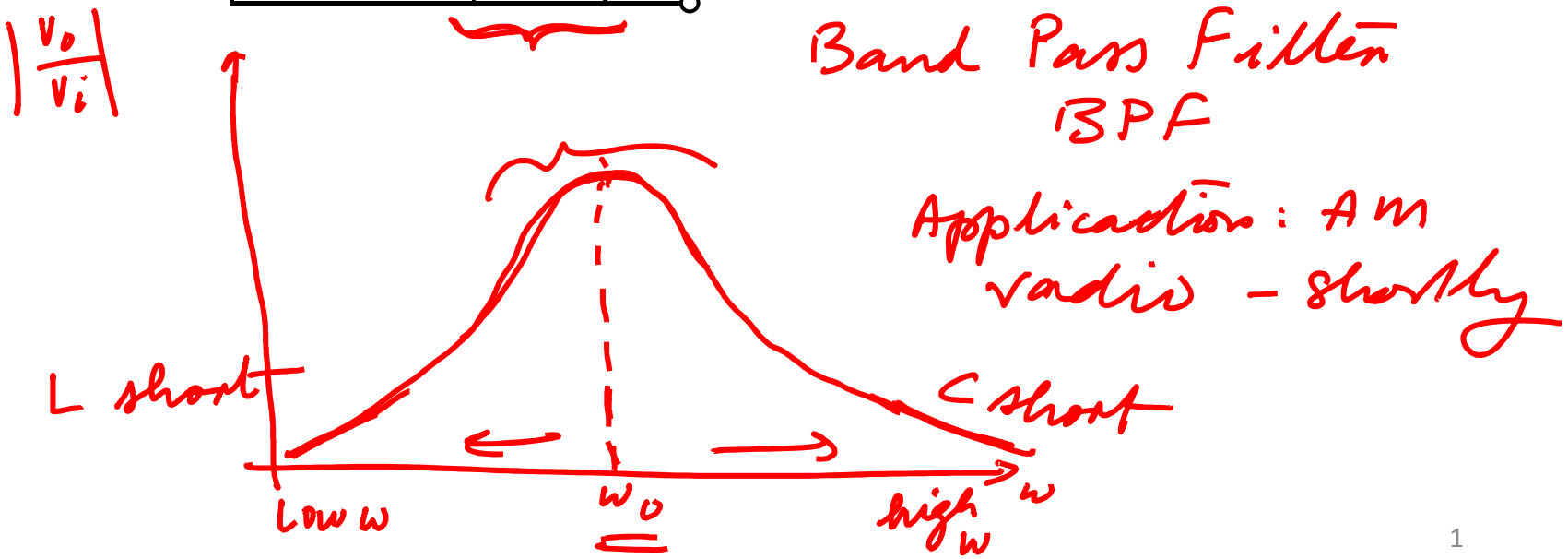
Conceptually, voltage drop across L exactly equal & out-opposite to drop across C. So called!

Another example:



Example:

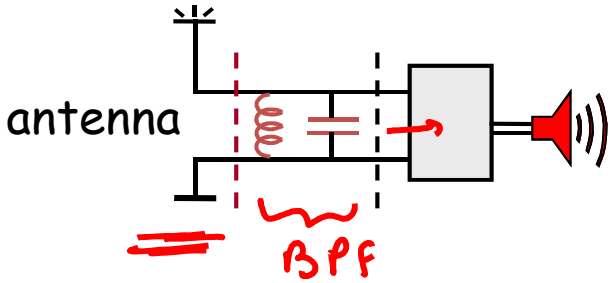


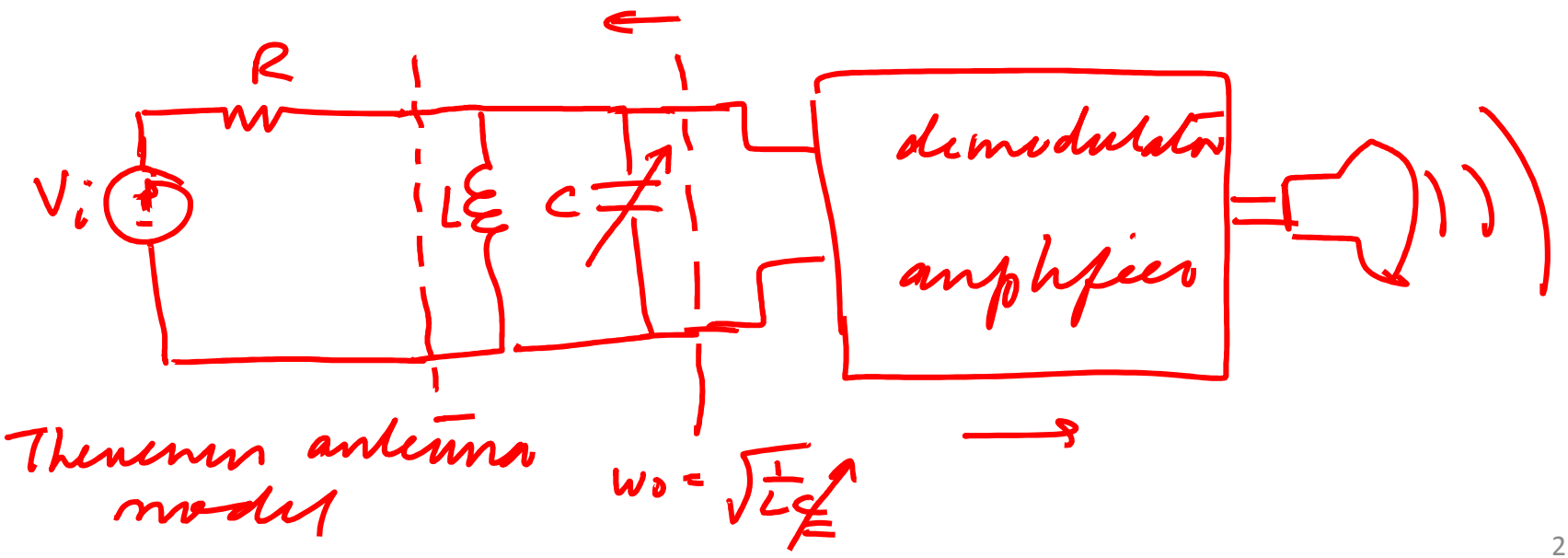


Band Pass Filter
BPF

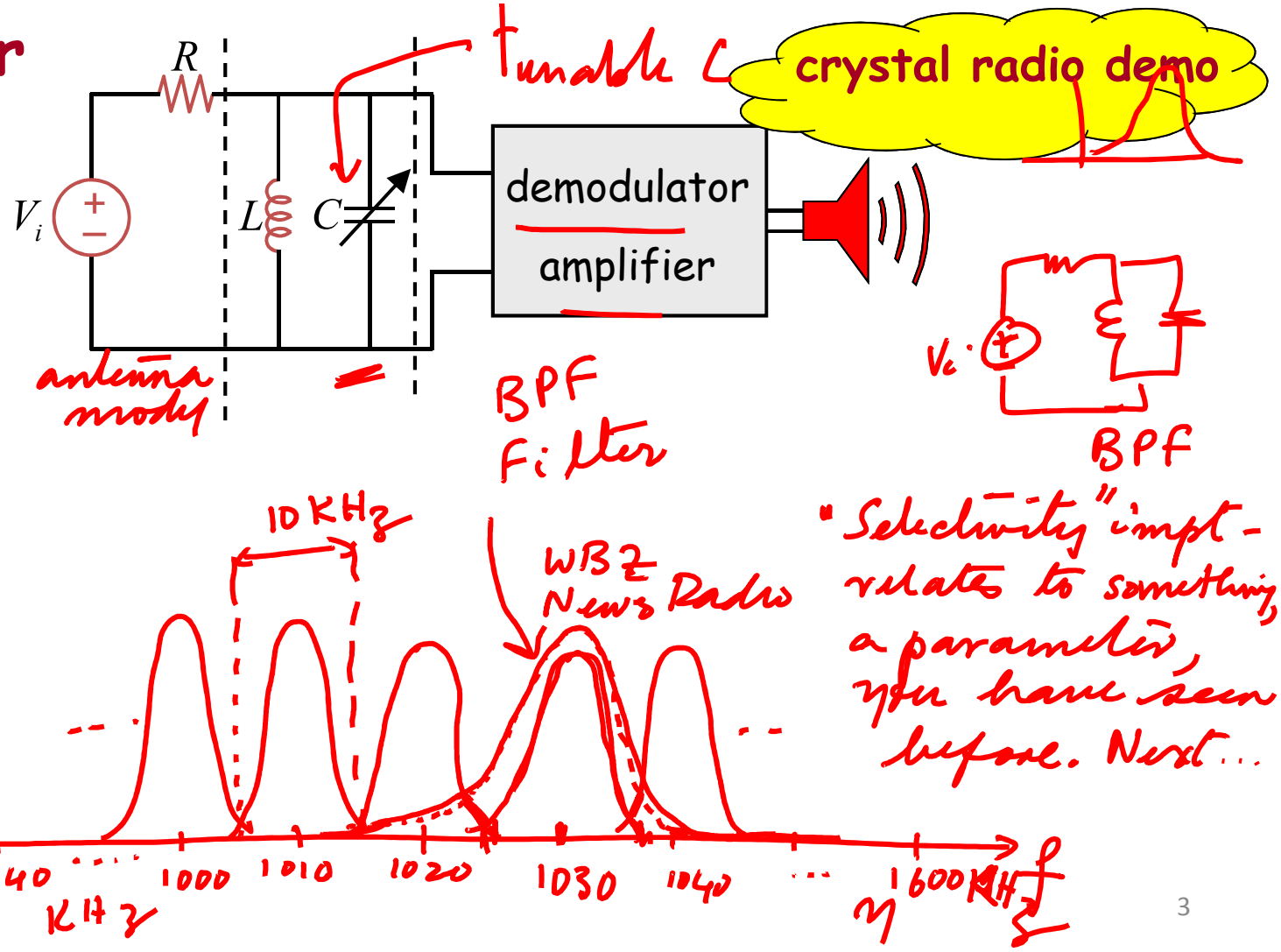
Application: AM radio - shortly

AM Radio Receiver

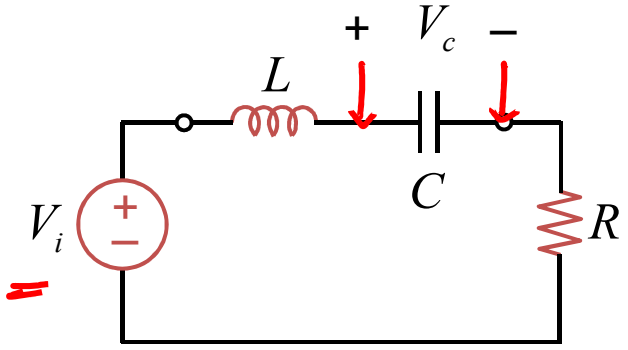




AM Receiver


R
V_i
L
C
tunable C
crystal radio demo
demodulator
amplifier
antenna mode
BPF
Filter
V_i
BPF
signal strength
10 KHz
WBZ News Radio
"Selectivity" impt - relates to something, a parameter, you have seen before. Next...
540
1000
1010
1020
1030
1040
1600 KHz
KHz
f

Next check out:

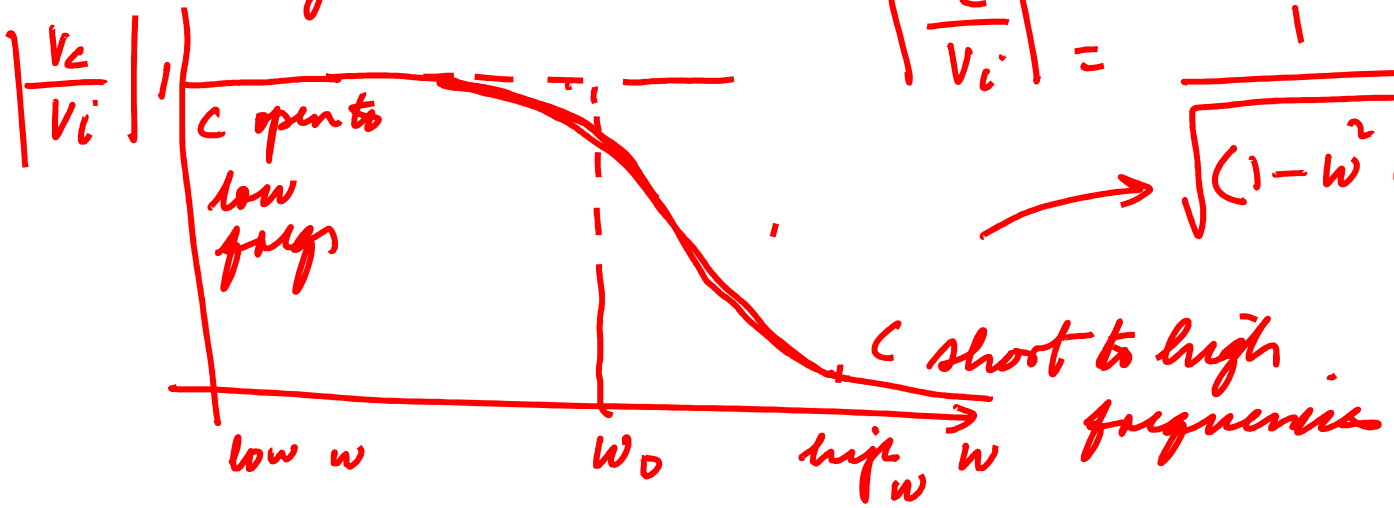


$$\frac{V_c}{V_i} = \frac{\frac{1}{j\omega C}}{j\omega L + \frac{1}{j\omega C} + R}$$

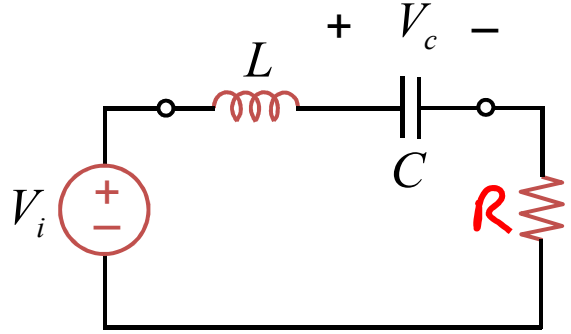
$$= \frac{1}{1 - \omega^2 LC + j\omega RC}$$

$$\left|\frac{V_c}{V_i}\right| = \frac{1}{\sqrt{(1-\omega^2 LC)^2 + (\omega RC)^2}}$$

Intuitively

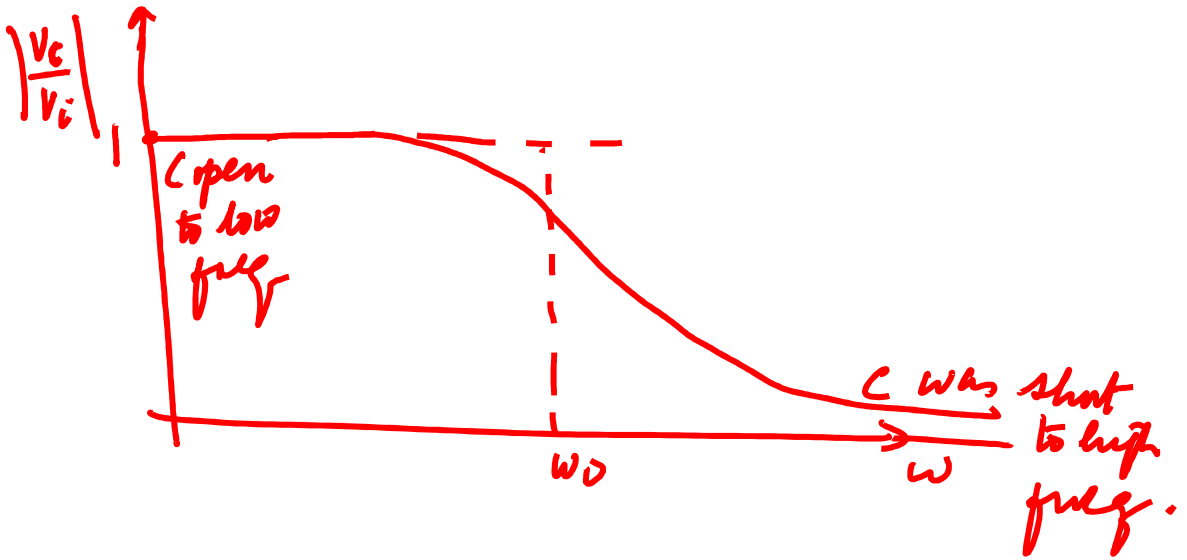


Check out:

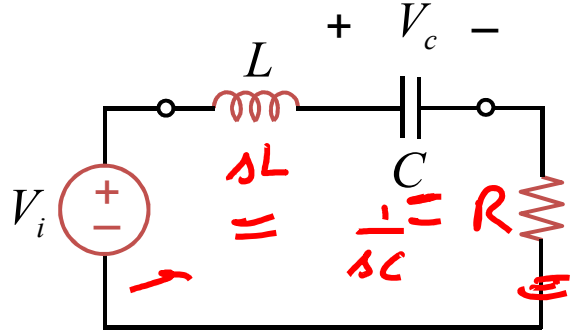


$$\omega_0 = \frac{1}{\sqrt{LC}}$$

$$2\zeta = \frac{R}{L}$$



Check out:



$$\frac{V_c}{V_i} = \frac{\frac{1}{sC}}{sL + \frac{1}{sC} + R} = \frac{1}{s^2LC + 1 + sRC}$$

$$= \frac{1/LC}{s^2 + \frac{1}{LC} + s\frac{R}{L}} = \frac{\omega_0^2}{s^2 + s\frac{R}{L} + \omega_0^2}$$

$$= \frac{\omega_0^2}{s^2 + 2\zeta s + \omega_0^2}$$

$\omega_0 = \frac{1}{\sqrt{LC}}$

$2\zeta = \frac{R}{L}$

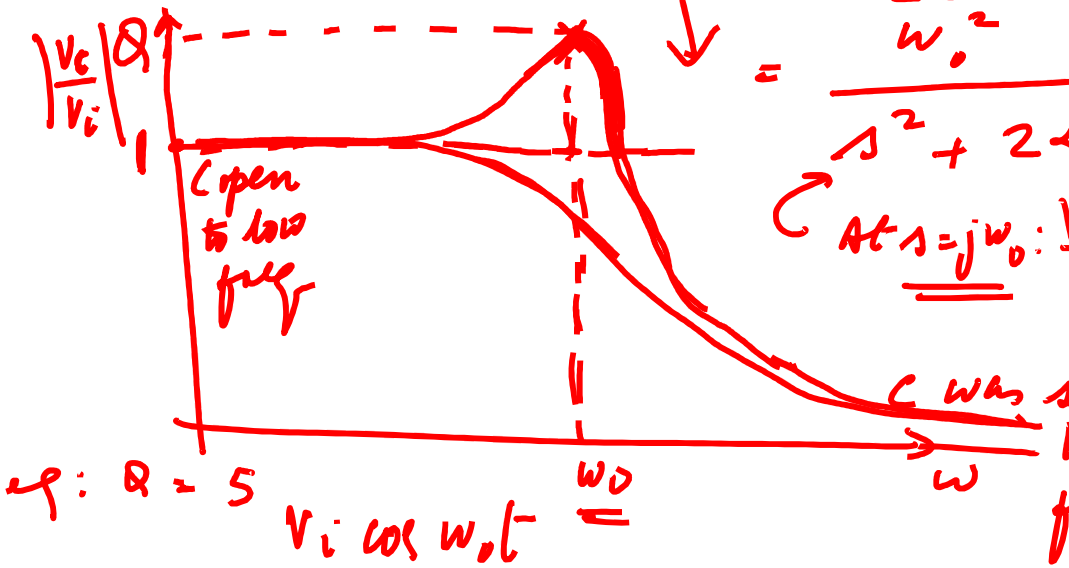
$s = j\omega$

At $s = j\omega$ ← at $\omega = \omega_0$

At $s = j\omega_0$: $\frac{V_c}{V_i} = \frac{\omega_0^2}{(j\omega_0)^2 + \omega_0^2 + 2\zeta(j\omega_0)}$

$\left|\frac{V_c}{V_i}\right| = \frac{\omega_0^2}{2\zeta\omega_0} = \frac{\omega_0}{2\zeta} = Q$! none of the others

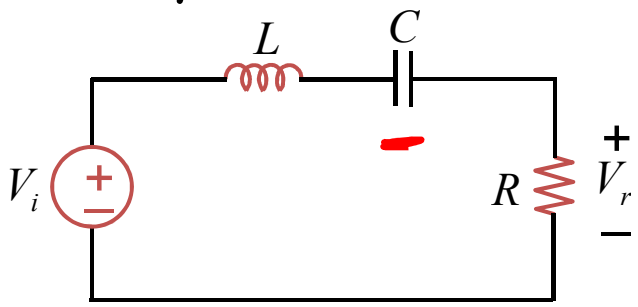
= Peakiness relates to Q



eg: Q = 5

Selectivity: Look at series RLC in more detail

Show you some fun stuff regarding Q!



Selectivity

Look at $\frac{\omega_0}{\Delta\omega}$ = $\frac{\text{resonant freq}}{\text{bandwidth}}$

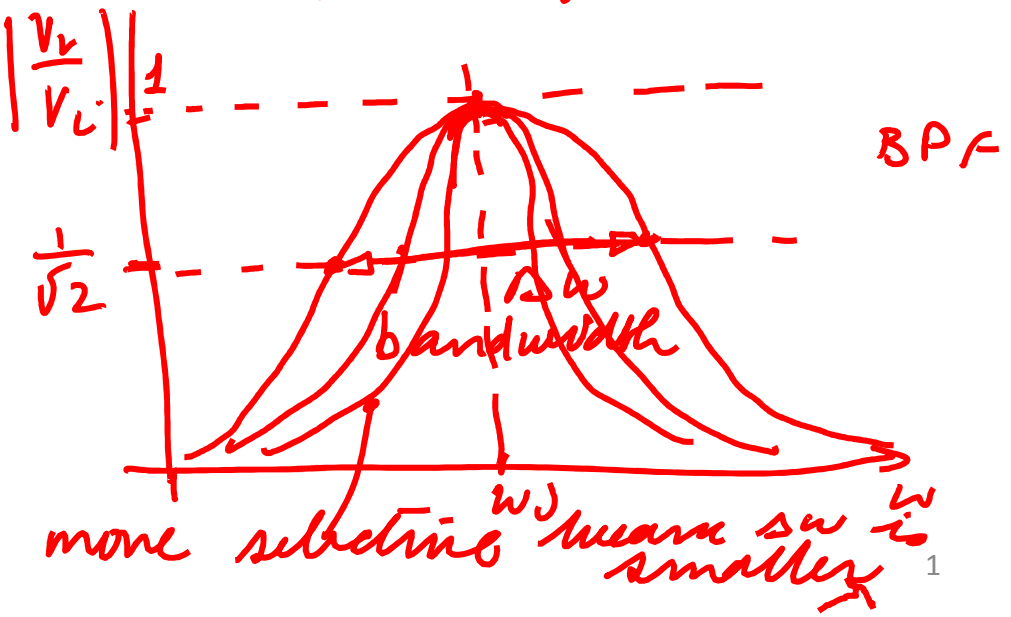
higher $\frac{\omega_0}{\Delta\omega}$, higher the selectivity

Recall

$$\frac{V_r}{V_i} = \frac{R}{R + j\omega L + \frac{1}{j\omega C}}$$

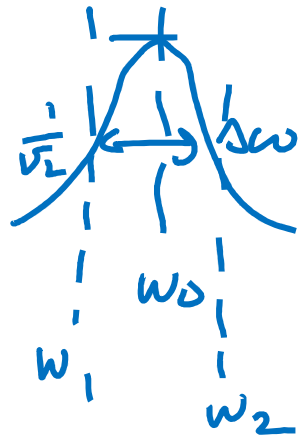
$\omega = \omega_0$

$$j\omega_0 L + \frac{1}{j\omega_0 C} = 0$$



Selectivity

Selectivity related to $\frac{\omega_0}{\Delta\omega}$



$$\omega_0 = \frac{1}{\sqrt{LC}}$$

$$\Delta\omega = ?$$

$$\frac{V_r}{V_i} = \frac{R}{R + j\omega L + \frac{1}{j\omega C}} = \frac{1}{1 + j\omega\frac{L}{R} - \frac{j}{\omega CR}} = \frac{1}{1 + j\left(\omega\frac{L}{R} - \frac{1}{\omega CR}\right)}$$

Selectivity

$$selectivity = \frac{\omega_o}{\Delta\omega}$$

$\Delta\omega$?



Note that abs magnitude is $\frac{1}{\sqrt{2}}$ when $\frac{V_o}{V_i} = \frac{1}{1 + j\left(\frac{\omega L}{R} - \frac{1}{\omega CR}\right)} \to 1$

$$\left|\frac{1}{1+j1}\right| = \frac{1}{\sqrt{2}} \quad , \quad \left|\frac{1}{1-j1}\right| = \frac{1}{\sqrt{2}}$$

$$= \frac{1}{1 \pm j1}$$

i.e. when $\frac{\omega L}{R} - \frac{1}{\omega CR} = \pm 1$

so $\omega^2 \frac{L}{R} - \frac{1}{CR} \mp \omega = 0$, or $\boxed{\omega^2 \mp \omega\frac{R}{L} - \frac{1}{LC} = 0}$

Roots of both equation are ω_2, ω_1

$$\omega_2 = \frac{R}{2L} + \frac{1}{2}\sqrt{\frac{R^2}{L^2} + \frac{4}{LC}} \quad , \quad \omega_1 = -\frac{R}{2L} + \frac{1}{2}\sqrt{\frac{R^2}{L^2} + \frac{4}{LC}}$$

$$\Delta\omega = \omega_2 - \omega_1 = \frac{R}{L}$$

Quality Factor Q makes an a grand reappearance!

So $\frac{\omega_0}{\Delta\omega} = \frac{\omega_0}{R/L} = \frac{\omega_0}{2\alpha} = Q$!

selectivity

$Q = \frac{\omega_0}{\Delta\omega}$

$\Delta\omega = \frac{R}{L}$

$\omega_0 = \frac{1}{\sqrt{LC}}$

$2\alpha = \frac{R}{L}$

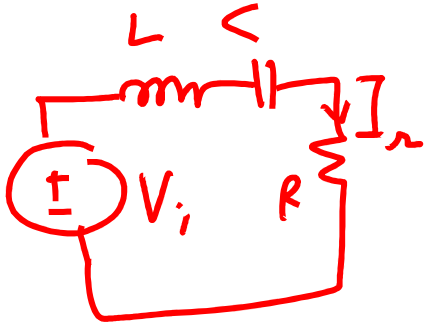
$Q = \frac{\omega_0 \cdot L}{R}$, $\omega_0 = \frac{1}{\sqrt{LC}}$

The low the R (for serie R), the sharper the peak, or the more selective the filter.

High Q ⟶ high selectivity

Quality Factor Q

Yet, another way of looking at Q:



$$Q = 2\pi \frac{\text{Energy stored in L and C}}{\text{Energy lost per cycle}}$$

$$= 2\pi \frac{\frac{1}{2} L |I_R|^2}{\frac{1}{2} |I_R|^2 R \frac{2\pi}{\omega_0}}$$ ← sum of L & C energy

$$= \omega_0 \frac{L}{R} \rightarrow \frac{\omega_0}{2\zeta} \rightarrow Q \text{ ! again}$$

Page 819 of the text book for ways of Looking at Q