6.002x

CIRCUITS AND ELECTRONICS

The Impedance Model

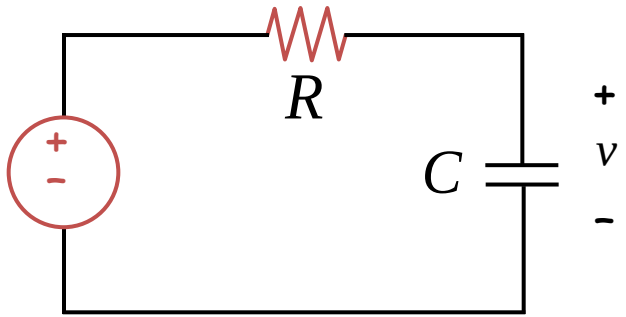


Reading: Section 13.3 from text

Review

- Sinusoidal Steady State (SSS)
  Reading 13.1, 13.2

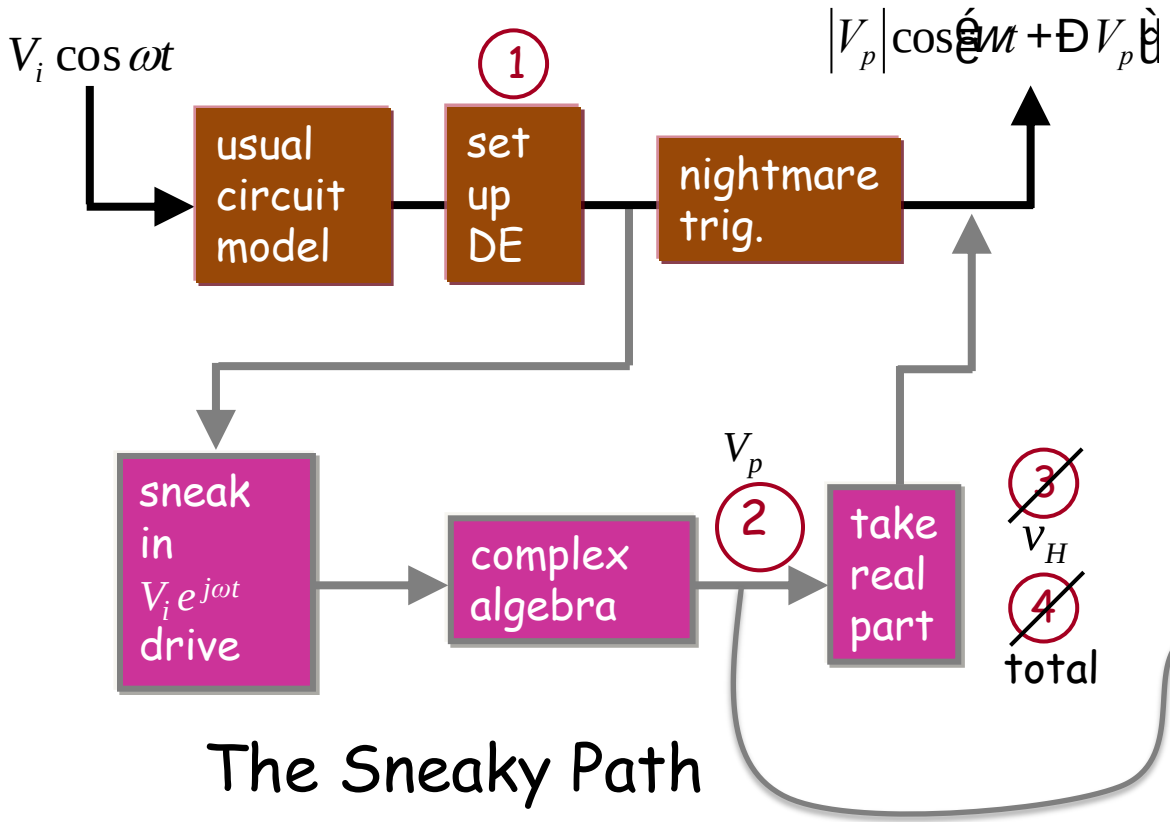
$$v_I = V_i \cos \omega t$$



SSS

- Focus on sinusoids.
- Focus on steady state, only care about v_P as v_H dies away.

Review Sinusoidal Steady State SSS Approach



V_p contains all the information we need:

$|V_p|$ Amplitude of output cosine

$\angle V_p$ Phase

$$\underbrace{V_p}\, e^{j\omega t} \qquad \frac{V_i}{1 + j\omega RC}$$

Review

Frequency response - magnitude

$$\frac{V_p}{V_i} = \frac{1}{1 + j\omega RC} = H(j\omega)$$

transfer function

$$V_p = \frac{V_i}{1 + j\omega RC}$$

$$v = |V_p| \cos(\omega t + \angle V_p)$$

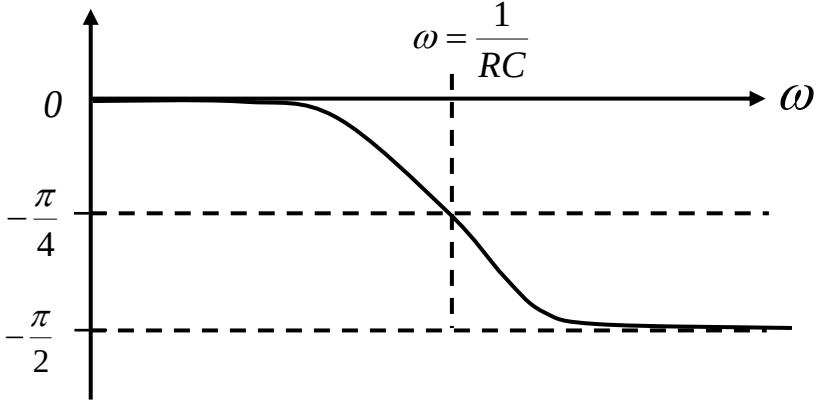


The Frequency View

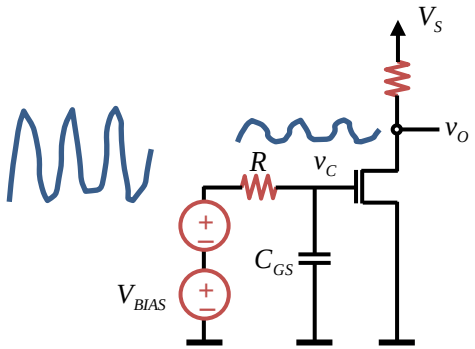
Review

Frequency response - phase

$$\angle \frac{V_p}{V_i} = tan^{-1}\left(\frac{-\omega RC}{1}\right)$$



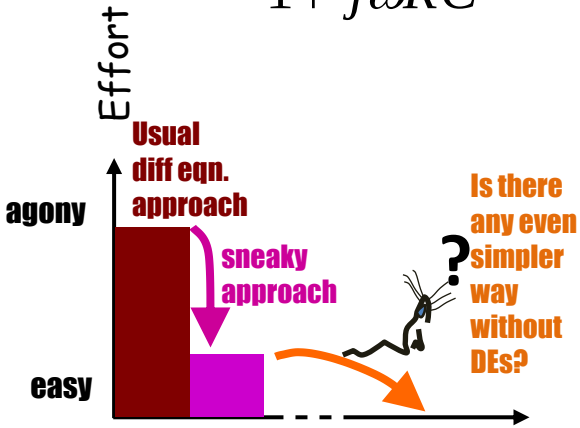
The Frequency View



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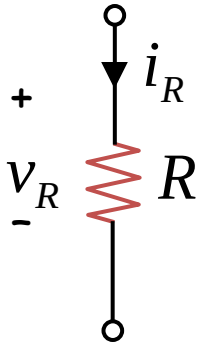


The Impedance Model

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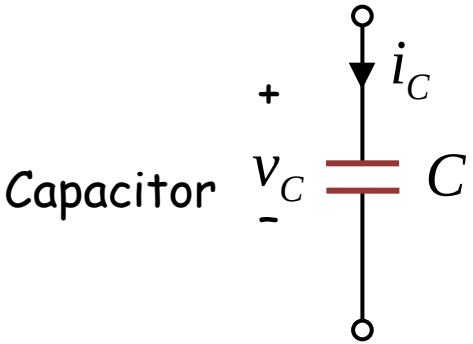
Consider resistor:

Resistor



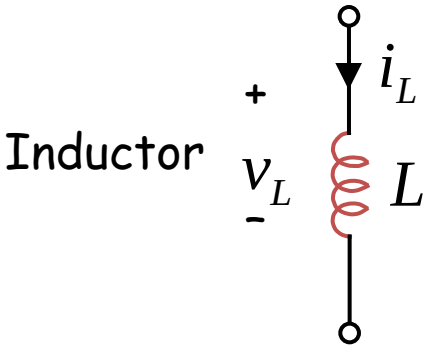
The Impedance Model -- Capacitor

Consider capacitor:



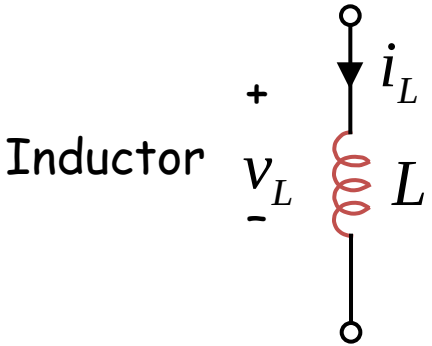
The Impedance Model -- Inductor

Consider Inductor:



The Impedance Model -- Inductor

Consider Inductor:



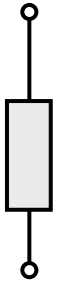
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The Impedance Model -- Summary

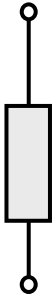
capacitor



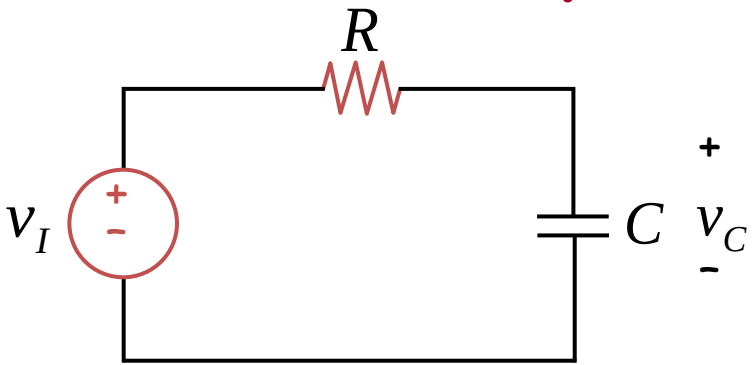
inductor



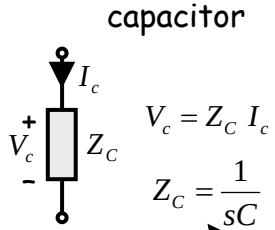
resistor



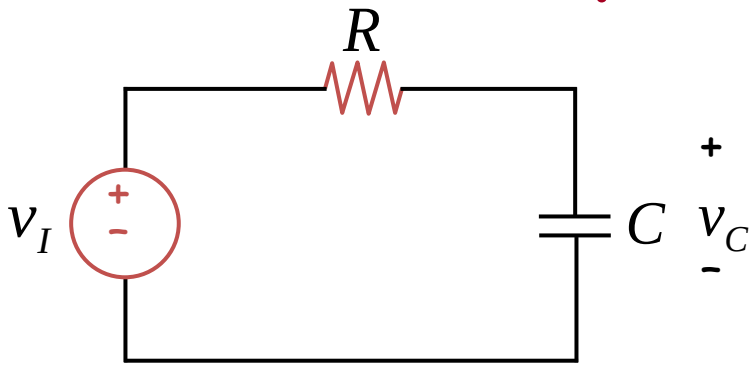
Back to RC example…



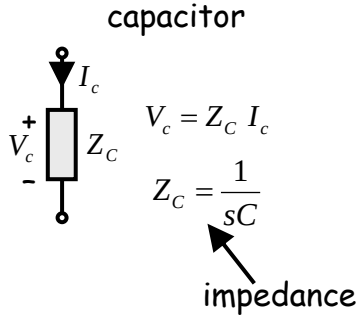
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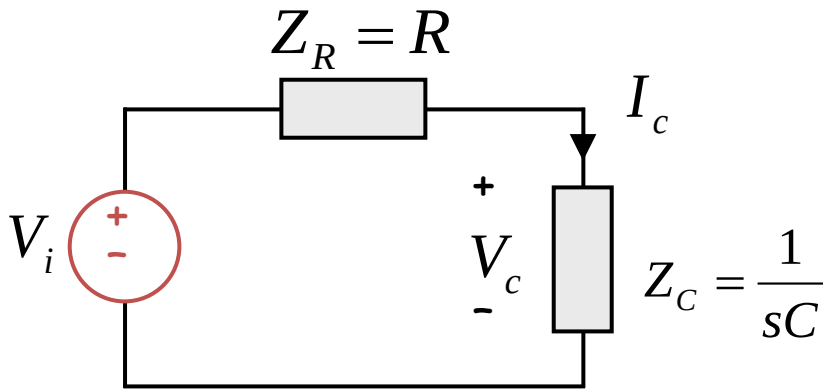
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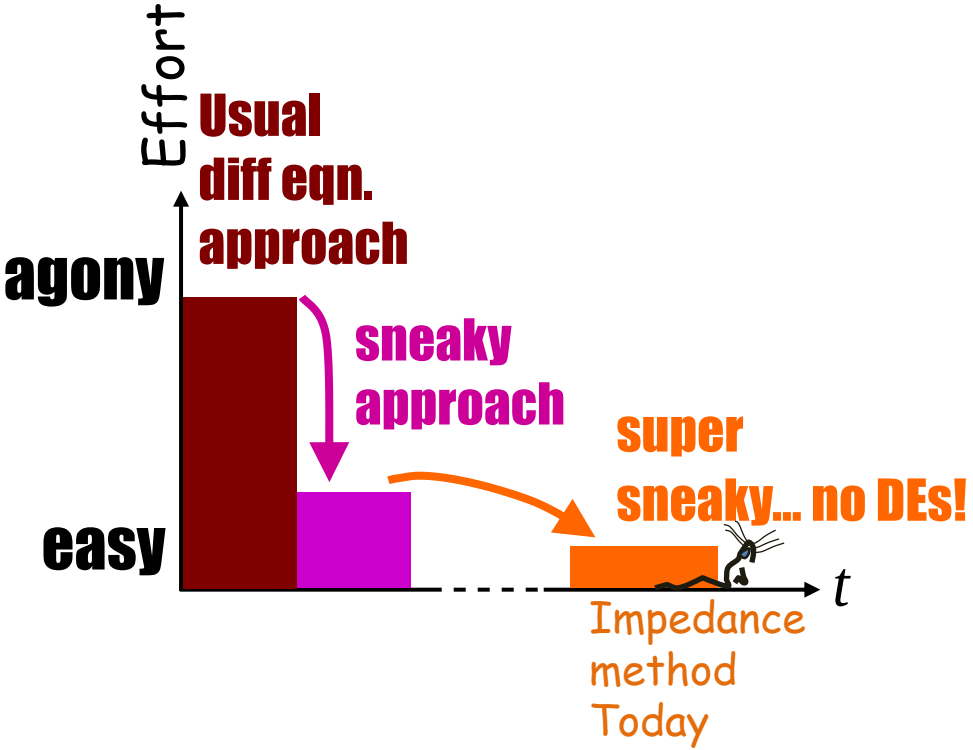
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Impedance model:



There you have it



Signal Notation

Impedance Method Summary

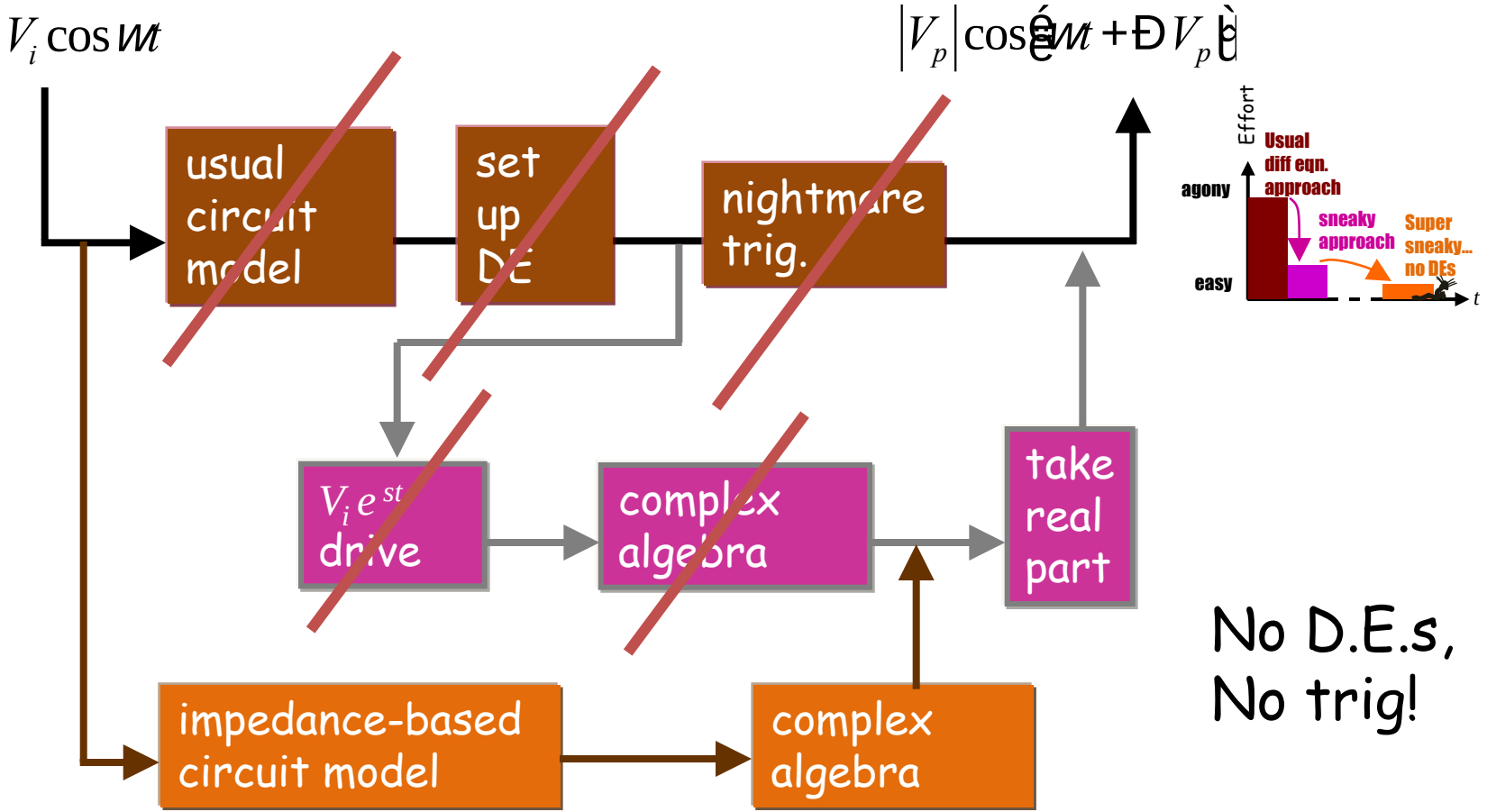
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The Big Picture…

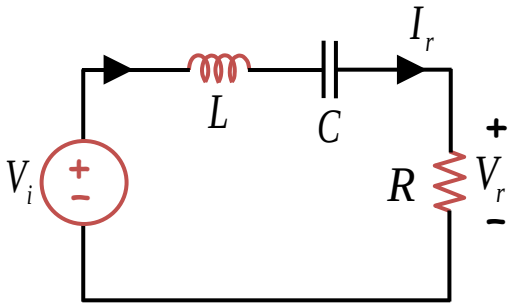


No D.E.s,
No trig!

Back to

Let's study this transfer function

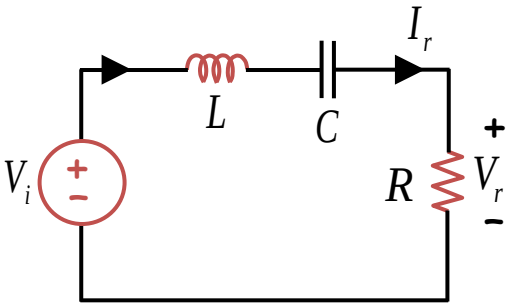
$$\frac{V_r}{V_i} = \frac{\frac{j\omega R}{L}}{\frac{1}{LC} + \frac{j\omega R}{L} - \omega^2}$$



Review complex algebra in Appendix C of textbook

Graphically

$$\left|\frac{V_r}{V_i}\right| = \frac{\omega RC}{\sqrt{\left(1-\omega^2 LC\right)^2 + (\omega RC)^2}}$$



Find limit

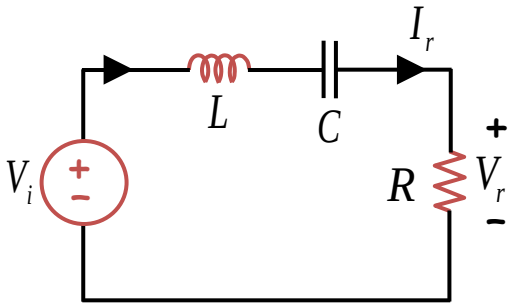
Low ω:

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Observe

Low ω:

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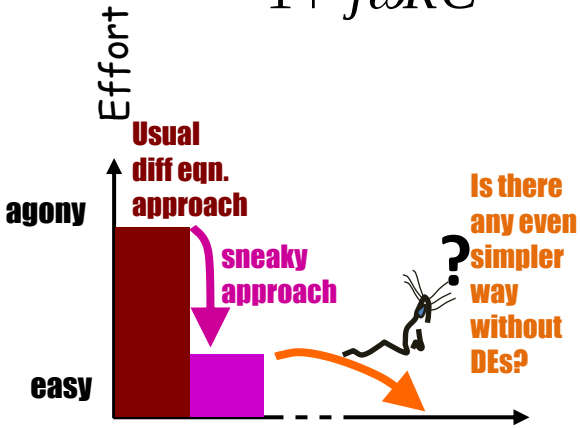
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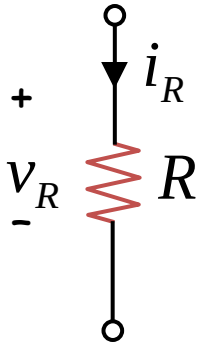


The Impedance Model

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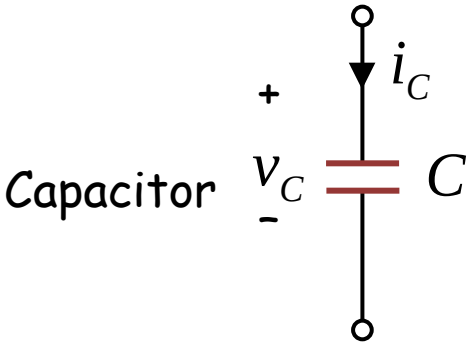
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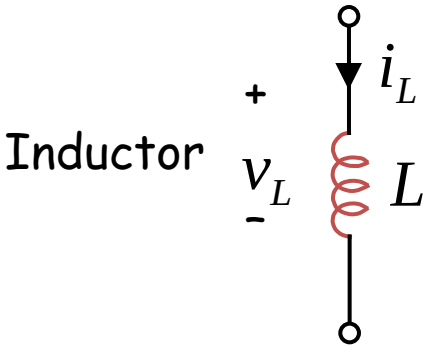
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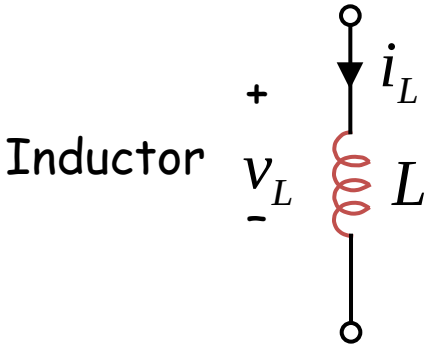
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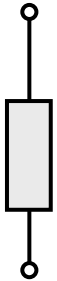
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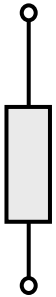
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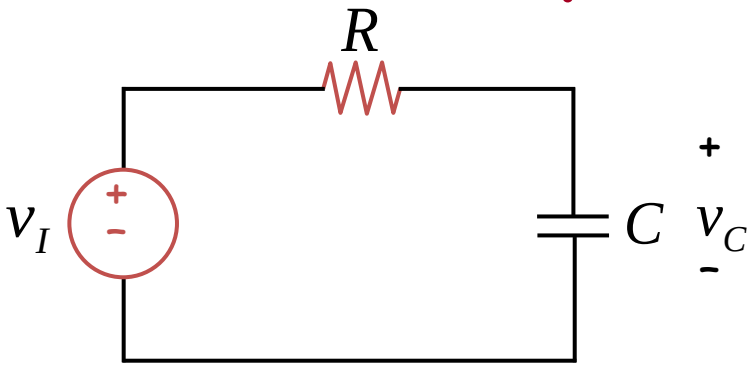
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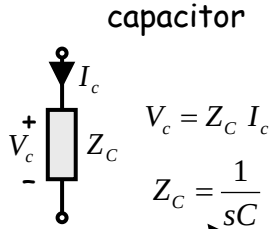
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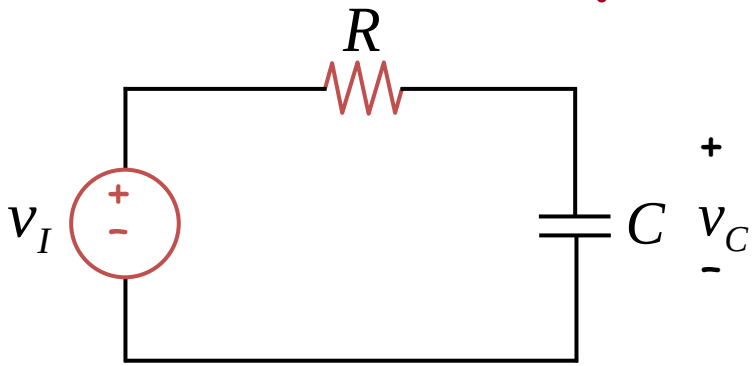
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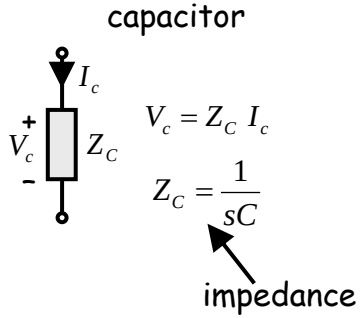
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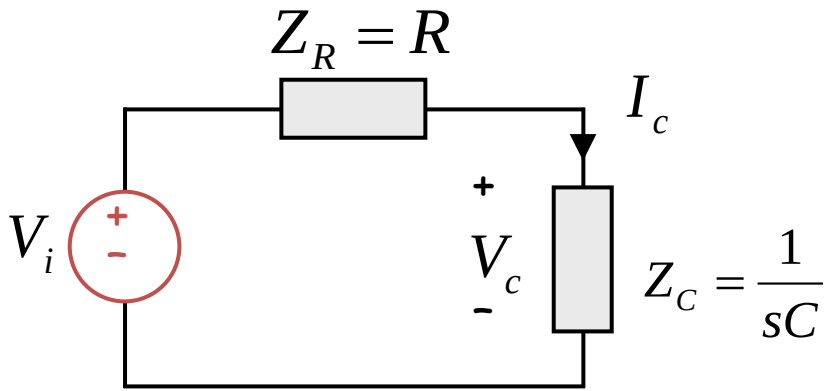
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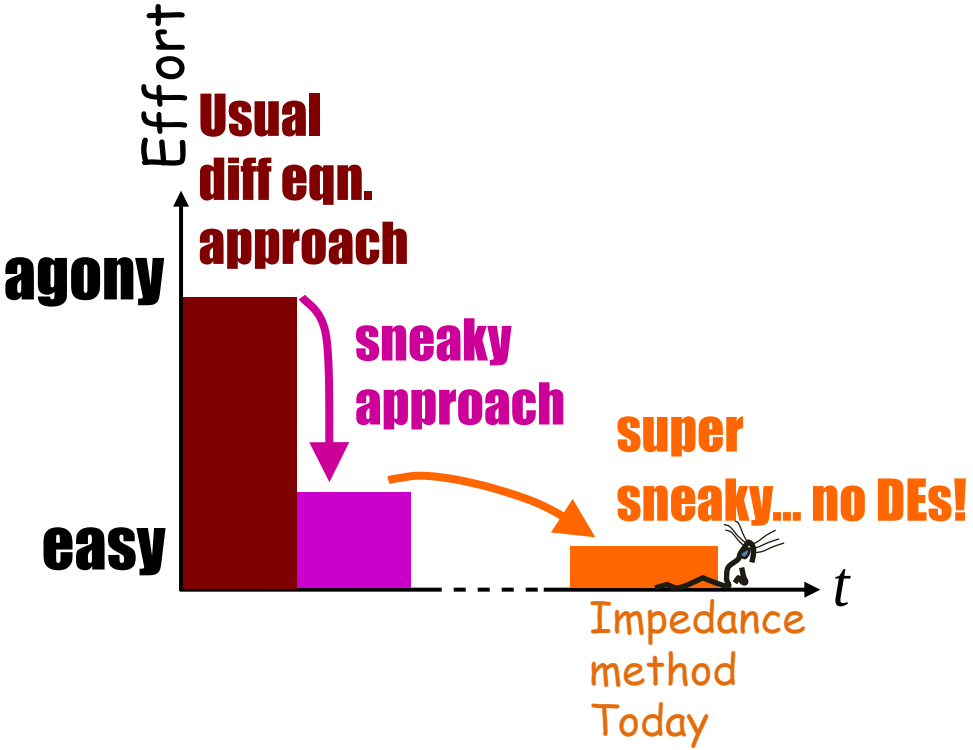
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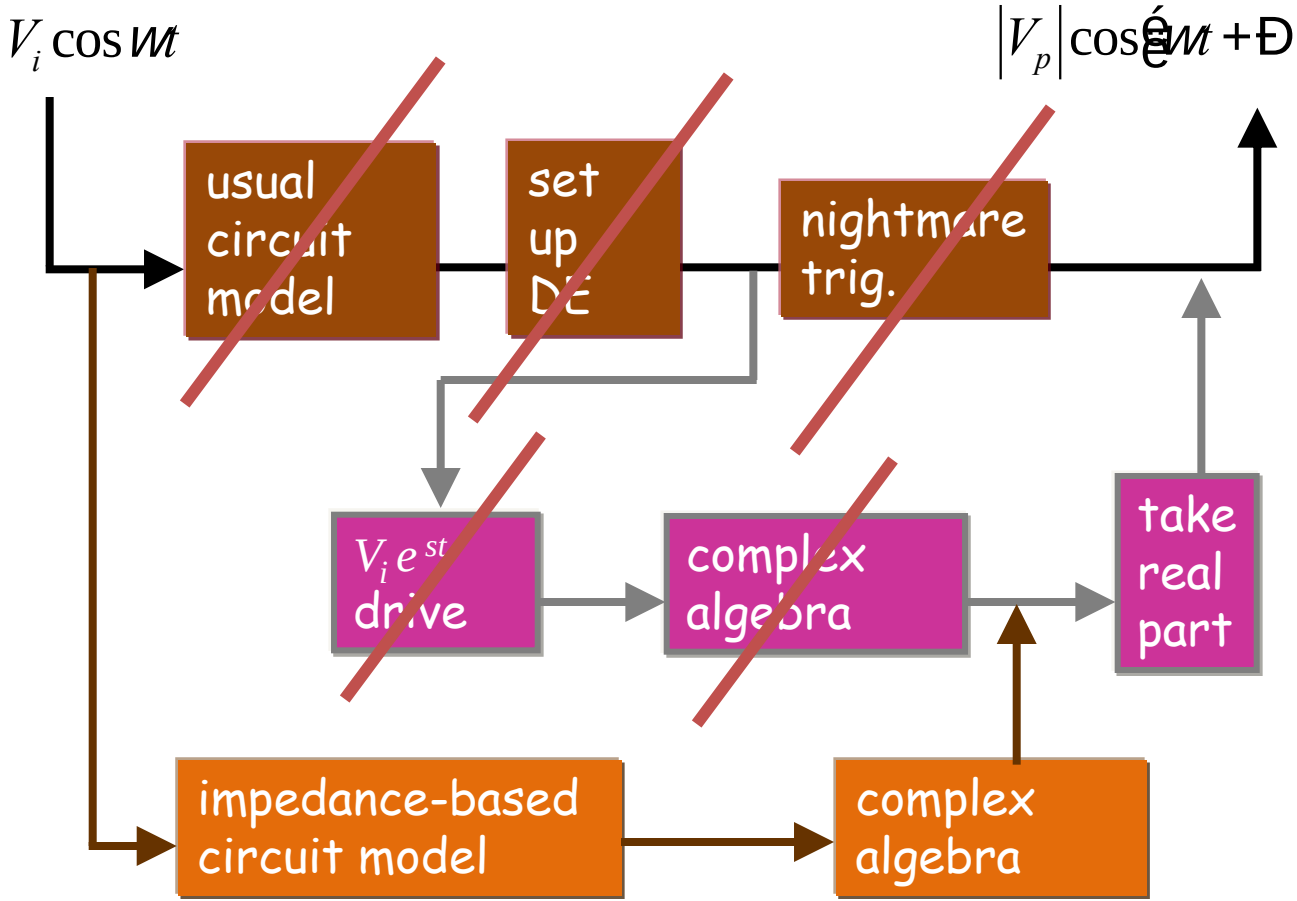
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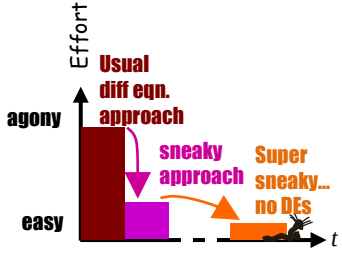
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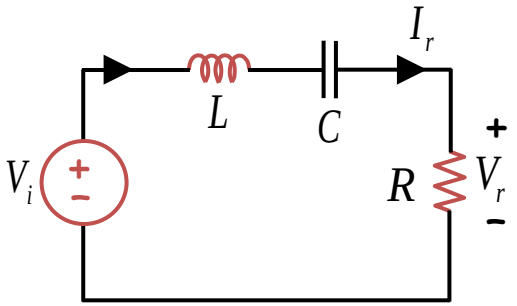




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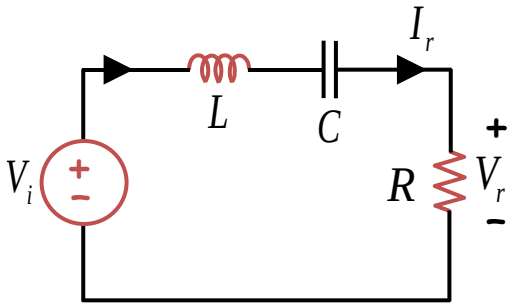
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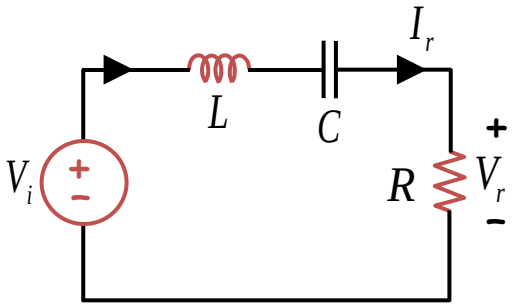
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