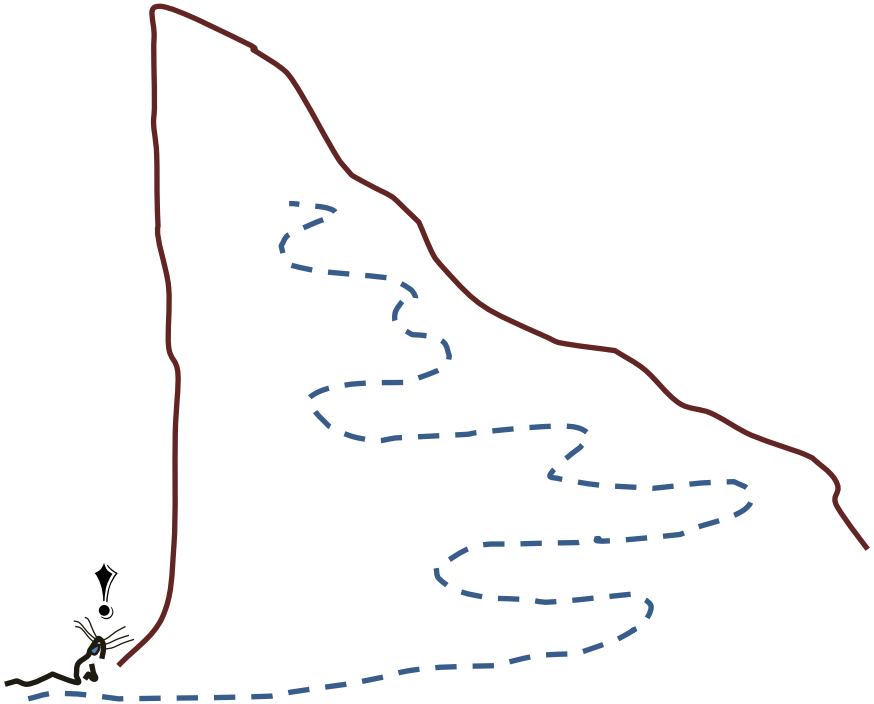
6.002x

CIRCUITS AND ELECTRONICS

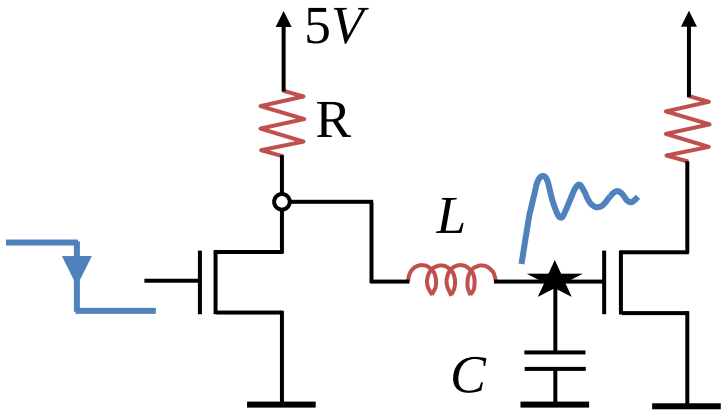
Sinusoidal Steady State

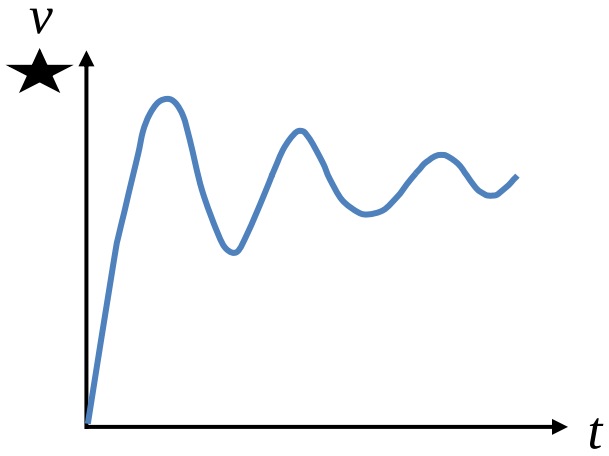


Reading Section 13.1 of the textbook

Review

- We now understand the why of:

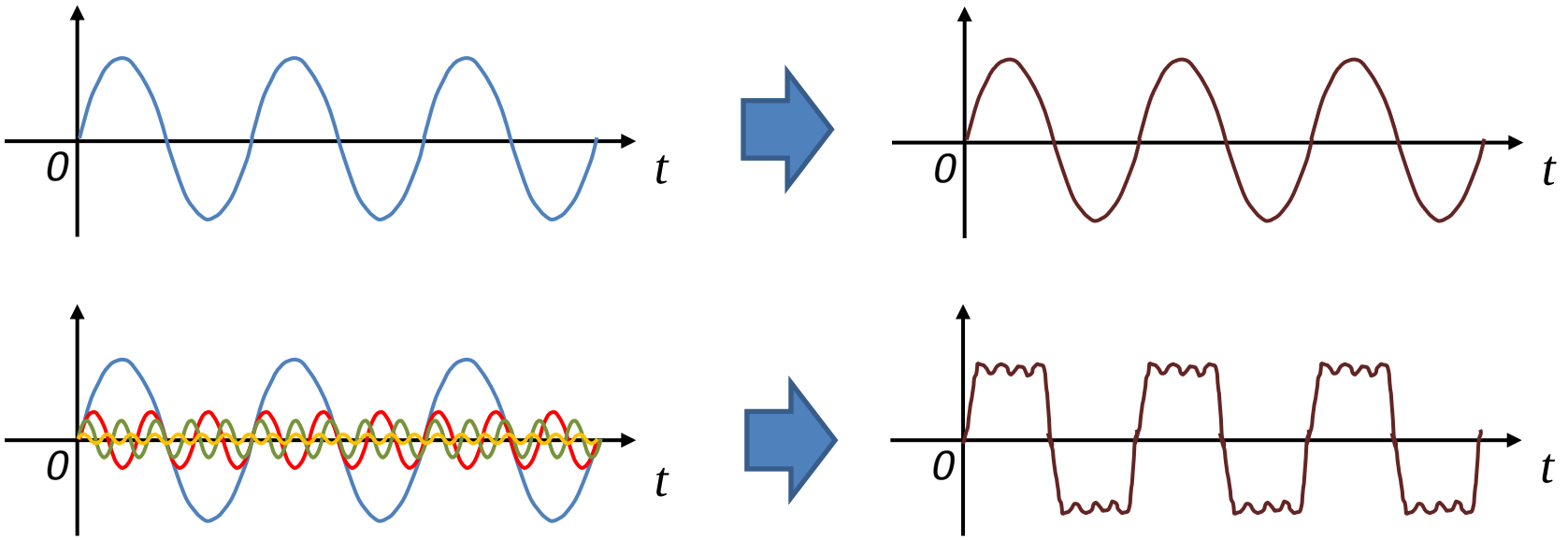




Today

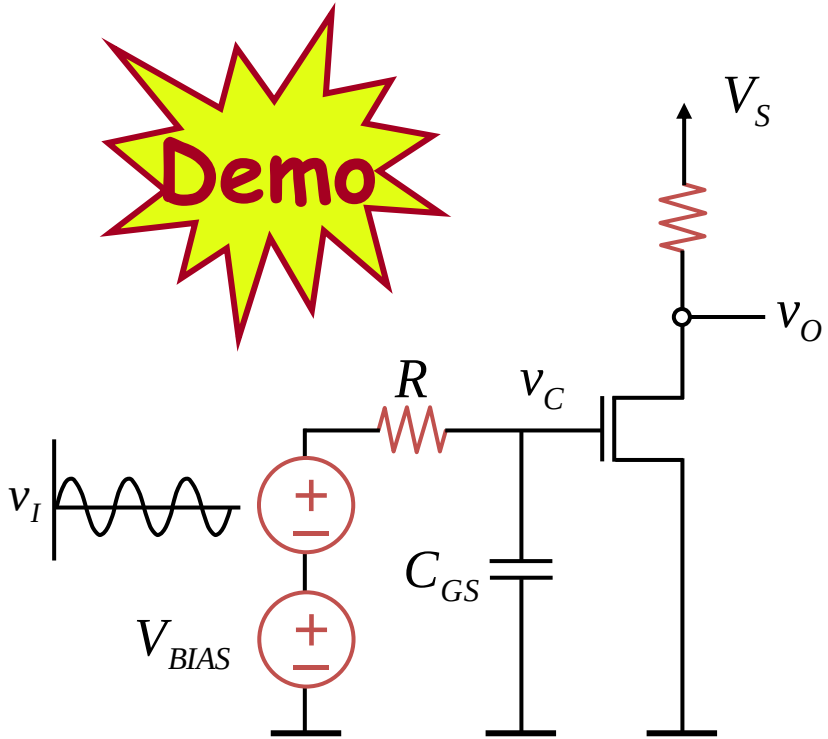
- Today, look at response of networks to sinusoidal drive.

  Sinusoids important because signals can be represented as a sum of sinusoids. Response to sinusoids of various frequencies -- aka frequency response -- tells us a lot about the system



Motivation

For motivation, consider our old friend, the amplifier:



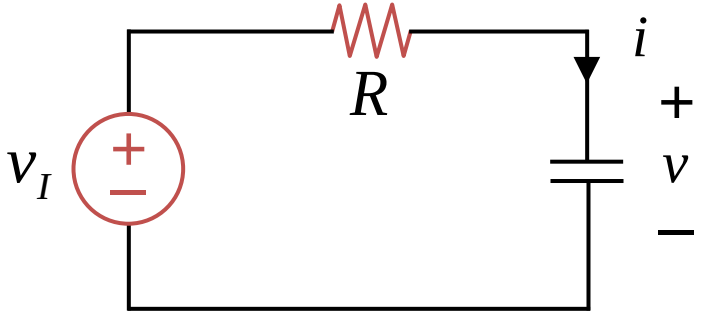
Need to study behavior of networks for sinusoidal drive.

Observe v_O amplitude as the frequency of the input v_I changes. Notice it decreases with frequency.

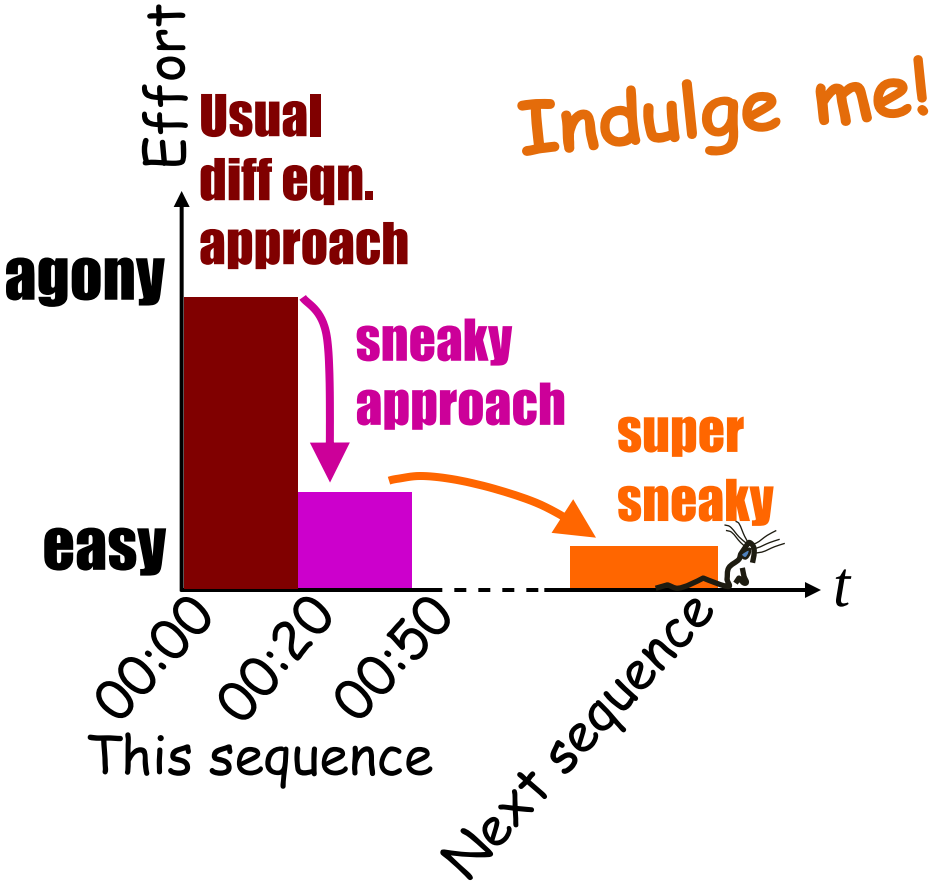
Also observe v_O shift as frequency changes (phase).

Sinusoidal Response of RC Network

Our Approach



Determine $v(t)$

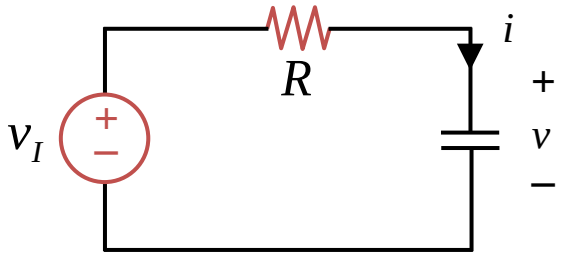


Indulge me!

Let's try the usual approach…



Usual approach…



Determine $v(t)$

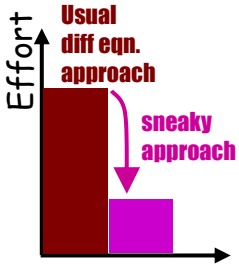


② Find v_P

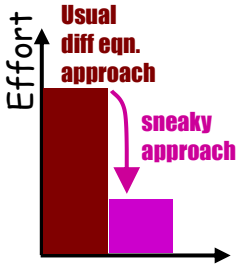
$$RC\frac{dv}{dt} + v = V_i \cos \omega t$$



Detour: Let's get sneaky!



Detour: Let's get sneaky!



Find particular solution to another input…

$$RC\frac{dv_{PS}}{dt}+v_{PS}=v_{IS}$$

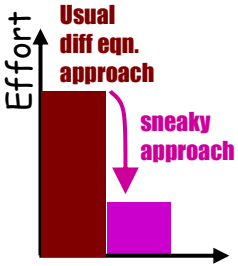
$$=V_i e^{st}$$

(s: sneaky :-))

Try solution $v_{PS}=V_p e^{st}$ **Find** V_P

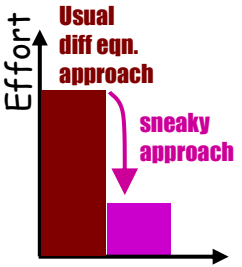
Sneaky detour: find particular solution to another input

$$V_p = \frac{V_i}{1 + sRC}$$



② Fourth try to find v_P... using the sneaky way $V_p = \frac{V_i}{1+sRC}$

$s = j\omega$



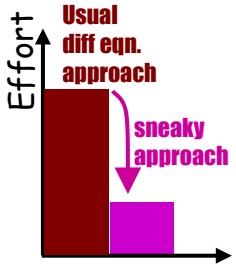
② Fourth try to find v_P...

v_P particular response to $V_i \cos\omega t$

v_{PS} particular response to $V_i e^{j\omega t}$

$$V_p = \frac{V_i}{1 + sRC}$$

$$s = j\omega$$



② Fourth try to find v_P...

v_P particular response to $V_i \cos\omega t$

v_{PS} particular response to $V_i e^{j\omega t}$

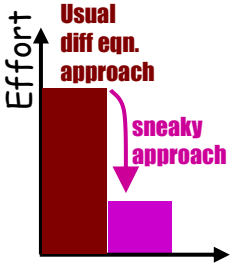
$$V_p = \frac{V_i}{1 + sRC}$$

$s = j\omega$

so,

complex

$$v_P = \mathrm{Re}\left[v_{PS}\right] = \mathrm{Re}\left[V_p e^{j\omega t}\right] = \mathrm{Re}\left[\frac{V_i}{1 + j\omega RC} \cdot e^{j\omega t}\right]$$



③ Find v_H

$$RC\frac{dv_H}{dt}+v_H=0$$

④ Find total solution

Sinusoidal Steady State

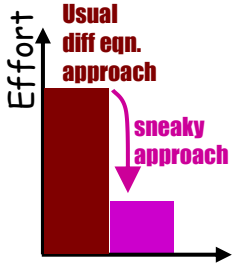
$$V_p = \frac{V_i}{1 + sRC}$$

$$s = j\omega$$

Sinusoidal Steady State – V_P says it all

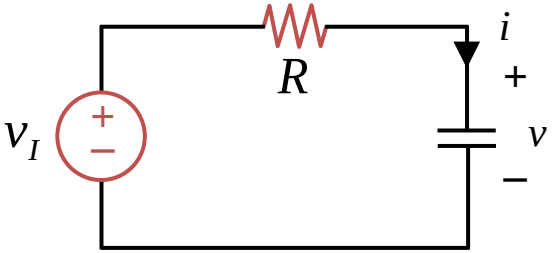
$$V_p = \frac{V_i}{1 + sRC}$$

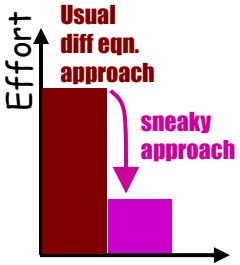
$$s = j\omega$$



Visualizing Sinusoidal Steady State

Summary of SSS approach – fits on one slide





Magnitude Plot

$$V_p = \frac{V_i}{1 + sRC}$$

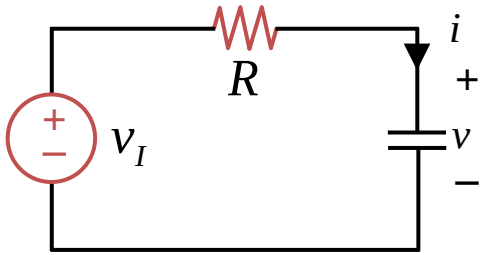
$$s = j\omega$$

Phase Plot

$$V_p = \frac{V_i}{1 + sRC}$$

$$s = j\omega$$

Preview of upcoming attractions: The next Aha moment!



1. Set up DE $RC\frac{dv}{dt}+v=V_i\cos\omega t$

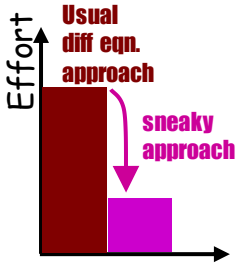
2. Apply sneaky input $V_i e^{st}$, then find particular solution $V_P e^{st}$

$$RC\frac{dv}{dt}+v=V_i e^{st}$$

3. Find $V_P e^{st}$

$$RC\frac{dV_p e^{st}}{dt}+V_p e^{st}=V_i e^{st}$$

$$V_p=\frac{V_i}{1+sRC} \qquad s=j\omega$$



4. Find v_P, which is steady state solution for v

$$v_P=|V_P|\cos(\omega t+\angle V_P)$$