6.002x CIRCUITS AND ELECTRONICS

Second-Order Systems



[Review complex algebra Appendix C in textbook]

Motivating Example – Slow Case

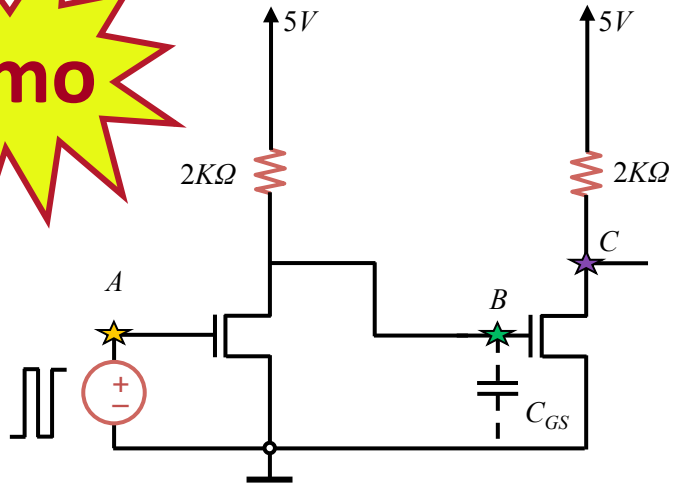


Our old friend, the inverter, driving another.

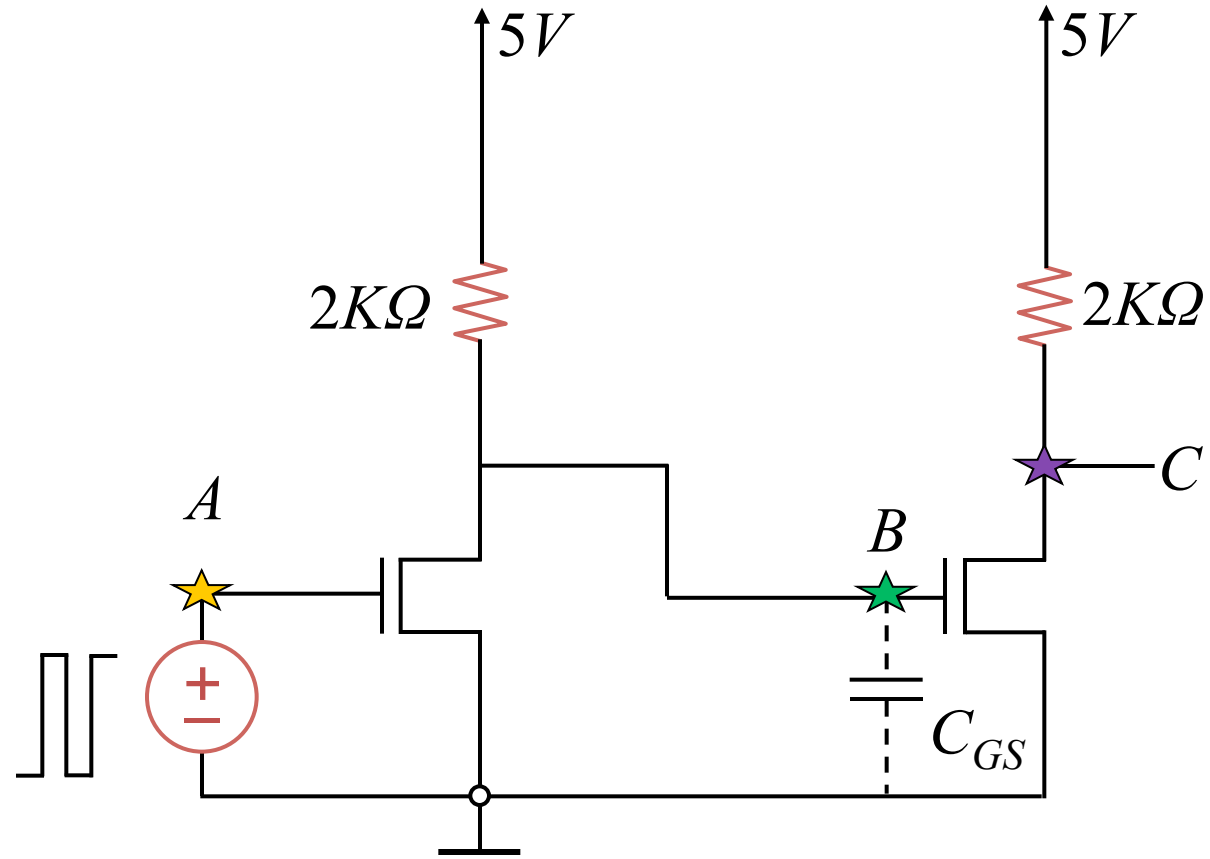
Observed Output - Slow



Demo

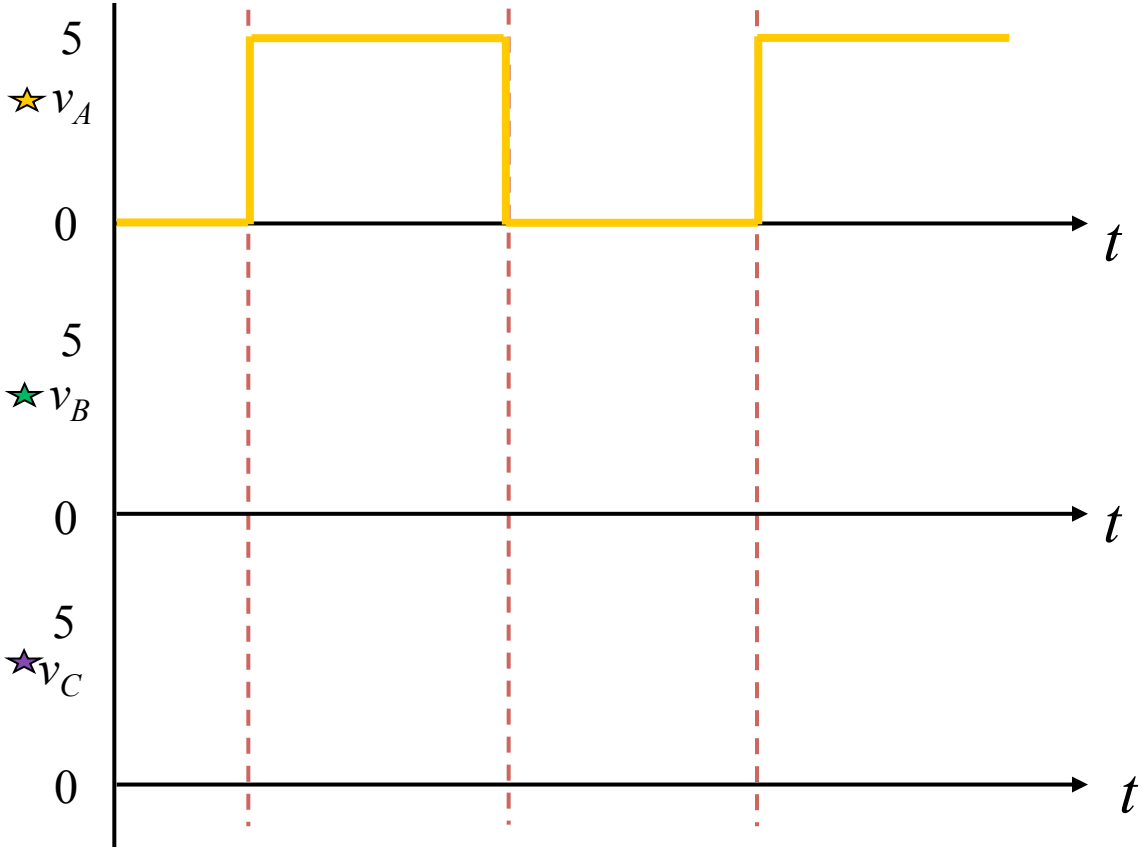


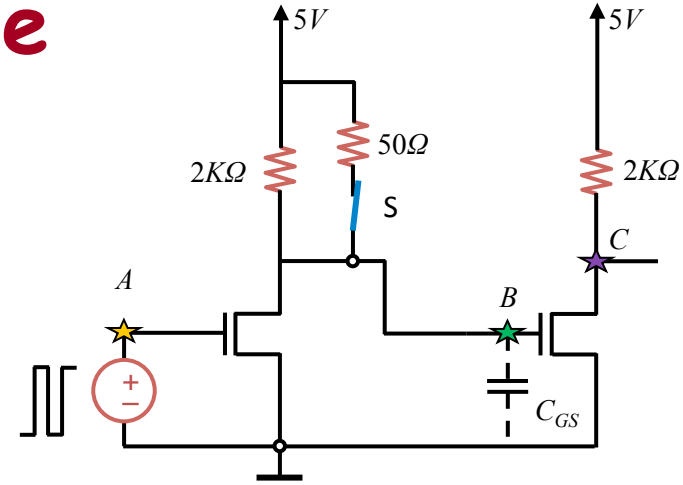
Motivating Example – Fast Case



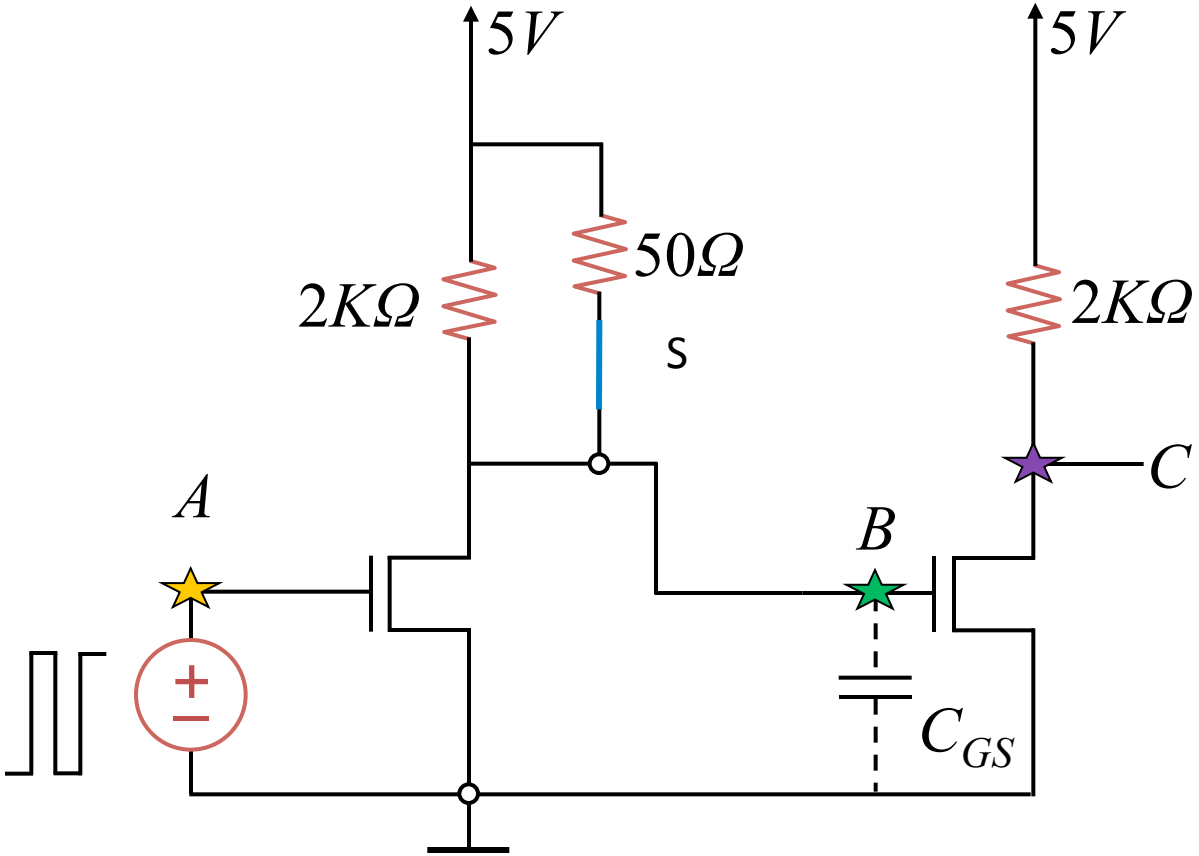
Demo

Observed Output – Fast Case



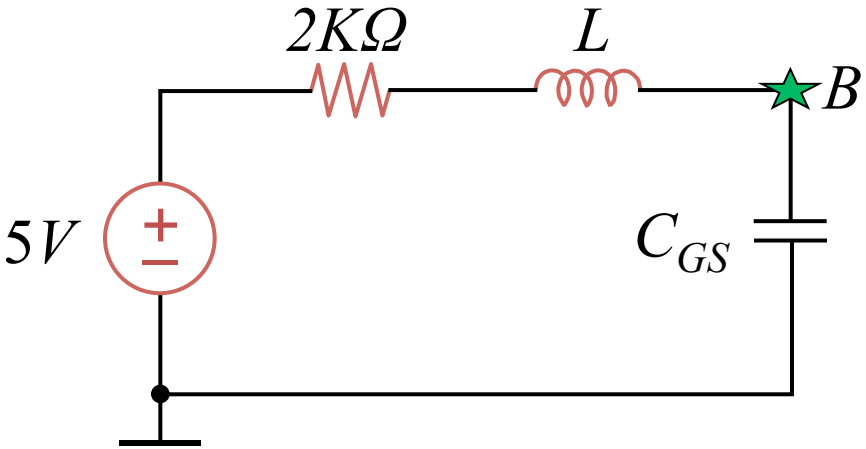


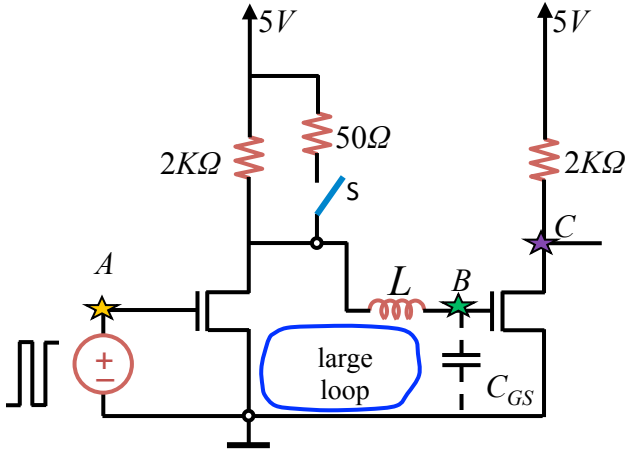
Fast Case – What's Really Going On



Second-Order Systems

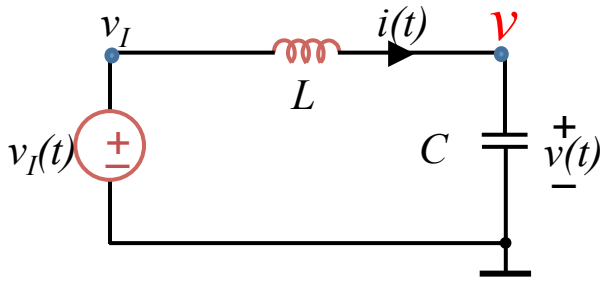
Relevant circuit:





First, let's analyze the LC network

Analyzing the LC network



Solving

Let's solve

1 Particular solution

1 Particular solution

$$LC\frac{d^2v_P}{dt^2}+v_P=V_I$$

② Homogeneous solution

(2) Homogeneous solution

Solution to $LC\frac{d^2 v_H}{dt^2} + v_H = 0$

Recall, v_H : solution to homogeneous equation (drive set to zero)

Four-step method:

(2A) Assume solution of the form*

$$v_H = Ae^{st} \quad , \; A, \; s \; = \; ?$$

*Differential equations are commonly solved by guessing solutions

② Homogeneous solution

$$LC\frac{d^2v_H}{dt^2}+v_H=0$$

③ Total solution

3 Total solution

Find unknowns from initial conditions.

$$v(t) = v_P(t) = v_H(t)$$

$$v(t) = V_I + A_1 e^{j\omega_o t} + A_2 e^{-j\omega_o t}$$

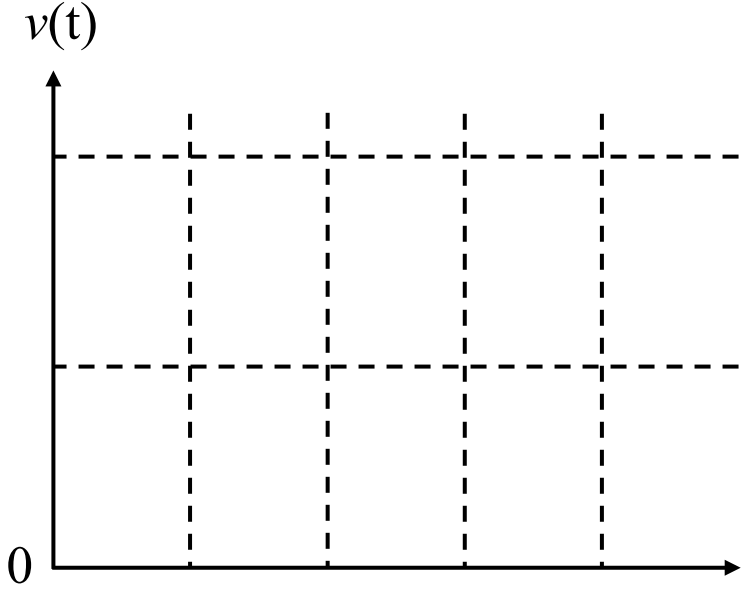
③ Total solution

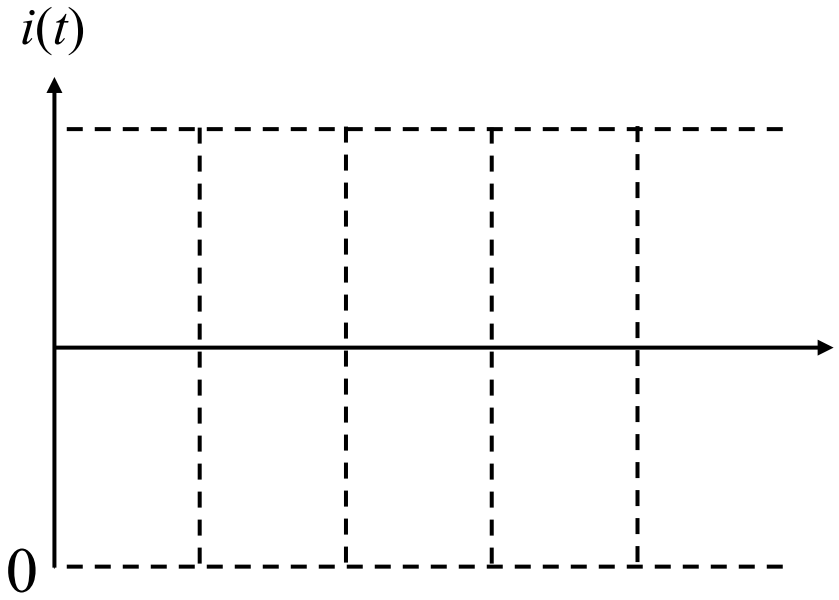
Remember Euler relation

$$v(t) = V_I - \frac{V_I}{2}\left(e^{j\omega_o t} + e^{-j\omega_o t}\right)$$

Plotting the Total Solution

$$v(t) = V_I - V_I \cos \omega_o t$$





Demo

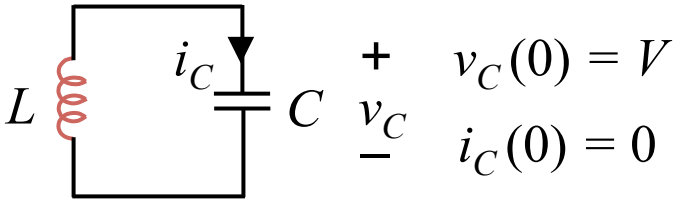
Summary of Method

1. Write DE for circuit by applying node method.
2. Find particular solution v_P by guessing and trial & error.
3. Find homogeneous solution v_H
   - A. Assume solution of the form Ae^{st}.
   - B. Obtain characteristic equation.
   - C. Solve characteristic equation for roots s_i.
   - D. Form v_H by summing $A_i e^{s_i t}$ terms.
4. Total solution is $v_P + v_H$, then solve for remaining constants using initial conditions.

Example

What if we have:

Example



We can obtain the answer directly from the homogeneous solution ($V_I = 0$).

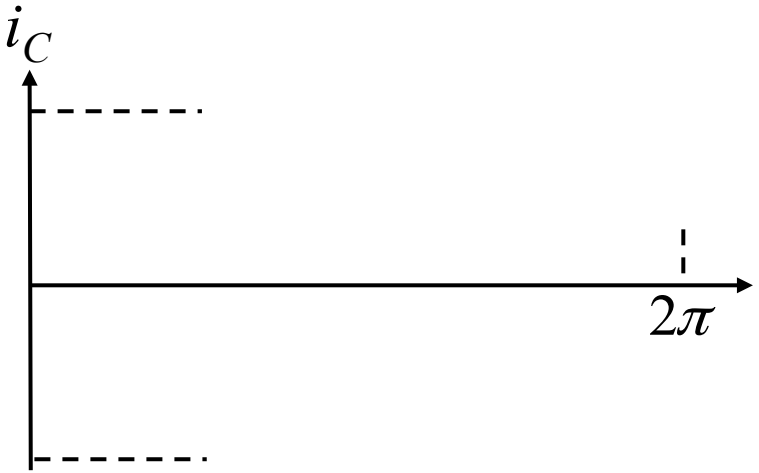
$$v_C(t) = A_1 e^{j\omega_o t} + A_2 e^{-j\omega_o t}$$

$$v_C(0) = V$$

$$i_C(0) = 0$$

Example



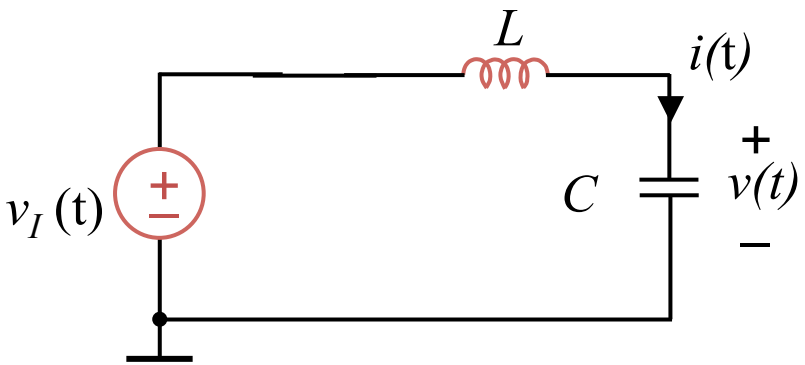


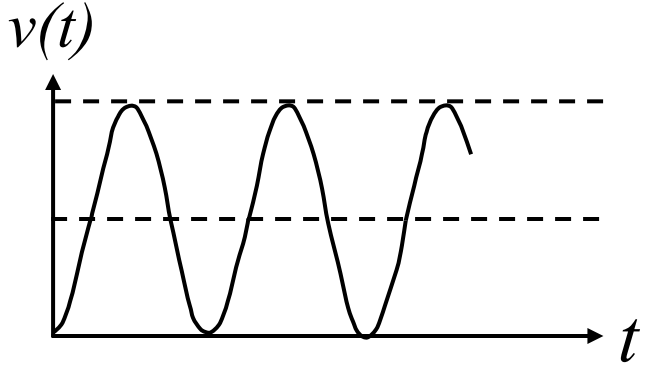
Energy





Next, introduce R: RLC Circuits





More in the next sequence! If you are impatient, see A&L Section 13.2