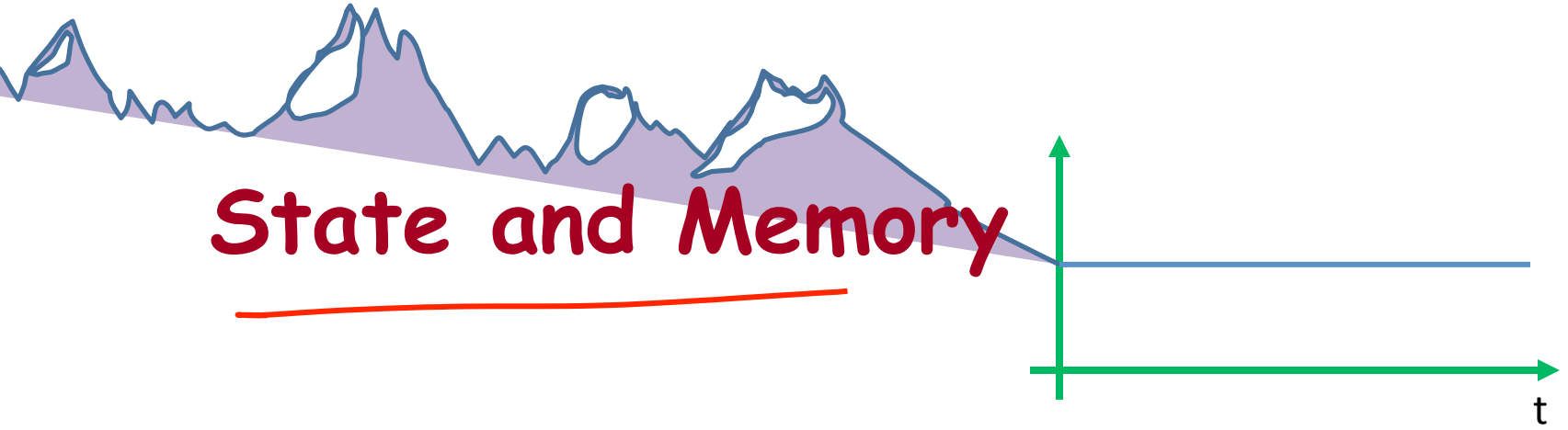
6.002x

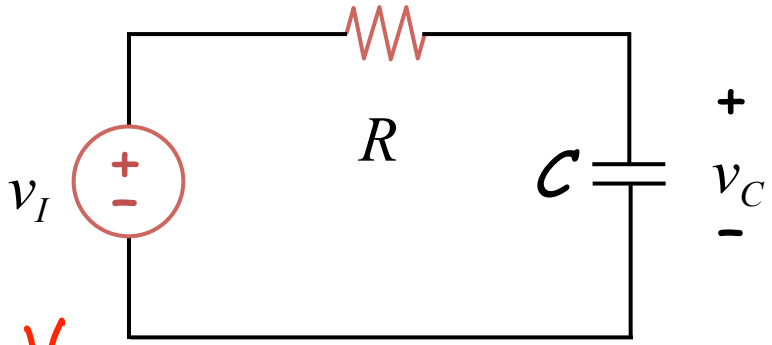
CIRCUITS AND ELECTRONICS



State and Memory

Review

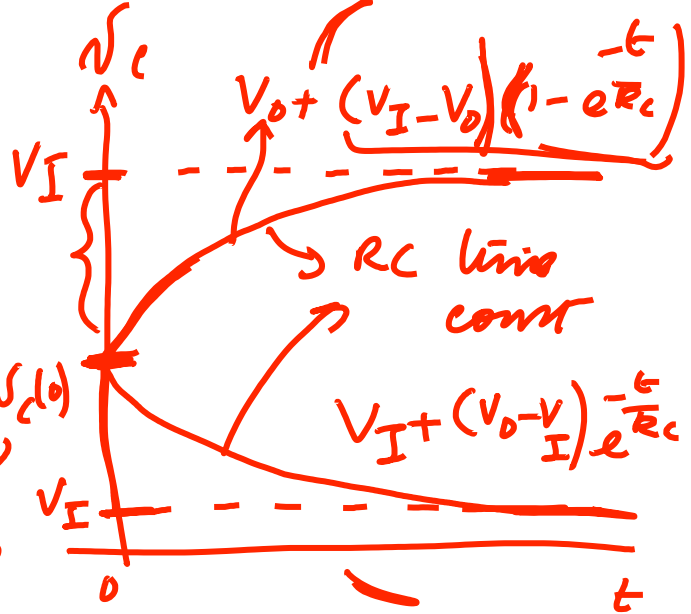
Recall



$v_I(t) = V_I$ for $t \geq 0$

$v_c(0) = V_0$

$$v_c = V_I + (v_c(0) - V_I)\, e^{\frac{-t}{RC}} \quad \text{— (1)}$$



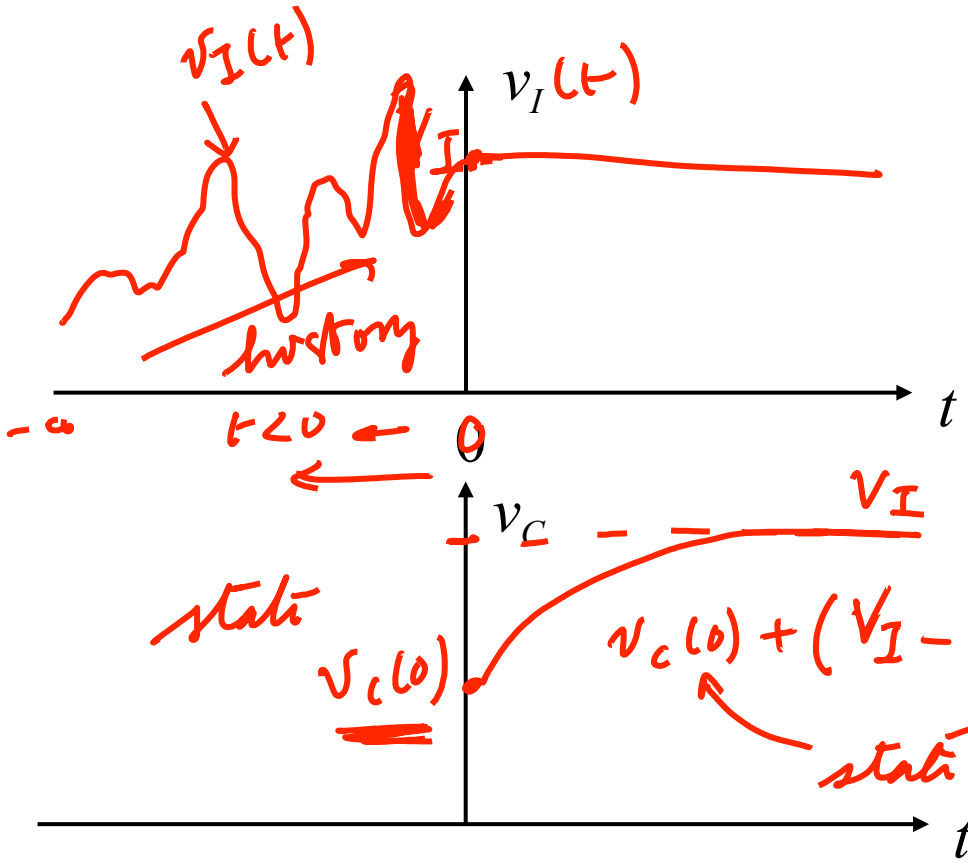
$(1 - e^{-t/RC})$

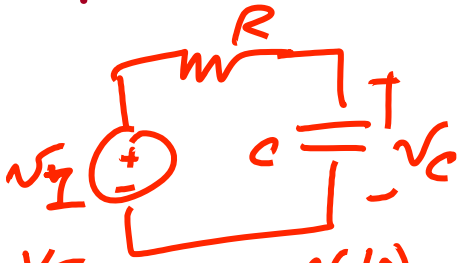
$e^{-t/RC}$

Reading: Section 10.3 and Chapter 11

This sequence will dwell on the memory property of capacitors

For the RC circuit in the previous slide





Notice that the capacitor voltage for $t \geq 0$ is independent of the form of the input voltage before $t=0$.

Instead, it depends only on the capacitor voltage at $t=0$, and the input voltage for $t \geq 0$.

State

State: Summary of past inputs relevant to predicting the future

$q = CV$

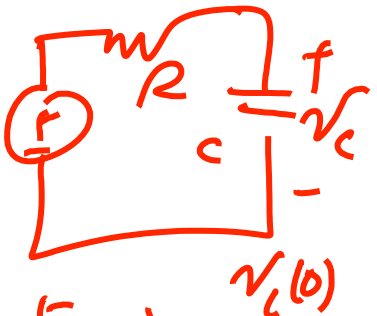
state variable

for linear capacitors, cap. voltage V is also a state variable

State

Back to our simple RC circuit

$$v_c = f(v_c(0), v_I(t))$$



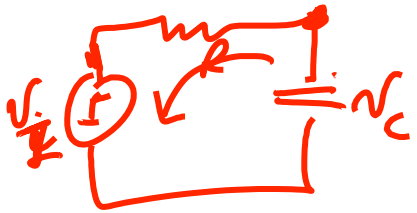
$$v_c(0) + (V_I - v_c(0))(1 - e^{-t/RC})$$

summarizes the past input relevant to predicting the future values of $v_c(t)$

ZIR and ZSR

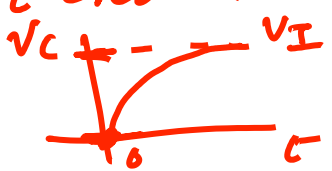
We are often interested in circuit response for

- Zero state $v_C(0) = 0$
- Zero input $v_I(t) = 0$



Correspondingly

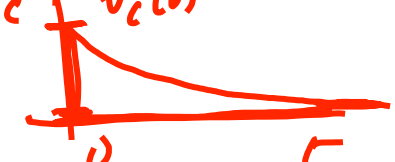
- Zero state response or ZSR



$$v_C = V_I\left(1 - e^{-t/RC}\right) \quad \text{— (2)}$$

$v_I(t) = 0$, $v_C(0)$ — ZIR

- Zero input response or ZIR



$$v_C = v_C(0)\, e^{-t/RC} \quad \text{— (3)}$$

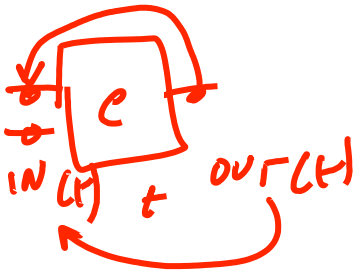
complete : $v_C = V_I + (v_C(0) - V_I)\, e^{-t/RC}$ — (1)

One application of STATE ➡ DIGITAL MEMORY

Why memory?

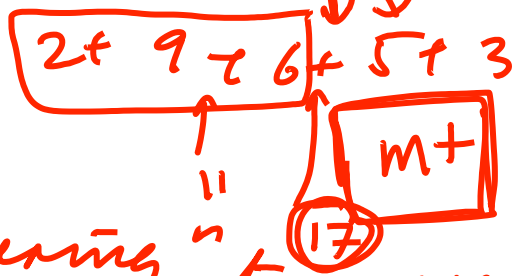
Or, Why is combinational logic insufficient:

Examples.



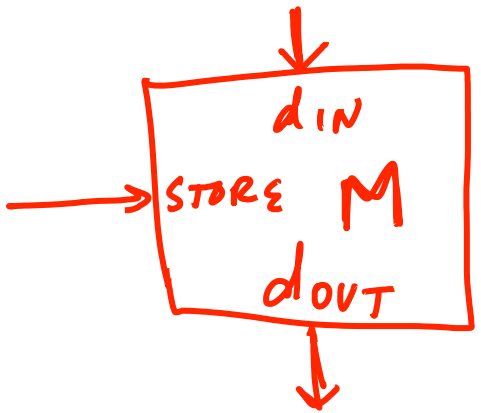
Total response = $2_{SR} + 2_{IR}$

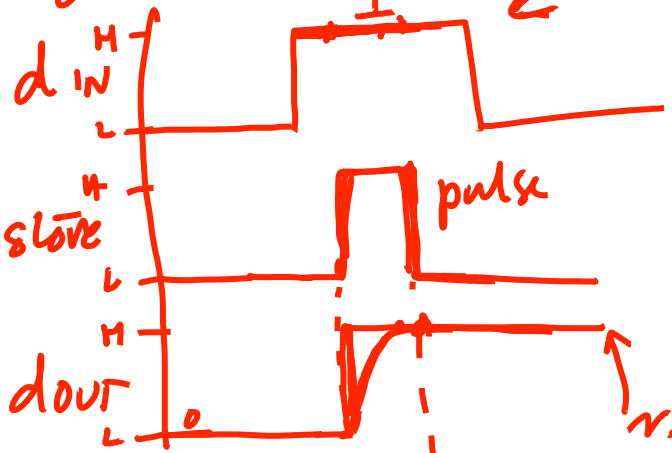
- Consider adding 6 numbers on your calculator

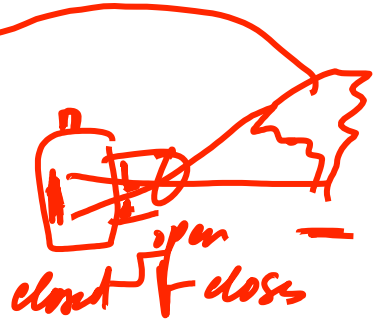


- "Remembering" transient inputs
- Storing data

Memory Abstraction







A 1-bit memory element

remembers the "1"

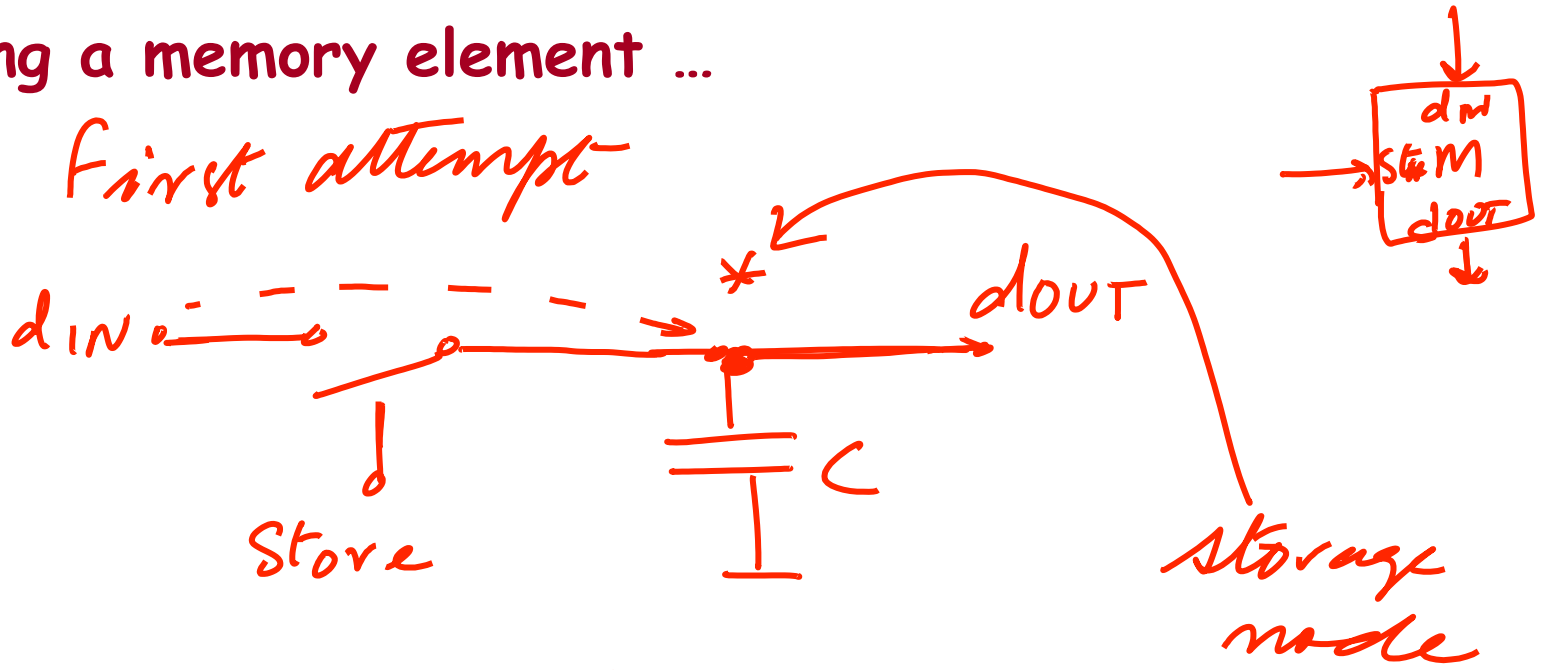
Remembers the input when "store" goes high.

Like a camera that records input when the user presses the shutter release button.

The recorded value is visible at d_{OUT}.

Building a memory element …

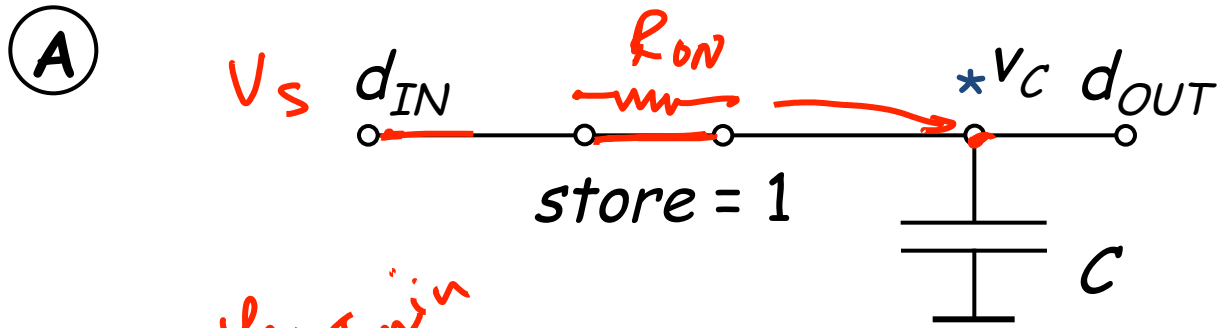
Ⓐ First attempt

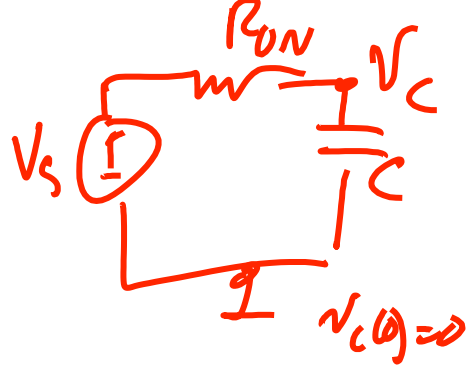


low - Switch open

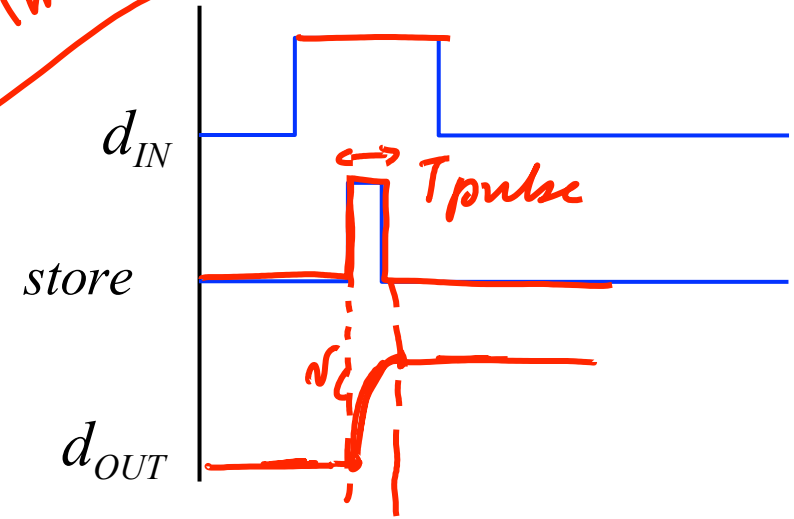
high - Switch closed

Minimum store pulse width





pulse with Tpulse → Tmin



$v_c(0) = 0$

$\tau = R_{ON} C$

$$v_c = V_S\left(1 - e^{-t/R_{ON}C}\right)$$

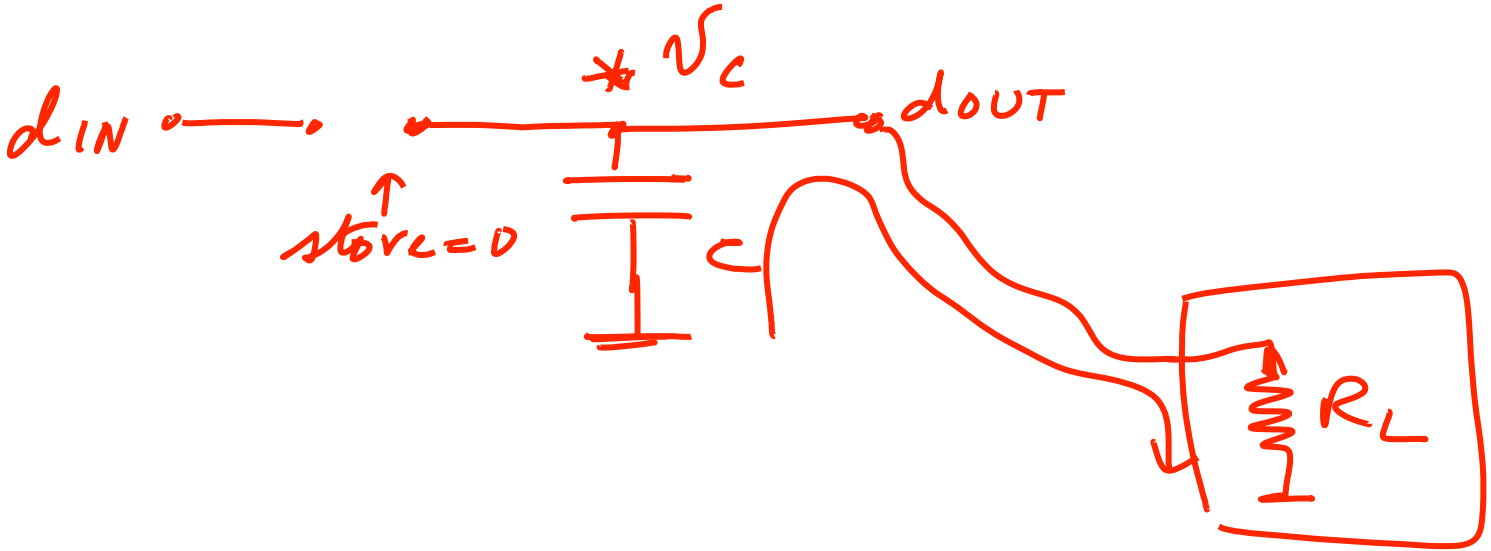
2SR

$$V_{OH} = V_S\left(1 - e^{-T_{min}/R_{ON}C}\right)$$

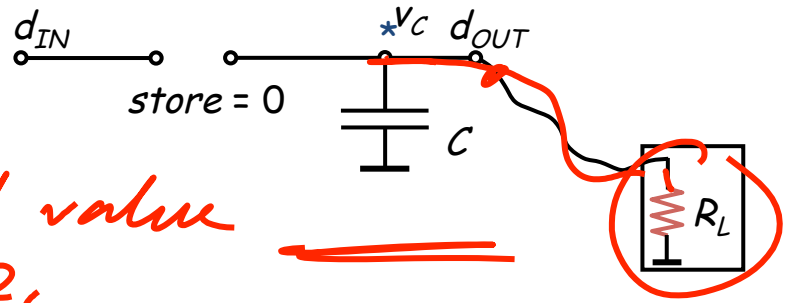
0V

Storage time

(A) first attempt



Storage time



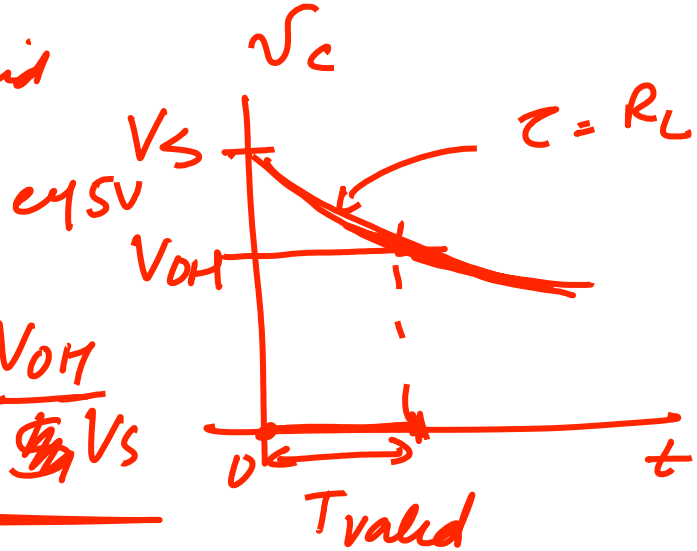
(A)

When store = 0, stored value leaks away through R_L

From (2) $\quad v_c = V_S e^{-\frac{t}{R_L C}} \leftarrow$ Tvalid

$\nearrow V_{OH}$

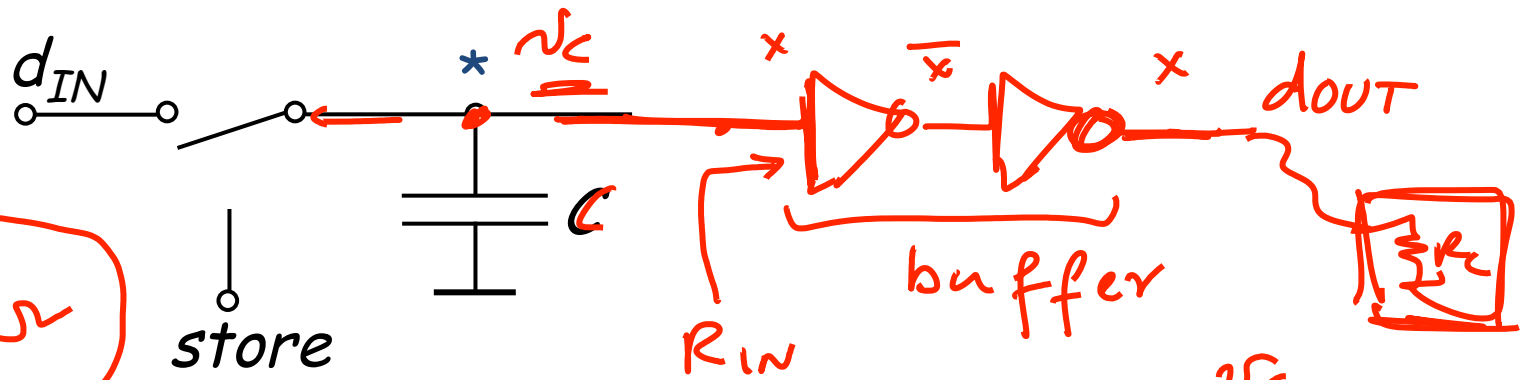
$$T_{valid} = -R_L C \ln \frac{V_{OH}}{V_S}$$



DOES NOT WORK

Building a memory element … second attempt

Ⓑ Second attempt → buffer

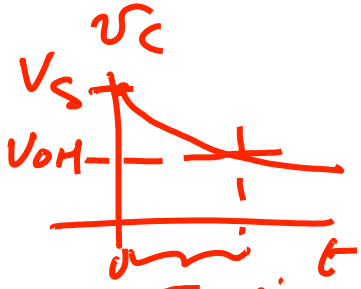


DRAM
$R_{IN} = 1\,G\Omega$
$C = 1\,fF$
$\tau = R_{IN} C$
$= 10^{12} \cdot 10^{-15}$
$= 10^{-3}\,s$
$= 1\,ms$

R_{IN} = Input resistance R_N

$T_{valid} = -R_{IN} C \ln \frac{V_{OH}}{V_S}$ //

$R_N >> R_L$

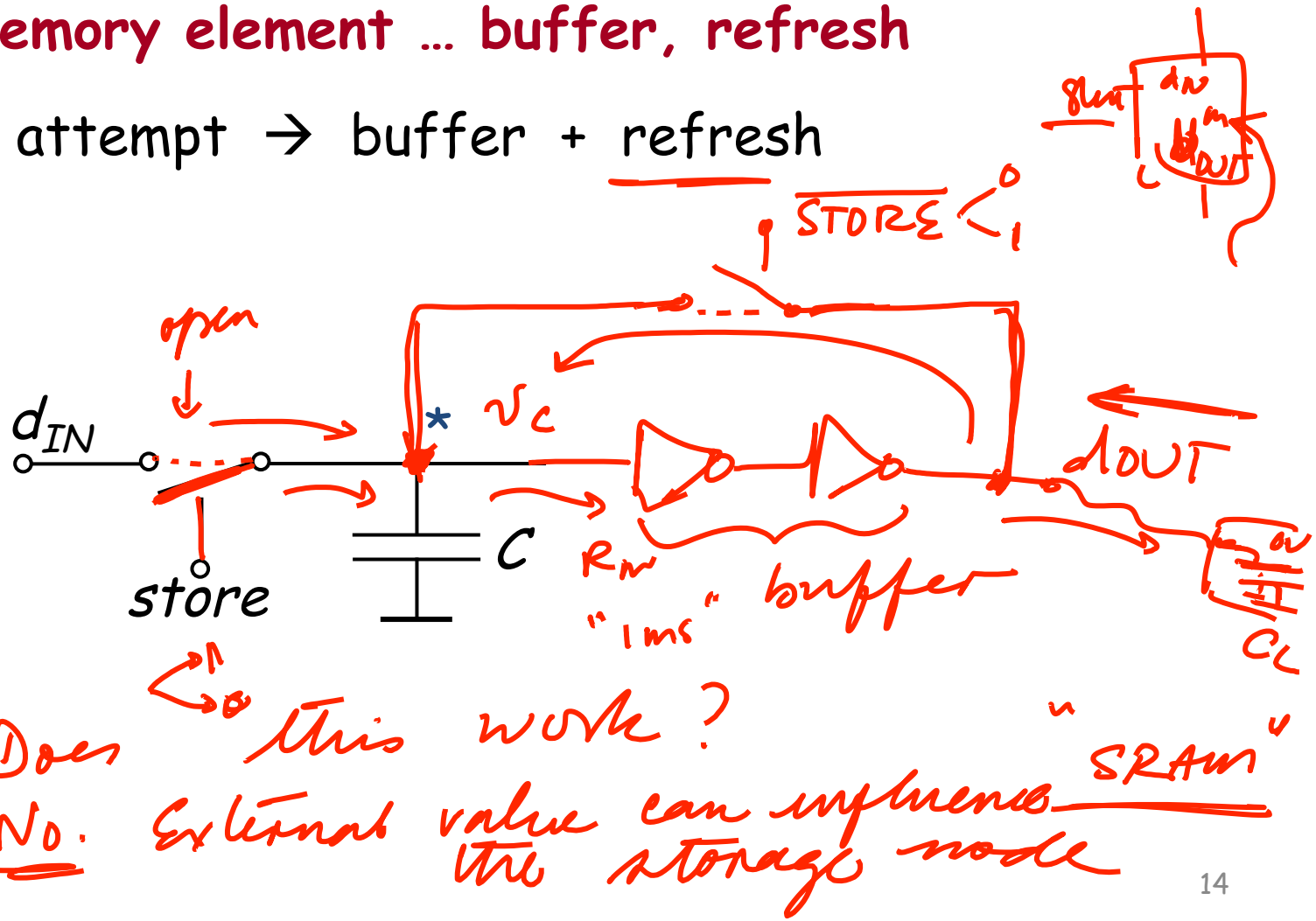


5V

Better! But it is still not perfect

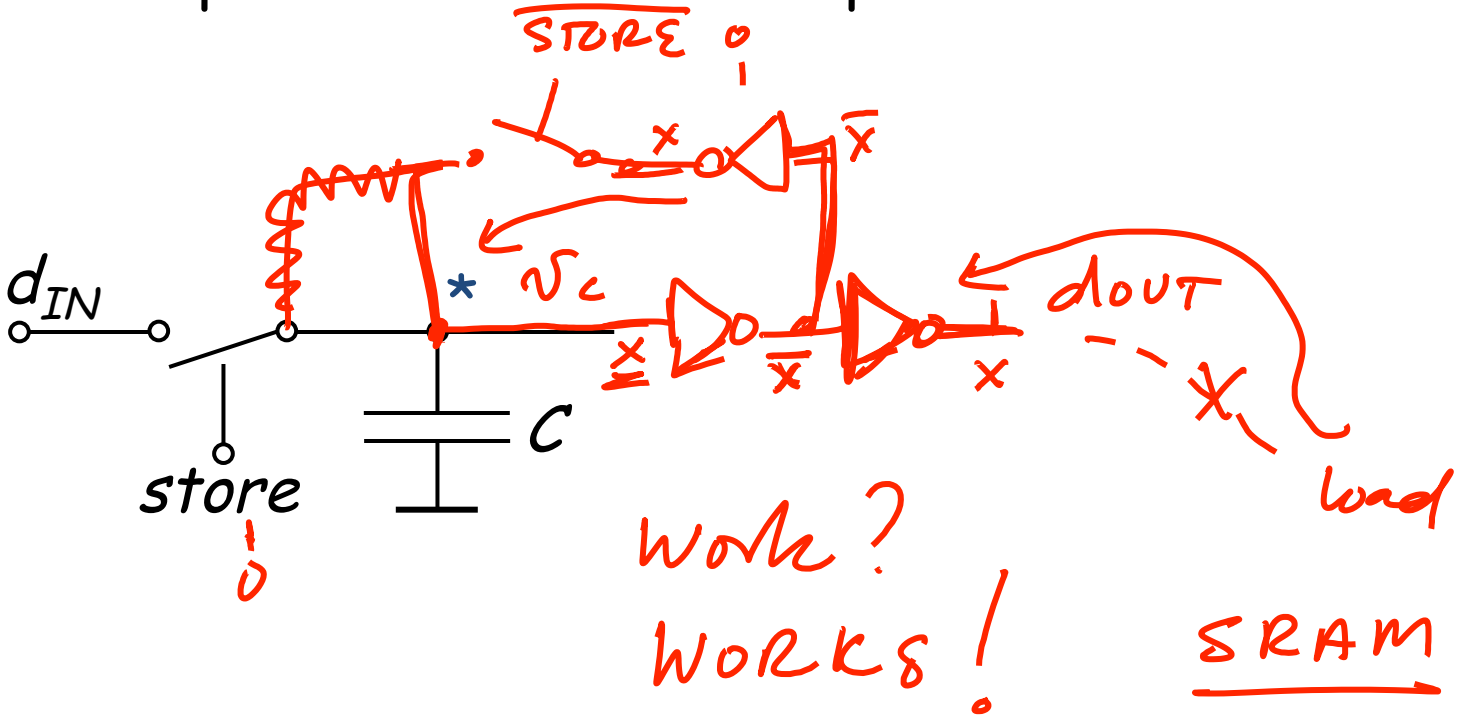
Building a memory element … buffer, refresh

Ⓒ Third attempt → buffer + refresh



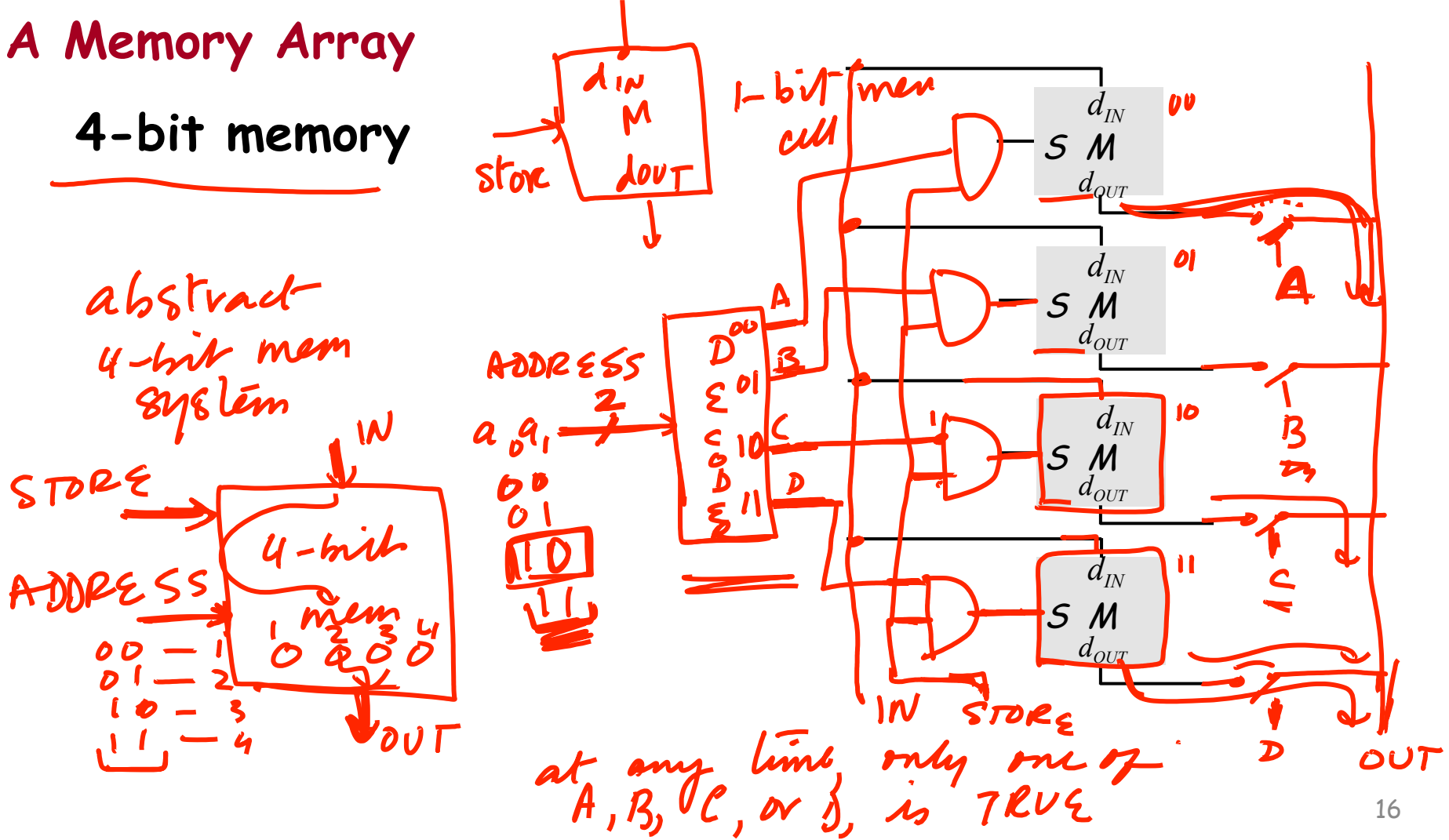
Building a memory element … buffer, decoupled refresh

Ⓓ Fourth attempt → buffer + decoupled refresh



A Memory Array

4-bit memory



Truth table for decoder

a_0	a_1	A	B	C	D
0	0	1	0	0	0
0	1	0	1	0	0
1	0	0	0	1	0
1	1	0	0	0	1

decoder

