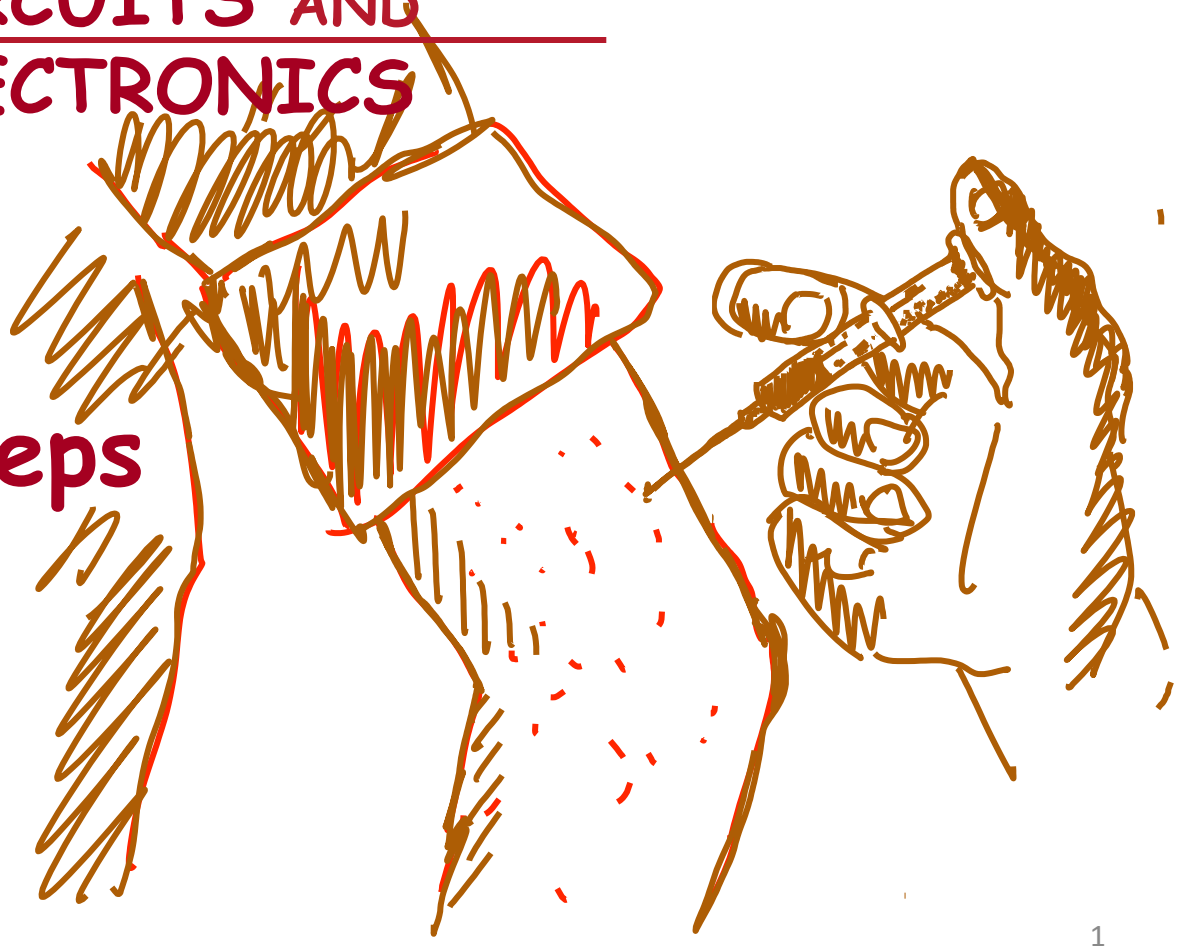
6.002x

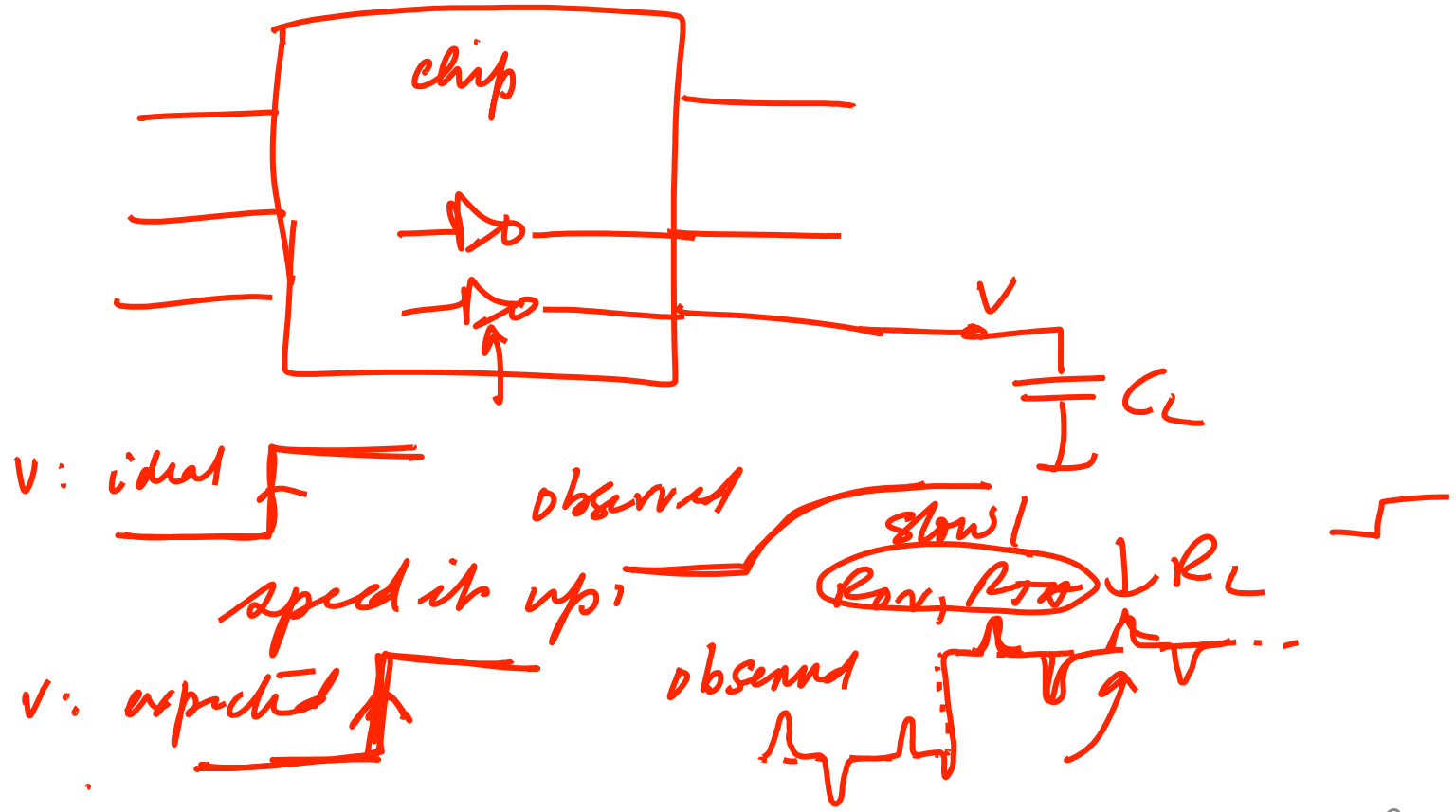
CIRCUITS AND ELECTRONICS

Impulses, Steps and Ramps



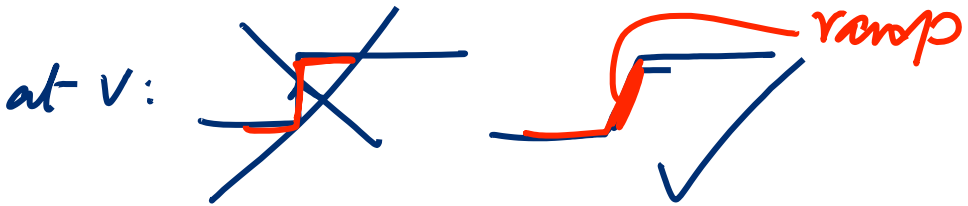
In our previous sequence…

Slower may be better



In our previous sequence…

6.002x expert said – make the transition slower





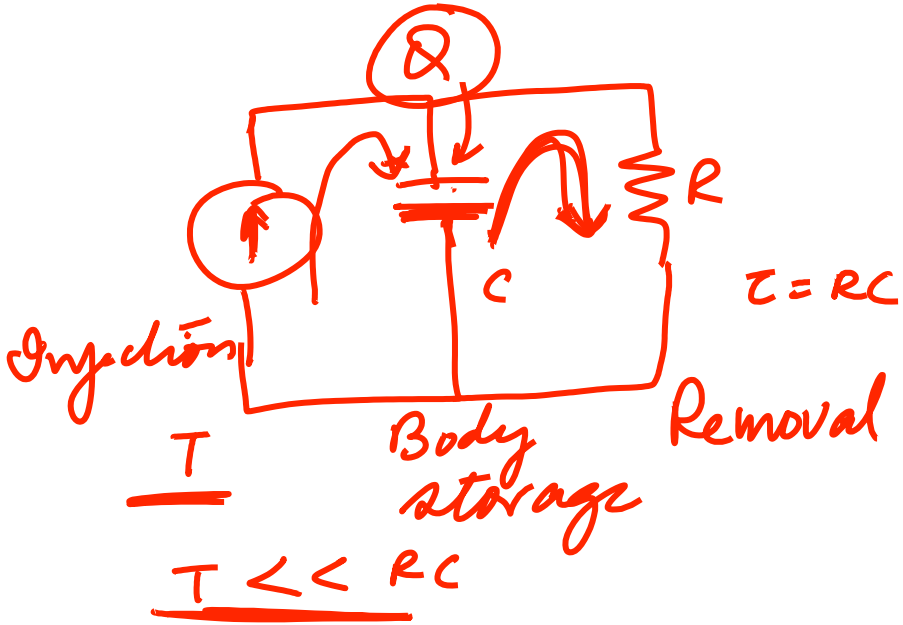
Solved – no glitches



We will see why here…

We will also understand how to model drug delivery





If the input duration is much shorter than system time constant, then shape or duration of input does not matter, only total amount of drug delivered.

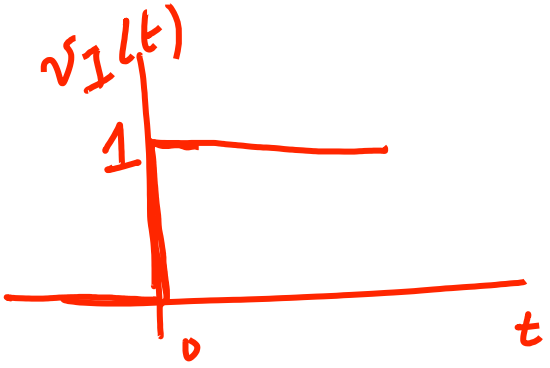
Model input as a short pulse → Impulse



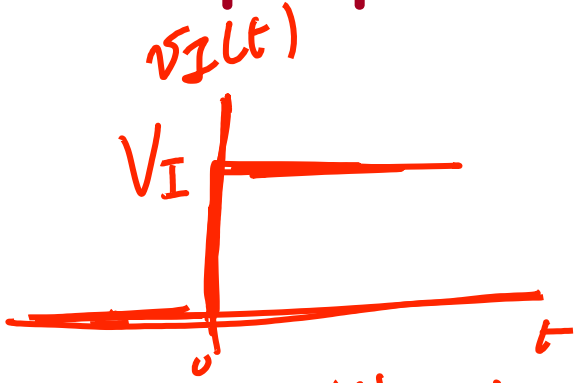
More on this soon…

First, let's understand a step input

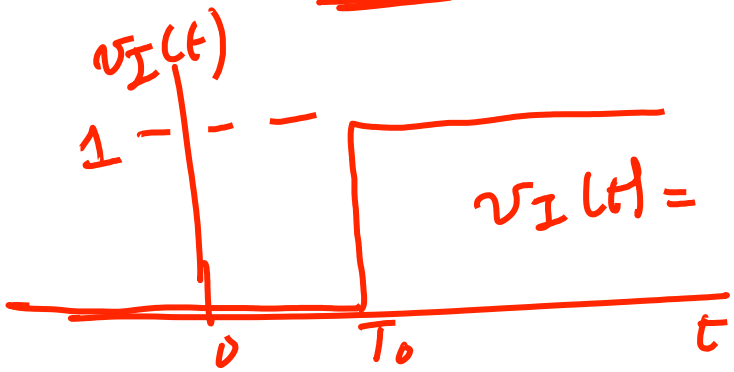
Steps
Impulse
pulse
ramp



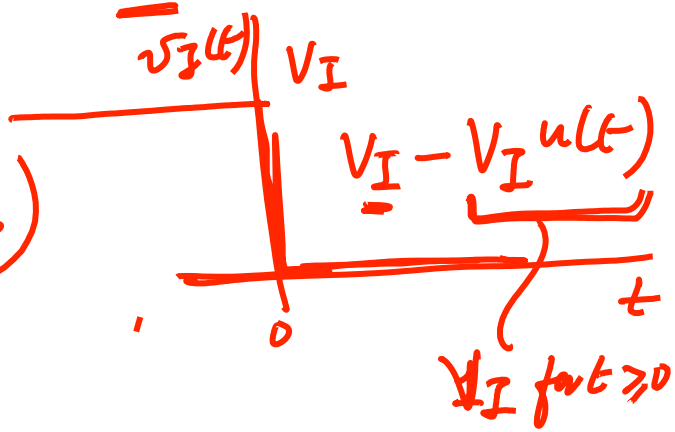
$v_I(t) = u(t)$



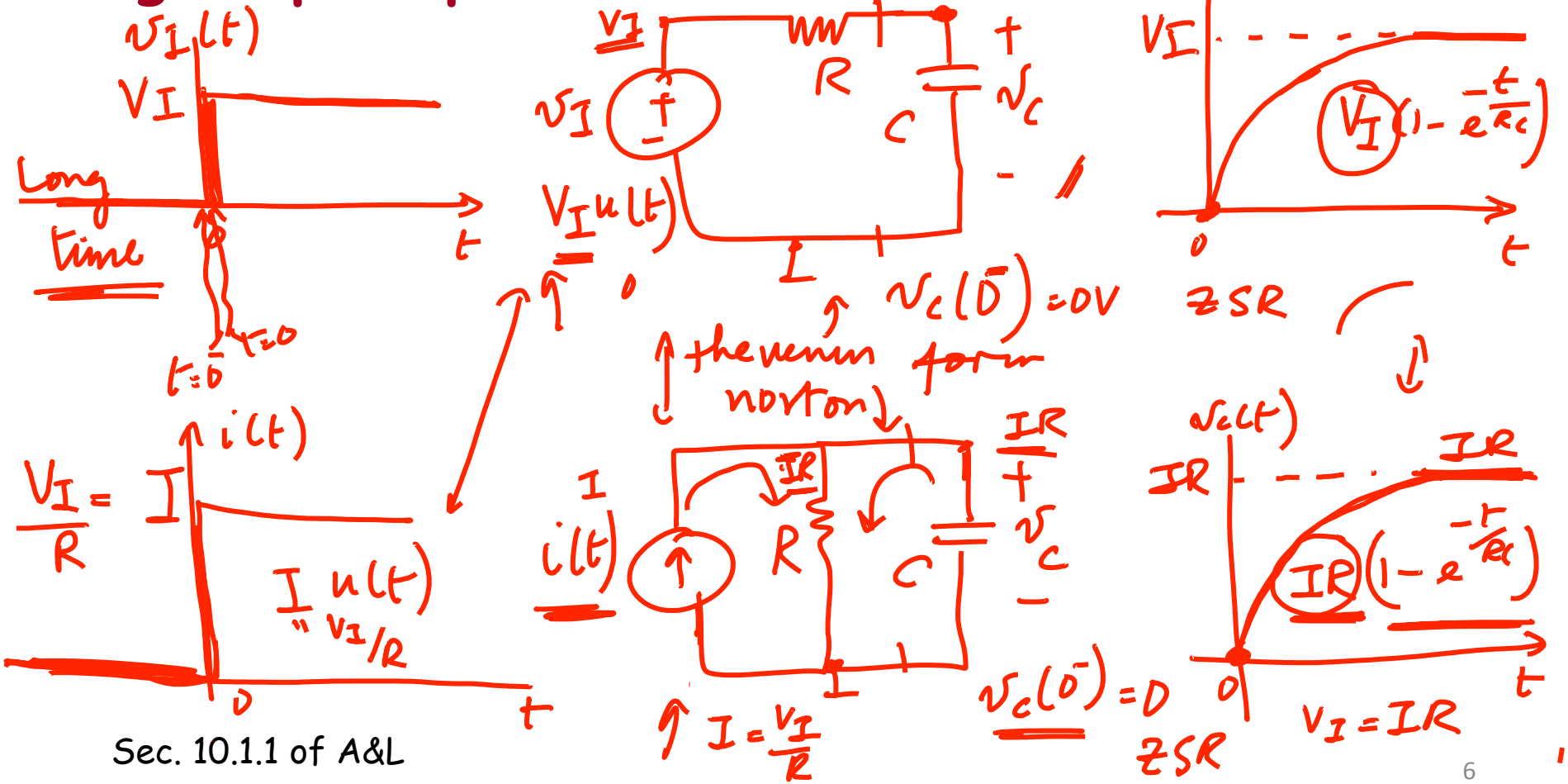
$v_I(t) = V_I u(t)$



$v_I(t) = u(t - T_0)$

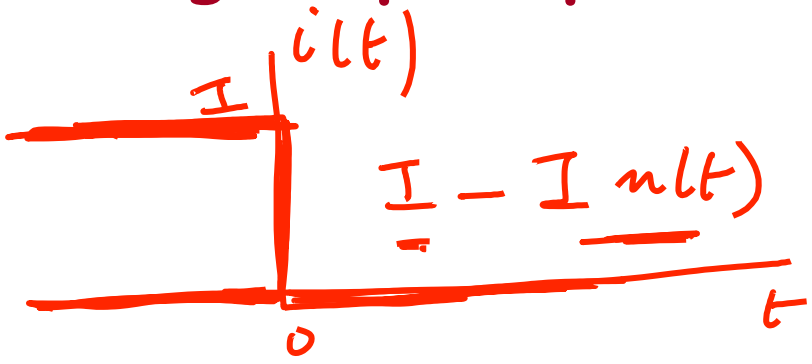


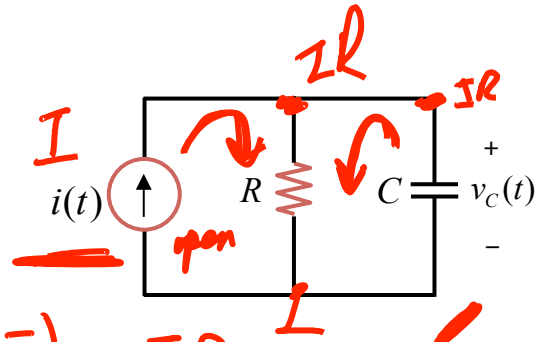
Rising step response of RC circuit



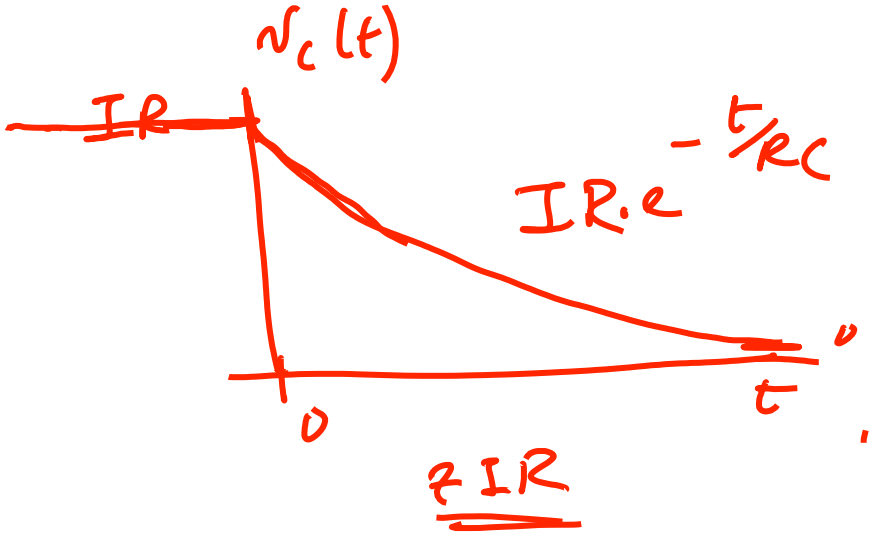
Sec. 10.1.1 of A&L

Falling step response of RC circuit





$v_c(0^-) = IR$



$2IR$

$e^{-t/RC}$

Rising step

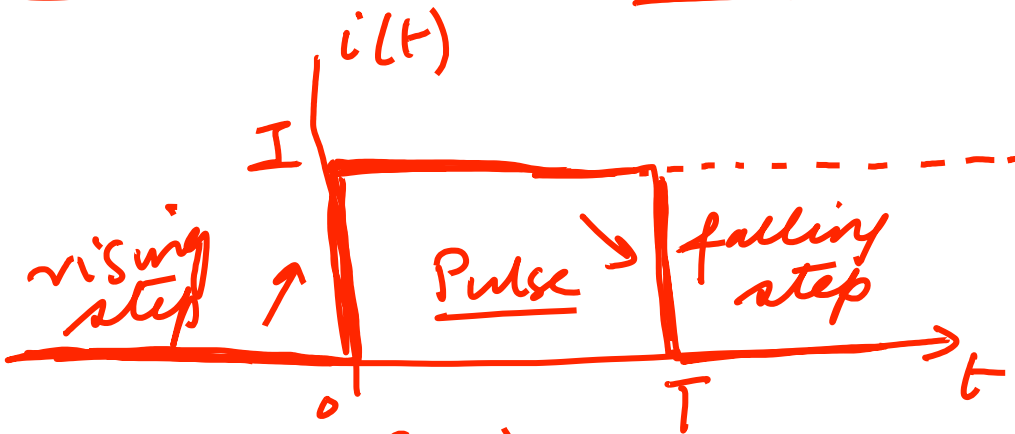
falling step

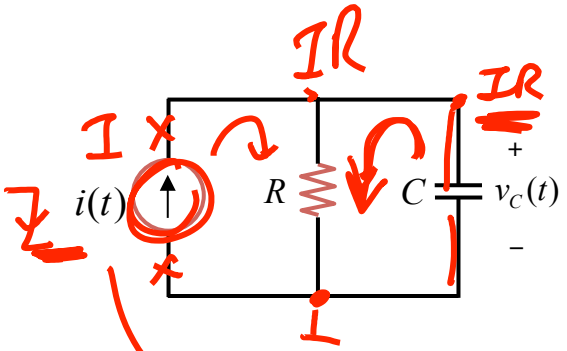
pulse

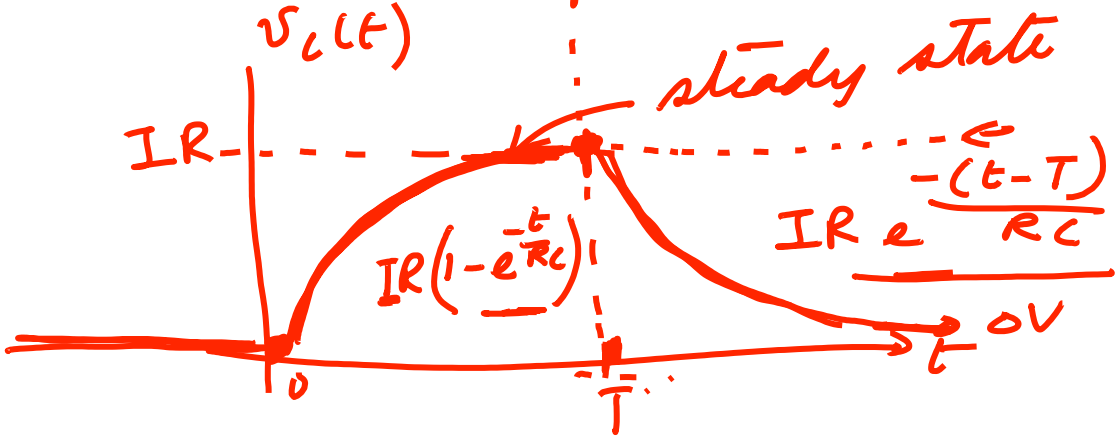
→ impulse

Pulse input:

Rising step followed by falling step (much later)!







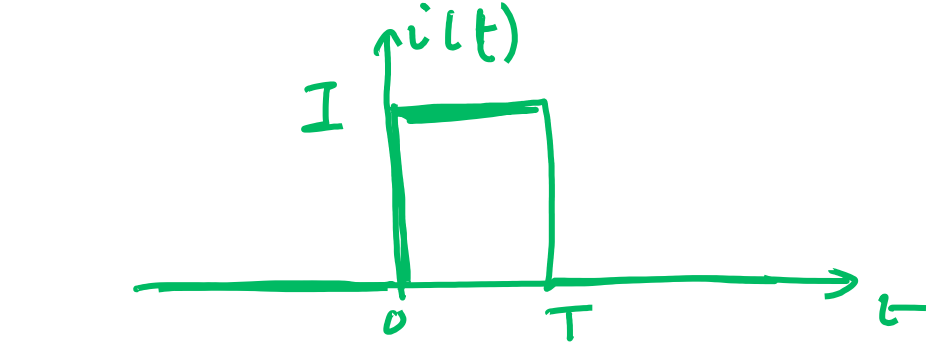
pulse width ↓ pulse wide

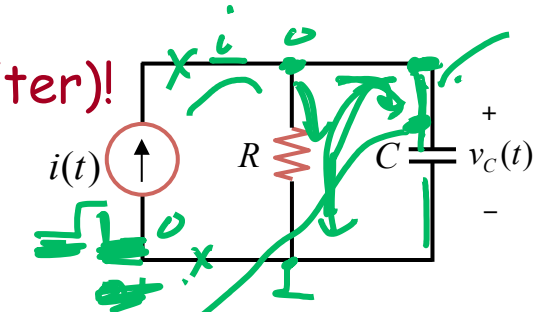
$T \gg RC$

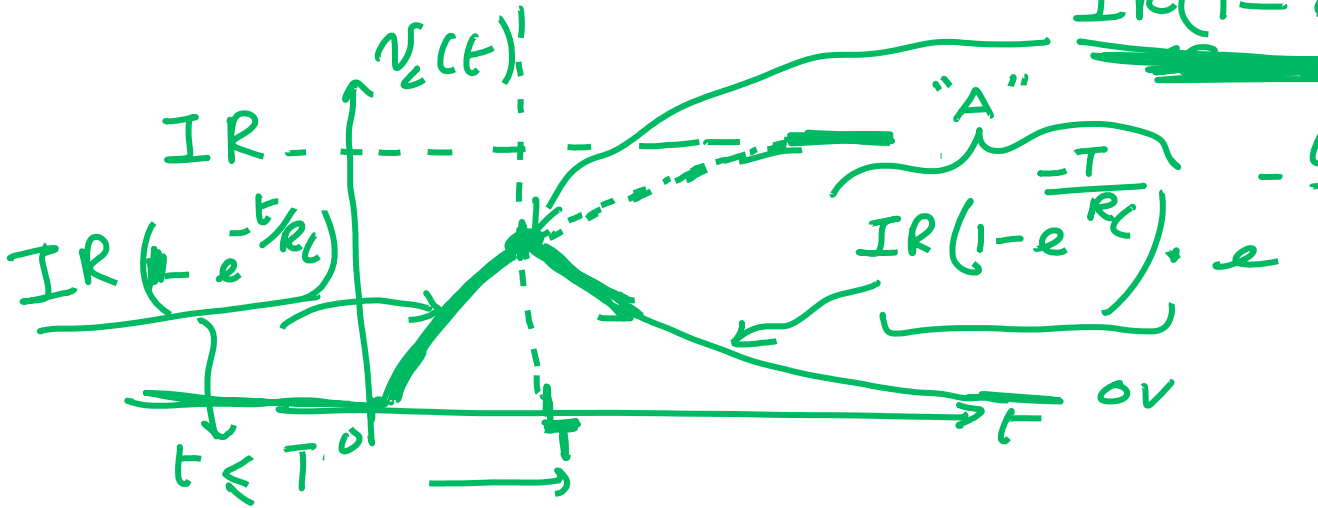
↑ time shifted by T

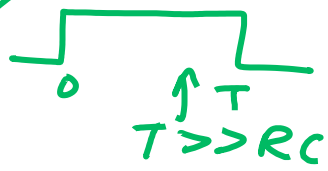
Short pulse input:

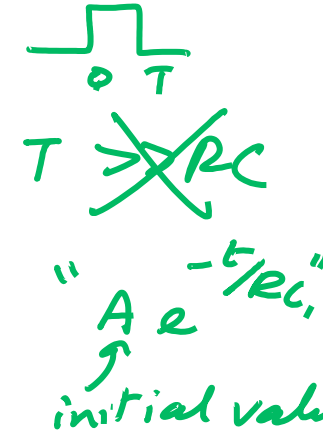
Rising step followed by falling step (shortly thereafter)!



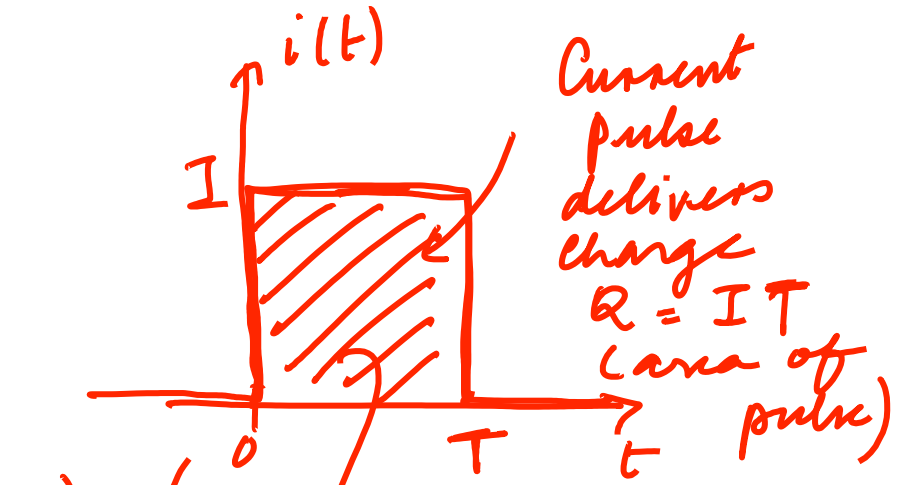




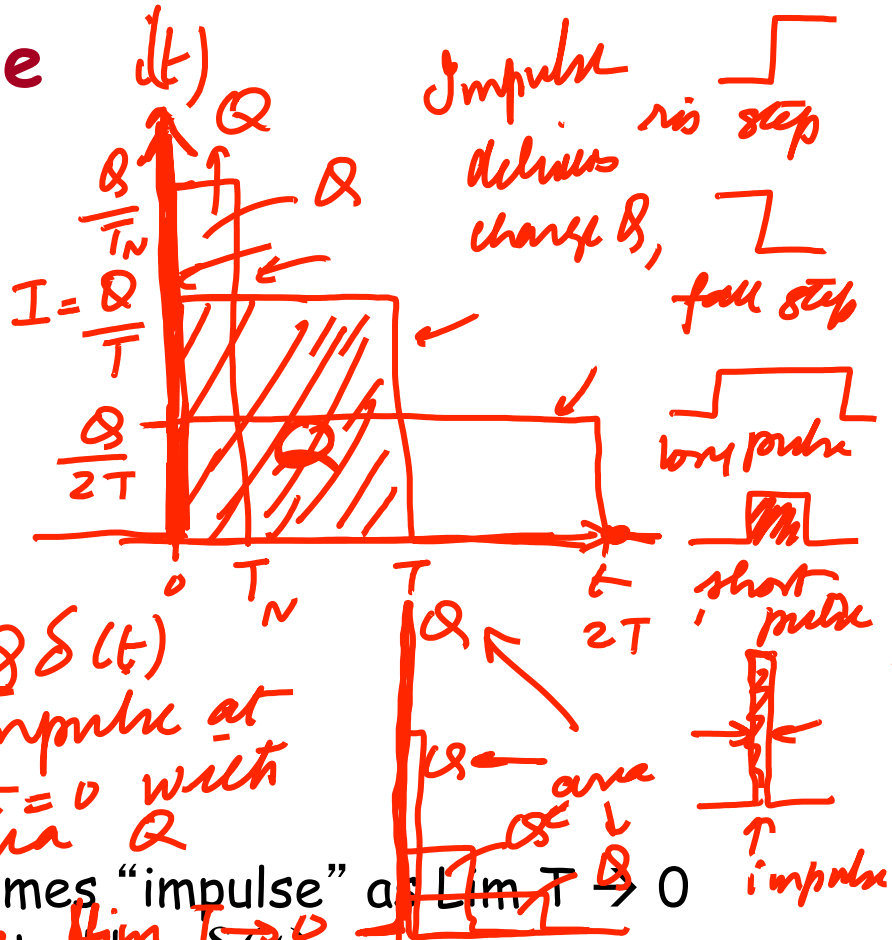




Let's Stare at Current Pulse

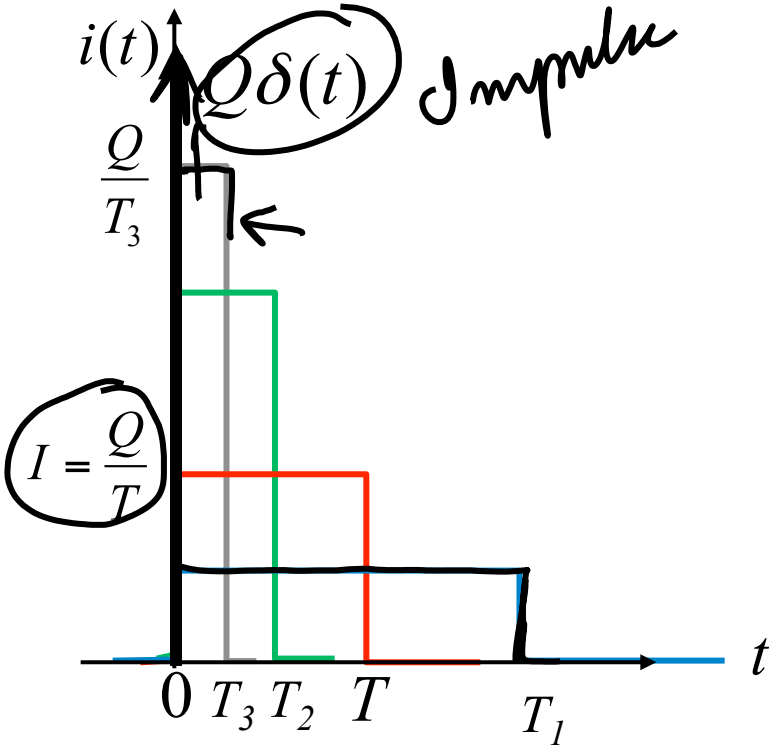


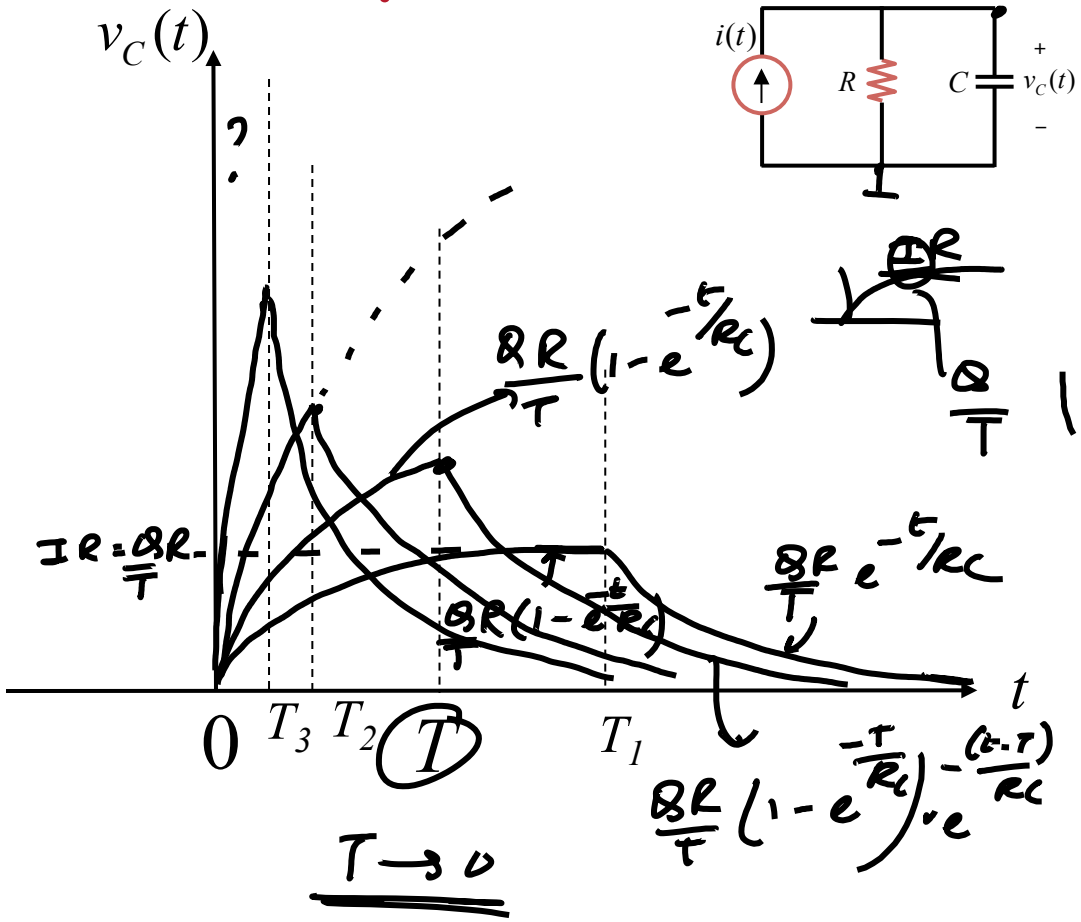




- Becomes "impulse" as Lim $T \to 0$
- Denoted by $\delta(t)$
- Impulse is an important signal type

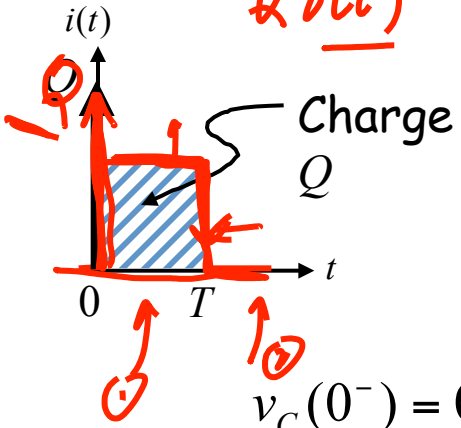
As the pulse gets narrower → impulse



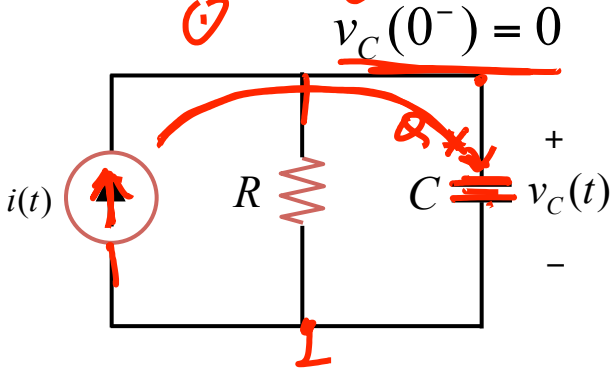


Becomes "impulse" as Lim T → 0
Denoted by $\delta(t)$
Impulse is an important signal type

Response to impulse – limit case



$Q\,\delta(t)$



$Q = v_c \cdot C$

① t: 0 – T

$v_c(t) = \frac{RQ}{T}\left(1 - e^{-\frac{t}{RC}}\right)$

T small

$v_c(T) = \frac{RQ}{T}\left(1 - \left(1 - \frac{T}{RC}\right)\right)$

$v_c(T) = \frac{Q}{C}$

all charge to capacitor, cap had no time to discharge.

② t > T

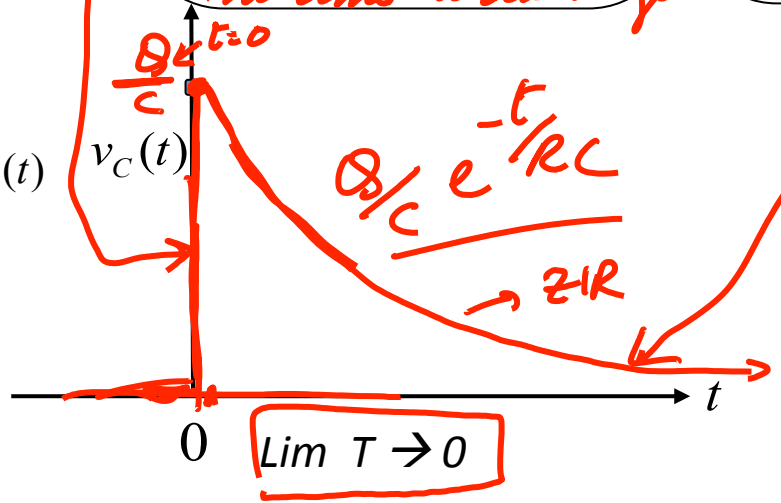
$v_c(t) = \frac{RQ}{T}\left(1 - e^{T/RC}\right) \cdot e^{-\frac{(t-T)}{RC}}$

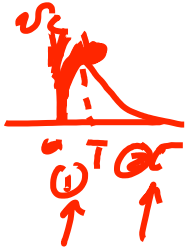
T small

$= \frac{RQ}{T}\left(1 - \left(1 - \frac{T}{RC}\right)\right) e^{-\frac{(t-T)}{RC}}$

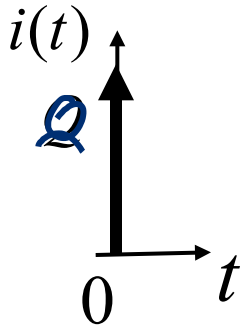
$v_c(t) = \frac{Q}{C} e^{-\frac{t}{RC}}$

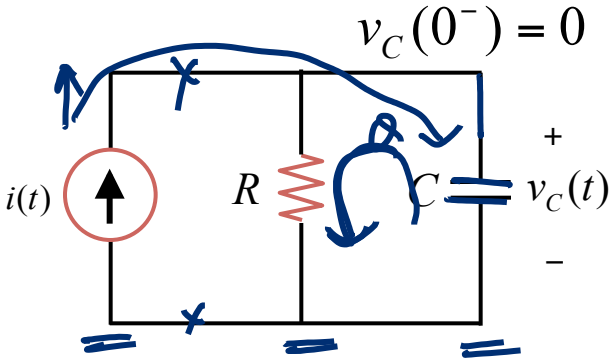
$T \to 0$

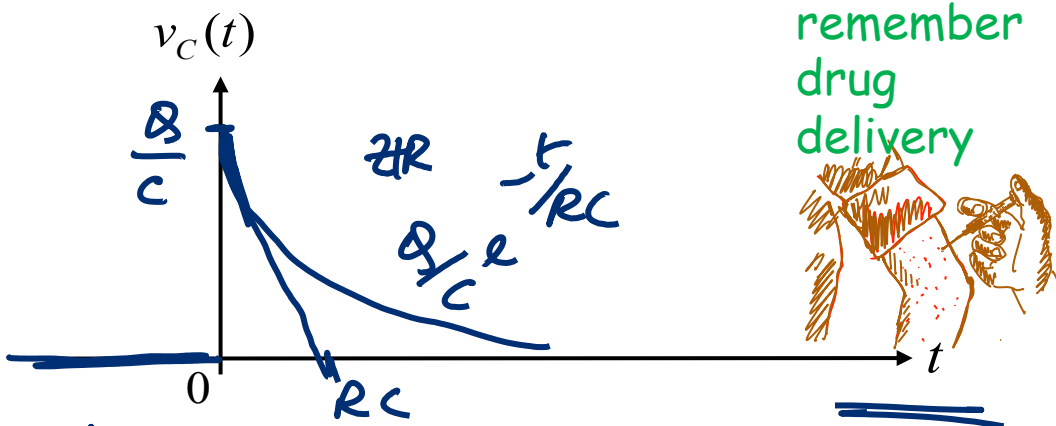




Response to impulse



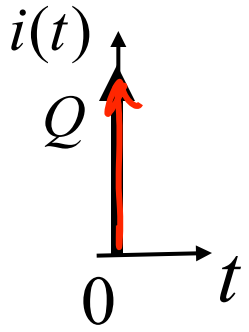


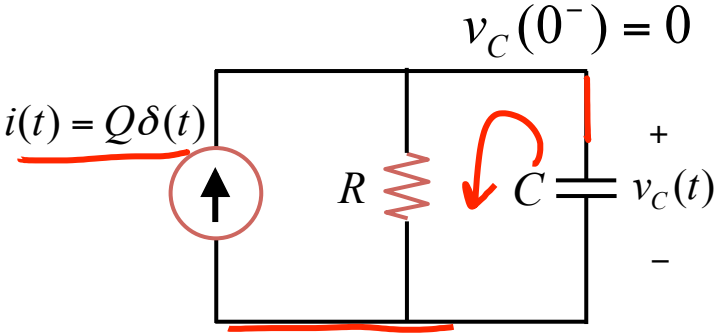


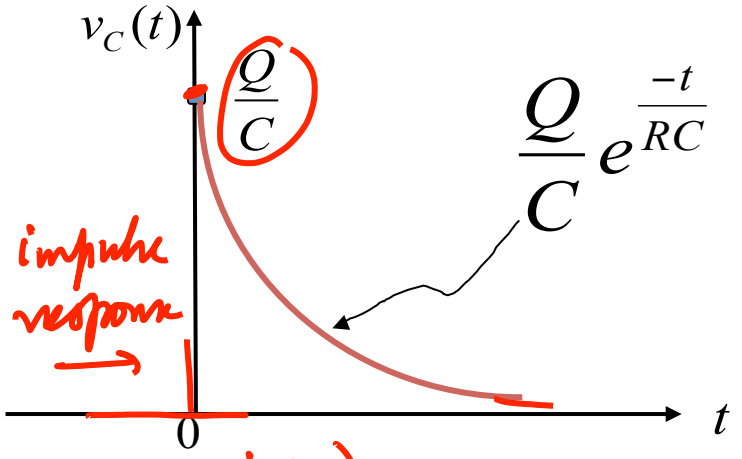
remember drug delivery

- $v_C(t)$ is largely independent of I, & details of the pulse (remember drug delivery – shape of pulse does not matter)
- Capacitor behaves like a short ckt. for changes that are much faster than the time constant RC
- All charge Q goes to capacitor. Recall for capacitor $v = Q/C$
- Impulse sets up initial conditions on capacitor.

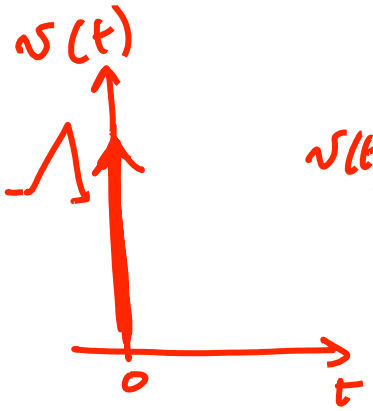
Current and voltage impulses

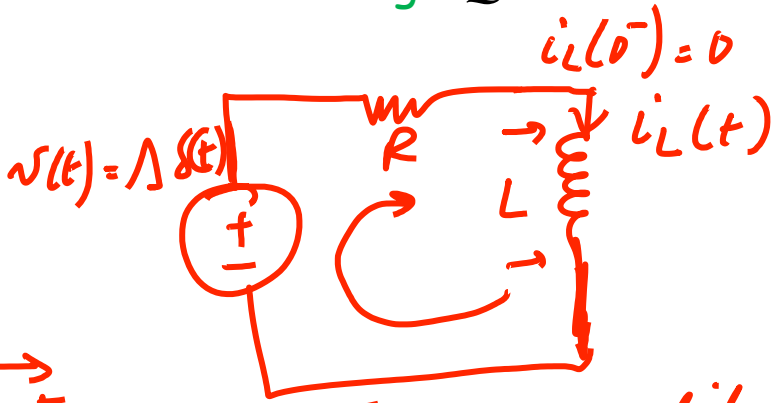


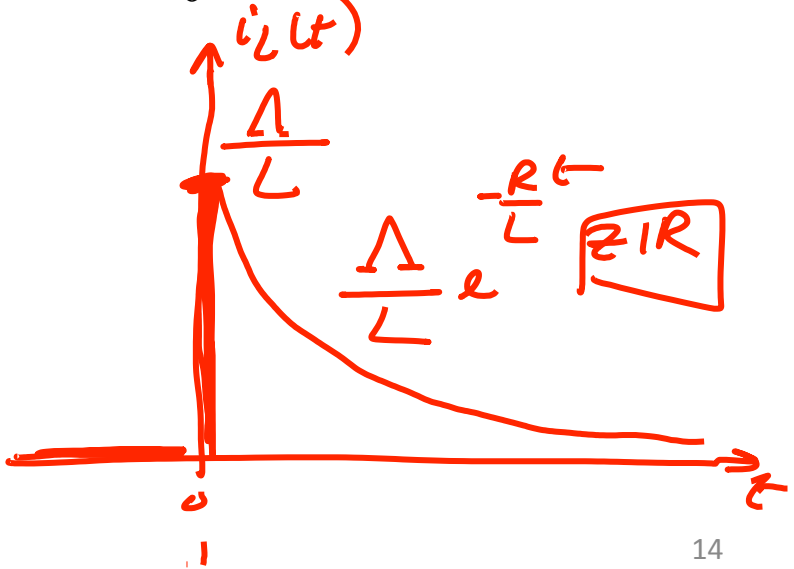




Current impulse delivers charge Q





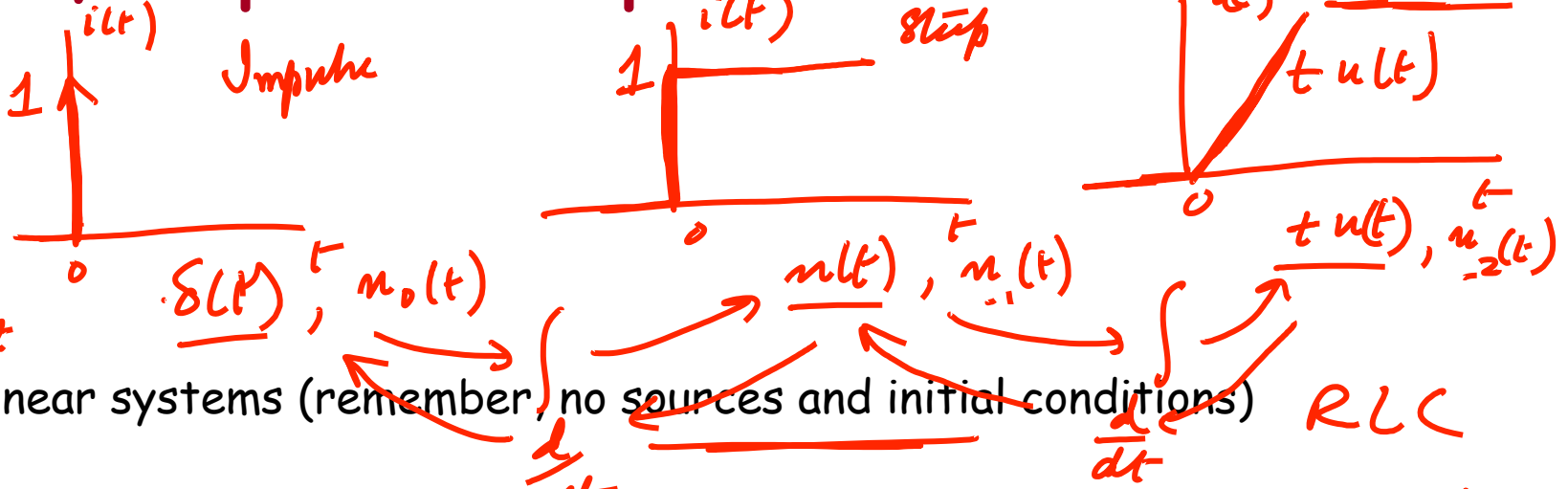


Voltage impulse delivers flux linkage Λ

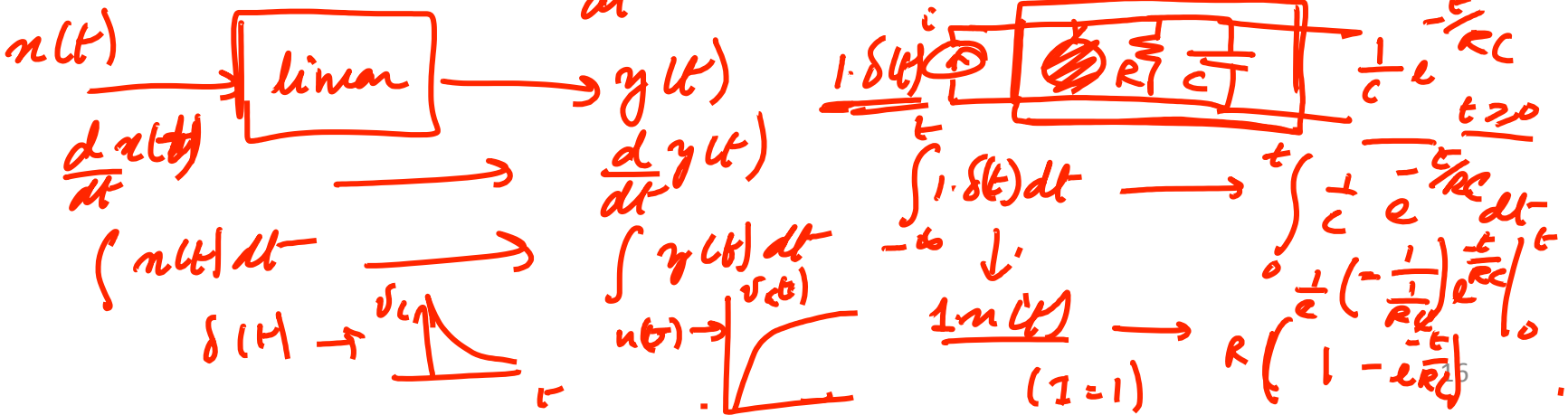
Superposition with initial conditions and sources

- Note that superposition does not apply when initial condition exist
- Superposition does apply when initial conditions are zero
- One way of solving circuits with multiple sources and multiple initial conditions:
  - Treat initial condition like source
  - Find response to each source or each initial condition ~~adding~~ acting alone and then sum all of these partial responses.

Impulses, steps and ramps

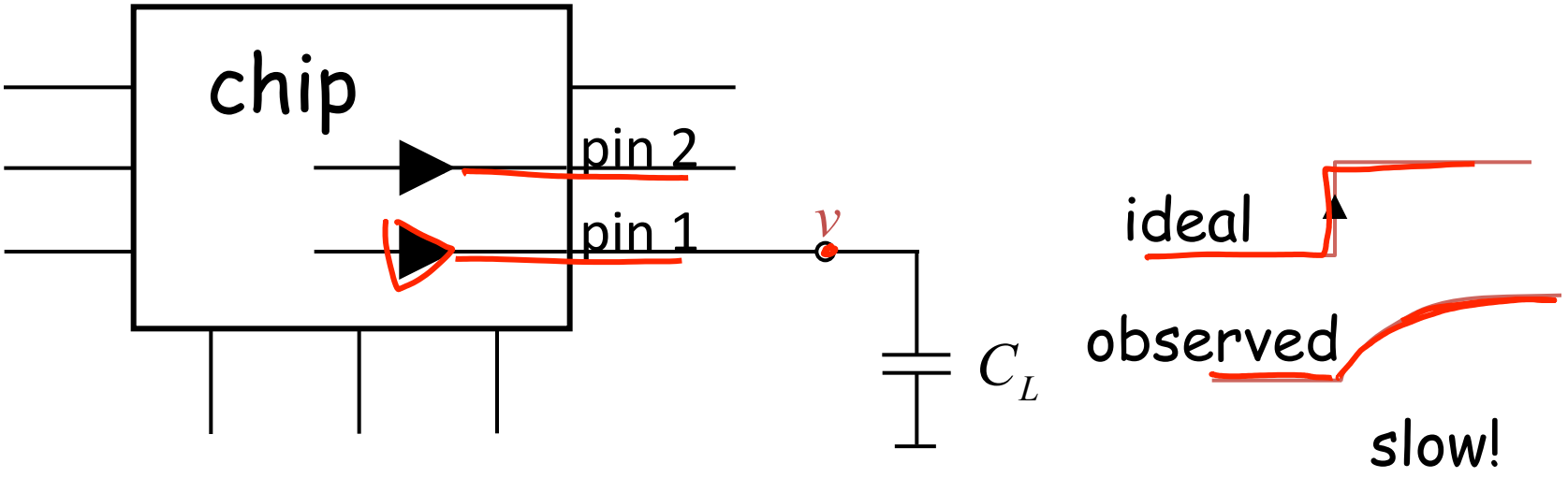


For linear systems (remember, no sources and initial conditions)

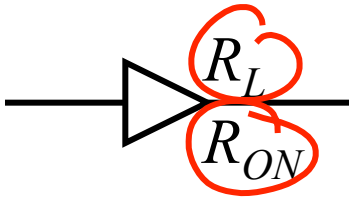


Back to:

Problem:



So the engineers decided to speed it up…

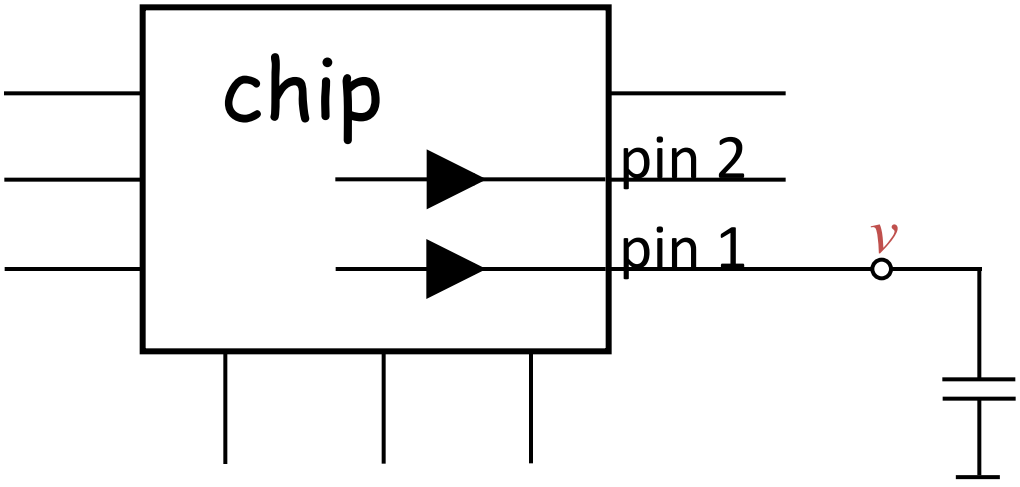


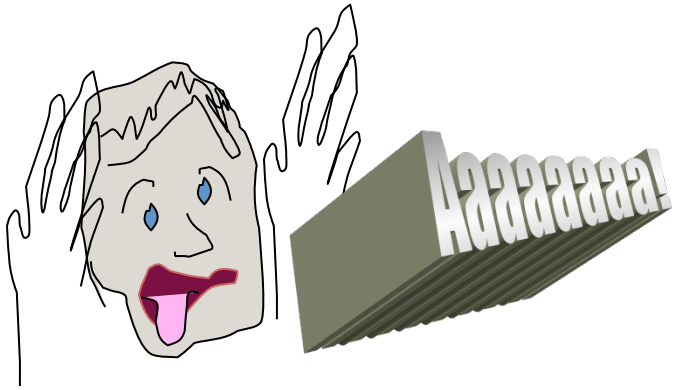
made R_L small

made R_{ON} small

Slower may be better

Problem: ... but, disaster!





v:

expected

small RC

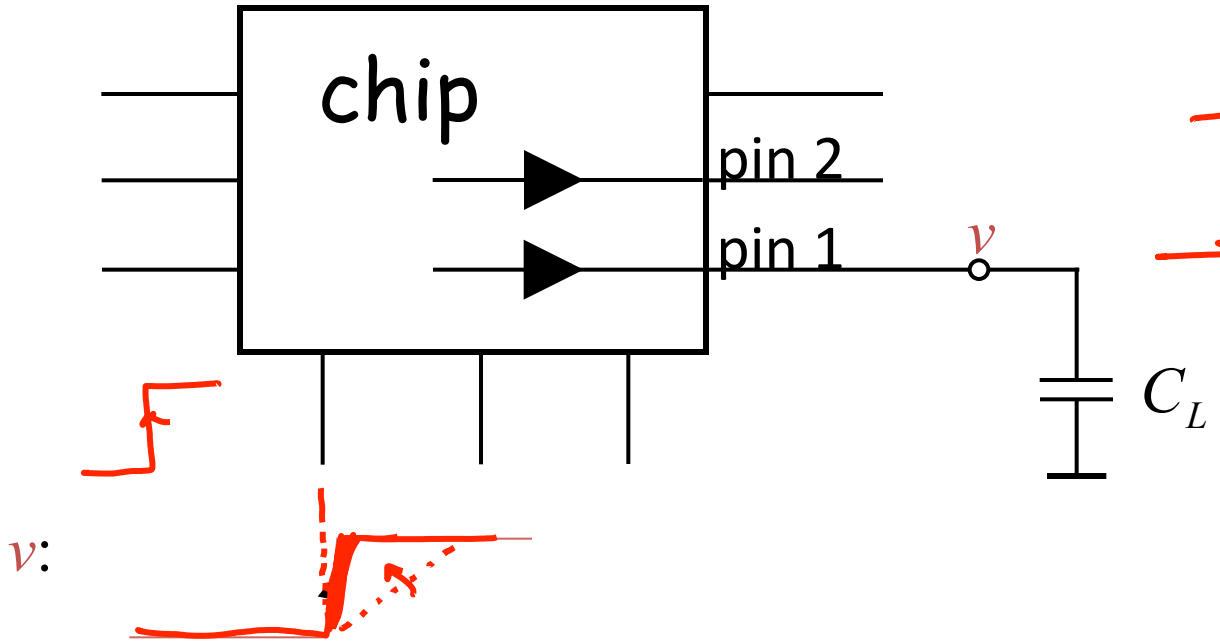
large RC

observed

V_{IL}

Slower may be better

... 6.002x expert saw the solution



→ ① why glitches

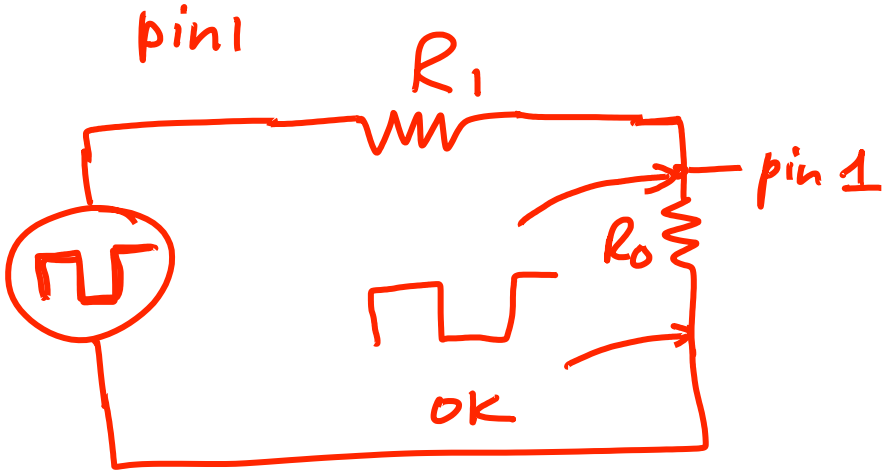
→ ② why ✓ show edges made glitches go away

We now have the tools to understand why

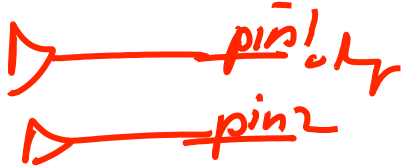
Slow the edges down!

Why? Consider ...

Case ①

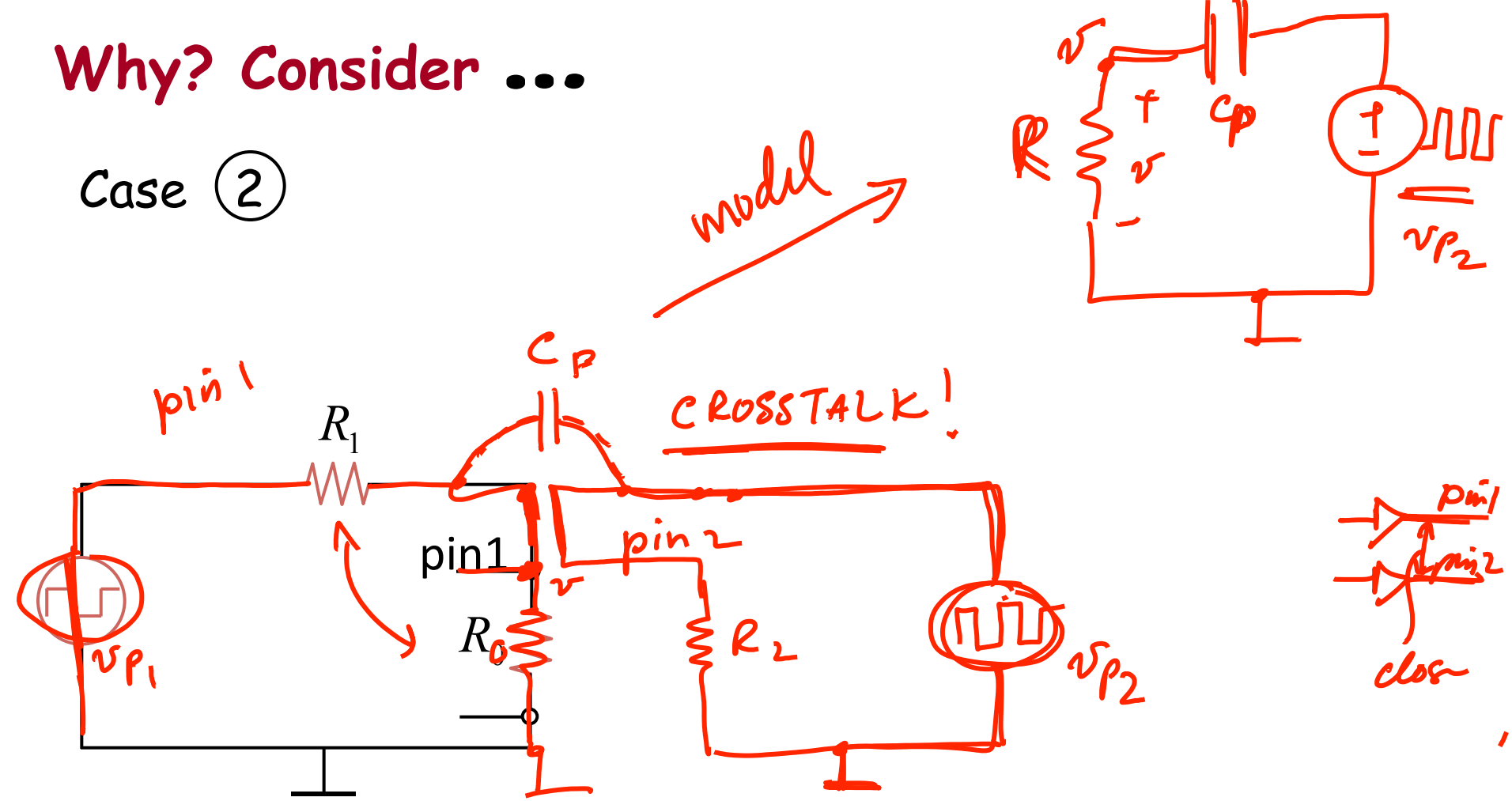


why glitches



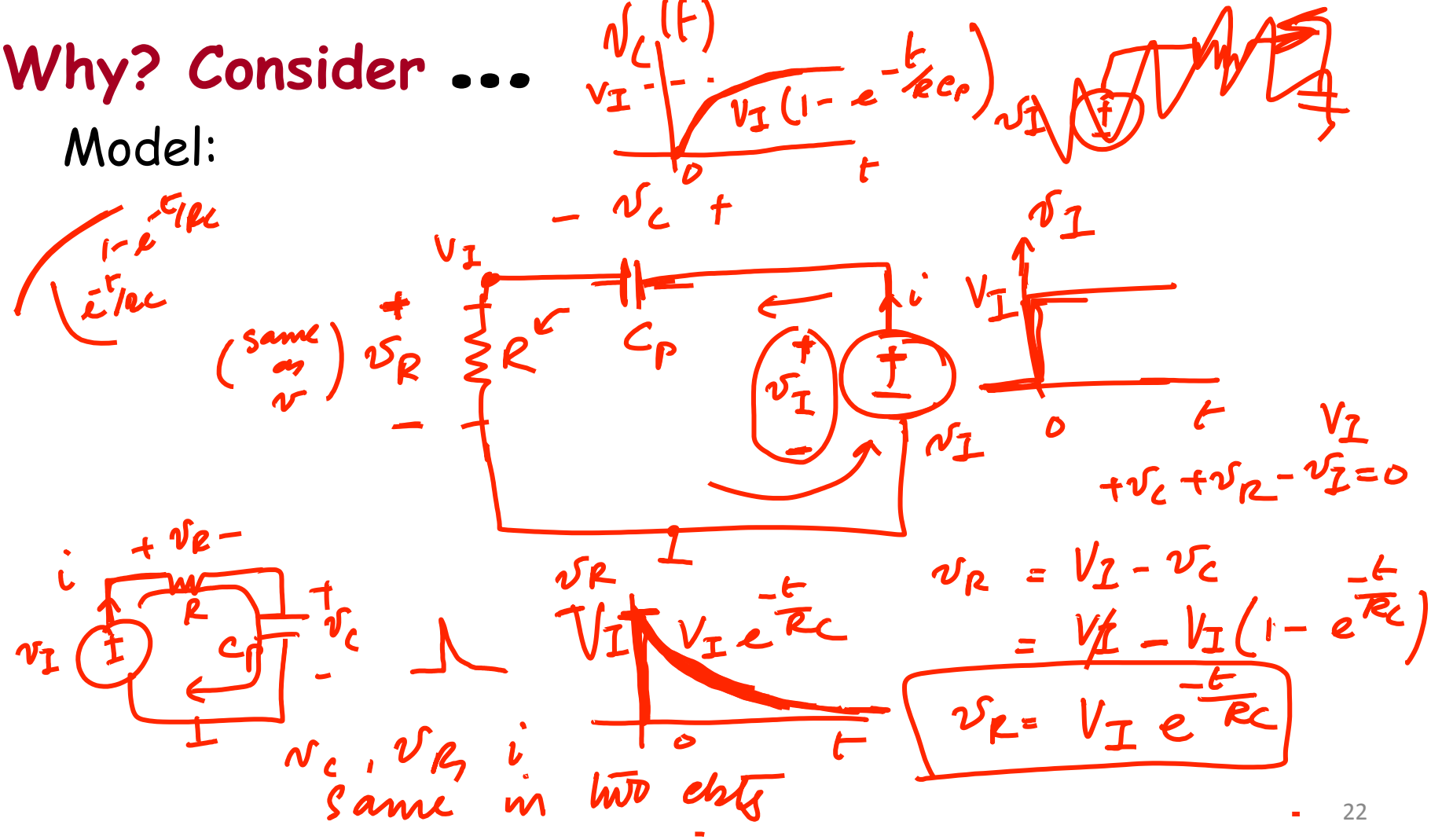
Why? Consider ...

Case ②



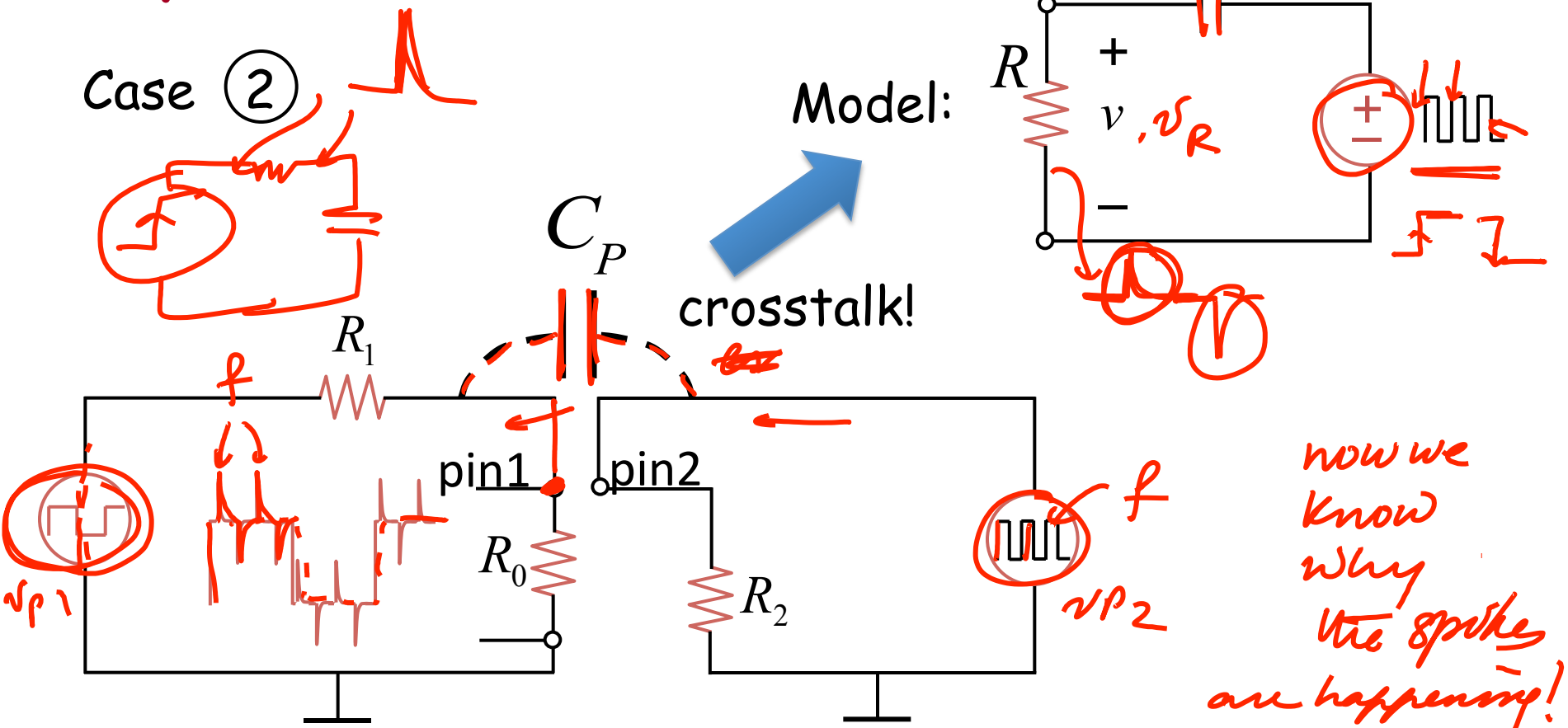
Why? Consider ...

Model:



Why? Consider ...

Case ②

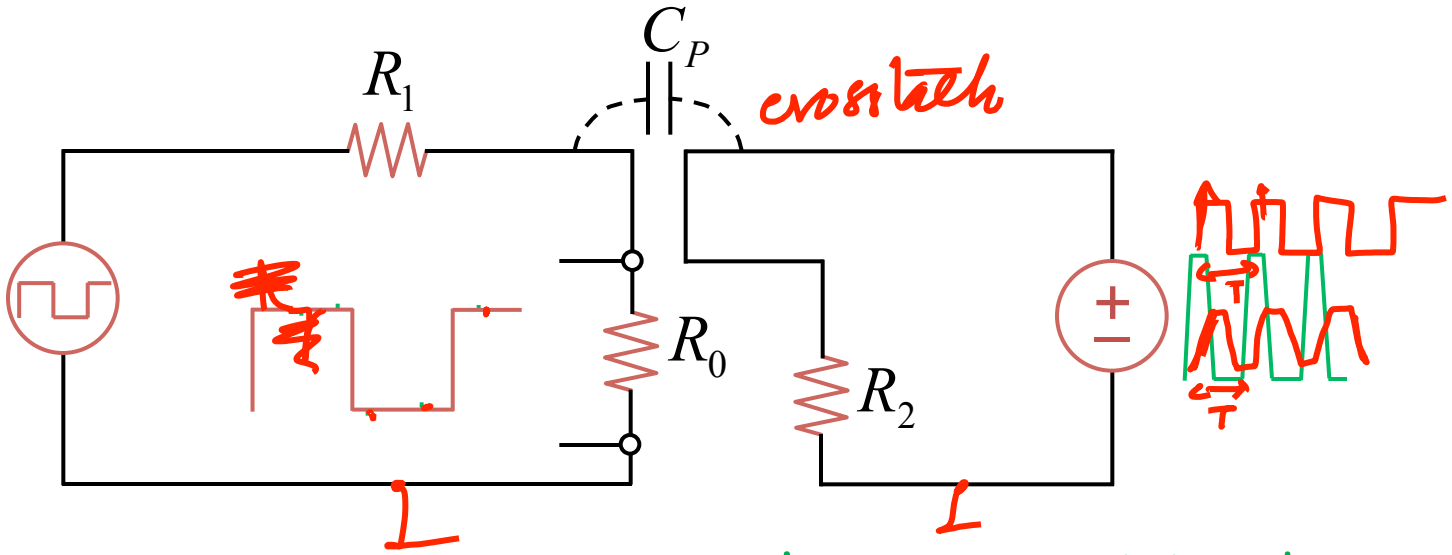


Model:

now we know why the spikes are happening!

Case ③

... 6.002x expert saw the solution

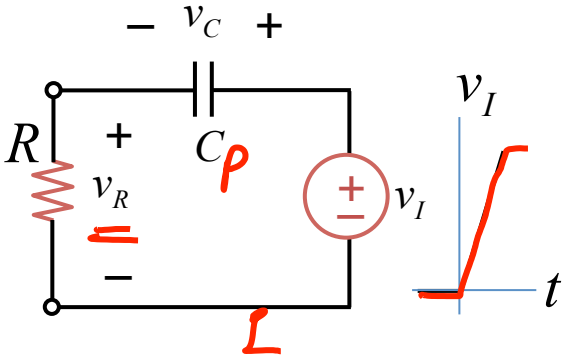


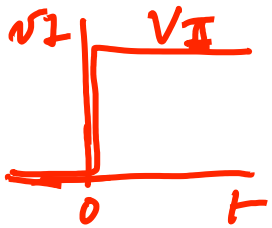
slower transitions!

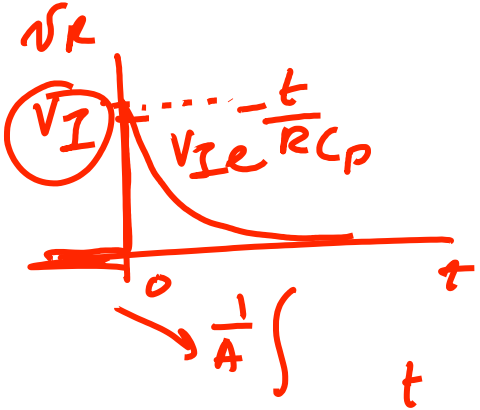
Demo

Why? Consider ...

Model:

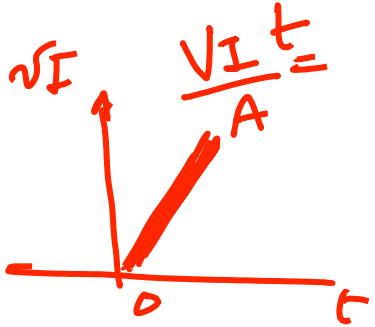






$\frac{1}{A}\int$

$\frac{1}{A}\int$



$$v_R = \frac{1}{A}\int_0^t V_I e^{-t/RC_p}\,dt$$

$$= -\frac{V_I}{A} RC_p \, e^{-t/RC_p}\Big|_0^t$$

$$v_R = \frac{V_I}{A} RC_p \left(1 - e^{-t/RC_p}\right)$$

